Recognition of Unit Segment and Polyline Graphs is $\exists\mathbb{R}$-Complete

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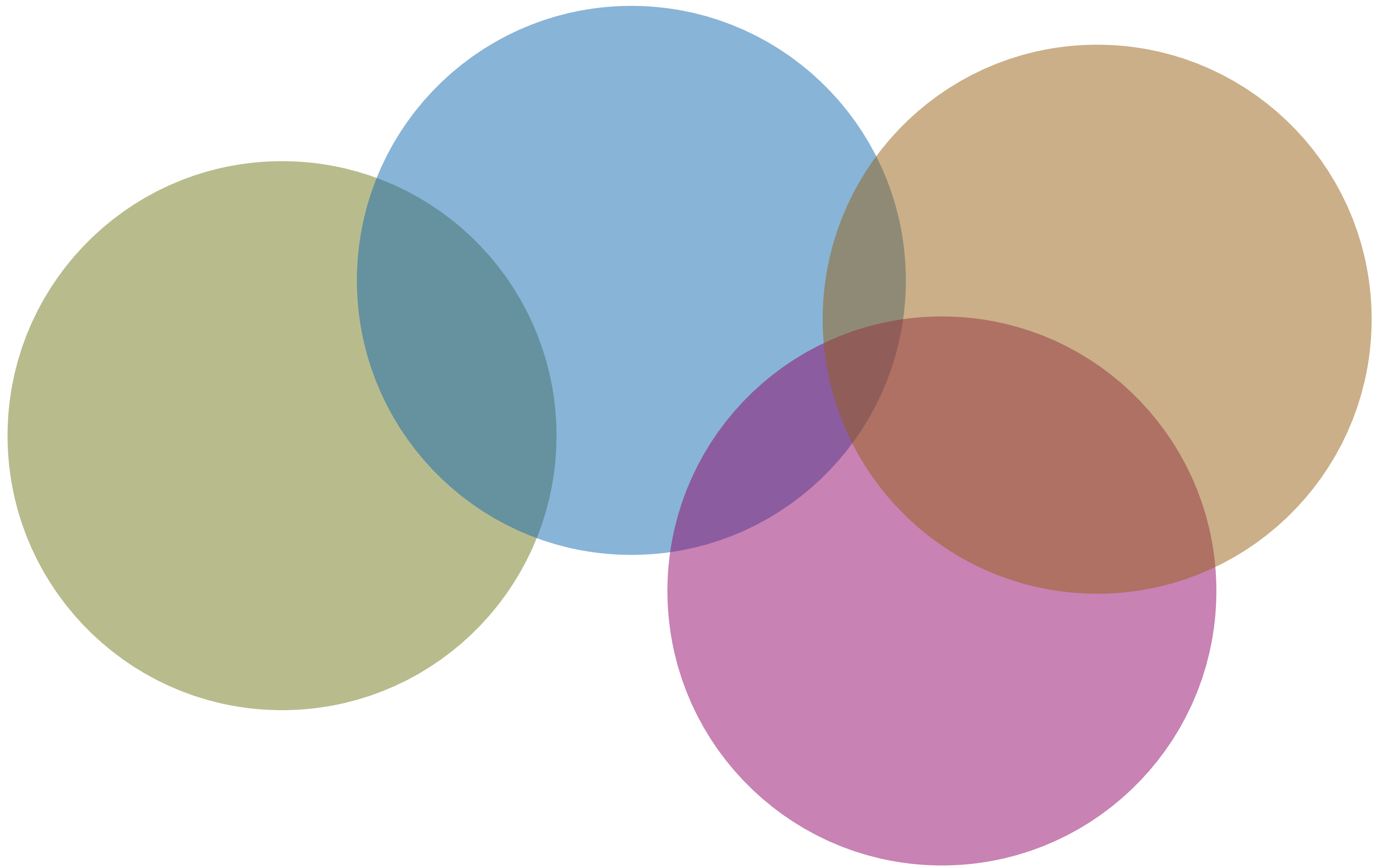
Intersection Graphs



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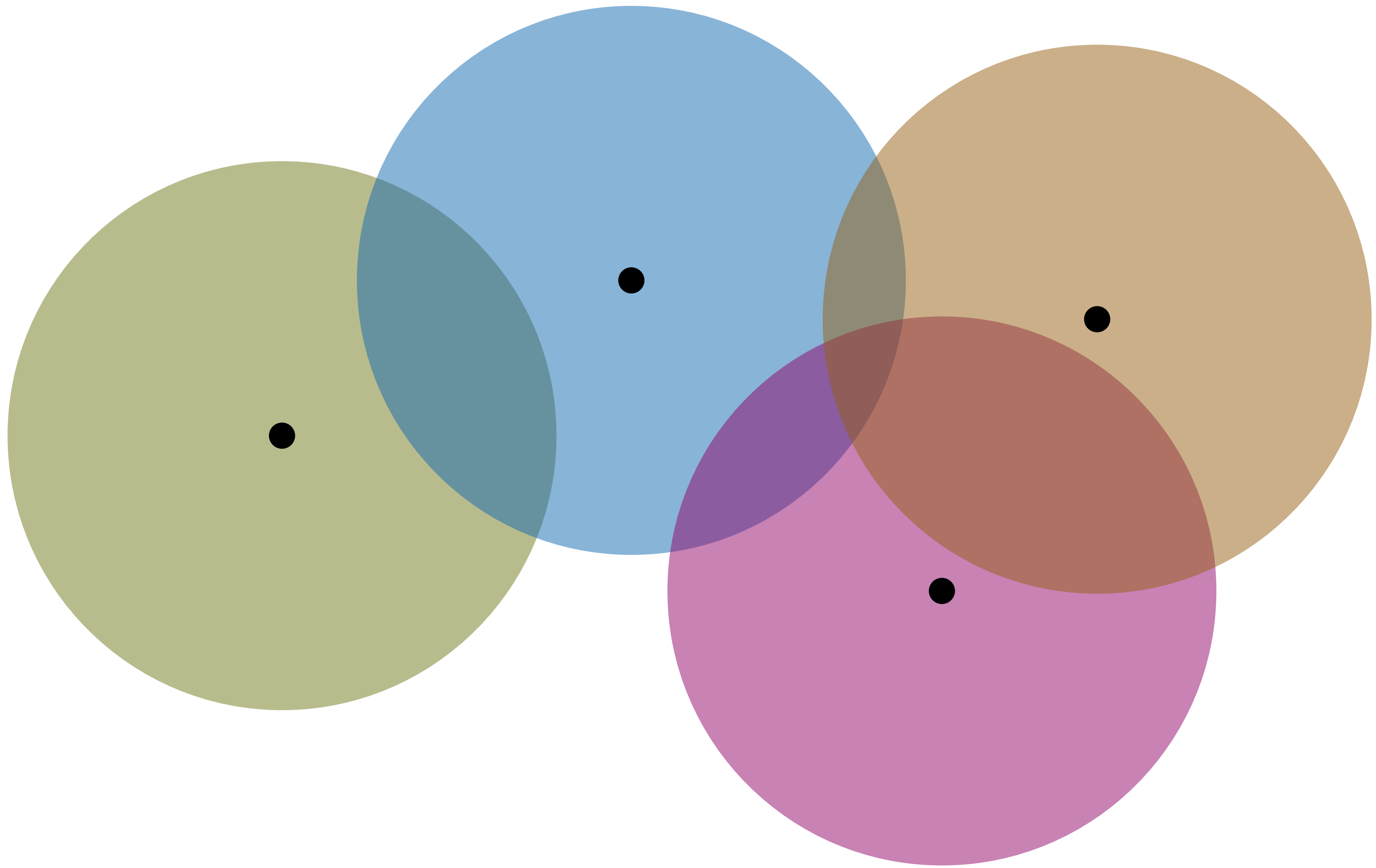

