Bounds on the Edge-length Ratio of 2-outerplanar Graphs

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Outline

Problem definition

State of the art and our contribution

Some technicalities

Open problems

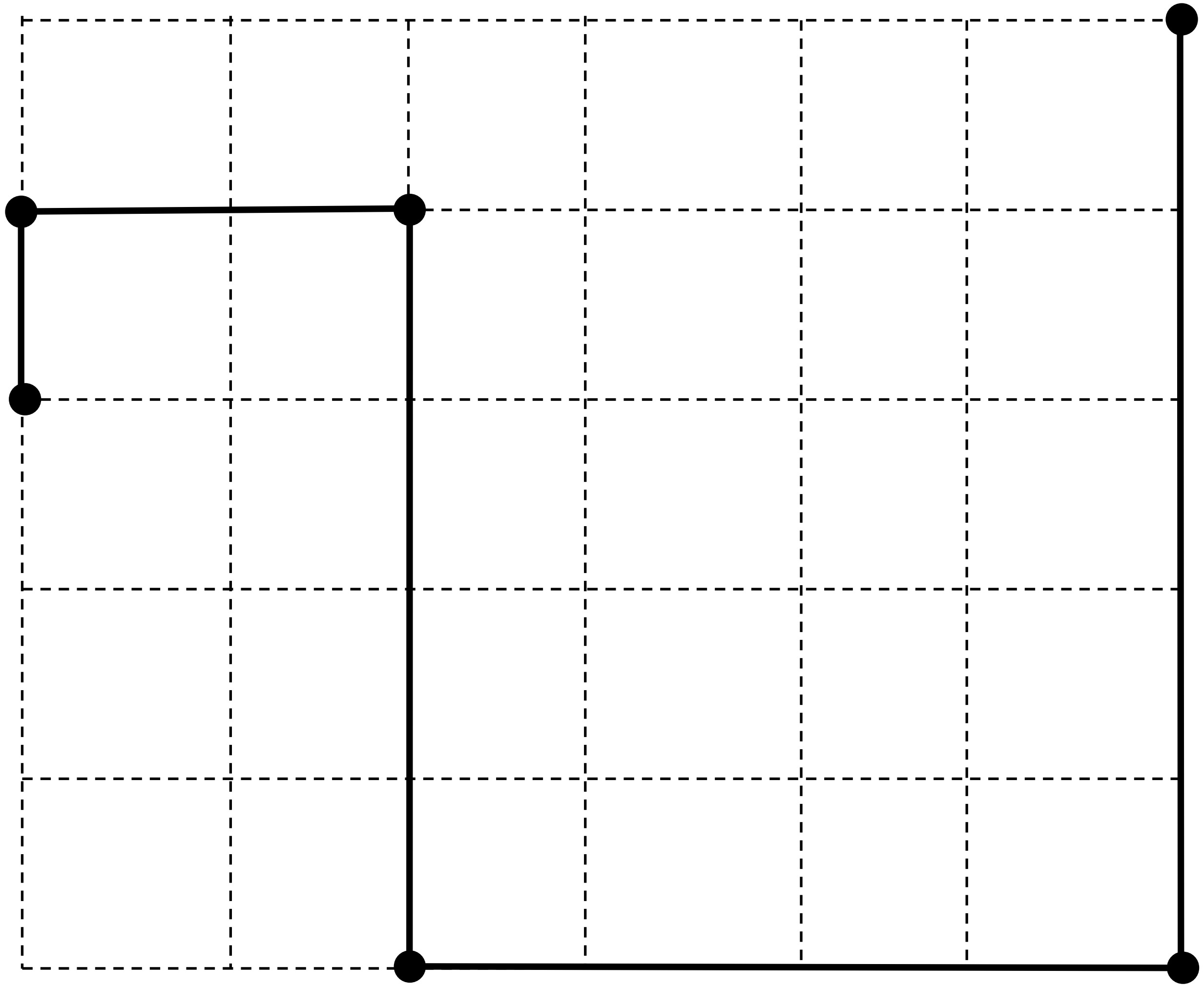
What is the (local) edge-length ratio?

Edge-length Ratio of a Planar Drawing

Let Γ be a planar **straight-line** drawing of a graph.

The **edge-length ratio** $\rho(\Gamma)$ of Γ is the maximum ratio between the lengths of every pair of its edges.

Example



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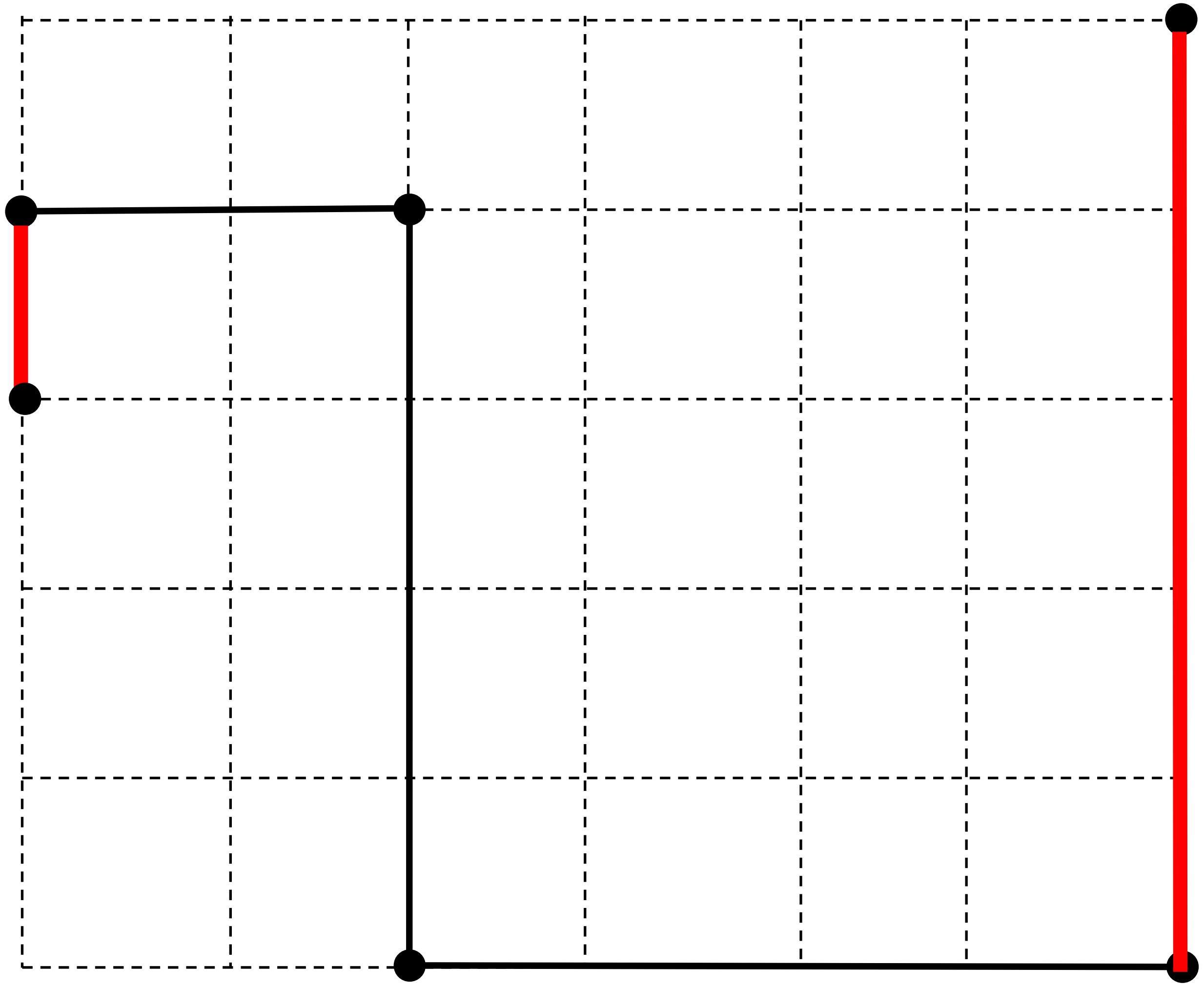


$\rho(\Gamma) = 5$

Edge-length Ratio of a Planar Graph

Let G be a planar **graph** and let $D(G)$ be the set of all planar straight-line drawings of G.

The **edge-length ratio** $\rho(G)$ of G is

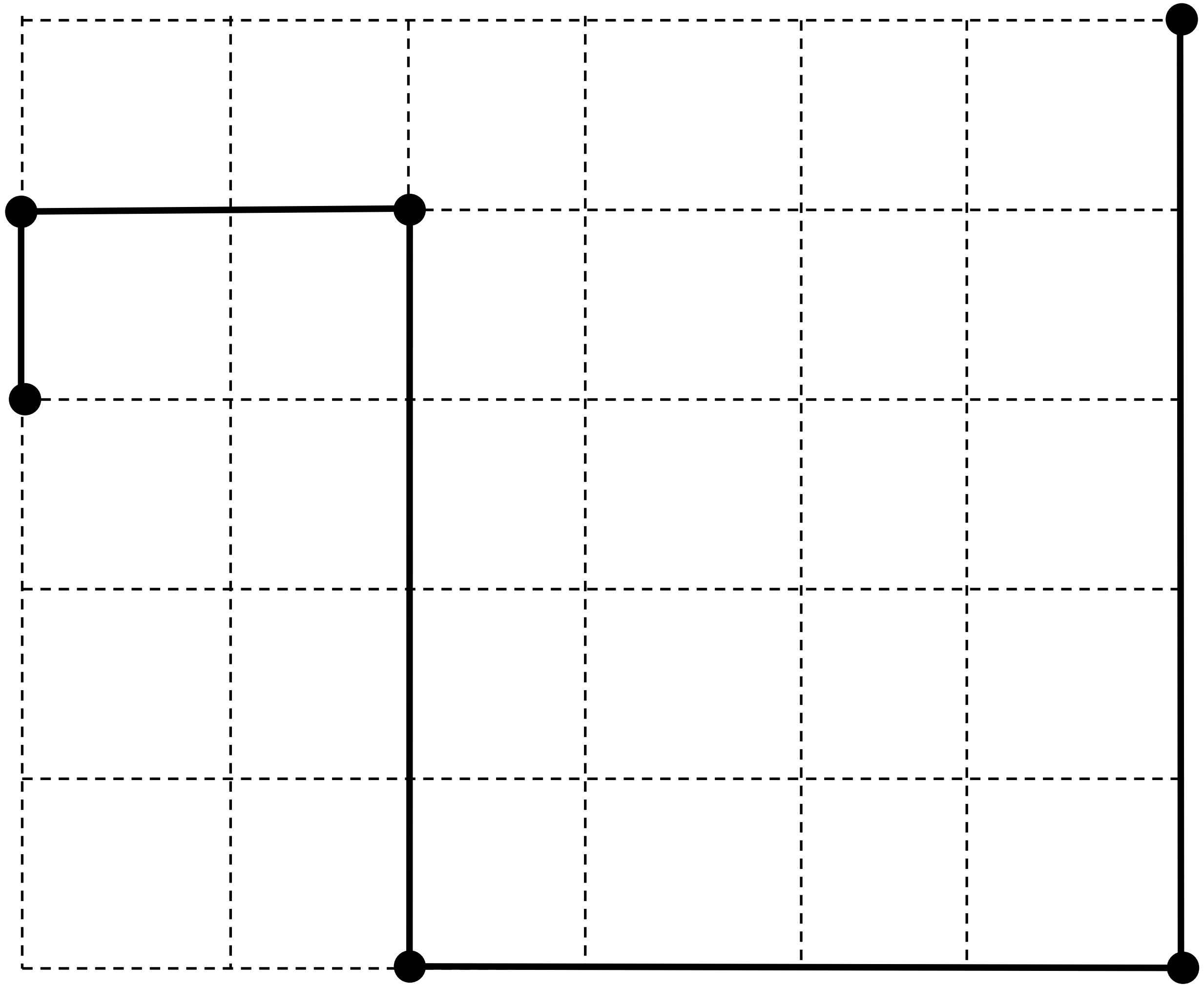
$$\rho(G) = \inf_{\{\Gamma \in D(G)\}} \rho(\Gamma)$$

