Covering line segments with drones: the minmax criterion [1]

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Motivation
Overview

- Fault detection
- Multiple drones
- Time cost minimization
- Recharge on a unique base station

Motivation

Problem Statement

Given a set of disjoint segments $\alpha = \{a_1, a_2, \cdots, a_n\}$ on a line.

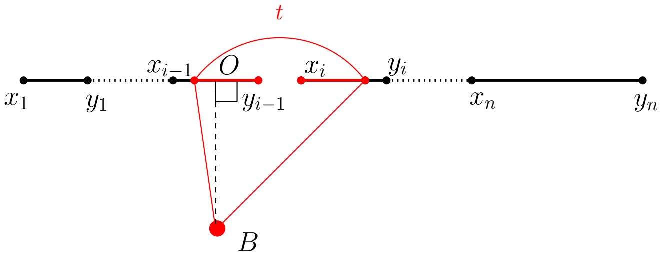

Figure: An example of a set α of intervals. A tour t (in red) is a part of the trajectory of one drone initialized and finished at B.

Motivation

Problem Statement

- $\ell(t)$ is the length of a tour t.
- $\ell(T) = \sum_{i=1}^{m} \ell(t_i)$ is the length of a collection of tours $T = \{t_1, \cdots, t_m\}$

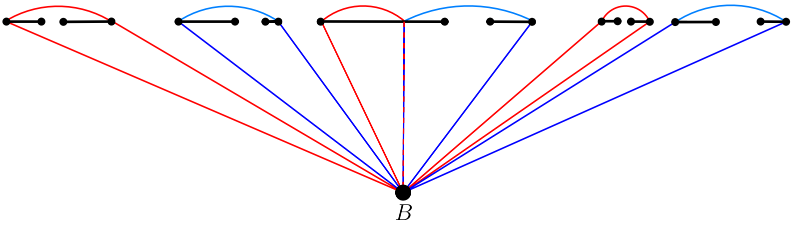

Figure: Covering segments on a line by two drones.

Motivation

Problem Statement

Problem

Minmax-k: compute a set of tours for each drone, $T_1, T_2, \cdots, T_k$*, such that:*

$$\alpha \subset T_1 \cup T_2 \cup \cdots \cup T_k \text{ and,}$$

$$\max_{j=1,\cdots,k} \ell(T_j) \text{ is minimized.}$$

Motivation

Observations

- The problem of minimizing the total length of a set of tours performed by a single drone (Minsum) is solved optimally in $O(n^2) + O(nm)$ with dynamic programming.
- The Minmax problem for two drones is NP-hard

NP-Hardness of min-max problem for two drones

Finite partition problem

Theorem

The min-max problem for two drones is NP-hard

- The finite partition problem is NP-hard: Given a finite set of positive integers $S = \{s_i\}_{i=1}^{n}$, determine if there is a subset $A \subset S$, such that $\sum_{s_i \in A} s_i = \sum_{s_j \in S \setminus A} s_j$.
- It is NP-hard also if the cardinality of A is half of S and S is even. (*two-way balanced partition problem*)
- *It is NP-hard also after applying a linear transformation to all the points of S.*

The NP-hardness of Minmax-2. Construction

For $i = 0, \cdots, 2n$, determine a sequence of right hand points y_i, with $c_i = d(y_i, B)$, that satisfies:

$$c_0 > \max\left(\frac{L}{2} - \frac{\epsilon L}{4n}, L - y_0 - \frac{L}{3}, \frac{\sqrt{5}L}{6}\right),$$

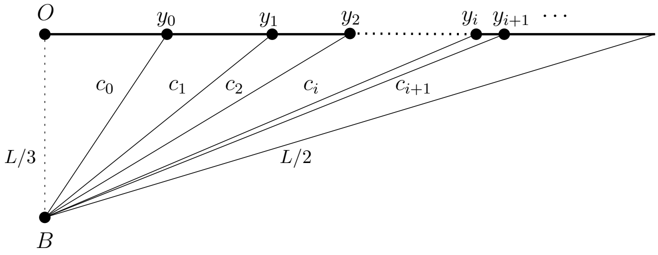

Figure: Right hand points y_i.

The NP-hardness of Minmax-2. Construction

For $i = 1, \cdots, 2n$, determine the sequence of right hand points x_i, with $b_i = d(x_i, B)$ that satisfies:

$$c_i + (y_i - x_i) + b_i = s'_i \in S' = \{Ks_i + 2c_{2n}\}_{i=1}^{2n},$$

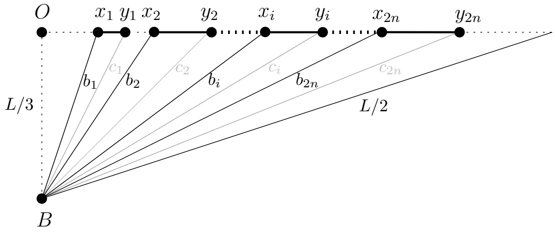

Figure: Left hand points x_i.