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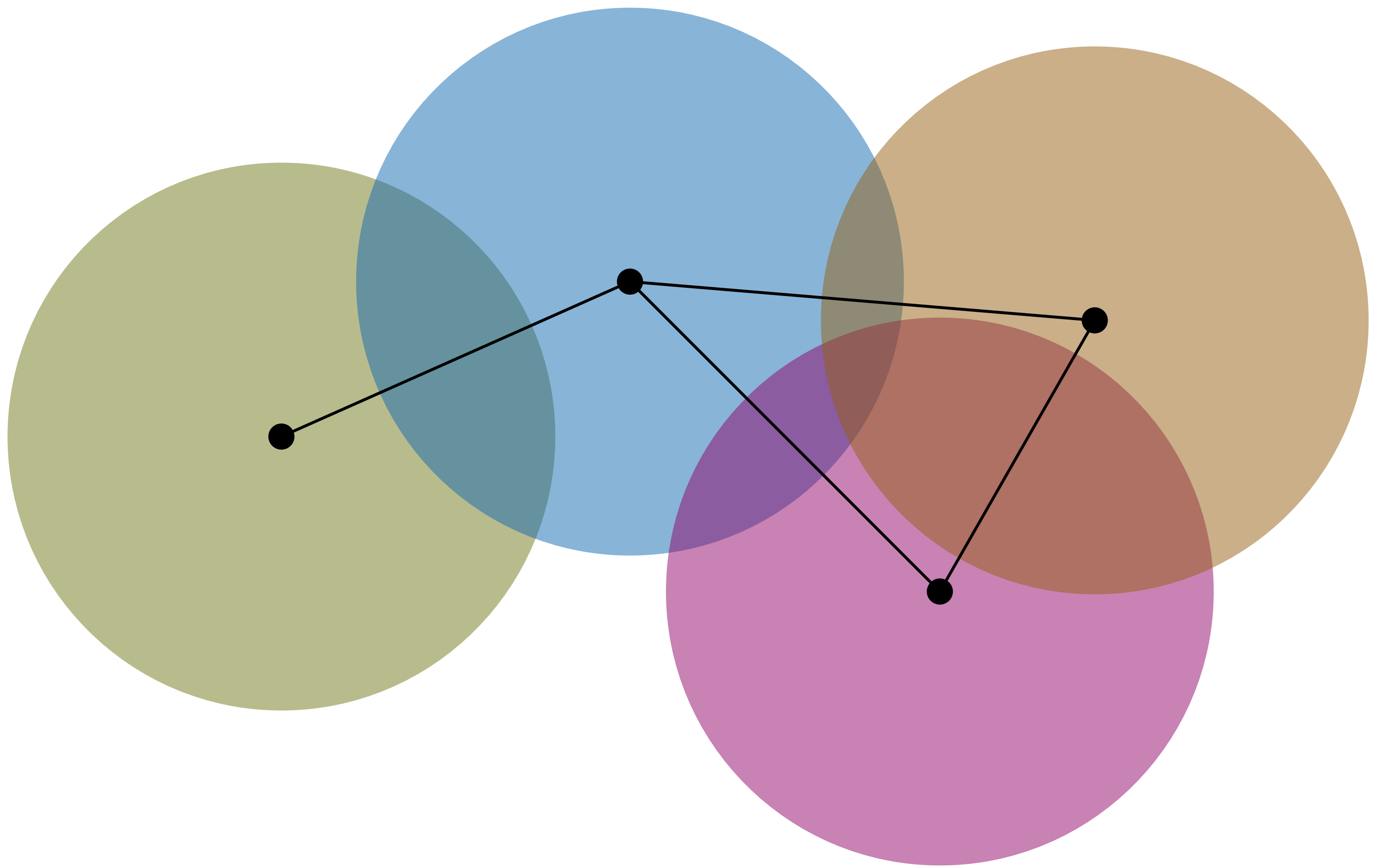

Intersection Graph Recognition

Definition: For a family $\mathcal{F}$ of subsets of $\mathbb{R}^2$, the problem $\textsc{Recognition}(\mathcal{F})$ is to determine whether a given graph $G = (V, E)$ is the intersection graph of some sets in $\mathcal{F}$.

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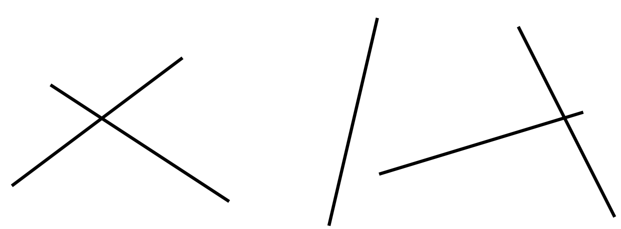
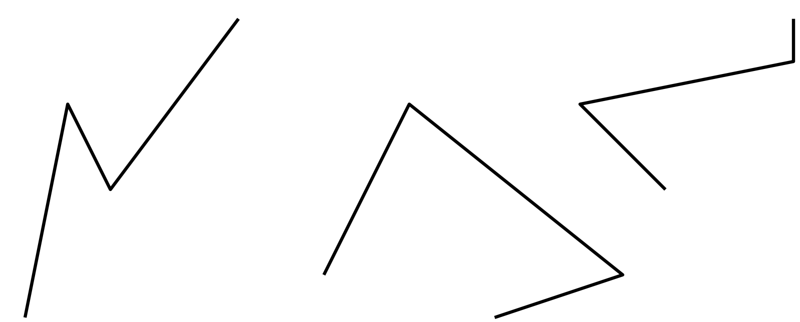
We consider the families of

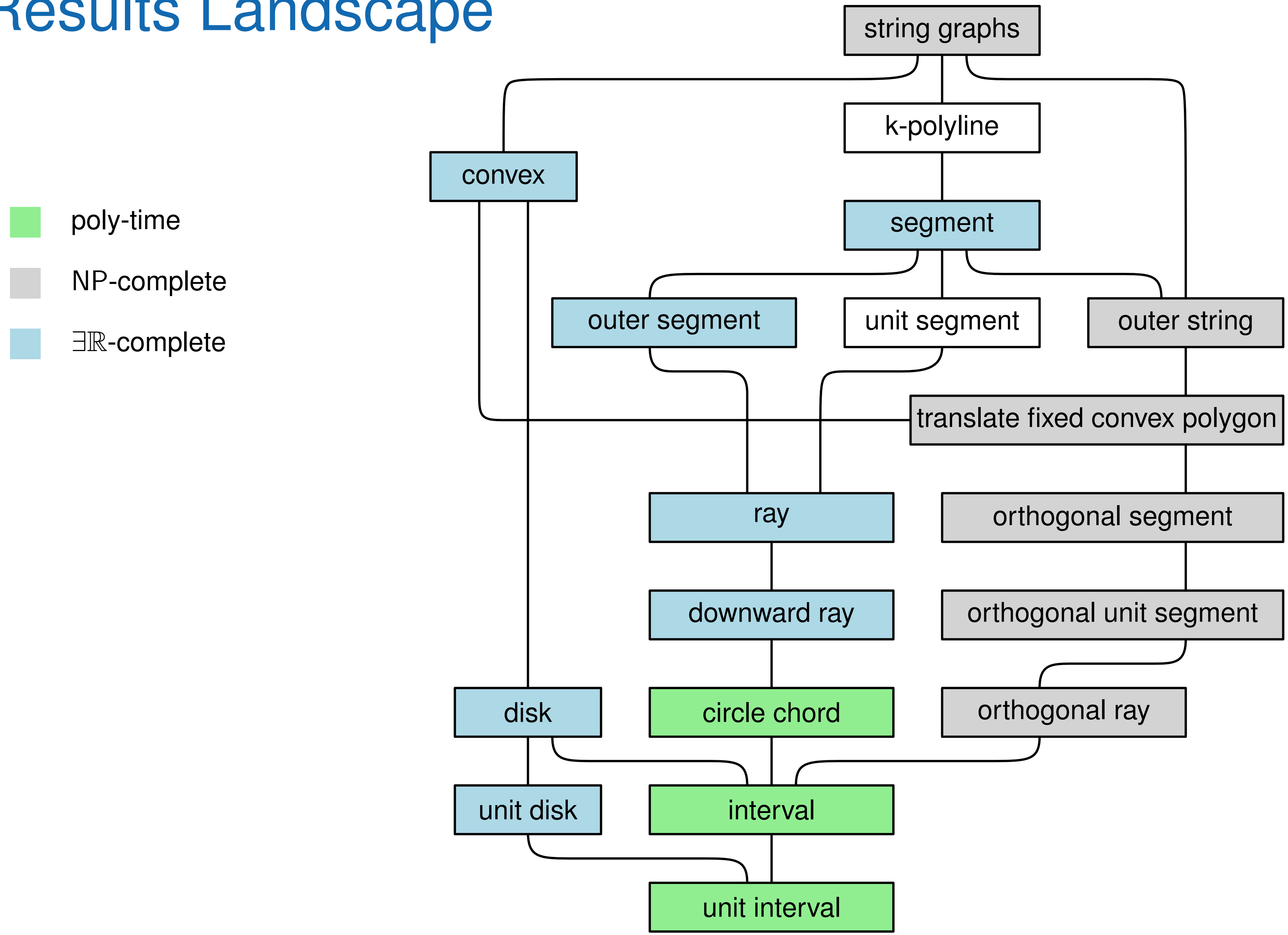


all unit segments in $\mathbb{R}^2$



all polylines with k bends (for any fixed k)