Local Edge-length Ratio of a Planar Drawing

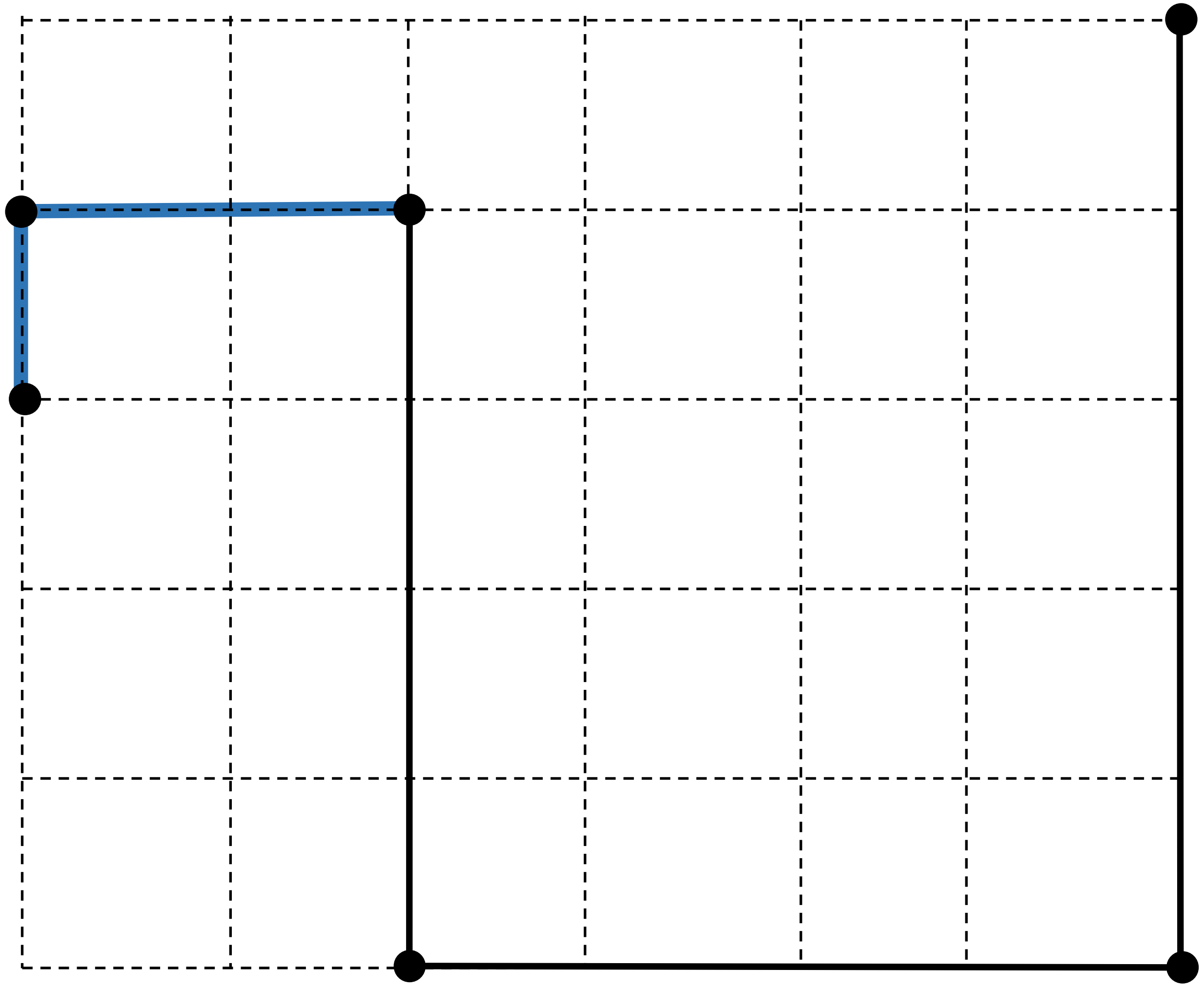
Let Γ be a planar **straight-line** drawing of a graph.

The **local edge-length ratio** $\rho_\ell(\Gamma)$ is the maximum ratio between the lengths of every pair of **adjacent edges** of of Γ.

Example



Example



$\rho_\ell(\Gamma)=2$

Local Edge-length Ratio of a Planar Graph

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$$\rho_\ell(G) = \inf_{\{\Gamma \in D(G)\}} \rho_\ell(\Gamma)$$

A Natural Question

Establish upper and lower bounds on the (local) edge-length ratios for various families of planar graphs.

Note: A lower bound on the local edge-length ratio is also a lower on the edge-length ratio; an upper bound on the edge-length ratio is also an upper bound on the local edge-length ratio.

What's known about these bounds?

Some recent results

ω(1) Lower Bounds:

The edge-length ratio over the class of n-vertex 3-trees is in $\Omega(n)$ [Borrazzo,Frati – 2020]

The edge-length ratio over the class of n-vertex 2-trees is in $\Omega(\log n)$ [Blazej, Fiala, L. - 2021]

Upper Bounds:

Let G be an n-vertex 2-tree: $\rho(G) \in O(n^{0.695})$ [Borrazzo,Frati – 2020]

Let G be an n-vertex 2-tree: $\rho_\ell(G) \leq 4$ [Blazej, Fiala, L. - 2021]

Let G be an outerplanar graph: $\rho(G) = 2$ [Lazard, Lenhart, L. - 2019]

Our Contribution

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- We prove that the local edge-length ratio over the class of n-vertex 2-outerplanar graphs is in $\Omega(\sqrt{n})$.

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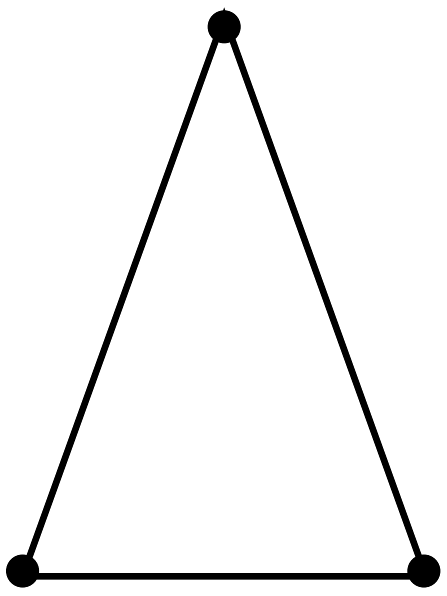
- We prove that the local edge-length ratio over the class of n-vertex 2-outerplanar graphs is in $\Omega(\sqrt{n})$.

- We study family of graphs having outerplanarity 2 for which $\rho(G) \in O(1)$(and hence $\rho_\ell(G) \in O(1)$). In the proceedings: Halin graphs.

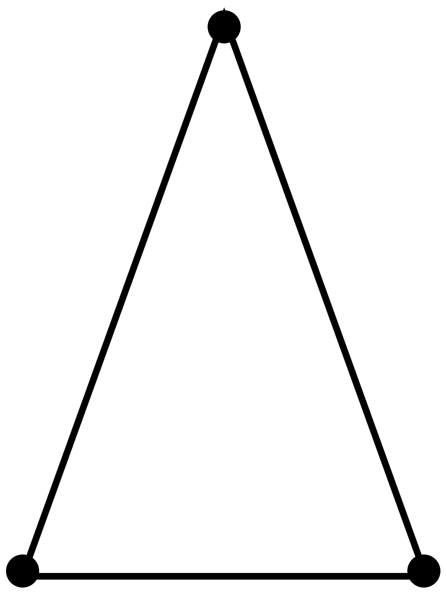
A Glance at the Technicalities: Lower Bound

Sketch of the Lower Bound (Fixed Embedding)



G_1

Sketch of the Lower Bound (Fixed Embedding)



