EuroCG 2024
13-15 March
Ioannina, Greece

STAR-FOREST DECOMPOSITIONS
OF CERTAIN COMPLETE GEOMETRIC GRAPHS

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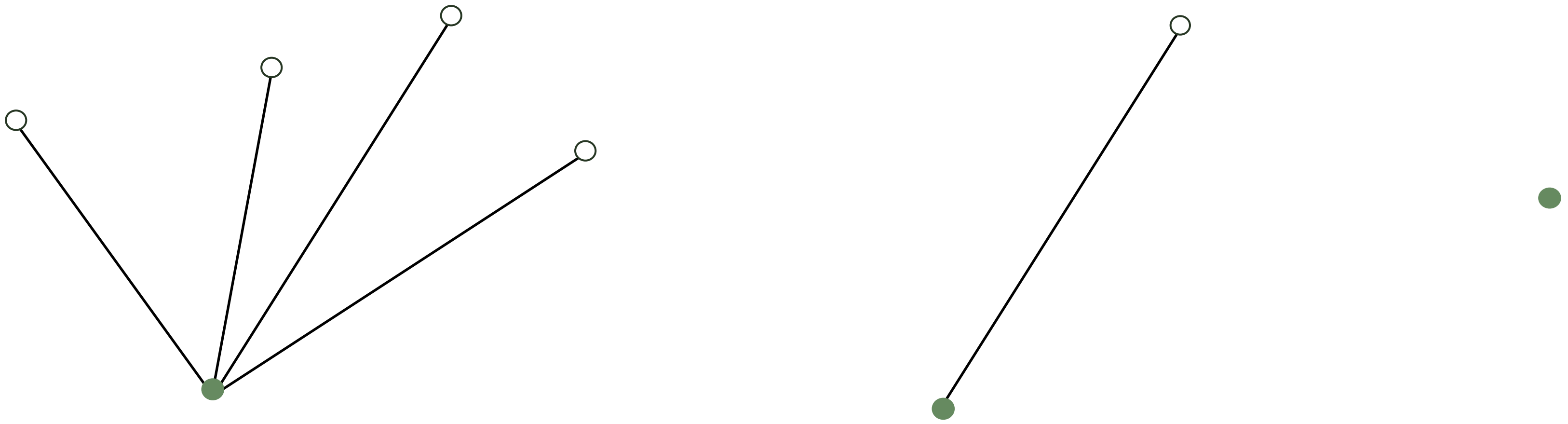
famnit

Ioannina 2024

BACKGROUND

DEFINITION:

*A **star** is a graph on k vertices with one vertex of degree k-1 (**center**) and n-1 vertices of degree 1. A **star-forest** is a forest in which every component is a star.*



BACKGROUND

DEFINITION:

*We say that a star-forest is **plane** if it is drawn in the plane without crossings.*

BACKGROUND

Decomposing G into minimal number of "P" subgraphs

P = Planar	~ Thickness
P = Forest	~ Arboricity
P = Star-forest	~ Star-arboricity
P = Plane Star-forest, G is geometric graph	~ Geometric star-arboricity

PREVIOUS RESULTS

THEOREM [AKIYAMA AND KANO]:

Let n ≥1. The complete graph with n vertices can be decomposed into at most $\left\lceil \frac{n}{2} \right\rceil + 1$ *star-forests and this bound is tight.*

THEOREM [PACH, SAGHAFIAN AND SCHNIDER]:

Let n ≥ 1. The complete convex geometric graph with n vertices cannot be decomposed into fewer than n – 1 plane star-forests.

CONTRIBUTION

CONJECTURE [PACH, SAGHAFIAN AND SCHNIDER]:

Let $n \geq 1$. There is no complete geometric graph K_n with n vertices that can be decomposed into fewer than $\left\lceil \frac{3n}{4} \right\rceil$ plane star-forests.

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CONJECTURE [PACH, SAGHAFIAN AND SCHNIDER]:

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ANSWER:

There exists a complete geometric graph on n vertices which can be decomposed into $\left\lceil \frac{2n}{3} \right\rceil$ plane star-forests.

