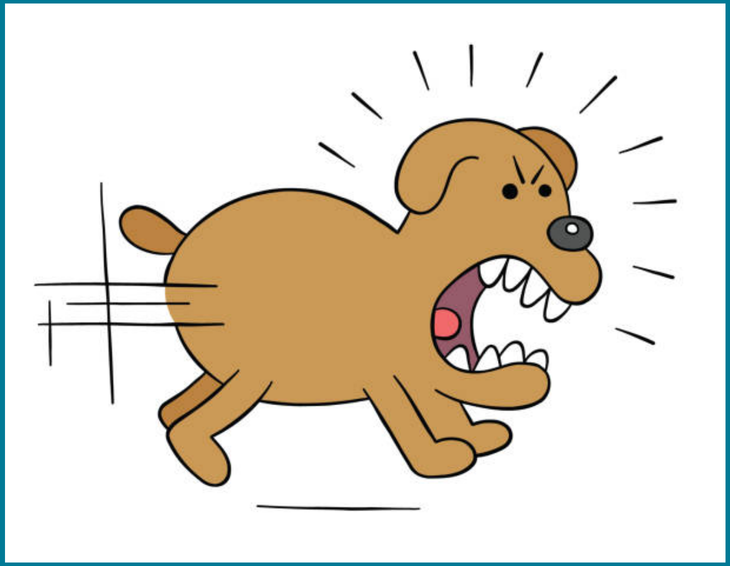
Barking Dogs:

A Fréchet distance variant for detour detection

Ivor van der Hoog, Fabian Klute,
Irene Parada, Patrick Schnider

EuroCG 2024



A hike in Perugia

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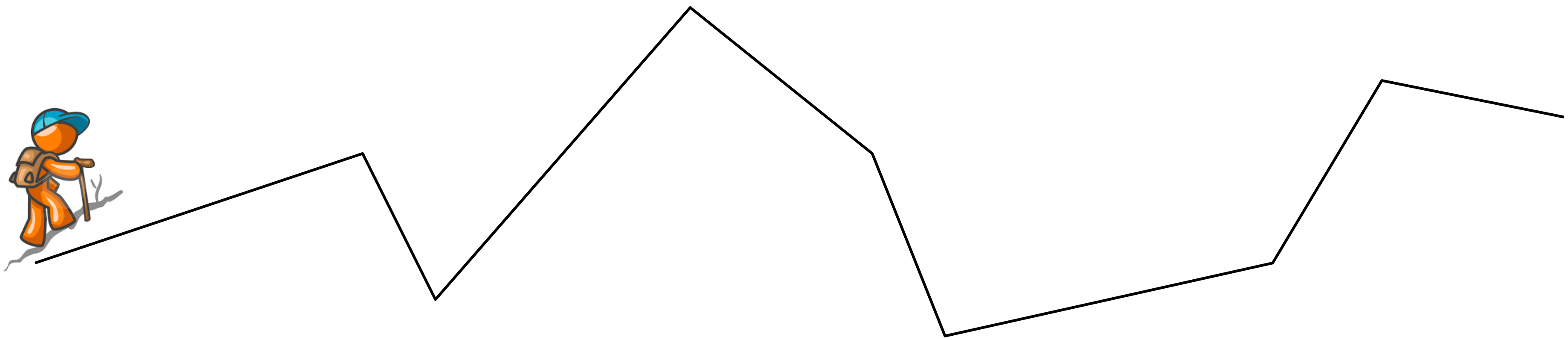

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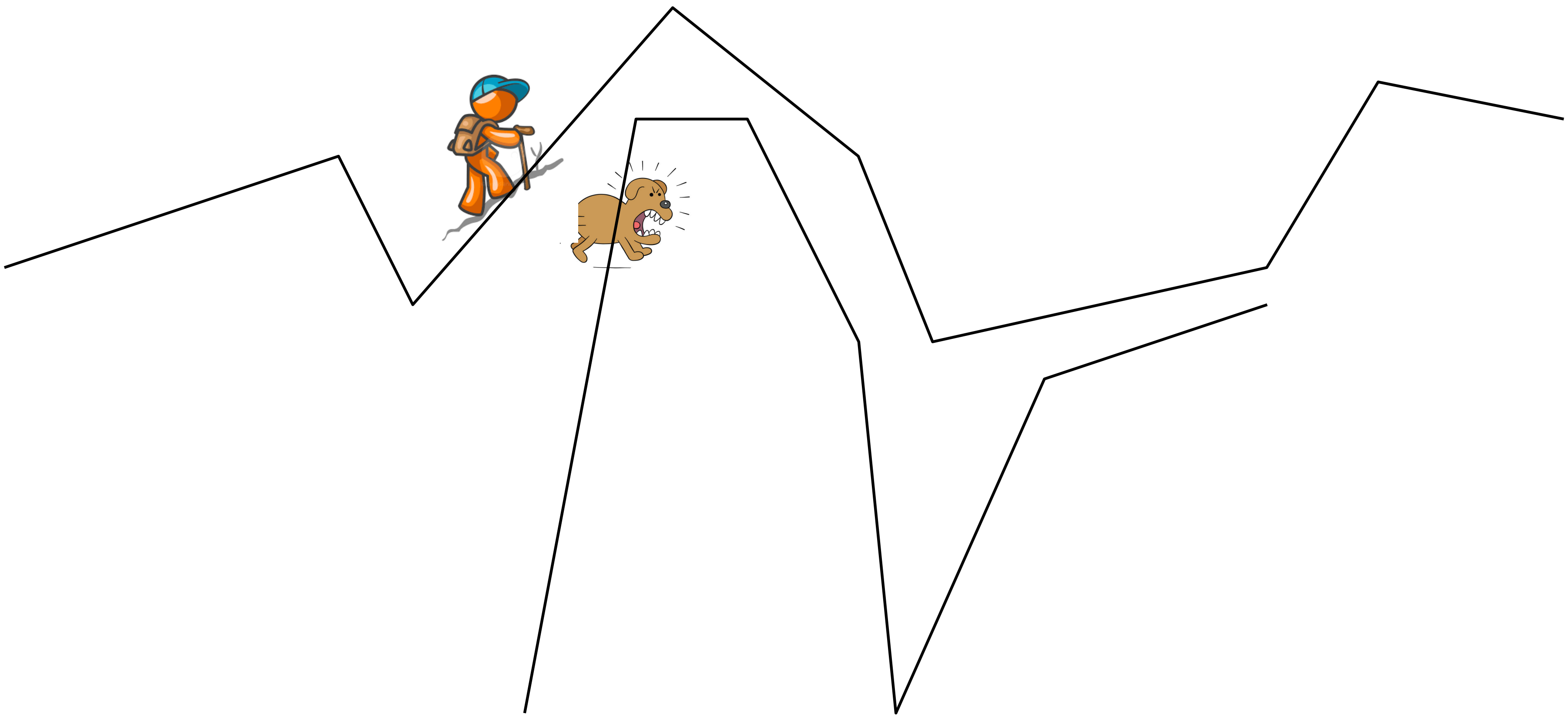