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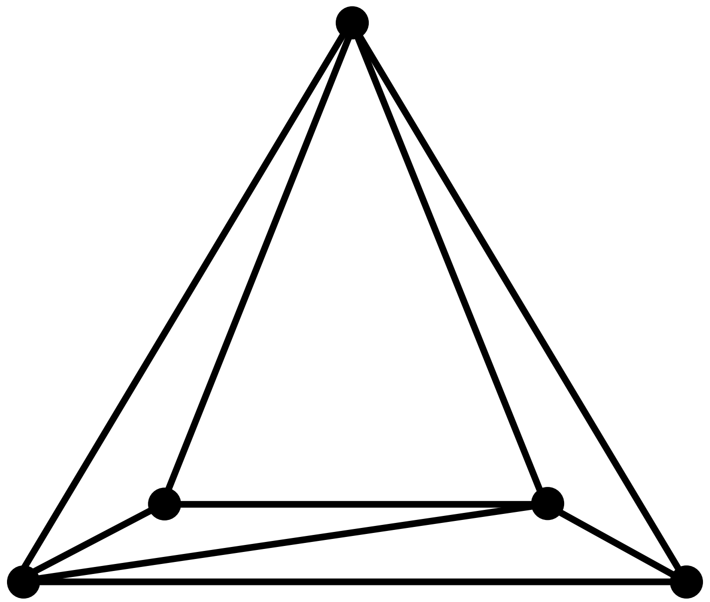
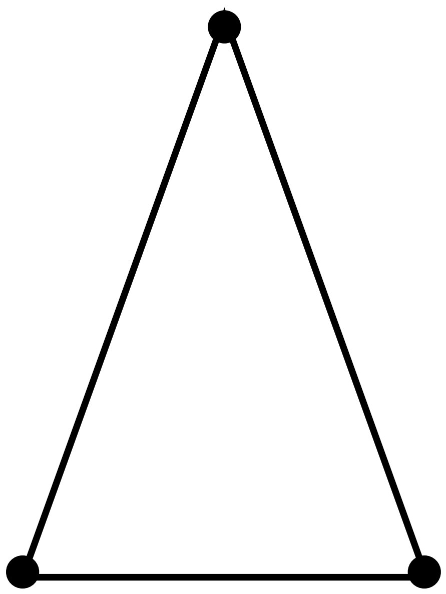
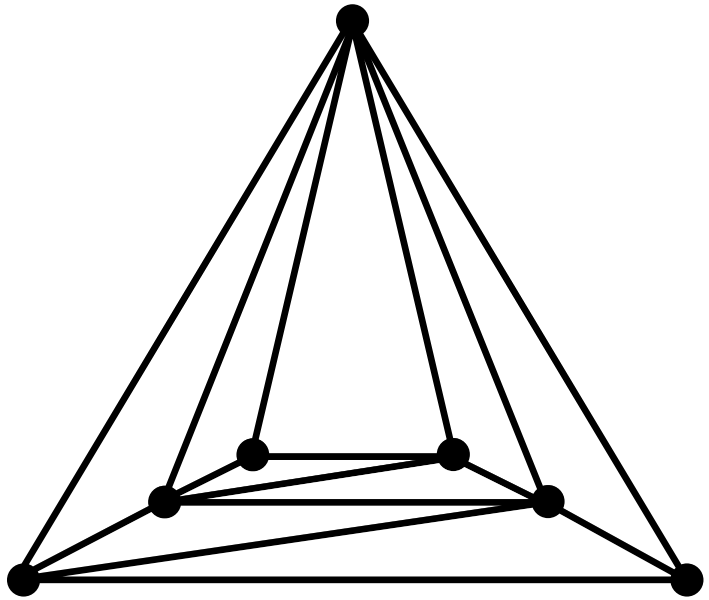


G_2

Sketch of the Lower Bound (Fixed Embedding)



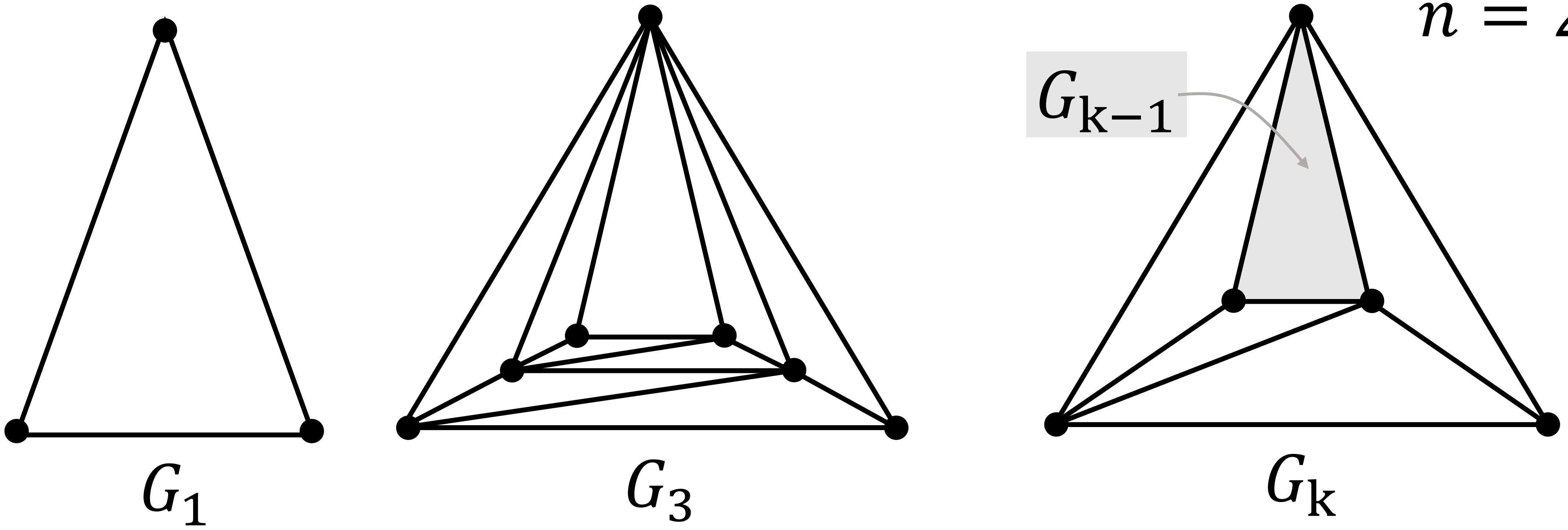
G_1



G_3

Sketch of the Lower Bound (Fixed Embedding)

$n = 2k + 1$



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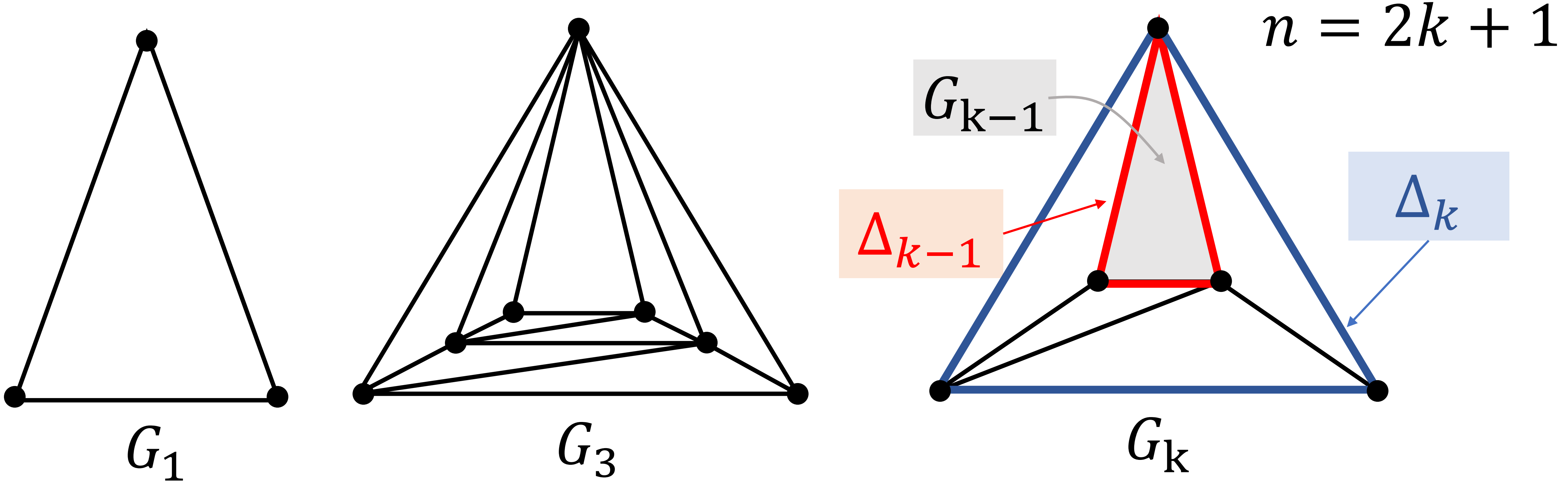
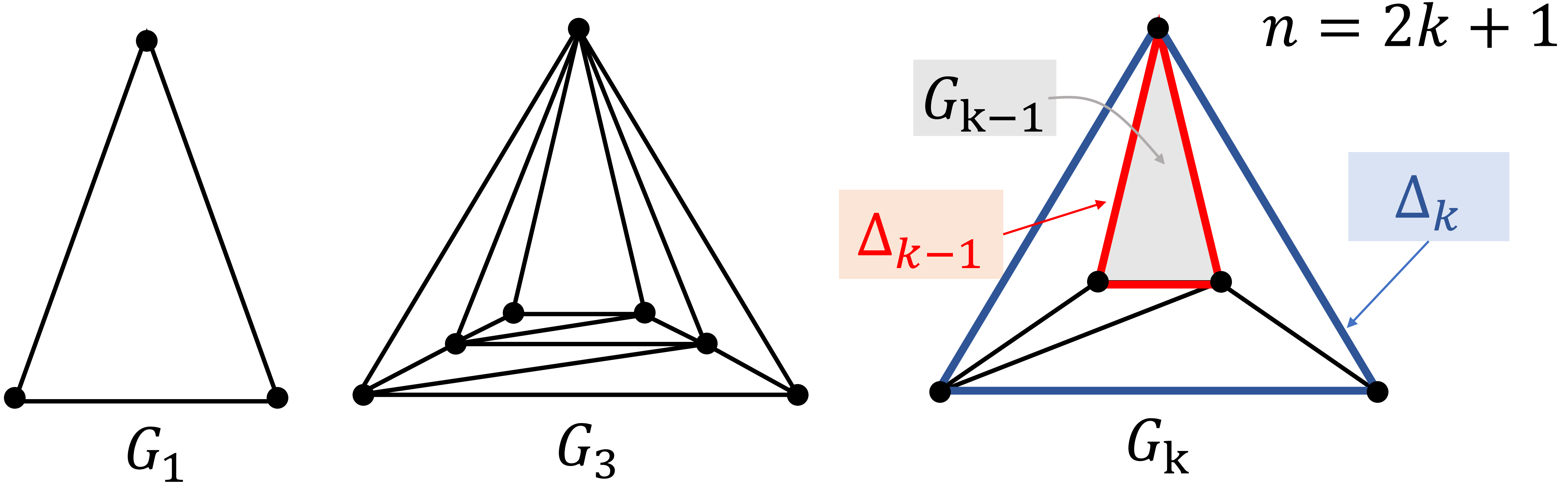