Results Landscape



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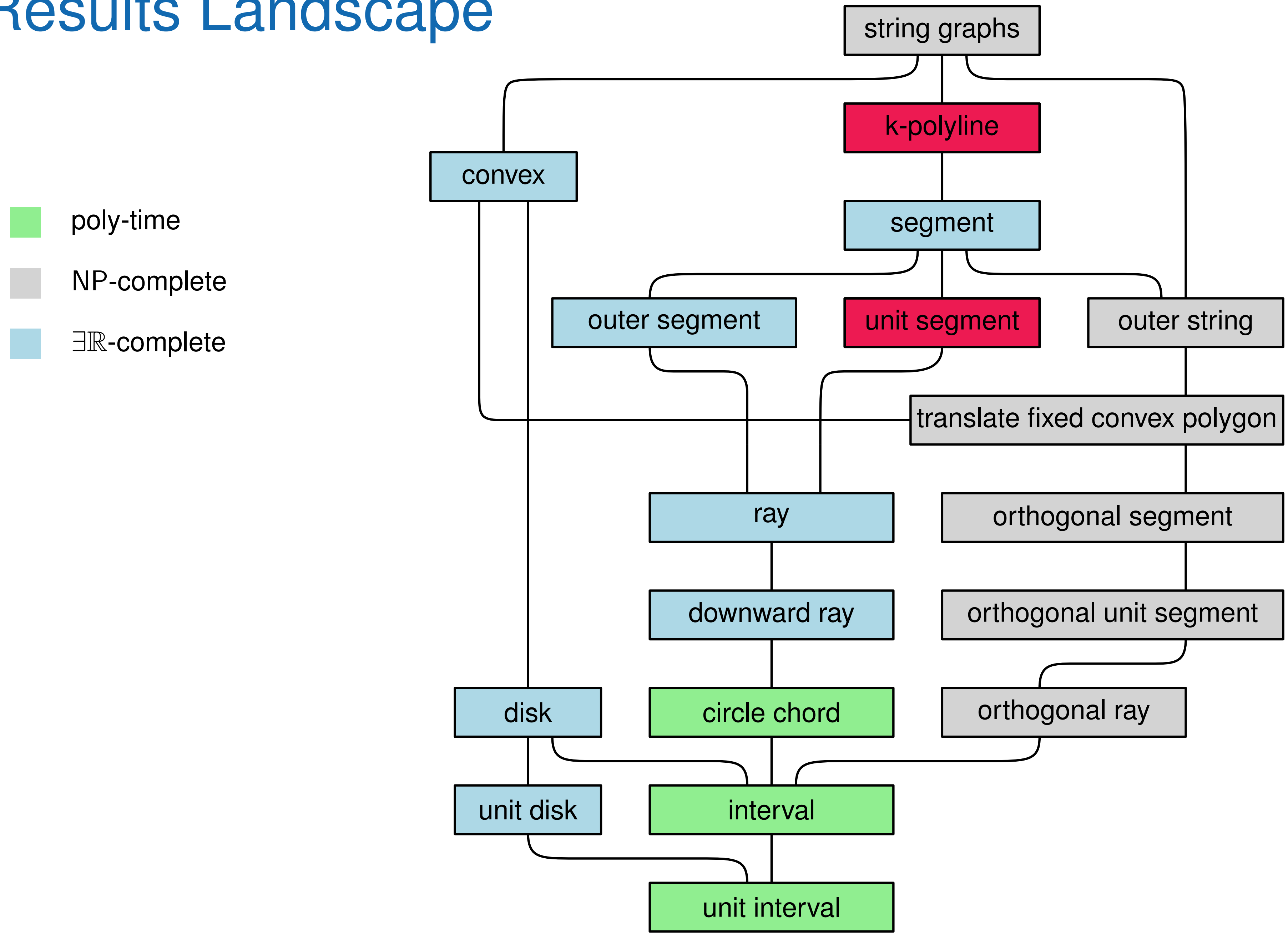