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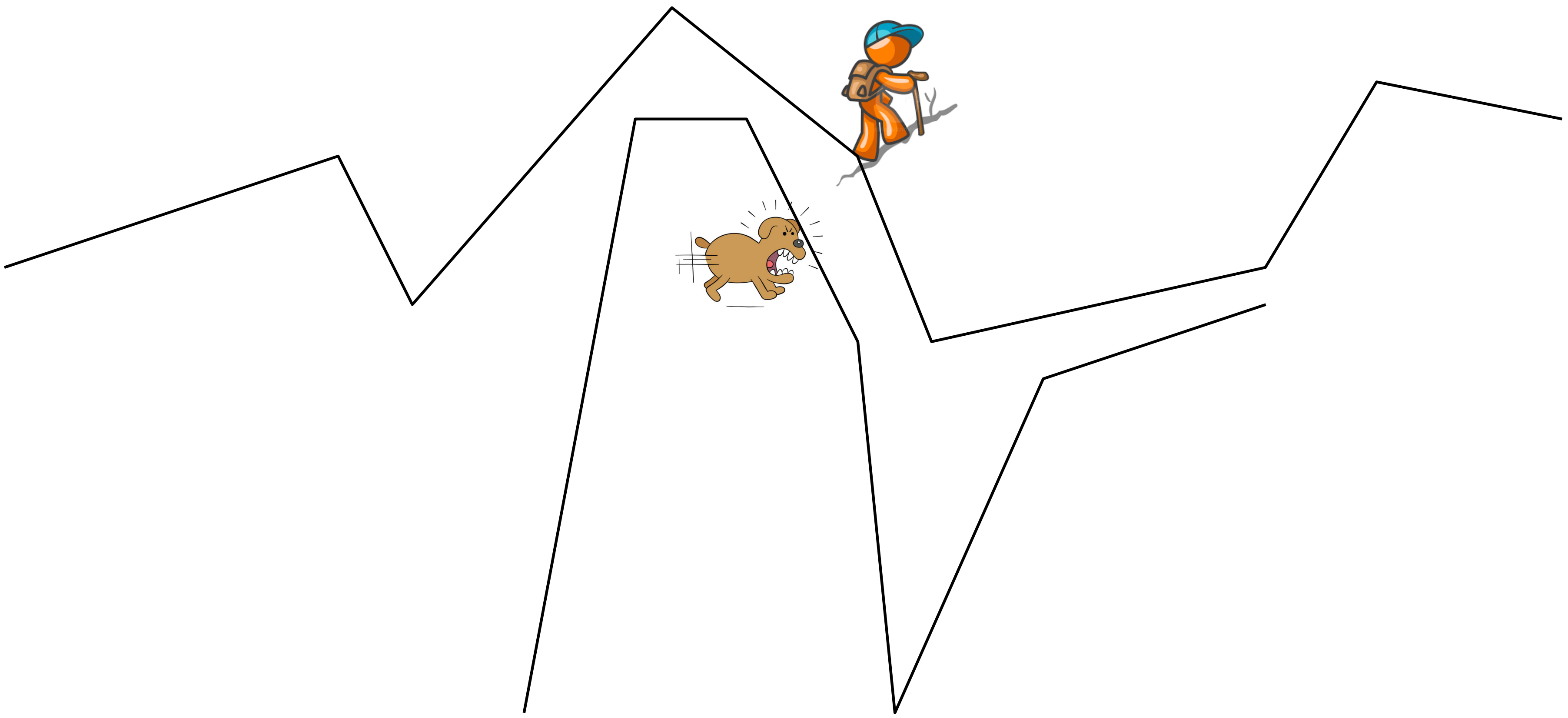

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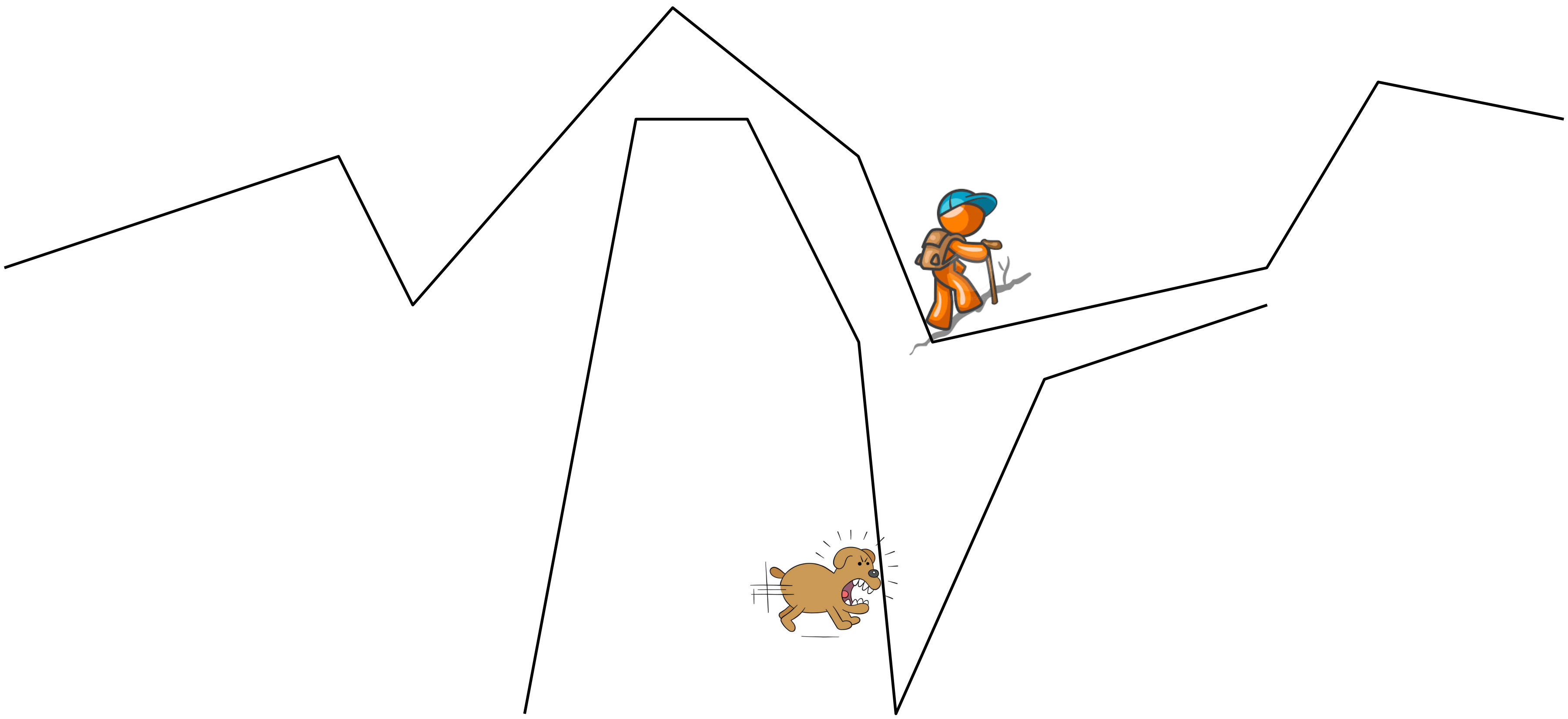

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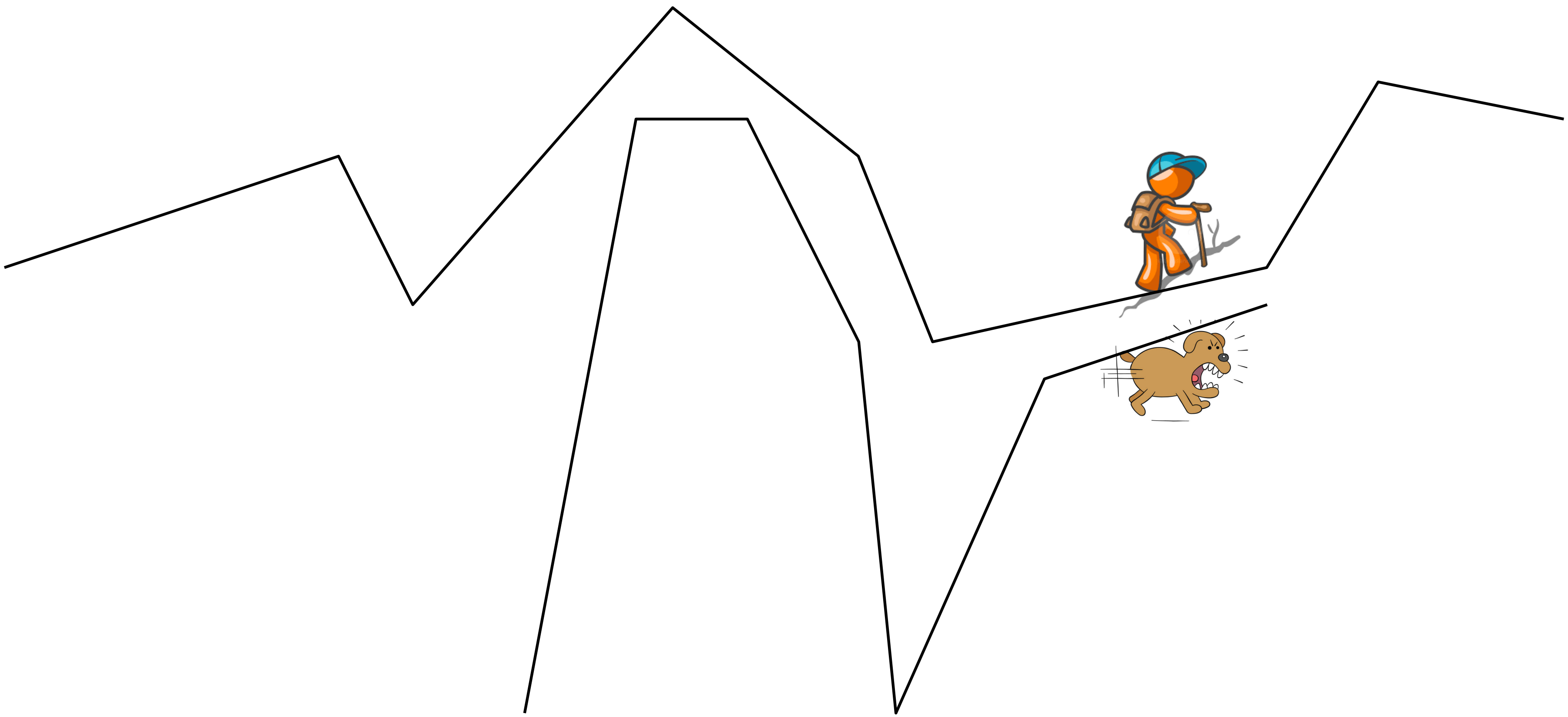