$P(\Delta_k) > P(\Delta_{k-1}) + 0.3$

Sketch of the Lower Bound (Fixed Embedding)



$$P(\Delta_k) > P(\Delta_{k-1}) + 0.3$$

$$P(\Delta_k) > 0.3 \cdot k$$

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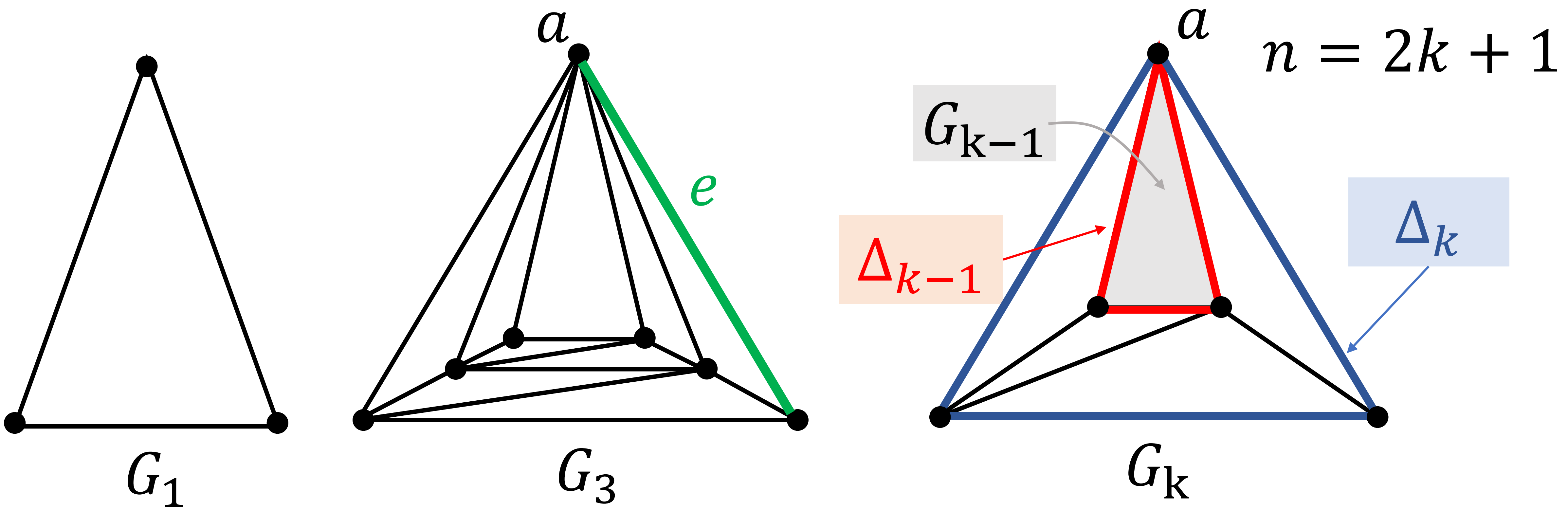

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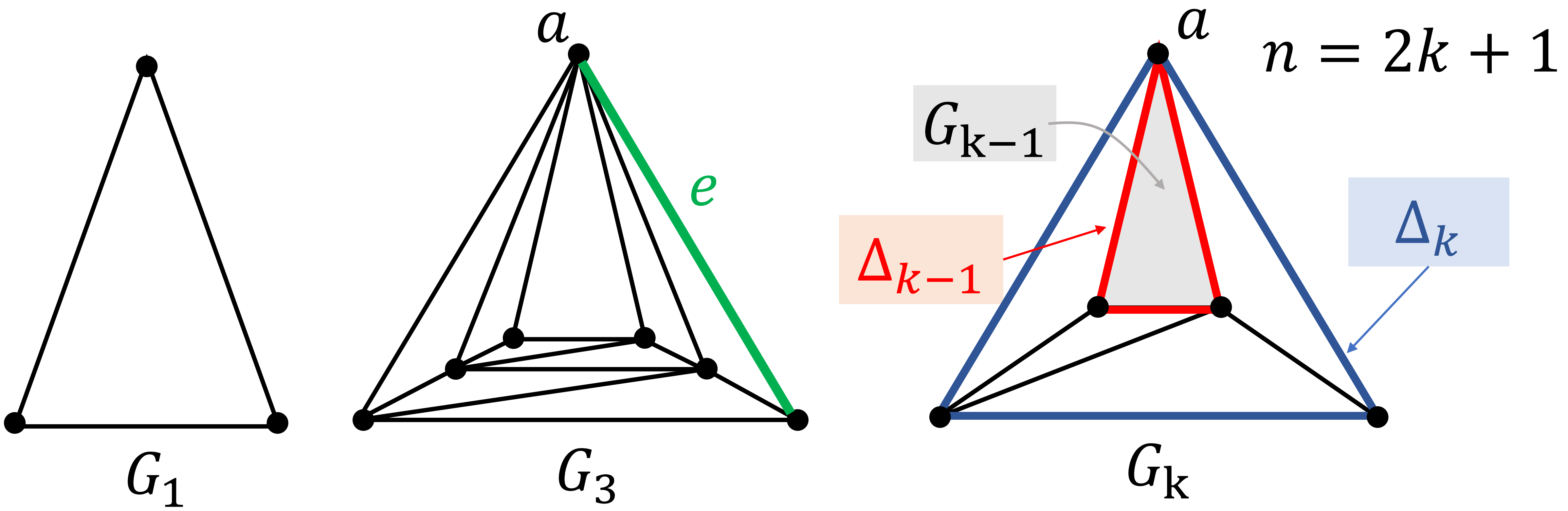
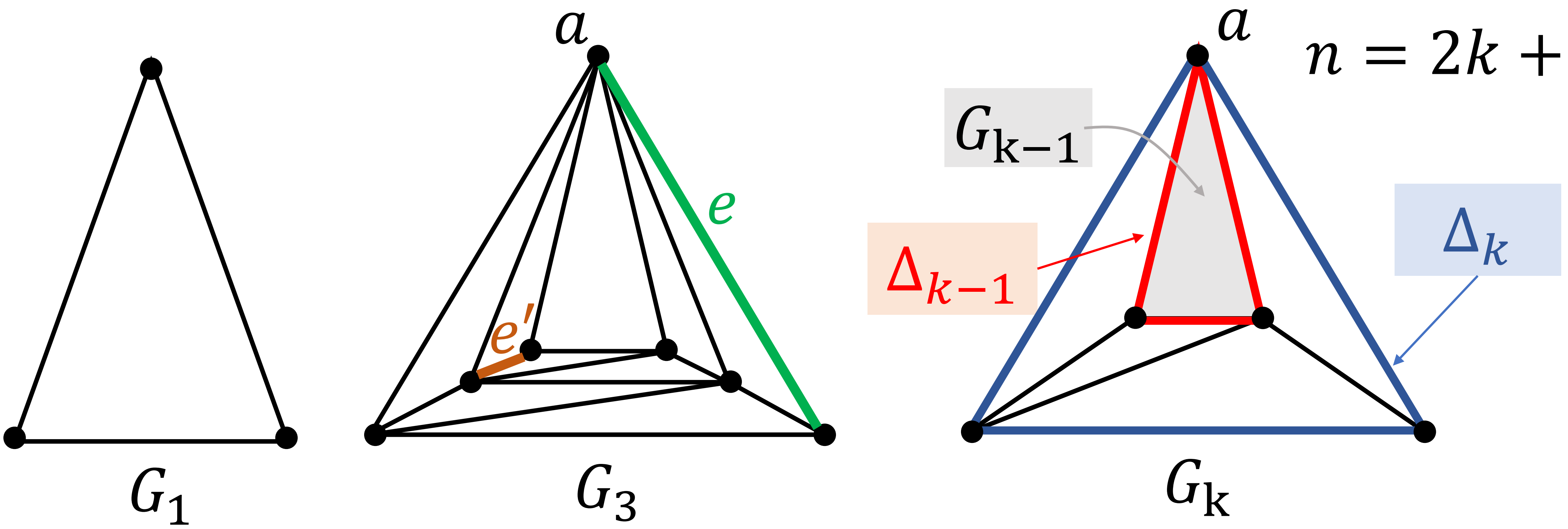


$P(\Delta_k) > P(\Delta_{k-1}) + 0.3$

$P(\Delta_k) > 0.3 \cdot k$

$$|e| \geq \frac{P(\Delta_k)}{4} > \frac{3k}{40}$$

Sketch of the Lower Bound (Fixed Embedding)

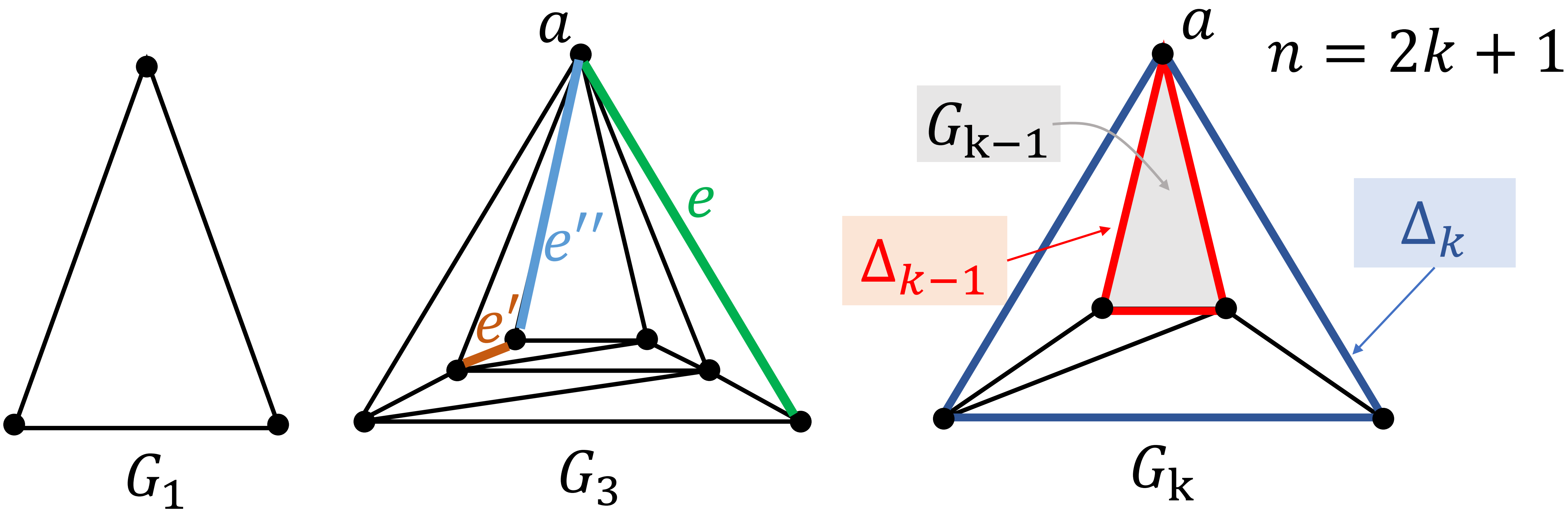


$n = 2k + 1$

$|e| > \frac{3k}{40}$

$|e'|=1$

Sketch of the Lower Bound (Fixed Embedding)