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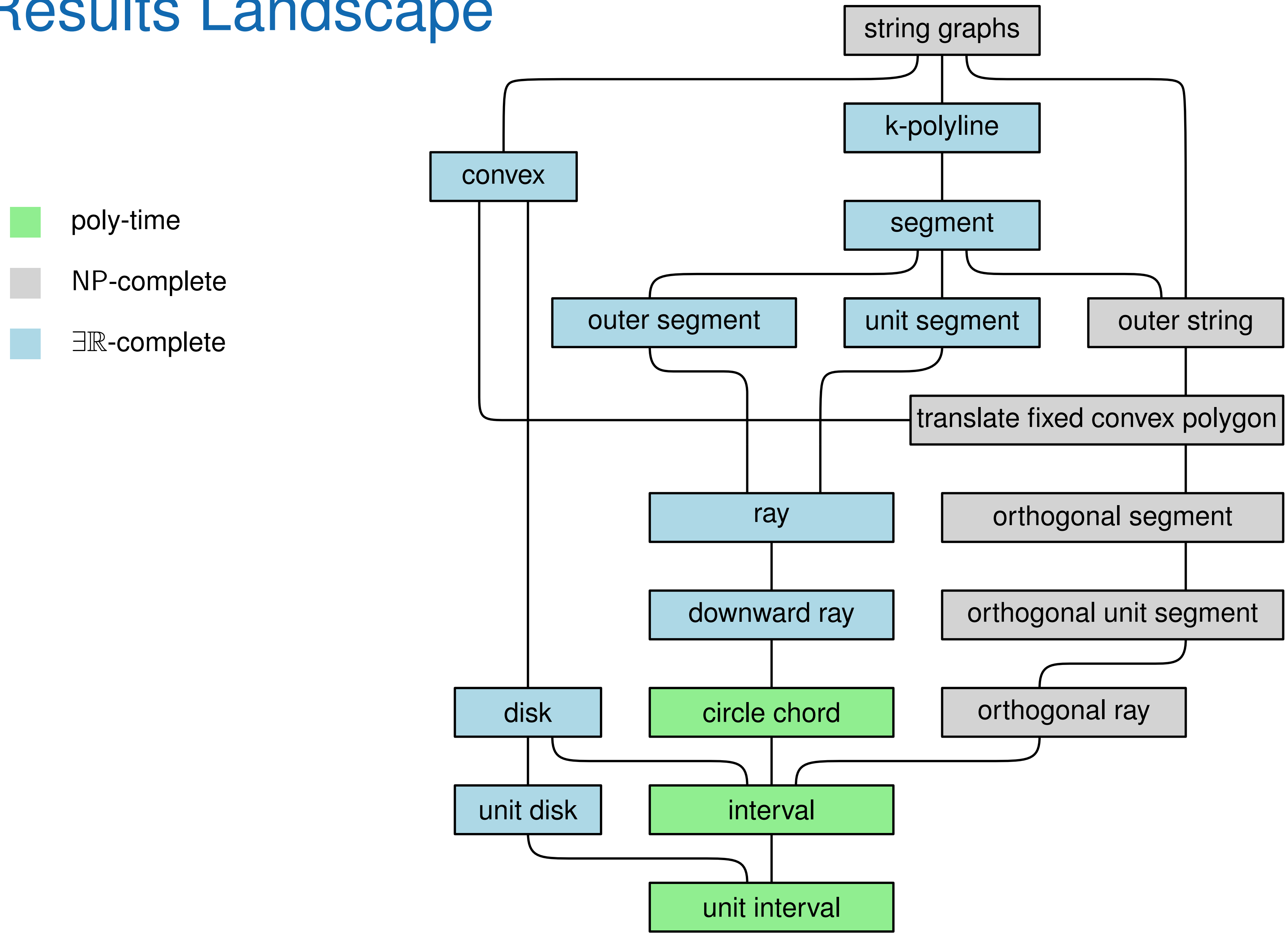
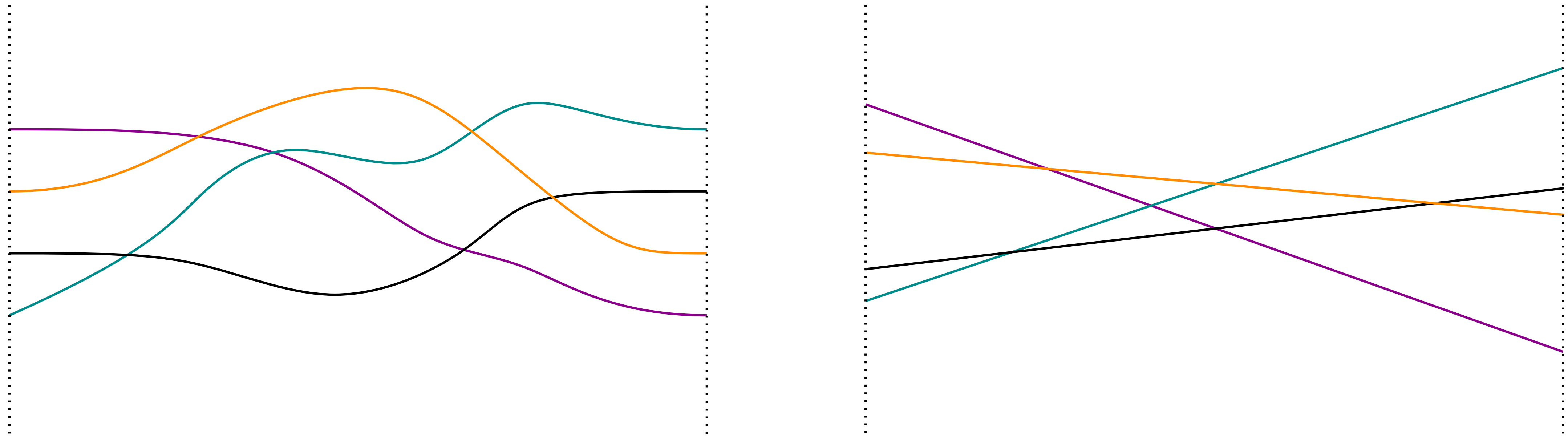
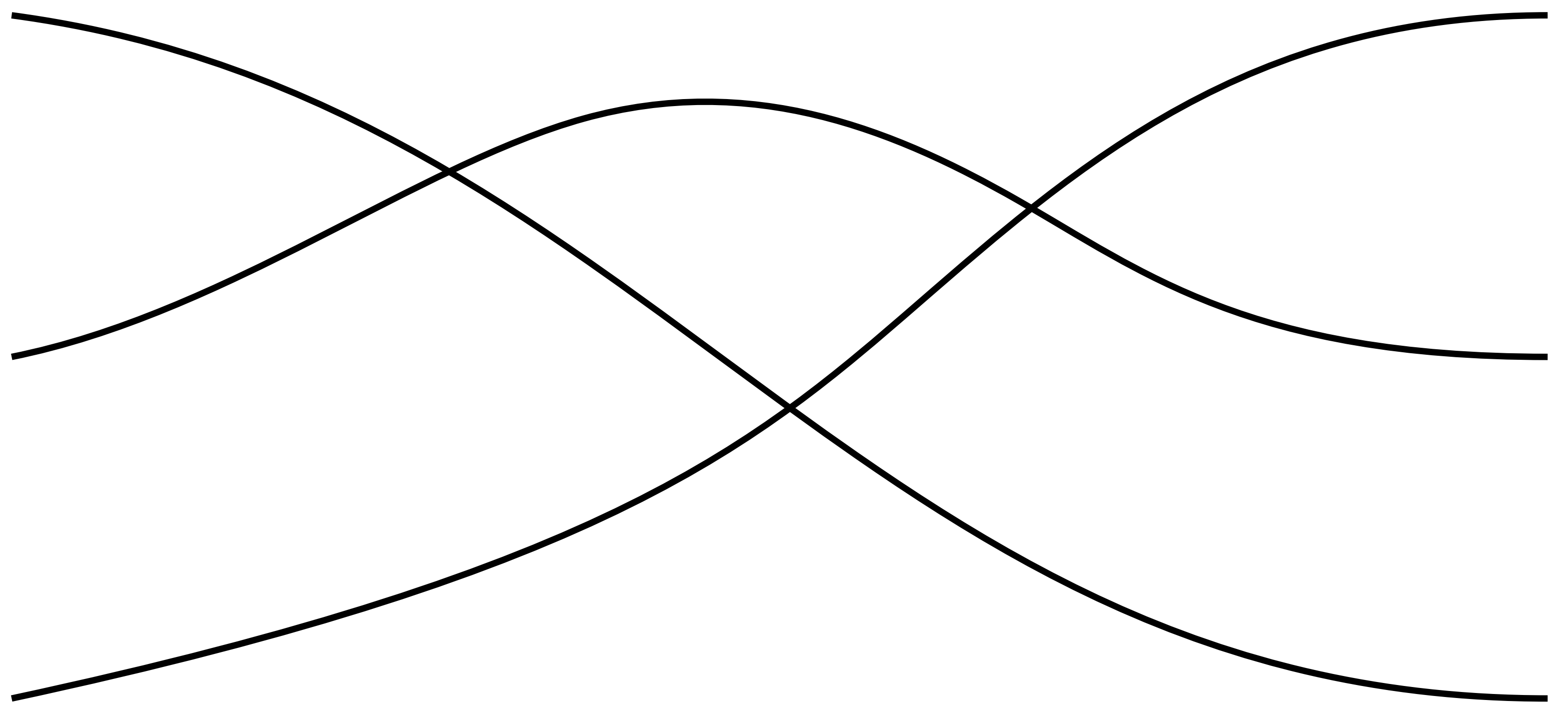


Proof Techniques: Hardness

Reduction from SIMPLESTRETCHABILITY:



Hardness for Unit Segments



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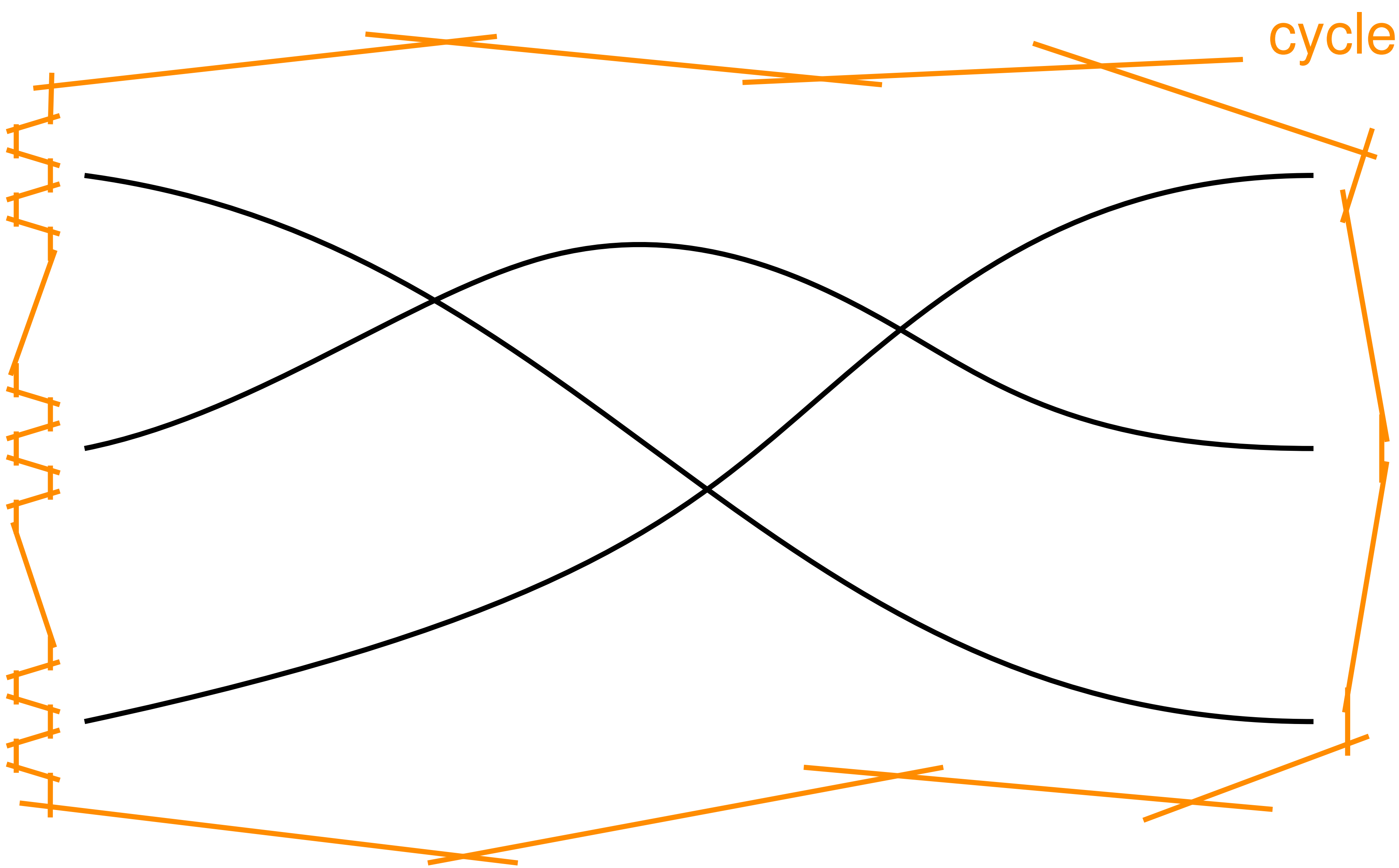

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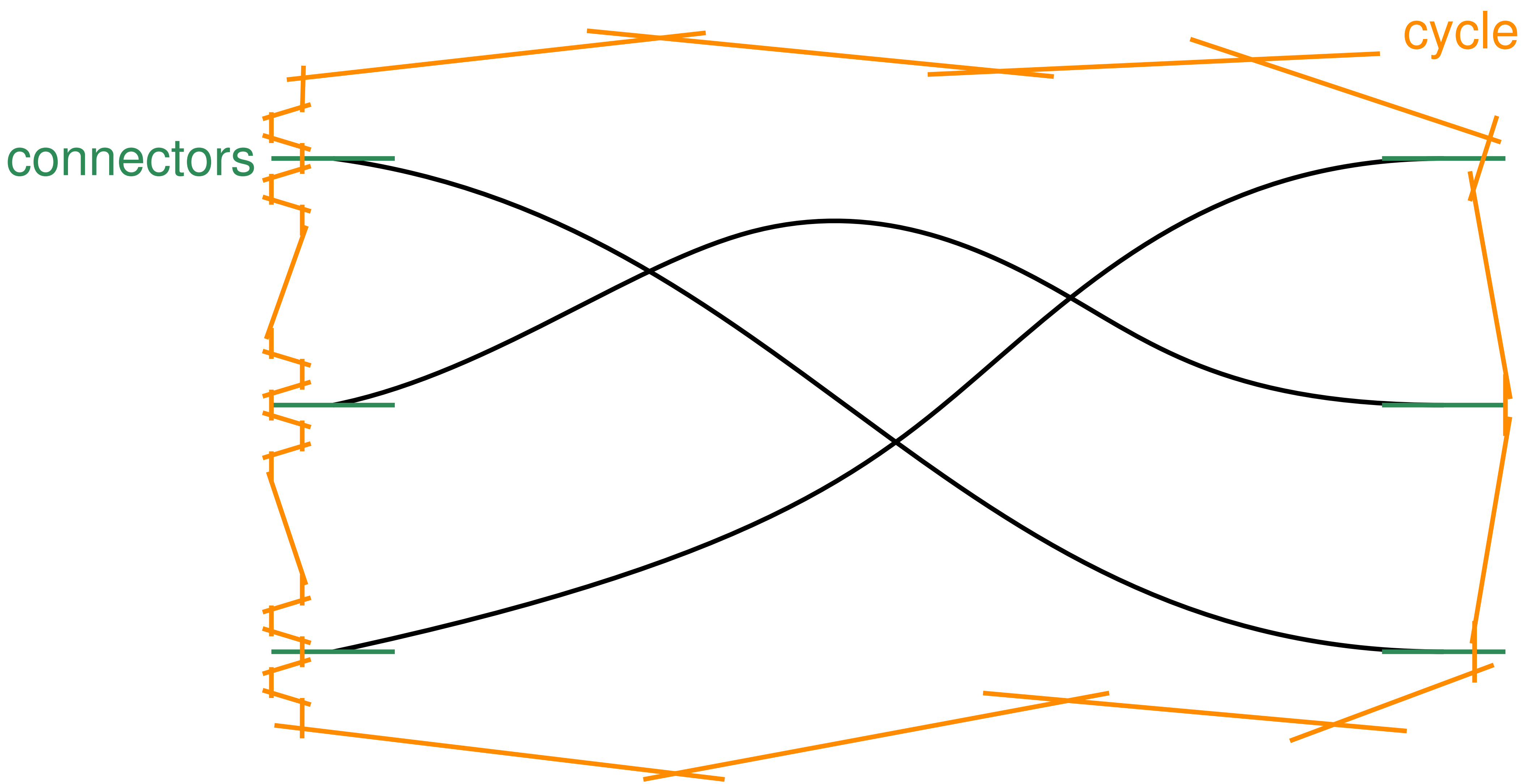