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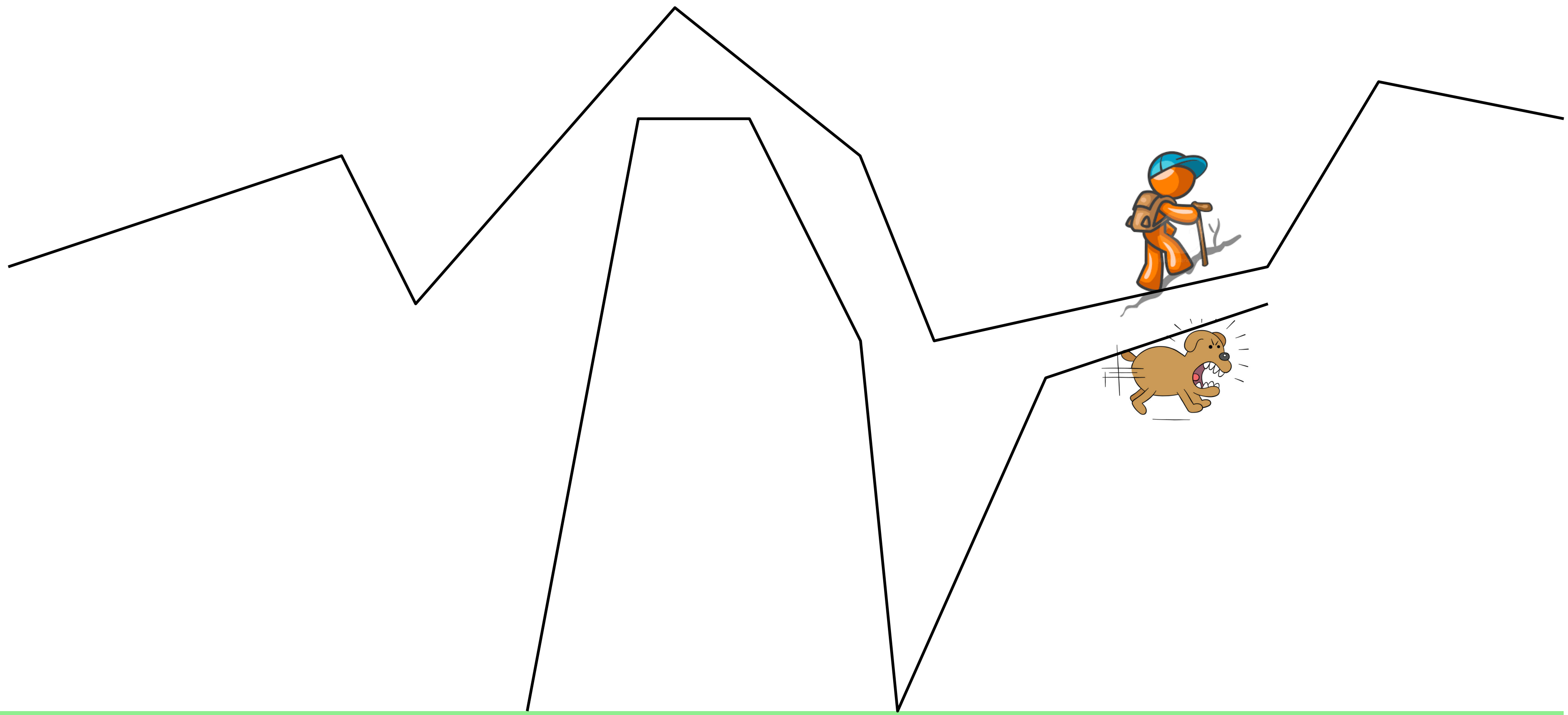

How can the angry dog maximize the time it barks at us?

Formal definition

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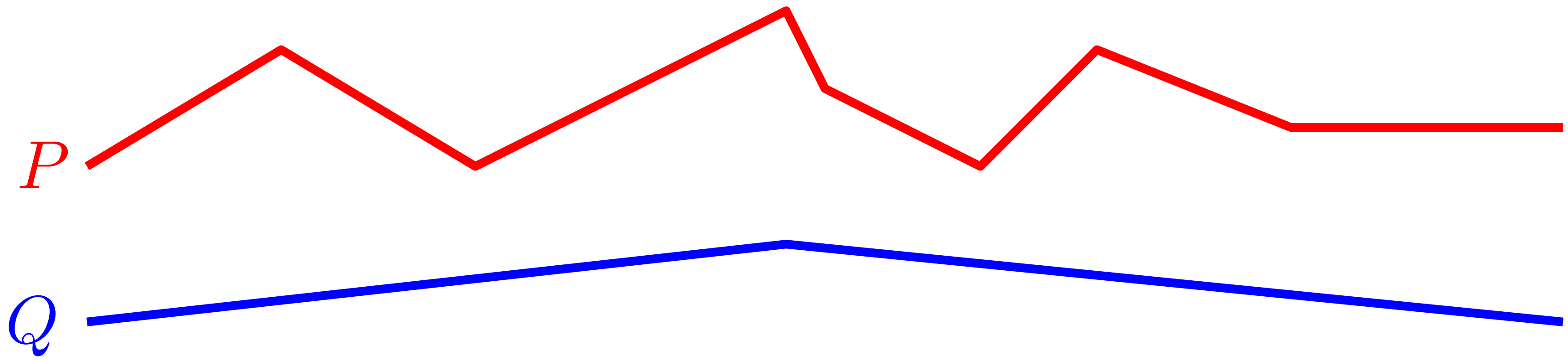
P, Q polygonal curves



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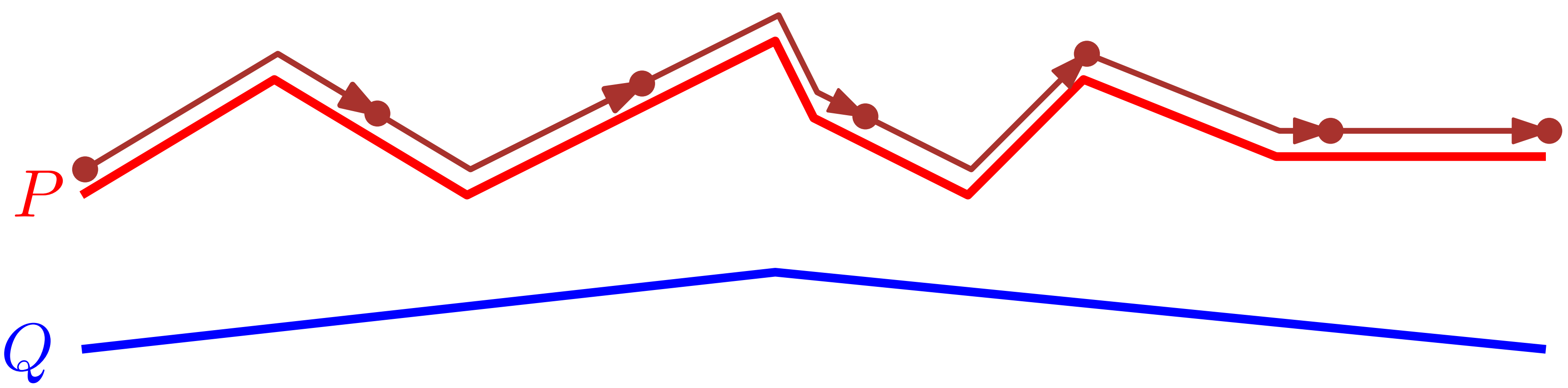
$f : [0, 1] \to P$ fixed (uniform) traversal of P