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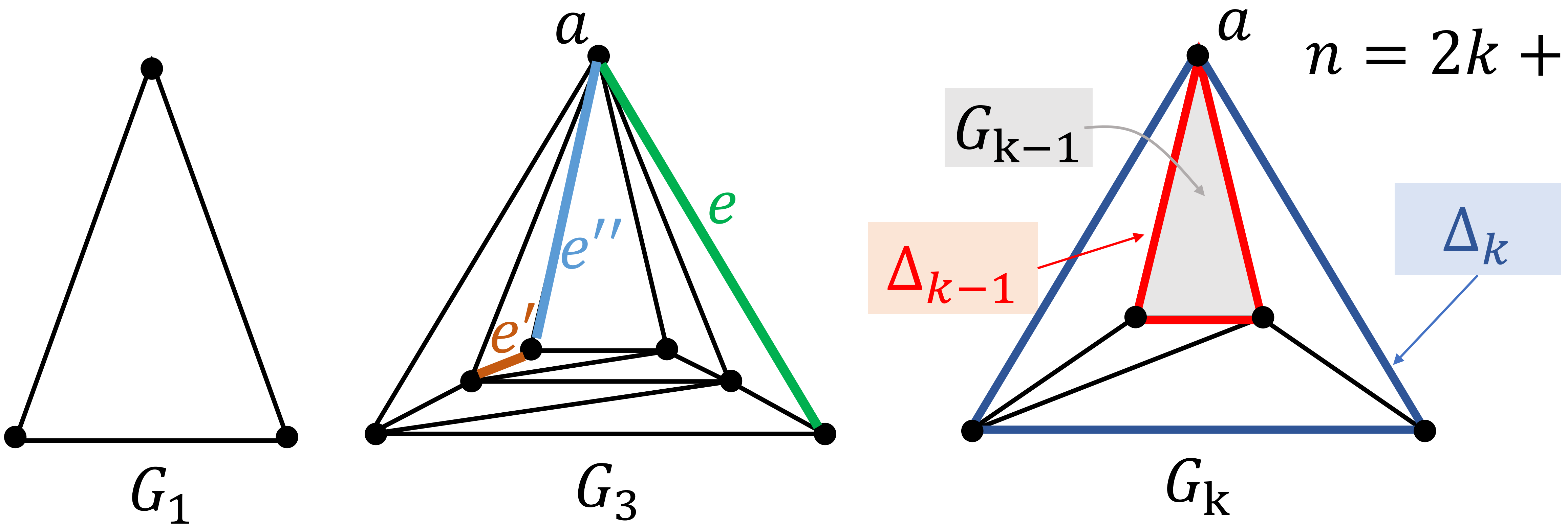

$n = 2k + 1$

$|e| > \frac{3k}{40}$

$|e'|=1$

$$\rho_\ell(\Gamma) \geq \frac{|e''|}{|e'|} = |e''|$$

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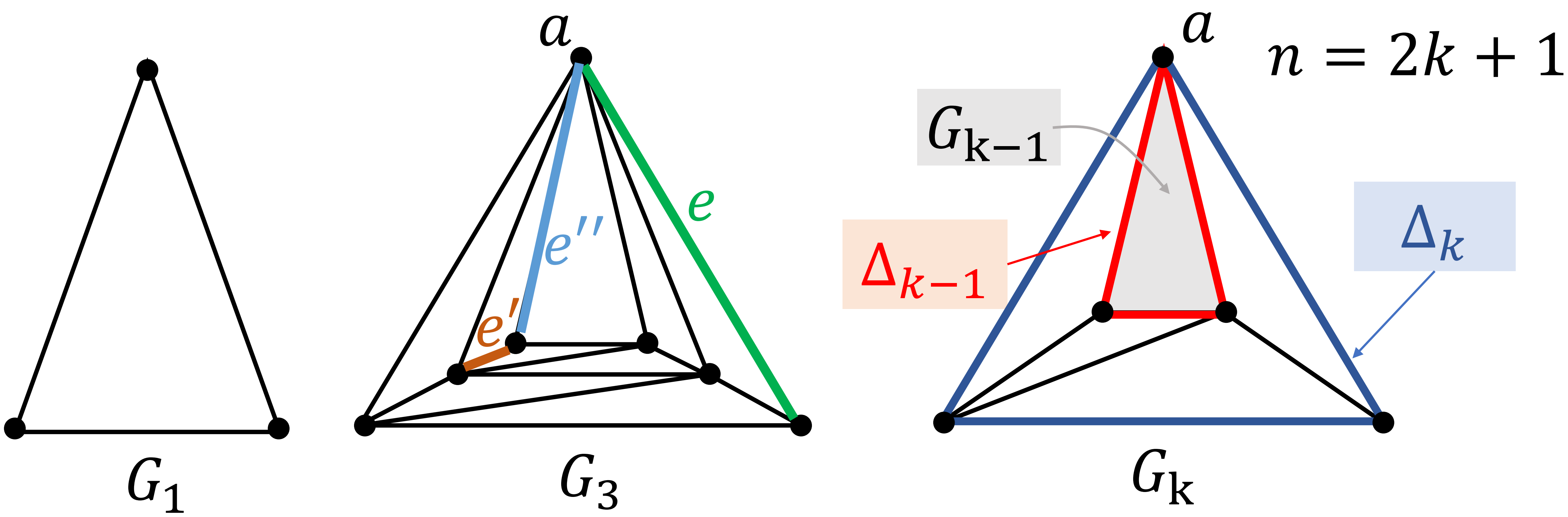


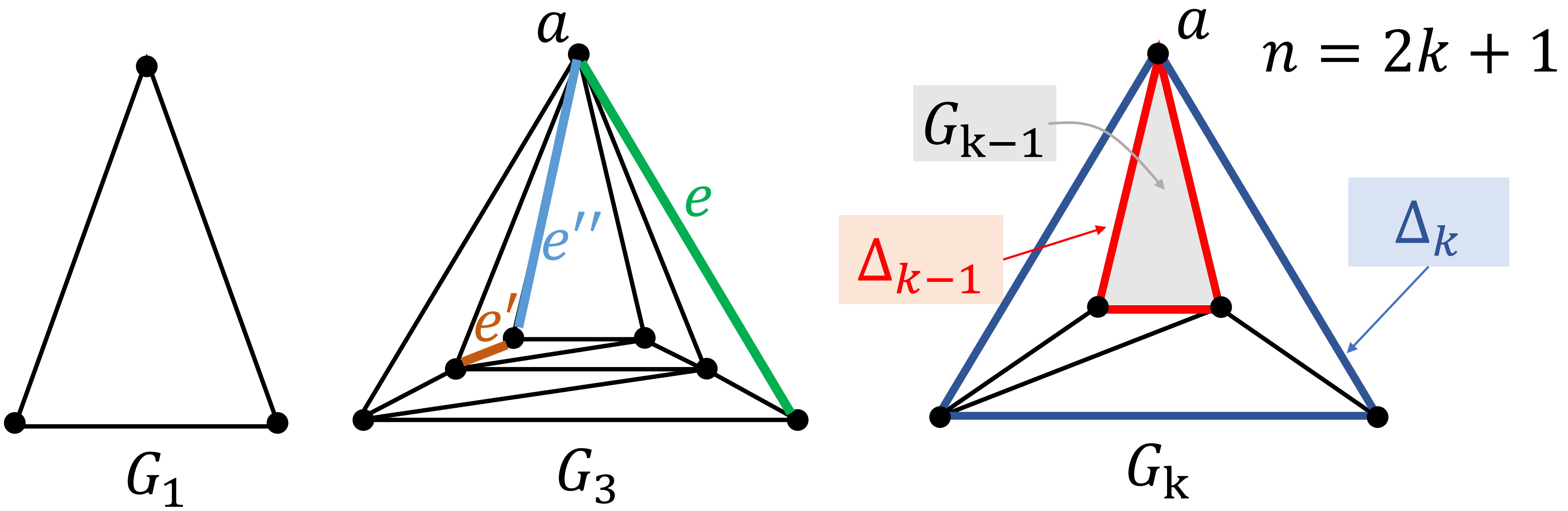
$$|e| > \frac{3k}{40}$$

$$|e'| = 1$$

$$\rho_\ell(\Gamma) \geq \frac{|e''|}{|e'|} = |e''|$$

$$\rho_\ell(\Gamma) \geq \frac{|e|}{|e''|} \geq \frac{|e|}{\rho_{\ell(\Gamma)}}$$

Sketch of the Lower Bound (Fixed Embedding)