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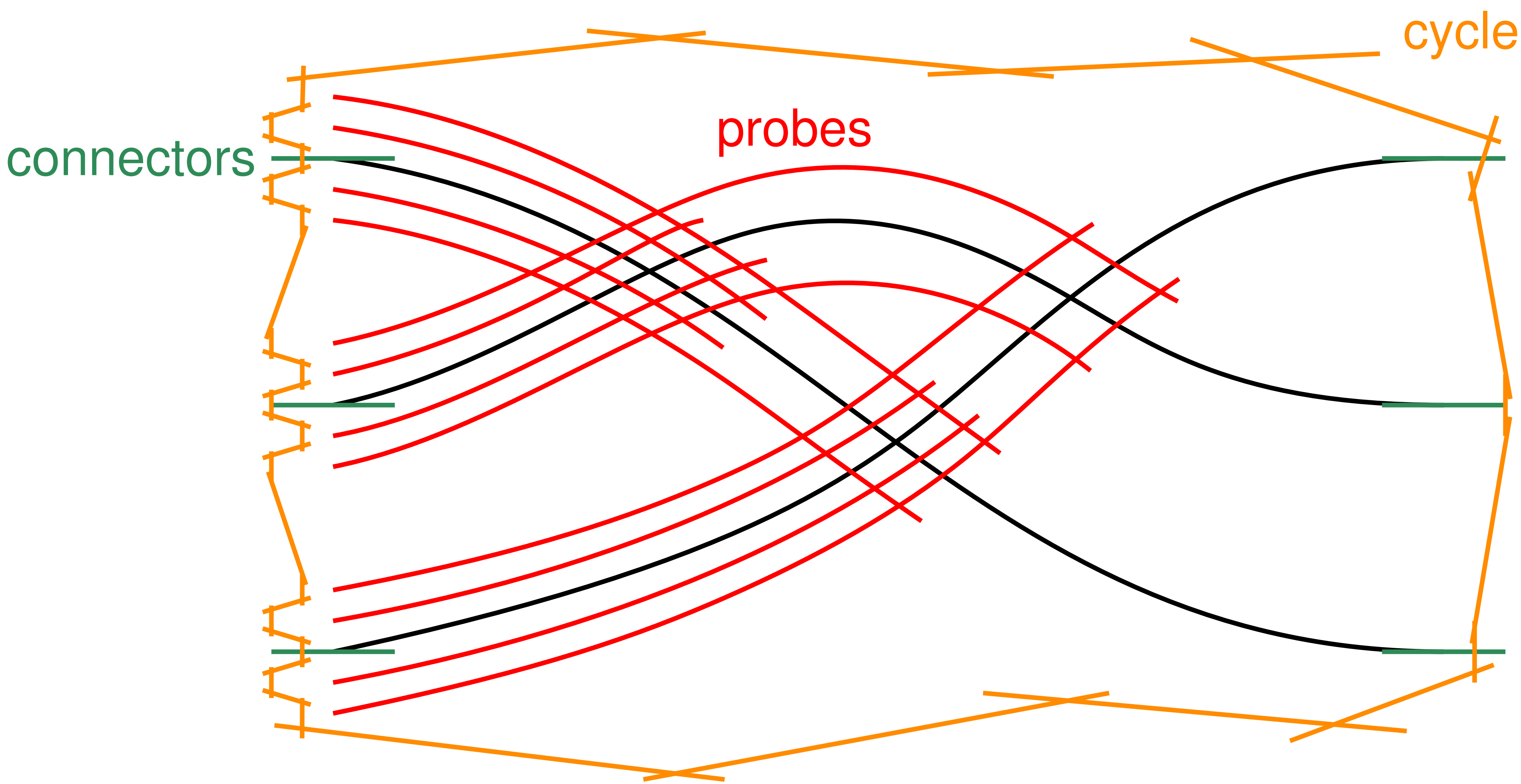

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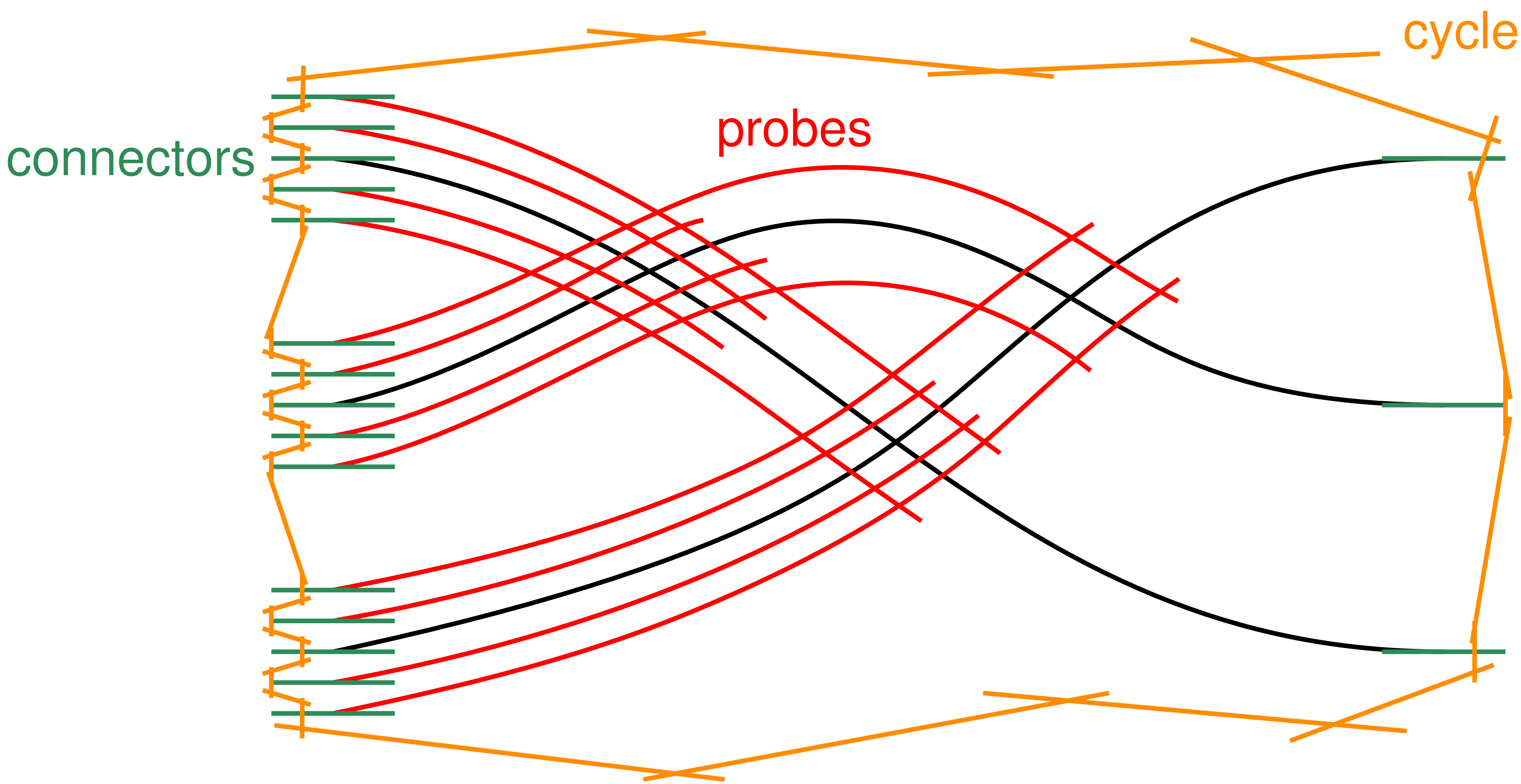

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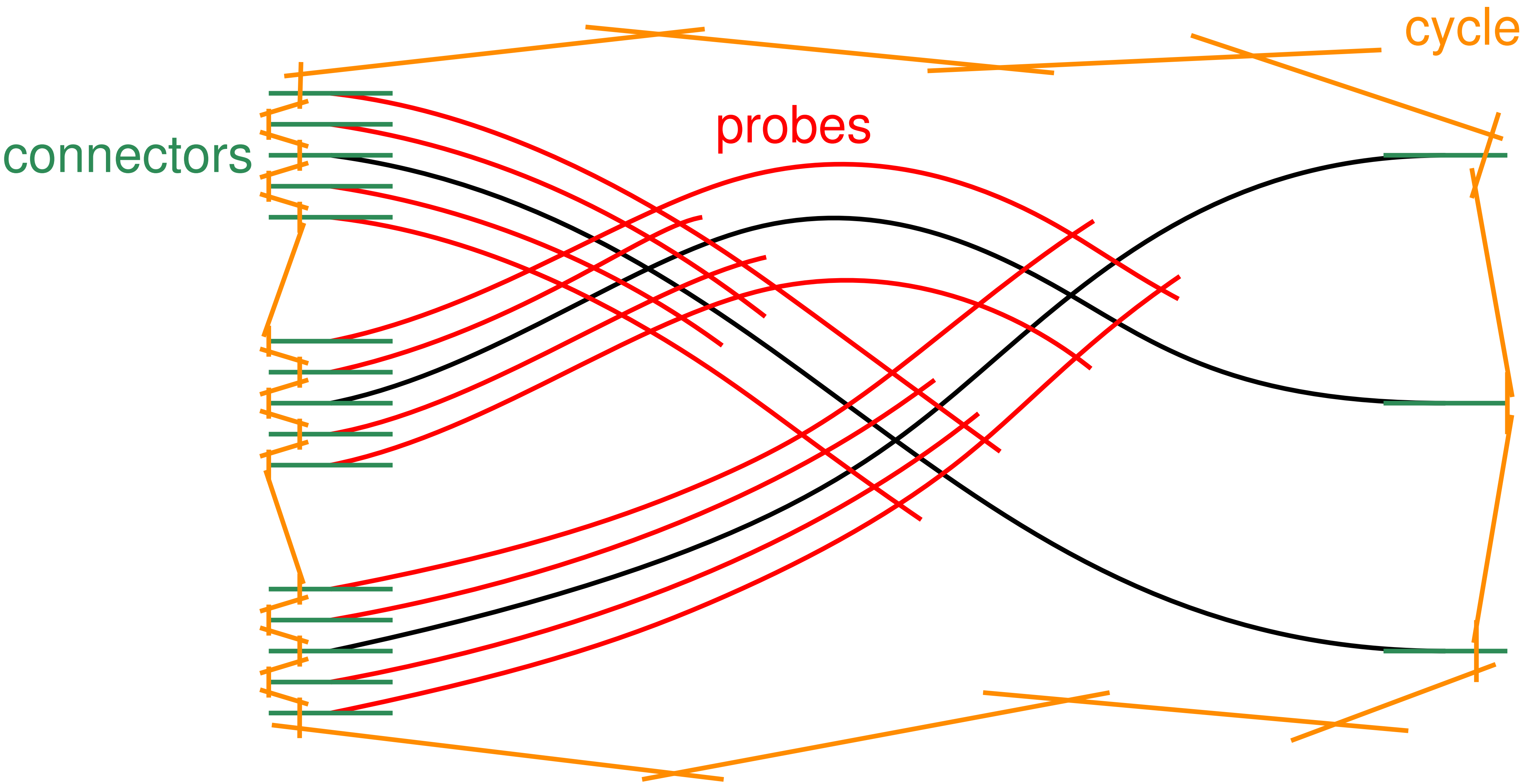