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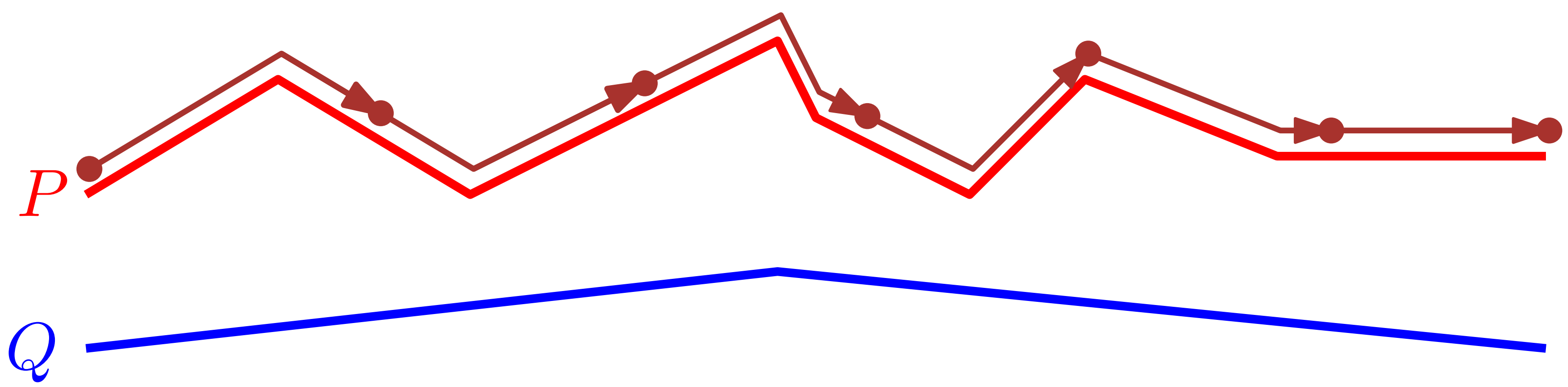

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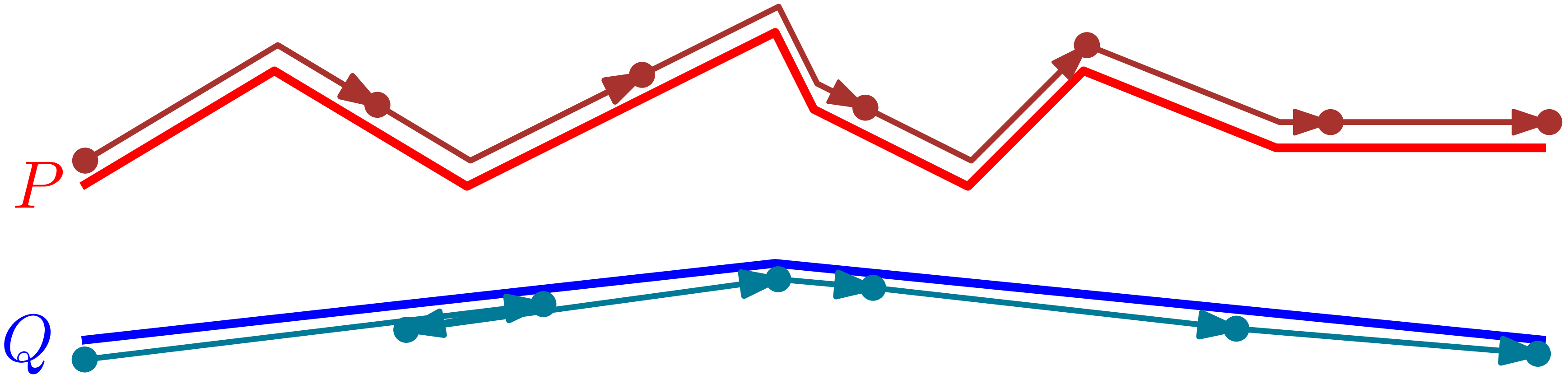

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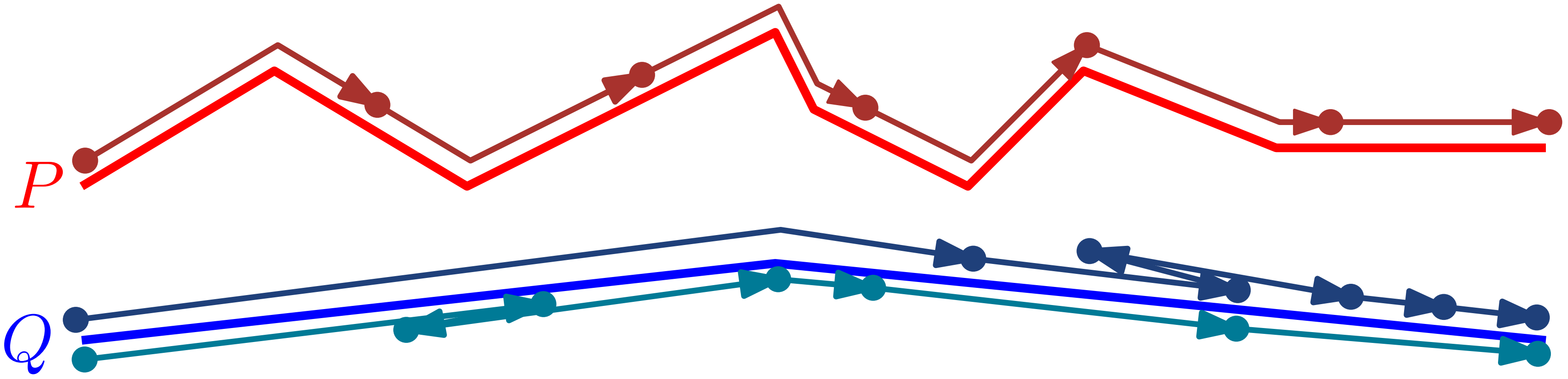

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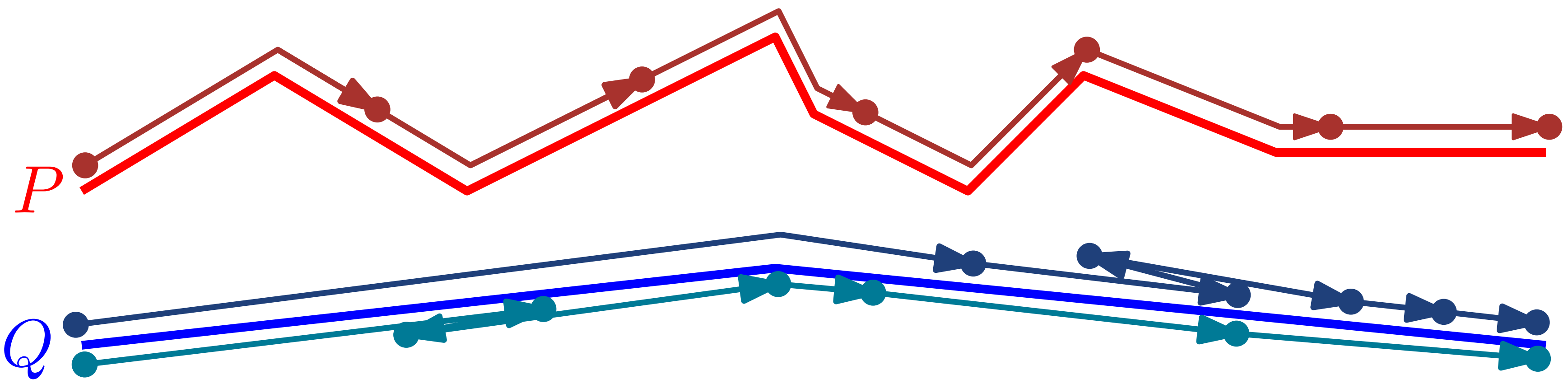