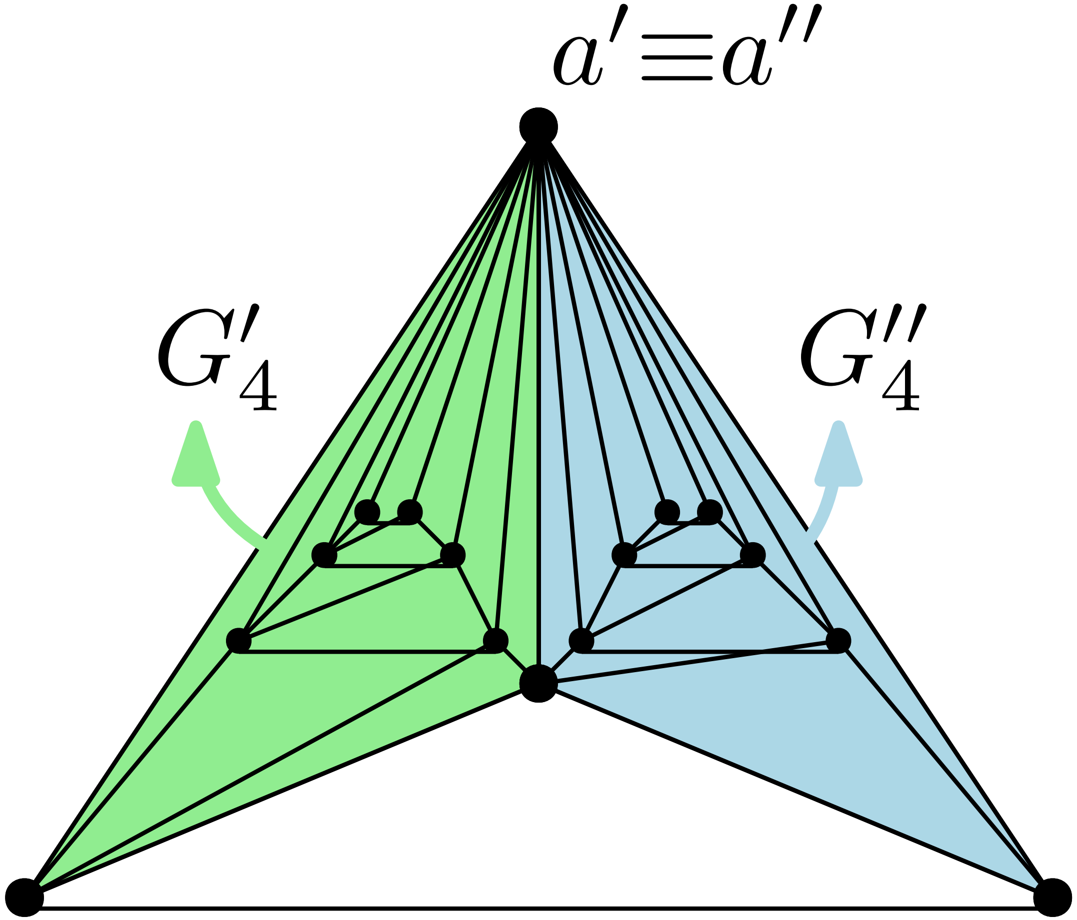


$$|e| > \frac{3k}{40}$$

$$\rho_\ell(\Gamma) \geq \frac{|e|}{\rho_{\ell(\Gamma)}}$$

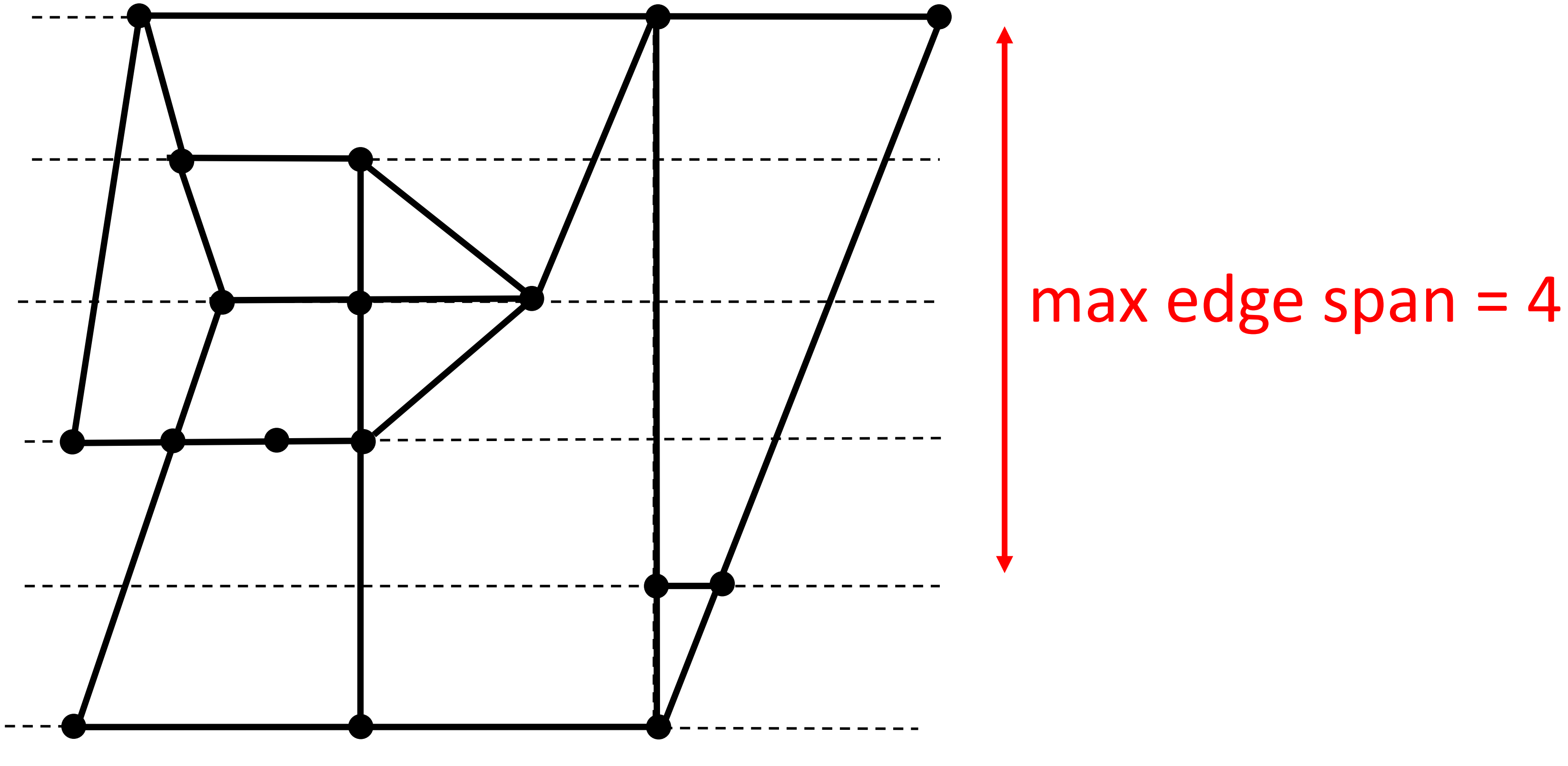
$$\rho_\ell(\Gamma) \geq \sqrt{\frac{3k}{40}}$$

Sketch of the Lower Bound (Variable Embedding)



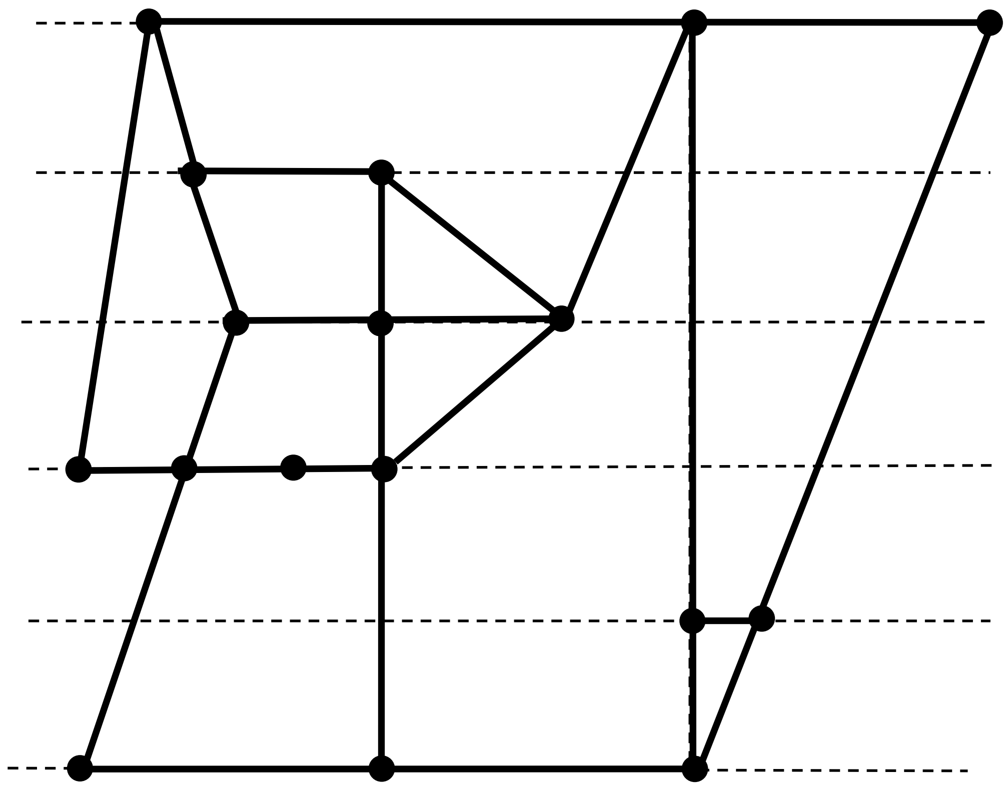
A Glance at the Technicalities: Upper Bound