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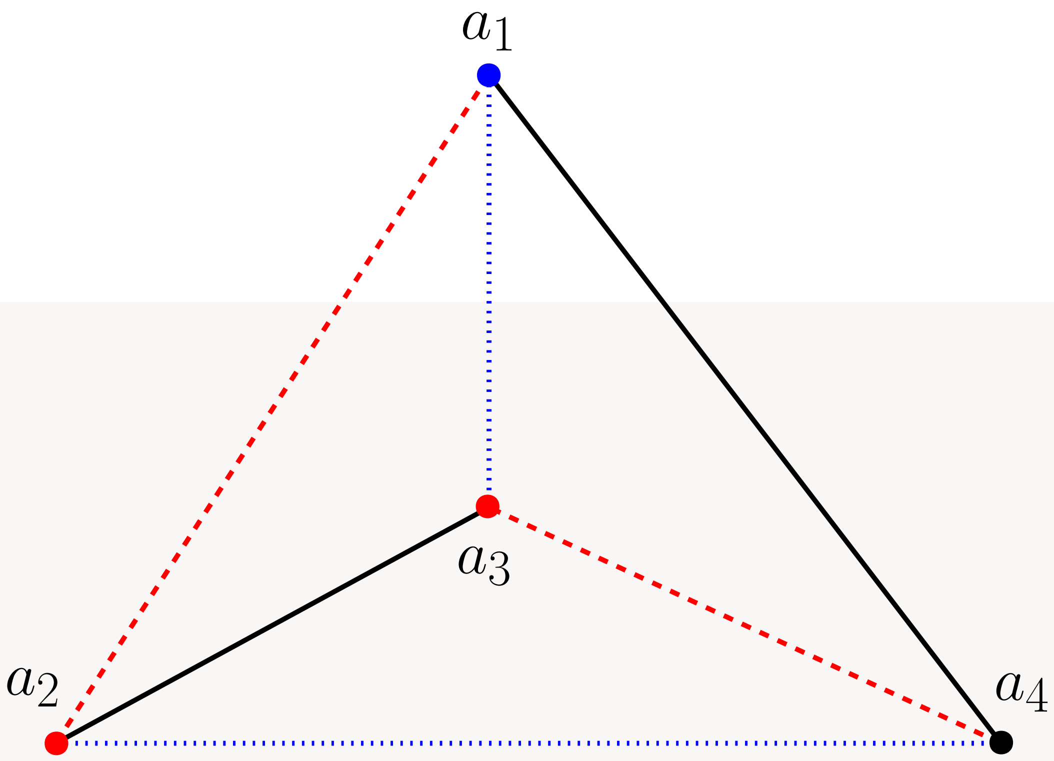
ANSWER:

There exists a complete geometric graph on n vertices which can be decomposed into ~~$\left\lceil \frac{2n}{3} \right\rceil$~~ *plane star-forests.*