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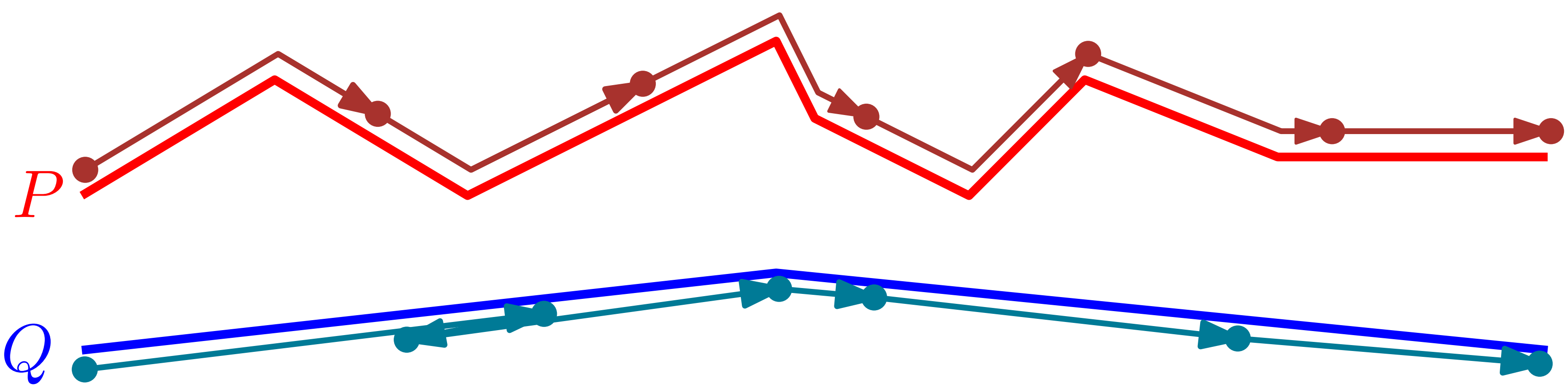

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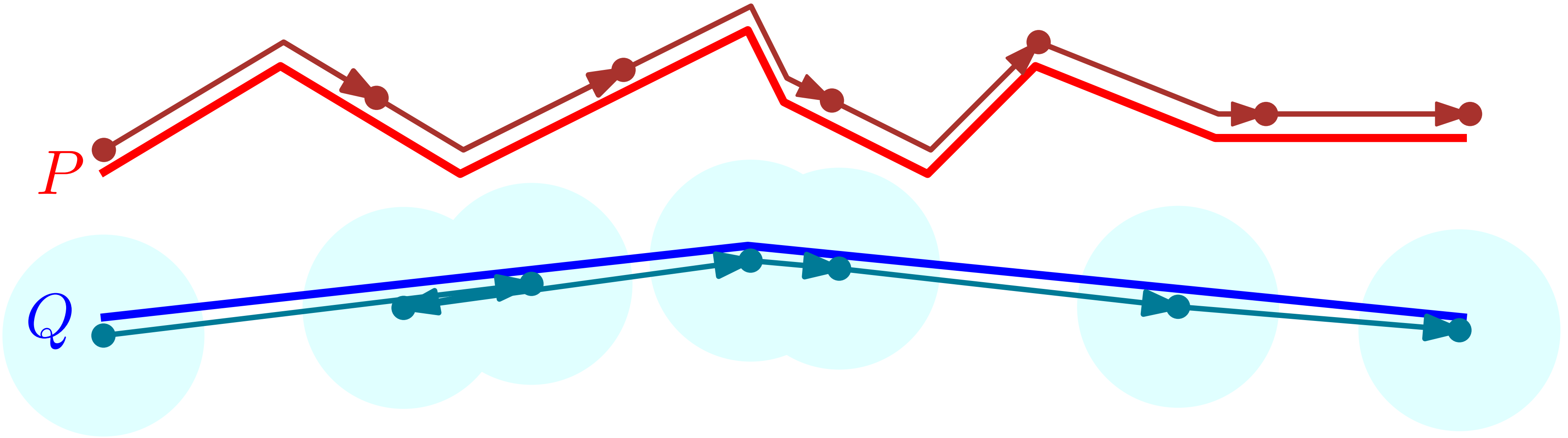

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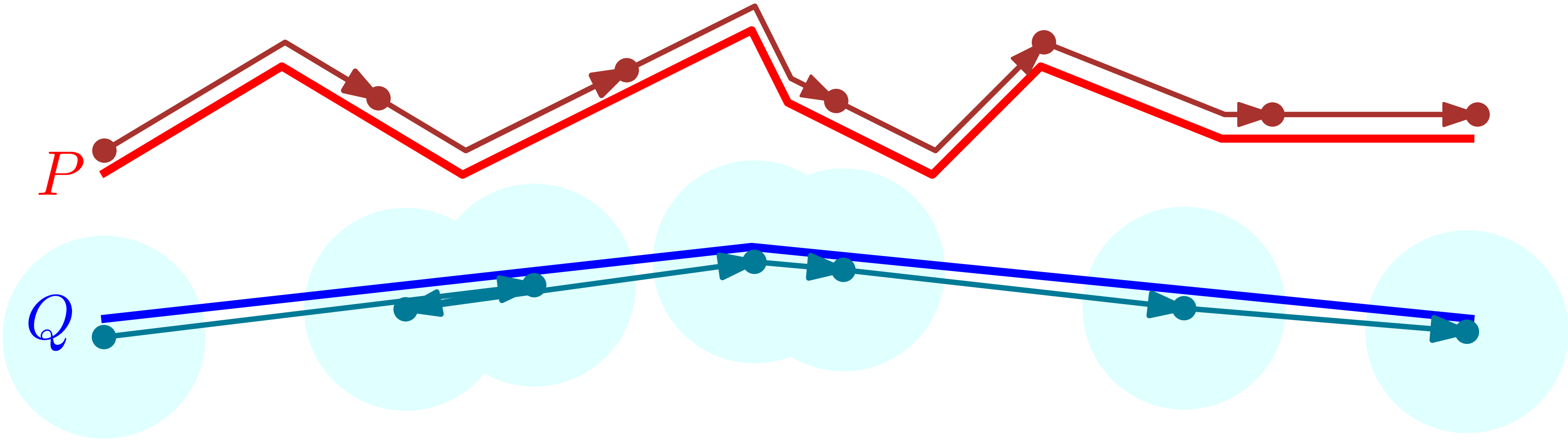

$$D_B(P, Q) = \int_{g' \leq s} \theta(f(t), g(t))dt$$

Detour detection

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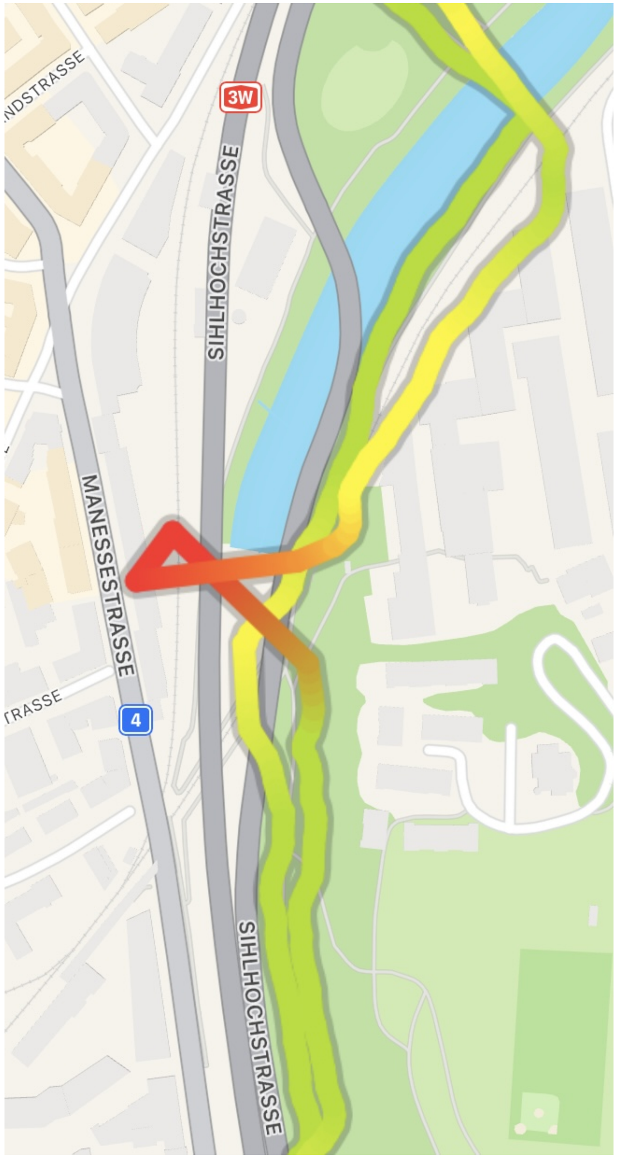