K-span Weakly Level Planarity

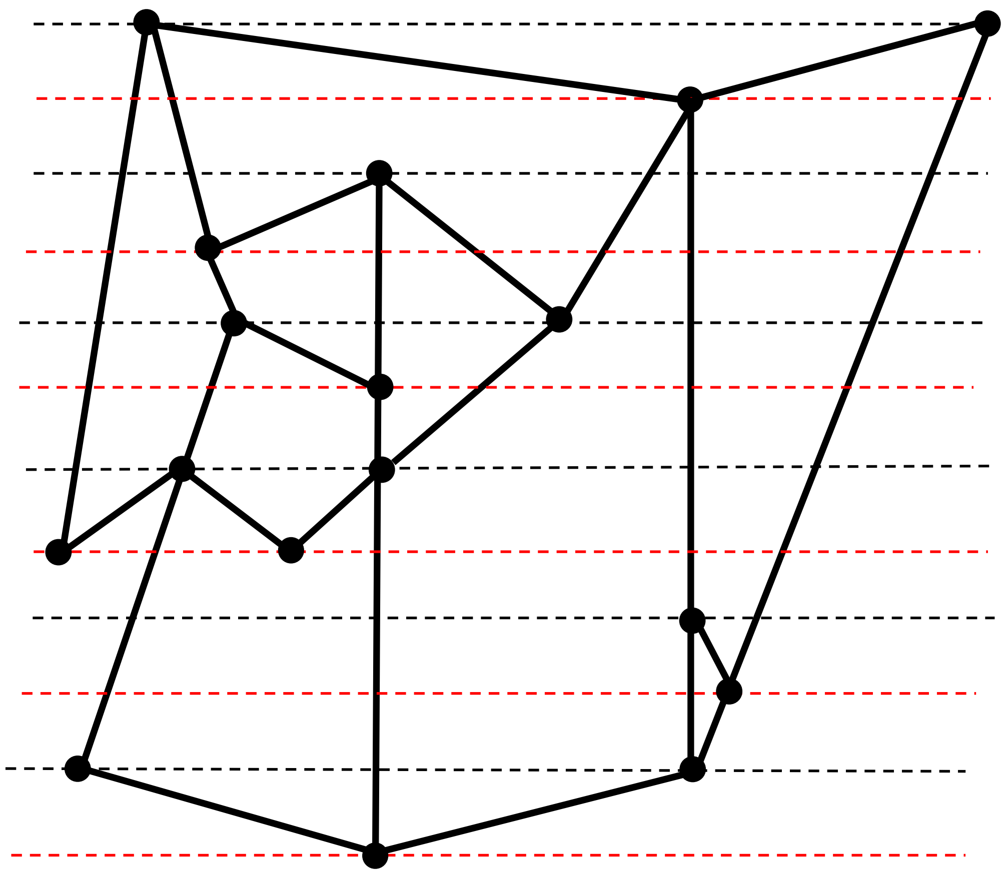


4-span weakly level planar drawing

From Weakly Level Planar to Level Planar

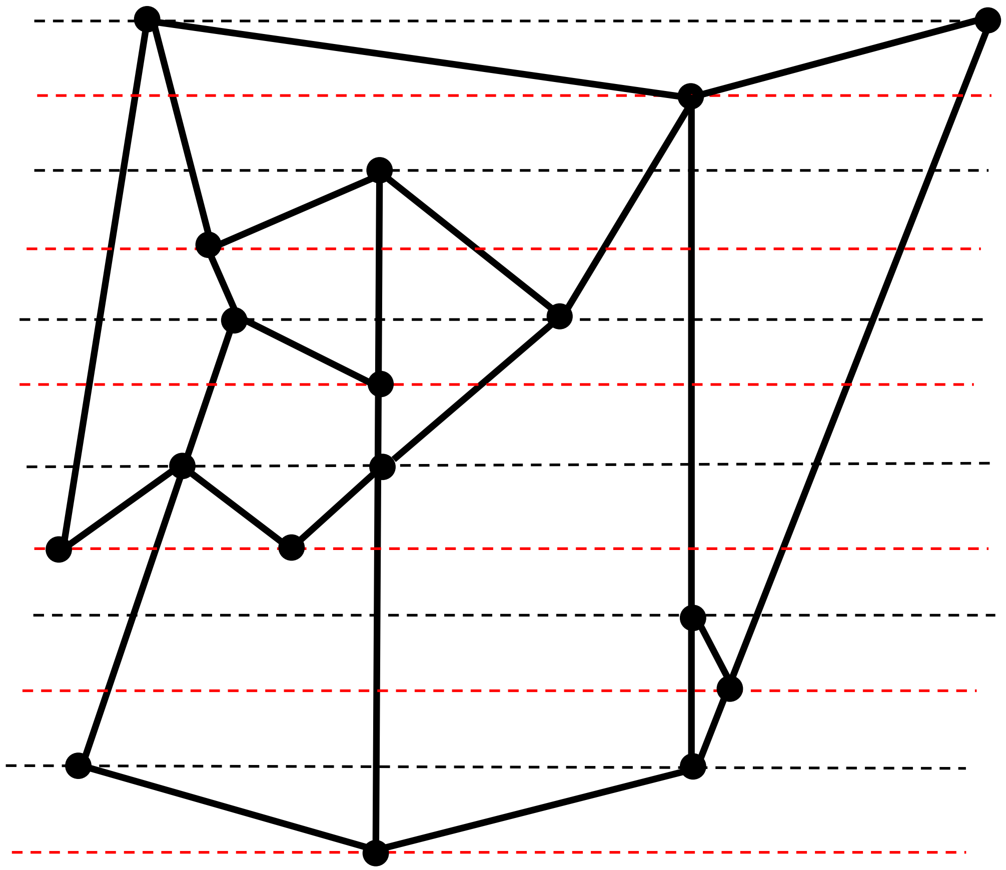


4-span weakly level planar drawing



9-span level planar drawing of the same graph

Edge-length Ratio and Level Planarity



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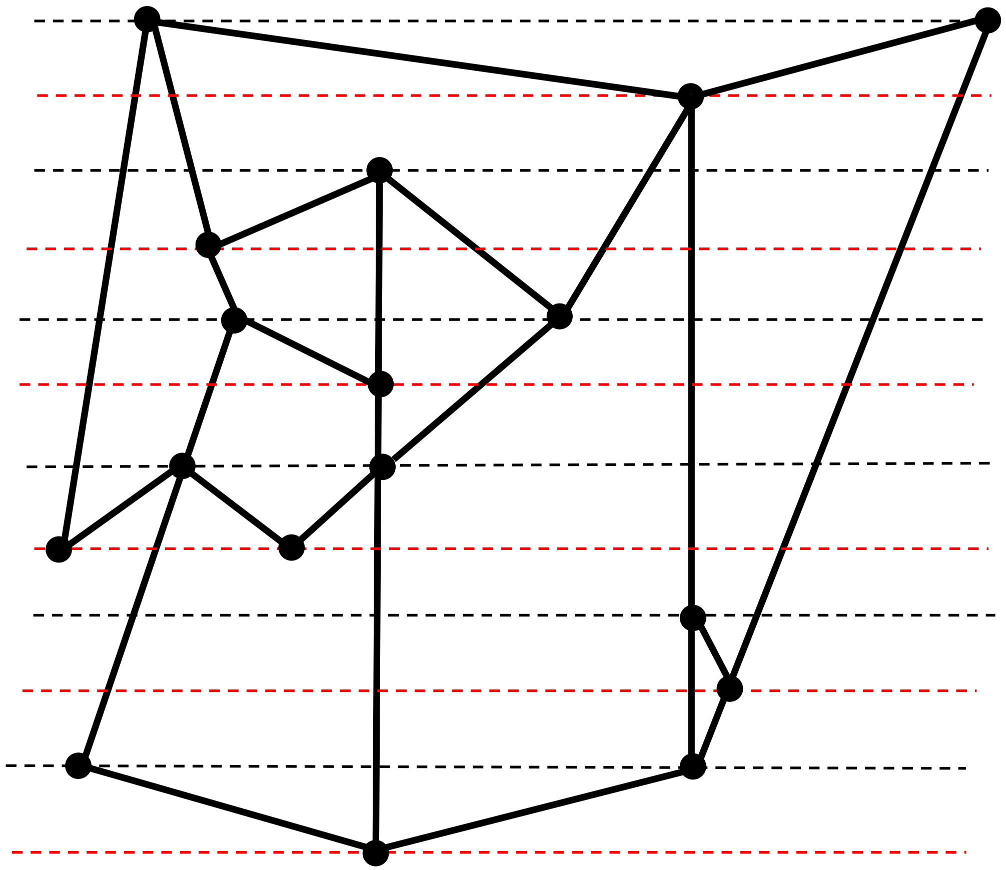
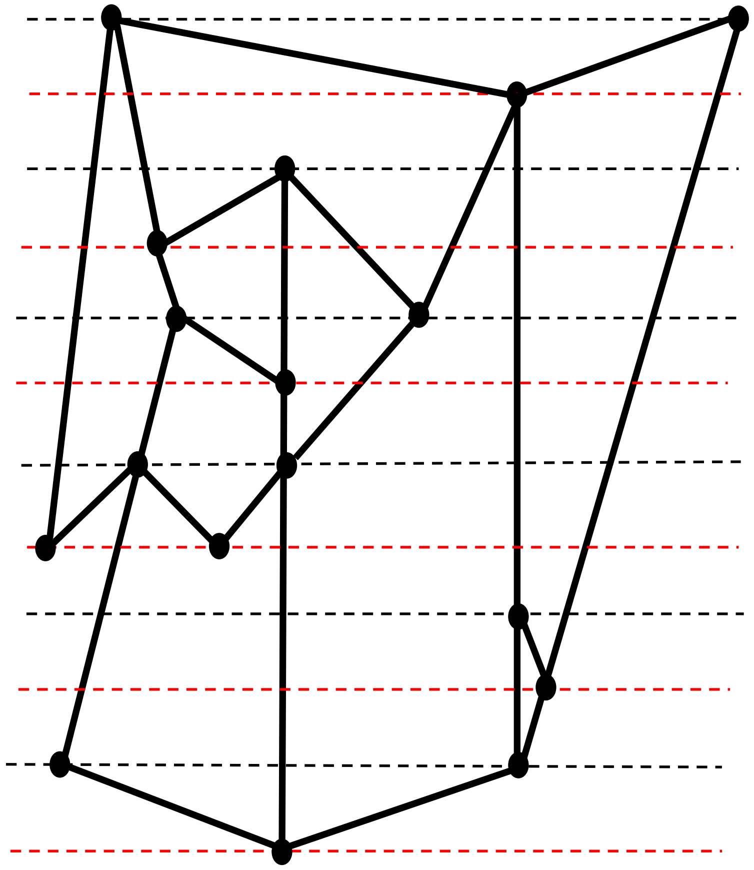
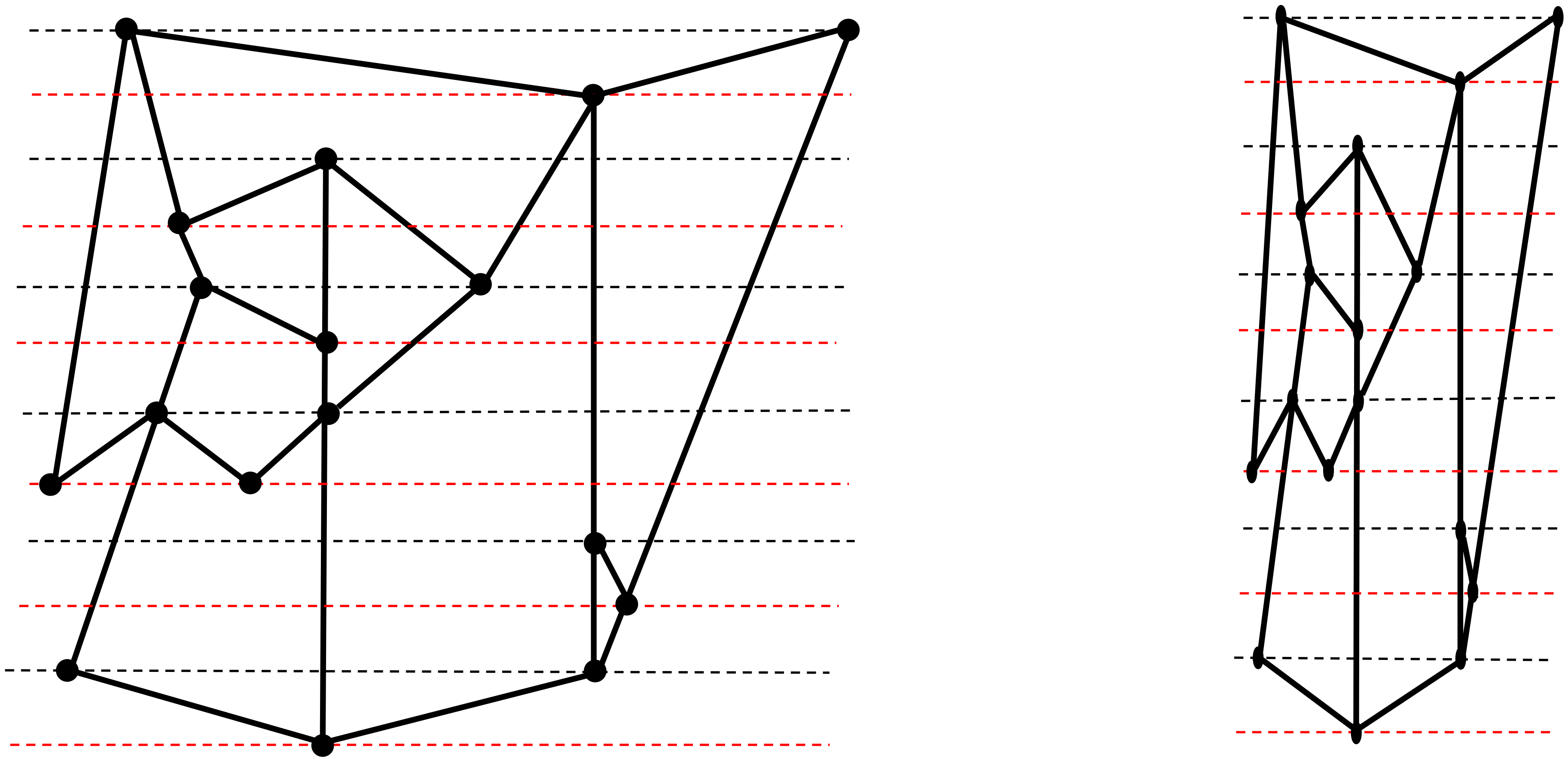
Edge-length Ratio and Level Planarity