$\left\lceil \frac{n}{2} \right\rceil + 1.$

FIRST CONSTRUCTION



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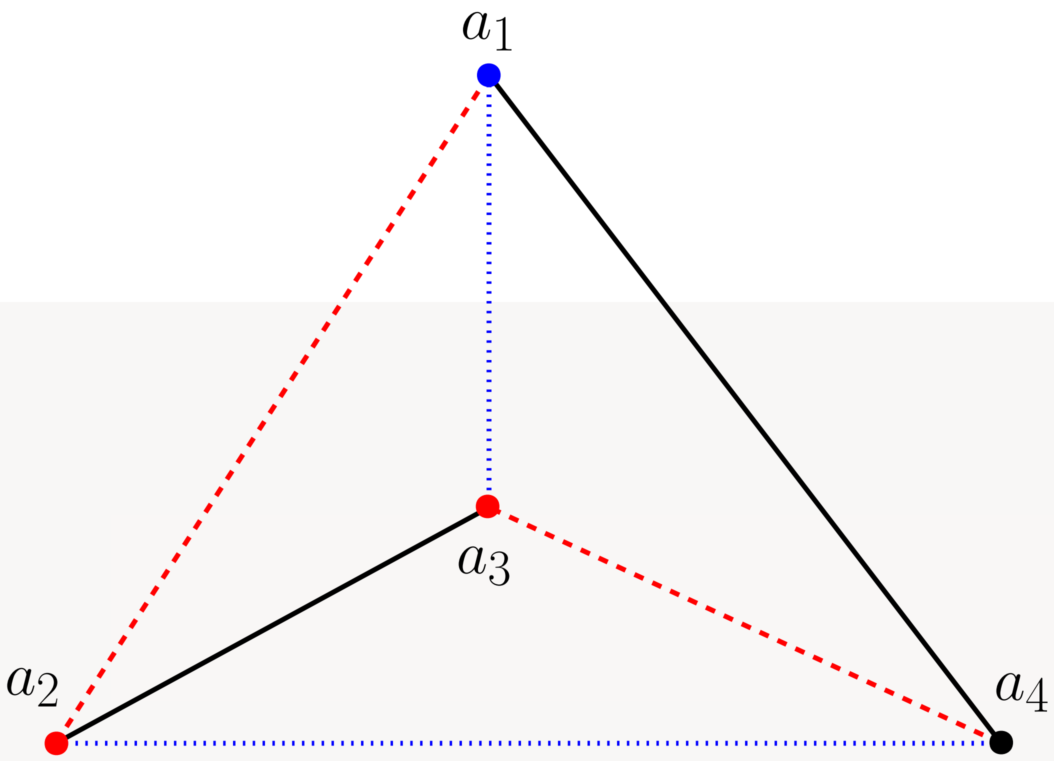
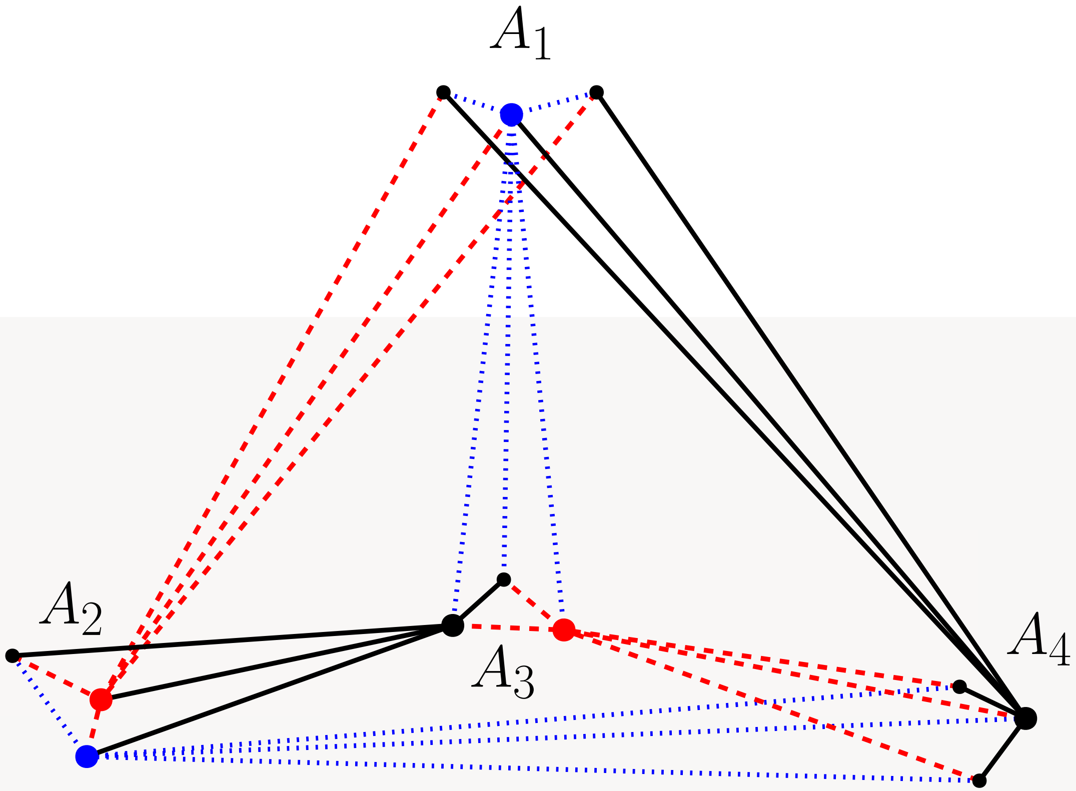