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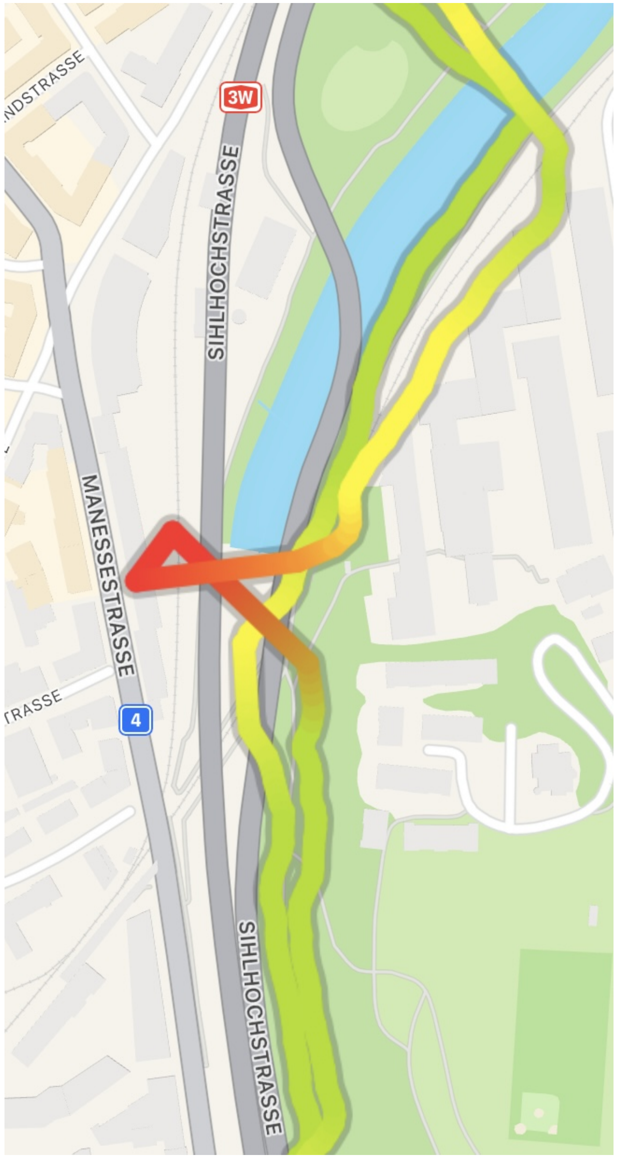
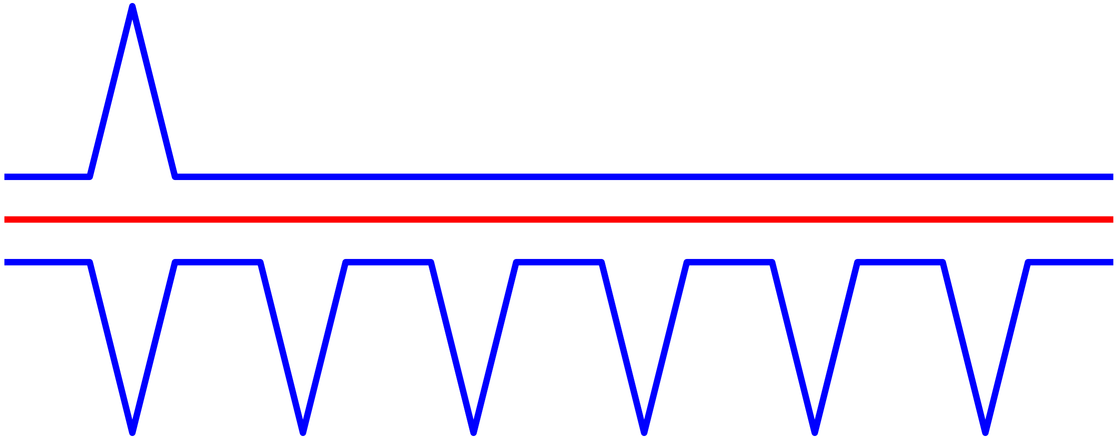


Fréchet can’t distinguish



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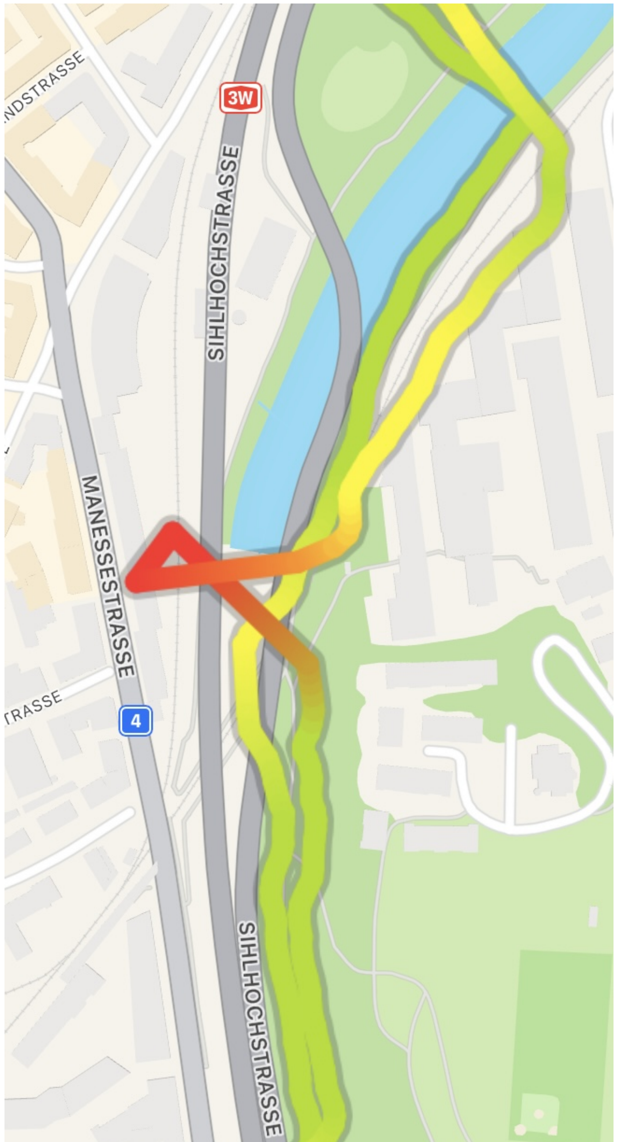

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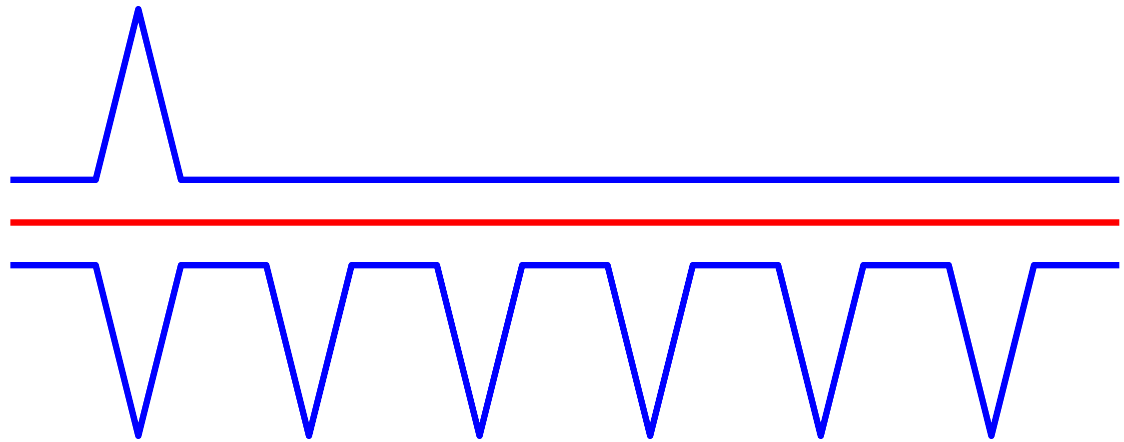
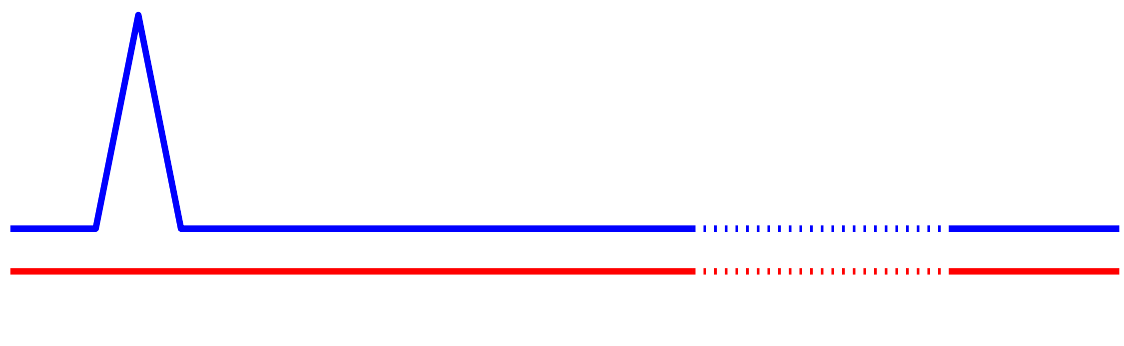