9-span level planar drawing of the same graph



Edge-length Ratio and Level Planarity



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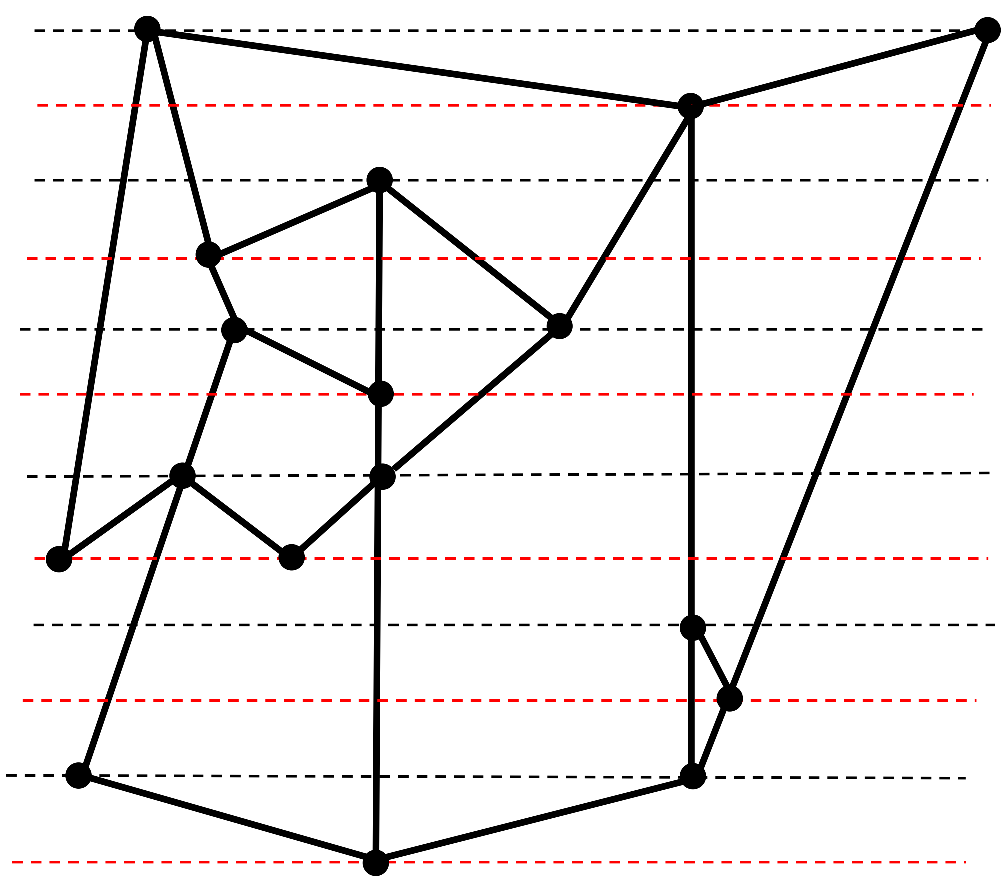

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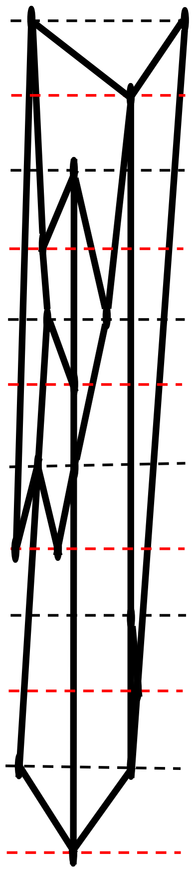
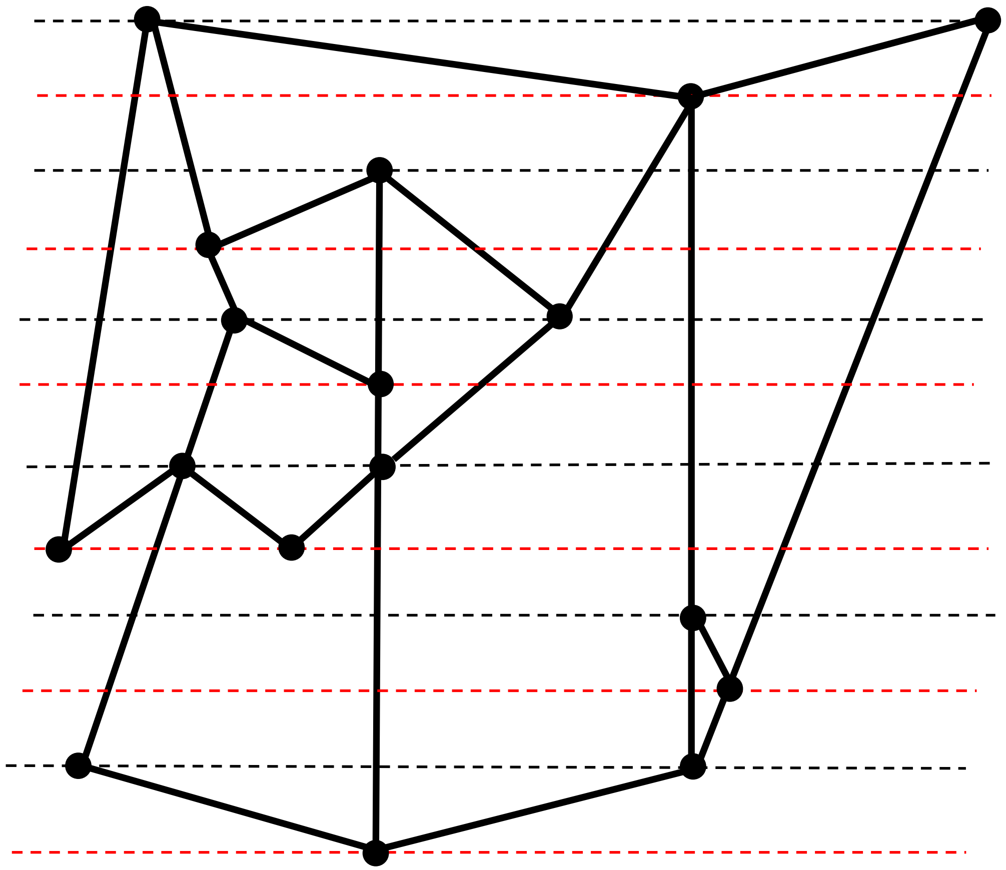


Edge-length Ratio and Level Planarity



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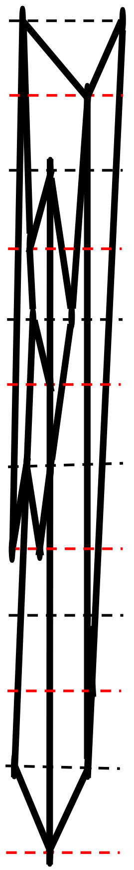
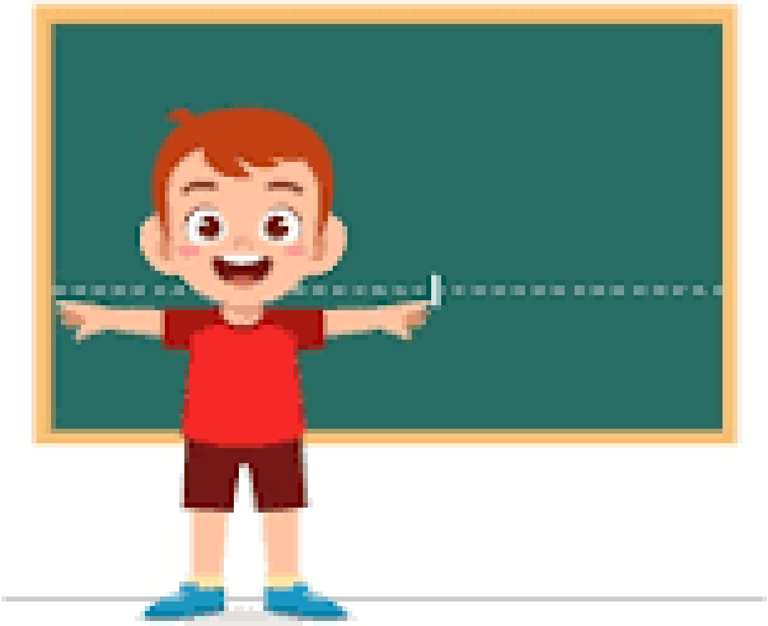


Γ

$\rho(\Gamma) \leq 9 + \varepsilon$
for any $\varepsilon > 0$

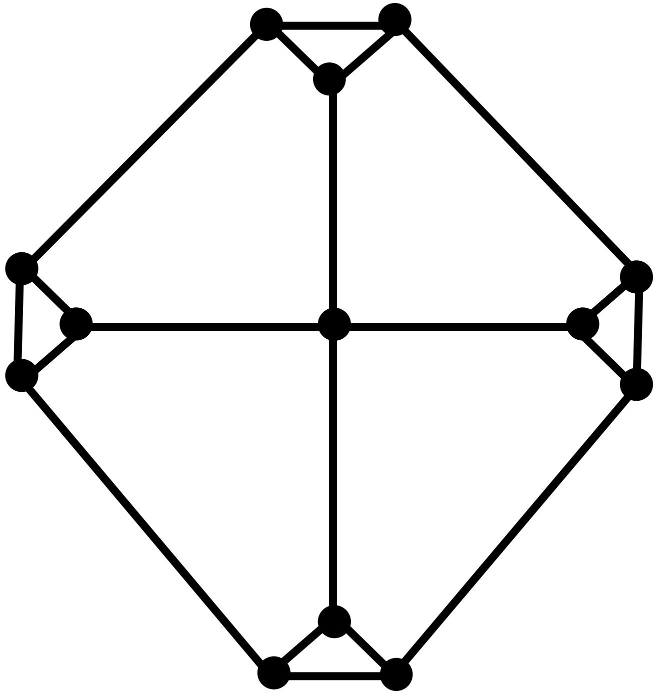
…moral of the story…..

Lemma: If a planar graph G admits a k-span weakly level planar drawing, then $\rho(G) \leq 2k + 1$



Halin Graphs

Theorem: Every Halin graph G different from K_4 admits a 1-span weakly level planar drawing. Hence, $\rho(G) \leq 3$



Open Problems

Is there an $\omega(\sqrt{n})$ lower bound for the local edge-length ratio?

Is the upper bound on the edge-length ratio of Halin graphs tight?

Investigate trade-offs between (local) edge-length ratio and other aesthetics, for example the angular resolution.

That's all, thank you!!