$G =$ intersection graph of these curves

Realizability $\Rightarrow$ Stretchability

If G is a unit segment intersection graph, then the pseudoline arrangement is stretchable.

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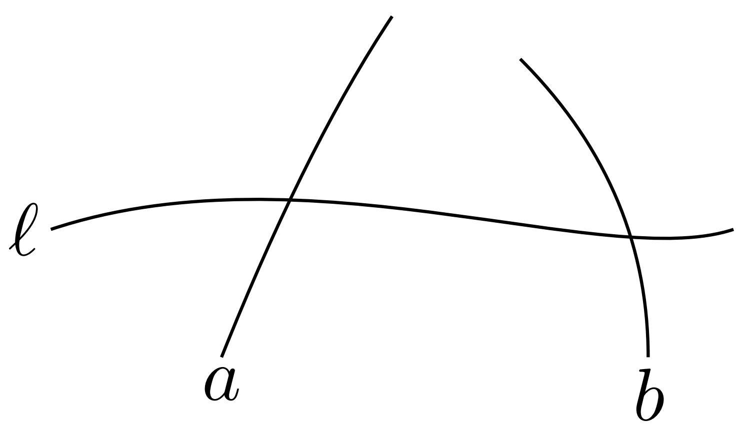

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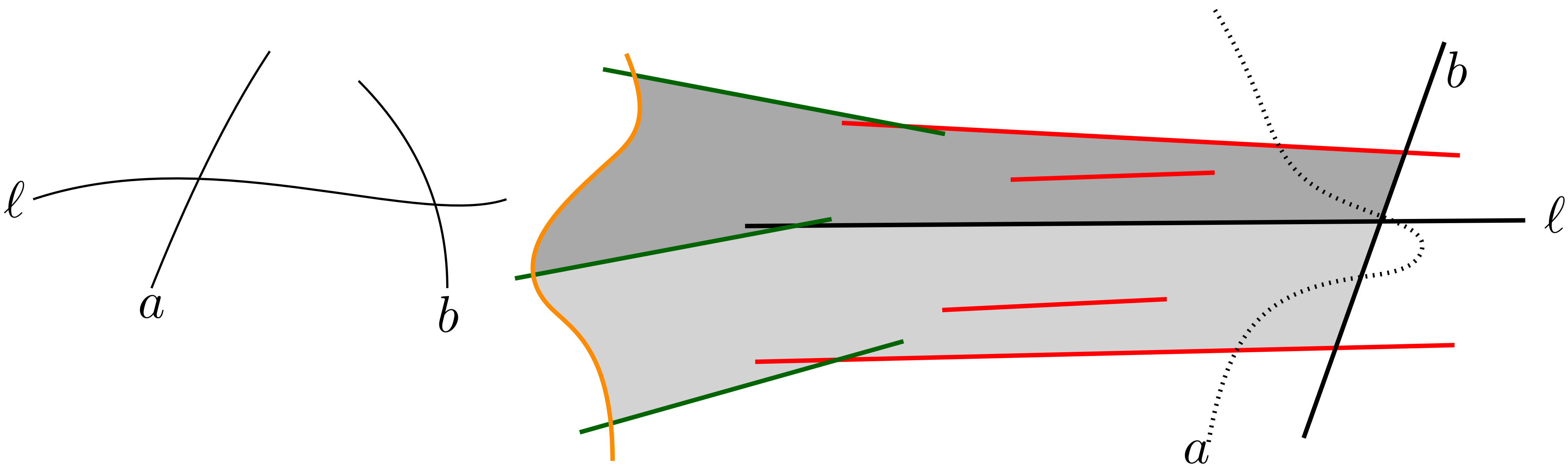

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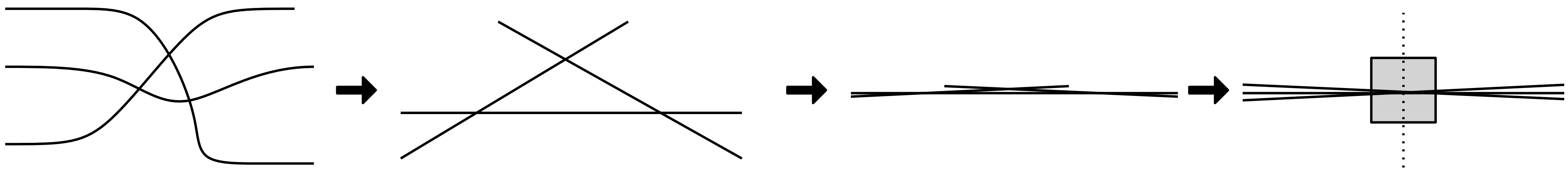

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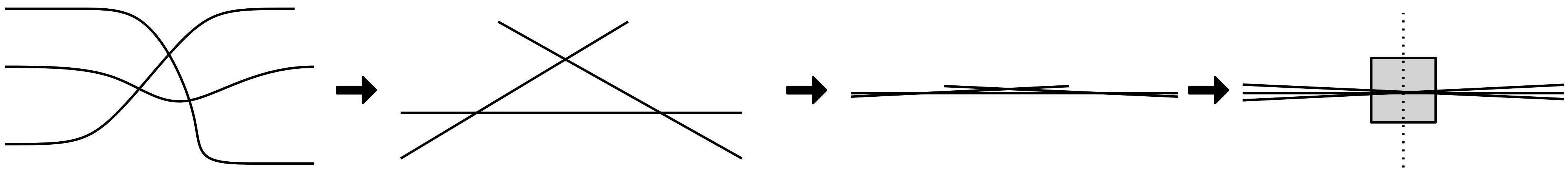