DTW can't distinguish



The discrete setting

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Traversals are discrete (both hiker and dog are frogs)

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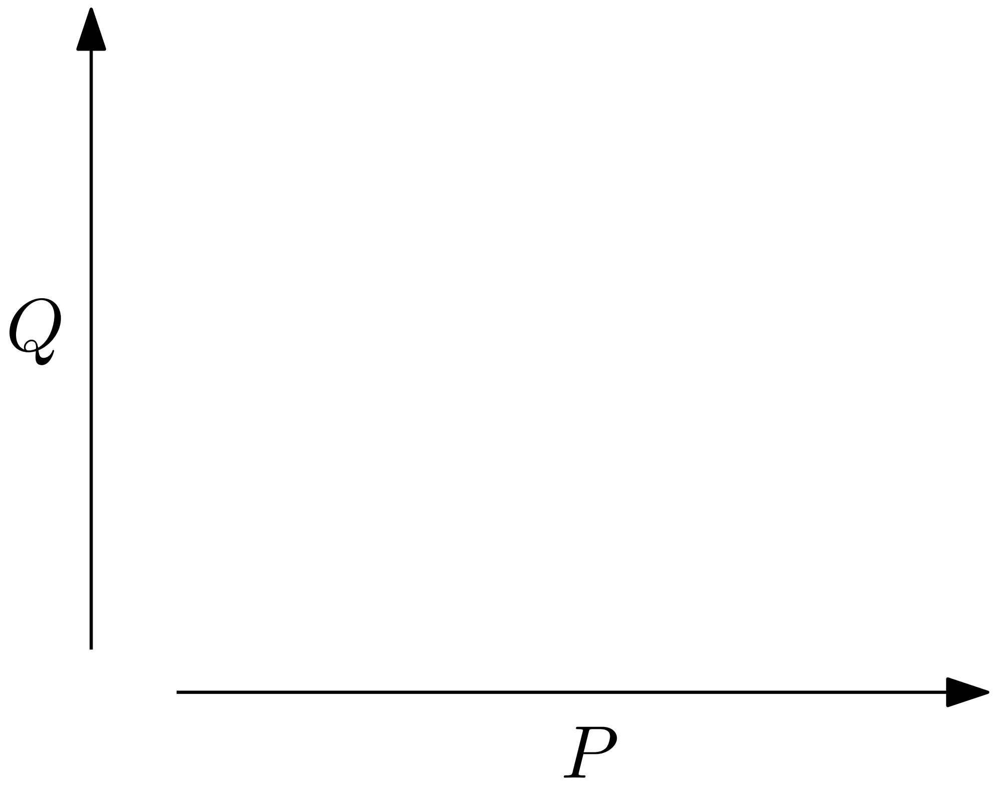

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Q

0	0	0	0	0	0	0	0	1	1
0	0	1	1	0	0	0	1	0	0
0	0	1	0	1	0	0	1	0	0
0	0	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	0	0	0
1	1	0	0	0	1	0	0	0	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	0	0	0	0	1

P

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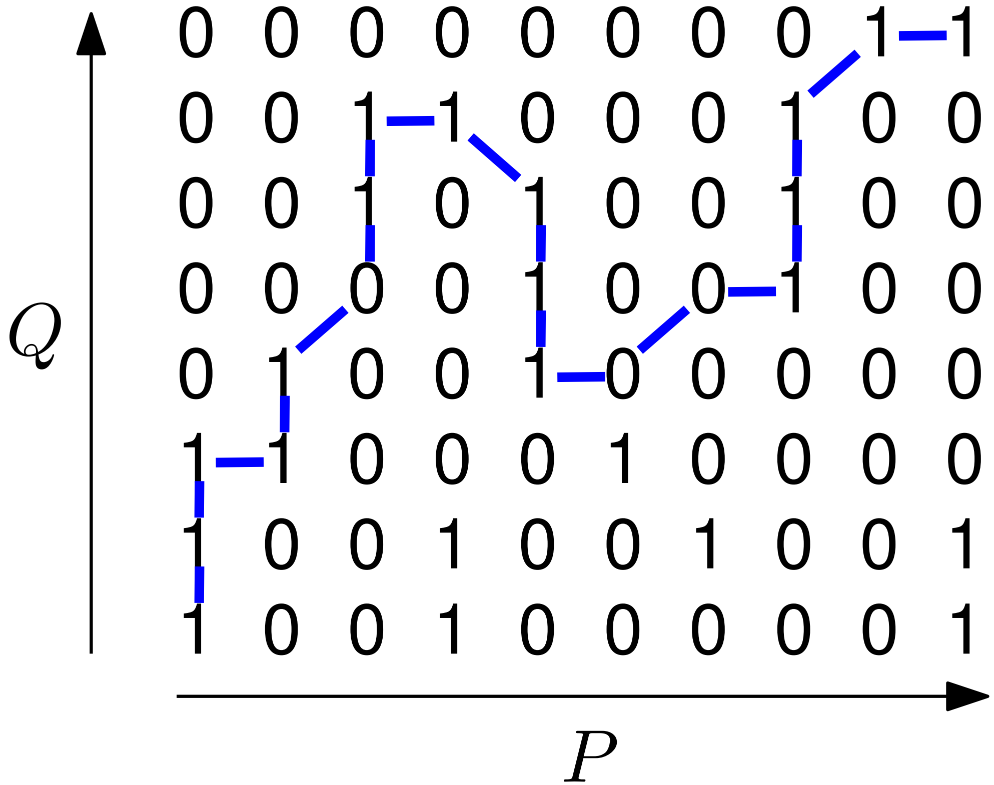

optimal traversal: lattice path minimizing number of zeros

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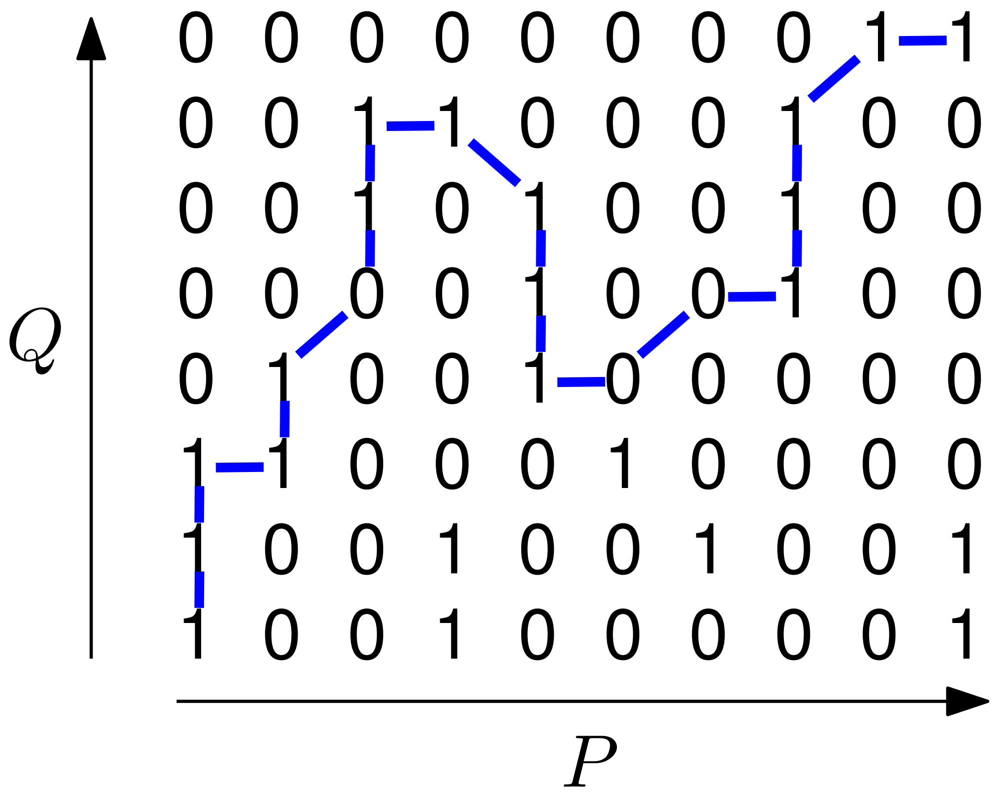