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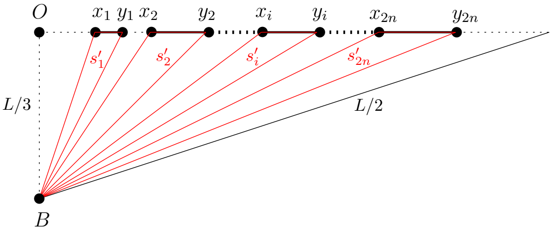

Figure: Left hand points x_i.

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The NP-hardness of Minmax-2. Construction. Step 4.

For $i = 1, \cdots, 2n$, draw the subsequence of right hand segments $[x_i, y_i]$ for i even, and left hand segments $[-y_i, -x_i]$ for i odd.

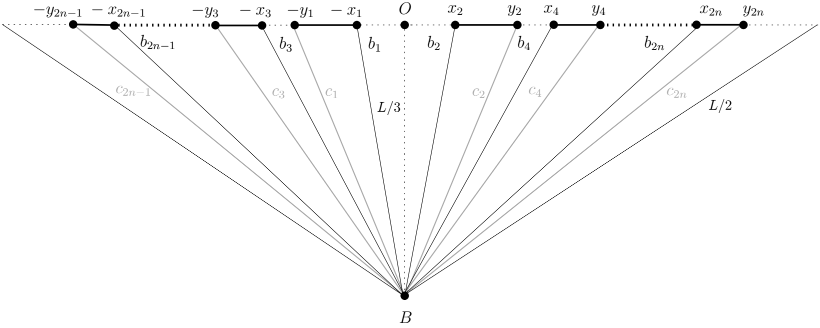

Figure: Final construction. A drone can not begin in one segment and finish in another.

The NP-hardness of Minmax-2 problem

In our construction:

- The solution of the Minsum problem for one drone:

$$T = \{t_i\}_{i=1}^{2n}$$

  each tour t_i covering exactly the i-th segment.

- The tours of any solution $\{T_1, T_2\}$ for the Minmax-2 problem correspond to tours t_i
- Solving this min-max problem for two drones would imply solving the balanced two-way finite partition problem.

Final conclusions about NP-Hardness

- The min-max problem for k drones would likely be NP-Hard, using the same ideas and the Multiway number partition problem.
- The finite partition problem for rational numbers is Strongly NP-Hard, so we conjecture that the min-max problem for two drones is strongly NP-Hard.

Idea of the algorithm
The one drone solution

Step 1. Apply the algorithm for the Minsum problem for one drone and get a solution T.

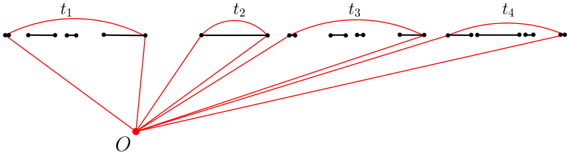

Figure: Optimal solution for the minimal total distance

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Idea of the algorithm

Greedy tour distribution

Starting from the farthest tour assign tours to each drone in such a way the total difference of the lengths does not exceed L

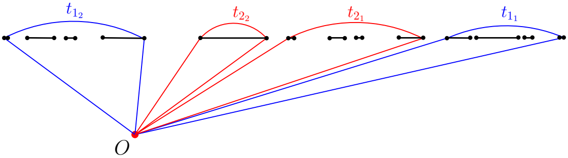


Figure: The lengths of the tours in each set is $|\ell(T_2) - \ell(T_1)| = aL$ with $0 \leq a \leq 1$

Properties of the algorithm

Output Sensitive

Theorem

Let $\{T_1^*, T_2^*\}$ *be any solution of the Minmax-2 problem and let* $\{T_1, T_2\}$ *be the final distribution of the* G2D-*algorithm. Assume* $\ell(T_1) \leq \ell(T_2)$ *and* $\ell(T_1^*) \leq \ell(T_2^*)$. *Then:*

a) $\ell(T_1) + \frac{aL}{2} \leq \ell(T_2^*) \leq \ell(T_1) + aL$

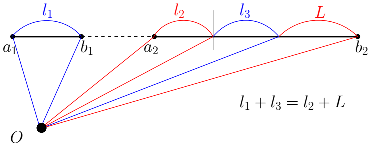
b) *If* $a = 0$, *then* $\{T_1, T_2\}$ *is optimal for the Minmax-2 problem.*

c) *If* $a \in (0, 1]$, *then* $\ell(T_2) \leq \Delta \cdot \ell(T_2^*)$, *with* $\Delta = \frac{\Gamma + 2}{\Gamma + 1}$ *and* $\Gamma = \frac{2\ell(T_1)}{aL}$.

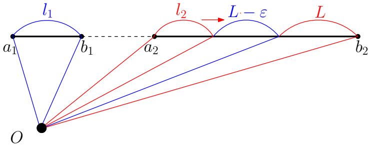
Future research
Cutting and enlarging

Future research:

- Looking for a better approximation algorithm for the problem.



(a) Cutting the tour.



(b) Enlarging a Tour.

Figure: Cutting and enlarging strategy for two segments

Future research

Future research:

- Considering horizontal segments located on a grid, which is the layout of tubes in solar power plants
- Looking for a good approximation for line segments of arbitrary orientations problem.

Thank you!