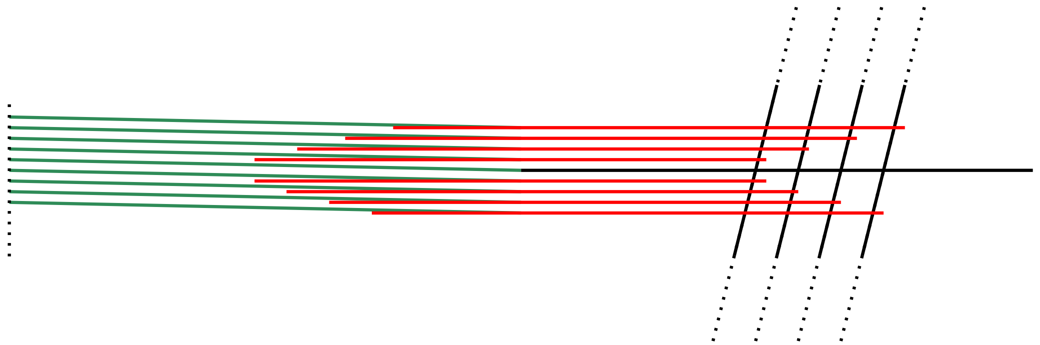

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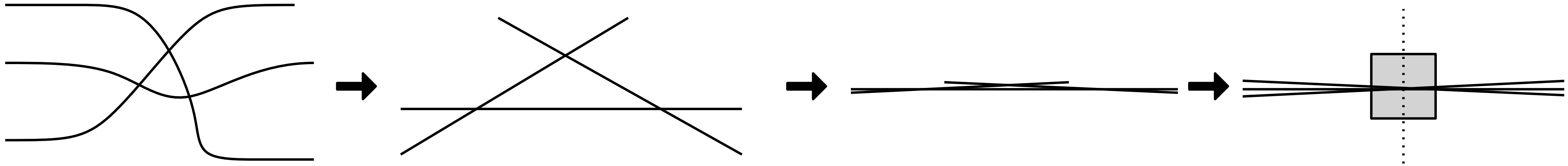
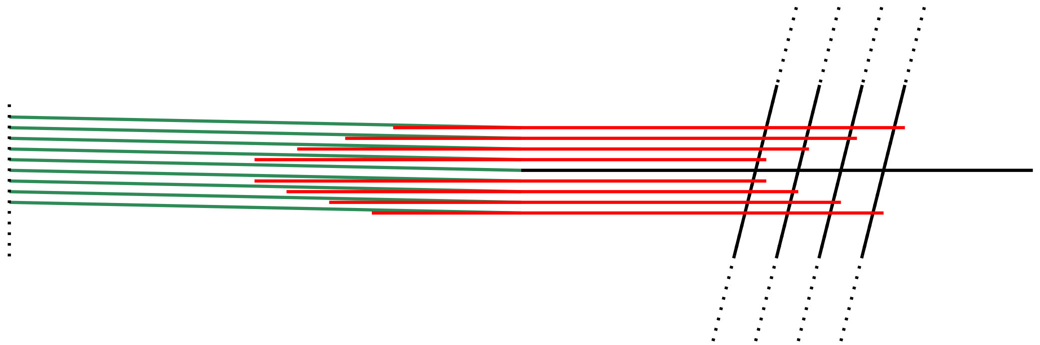
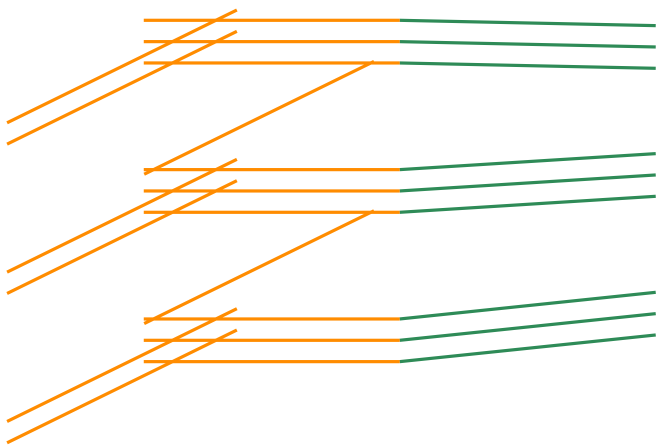
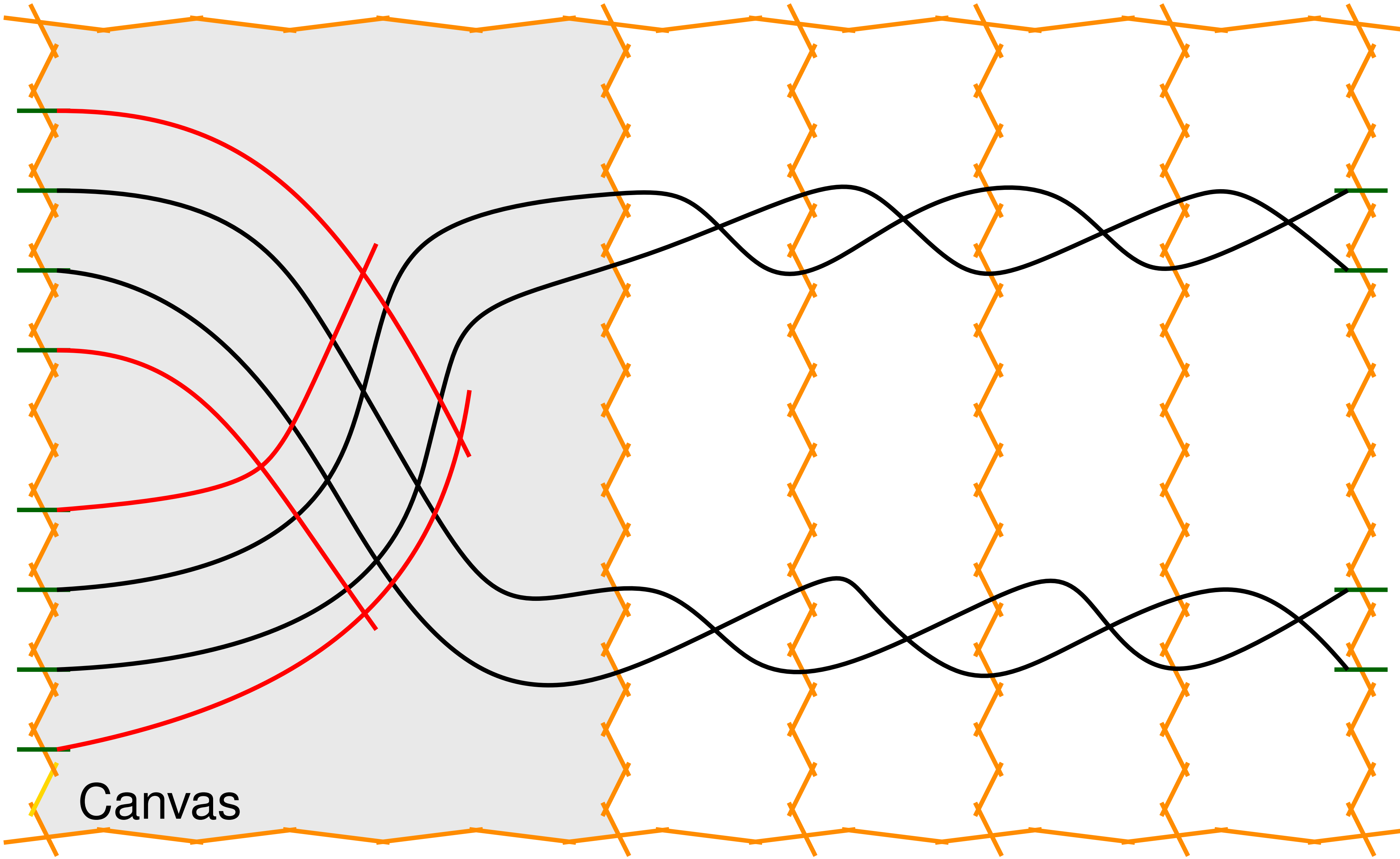






Hardness for k-Polylines



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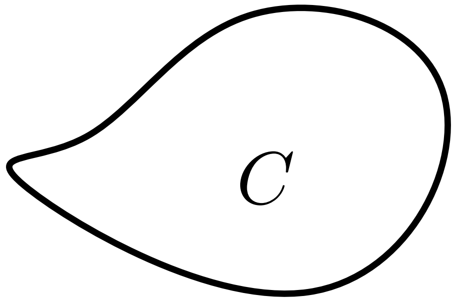

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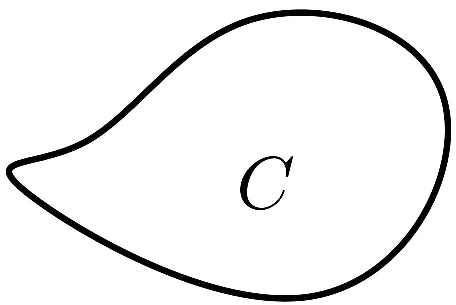

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For which shapes C is recognition of these families $\exists\mathbb{R}$-complete?