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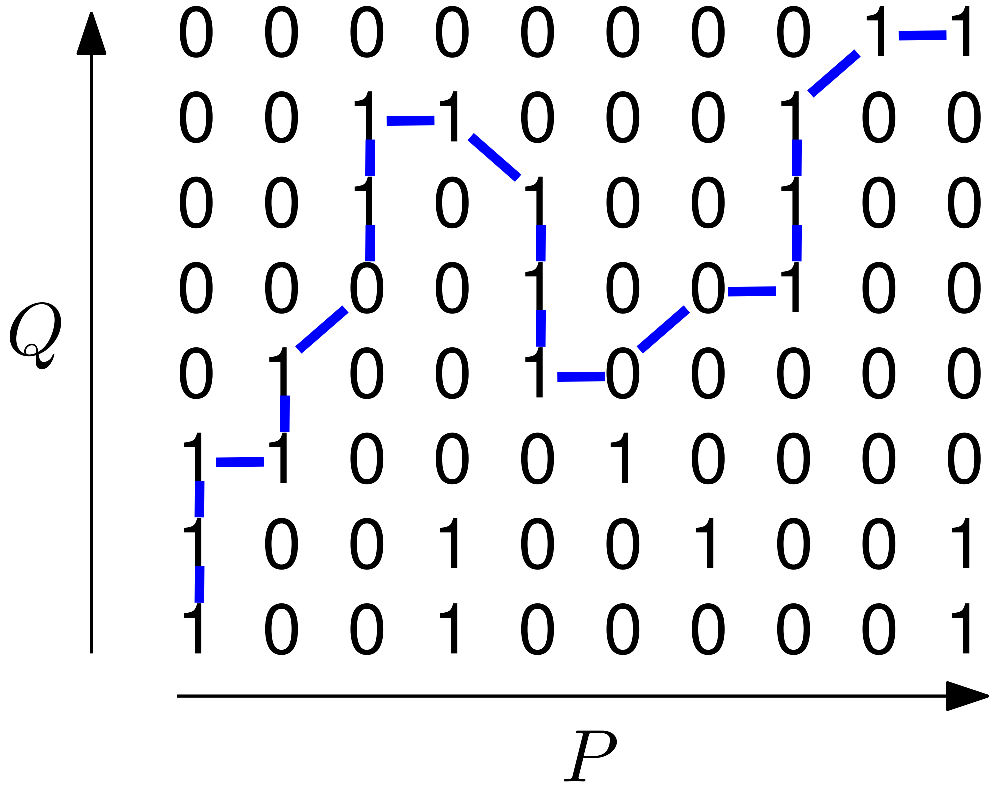

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$R_i(j_1, j_2)$: cost of horizontal path

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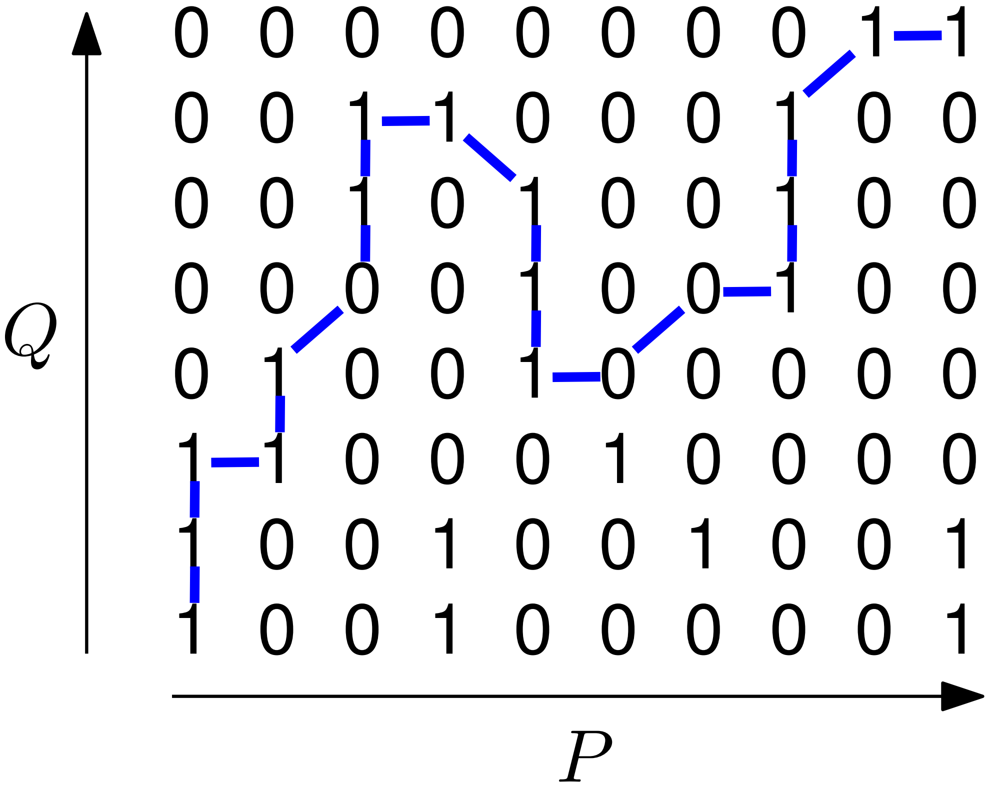

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$R_i(j_1, j_2)$: cost of horizontal path
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$F_\delta(i, j)$: min. cost to (i, j) if last step was $\delta \in \{\uparrow, \nearrow, \rightarrow, \searrow, \downarrow\}$

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$$F_d(i,j) = \begin{cases} \min\{C_j(i-k,i) + F_\delta(i-k,j) \mid \delta \in \{\nearrow, \rightarrow, \searrow\} \wedge k \in [1,s]\} & \text{if } d = \uparrow \\ F(i-1,j-1) + w(v_{i-1,j-1}) & \text{if } d = \nearrow \\ \min\{R_i(j-k,j) + F_\delta(i,j-k) \mid \delta \in \{\uparrow, \nearrow, \searrow, \downarrow\} \wedge k \in [1,s]\} & \text{if } d = \rightarrow \\ F(i+1,j+1) + w(v_{i+1,j+1}) & \text{if } d = \searrow \\ \min\{C_j(i+k,i) + F_\delta(i+k,j) \mid \delta \in \{\nearrow, \rightarrow, \searrow\} \wedge k \in [1,s]\} & \text{if } d = \downarrow \end{cases}$$

Q

$$\begin{matrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$

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$$\begin{array}{c|cccccccccc} & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ Q & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline & & & & & P & & & & & \end{array}$$

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Dynamic programming + right data structures

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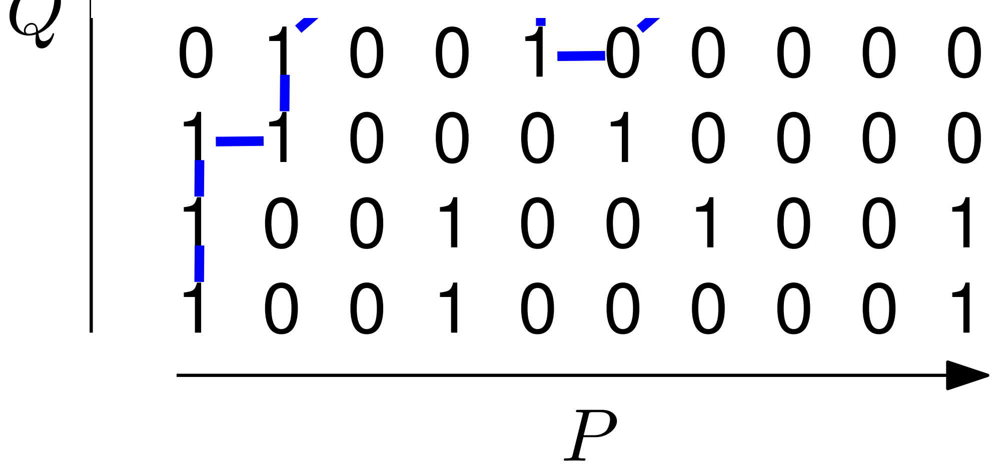

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Runtime $O(nm \log s)$

Other settings

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Semi-discrete setting:
Traversal for hiker is discrete, traversal for dog is continuous.

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$O(n^4 m^3 \log(nm))$ time algorithm

For $n = m$ no $O(n^{2-\varepsilon})$ algorithm exists, assuming SETH.

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Thank you!