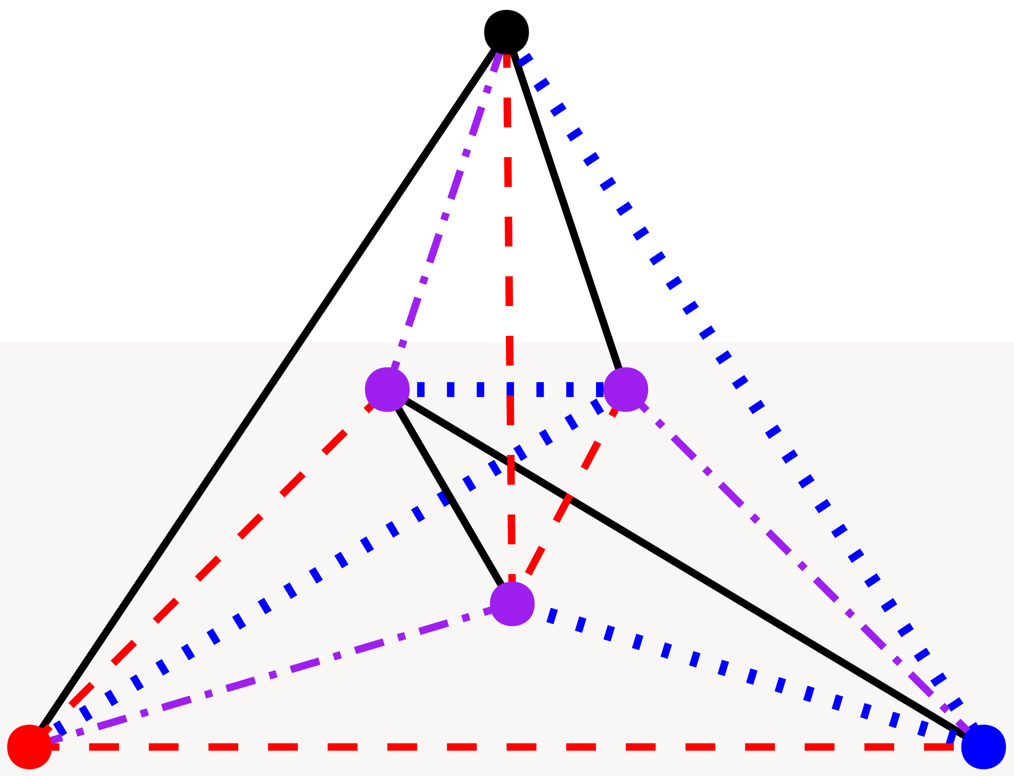
FIRST CONSTRUCTION

THEOREM:

Let c $\epsilon \left(\frac{1}{2}, 1\right)$ *be a constant.*

If there is a complete geometric graph on n_0 *points which can be partitioned into* cn_0 *plane star-forests, in such a way that each vertex is a center of at least one tree, then for each integer* $k \geq 1$*, there exists a complete geometric graph on* kn_0 *points that can be partitioned into* ckn_0 *plane star-forests.*

FIRST CONSTRUCTION



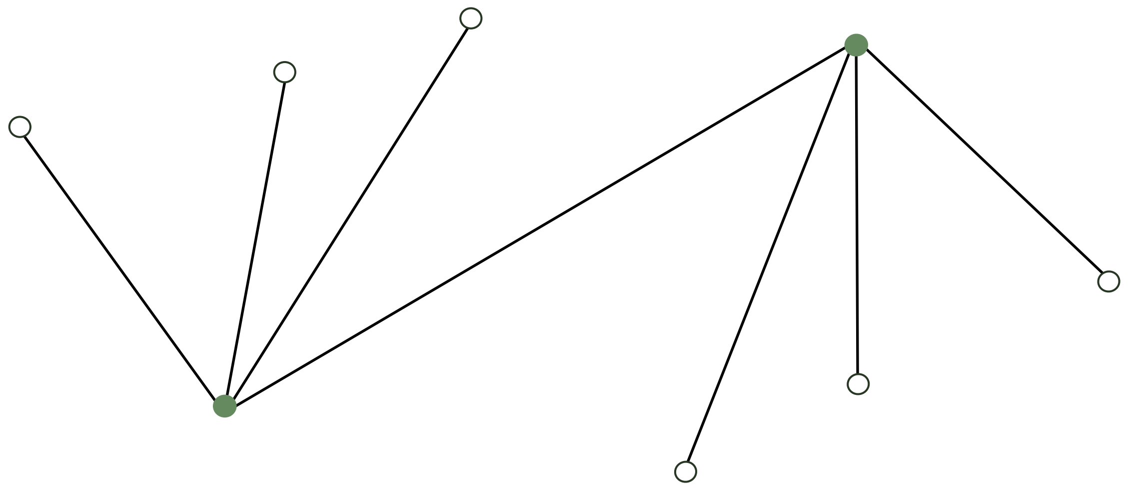
COROLLARY:

For every $k \in \mathbb{N}$ *there is a complete geometric graph on n=6k vertices which can be decomposed into* $\frac{2n}{3}$ *plane star-forests*

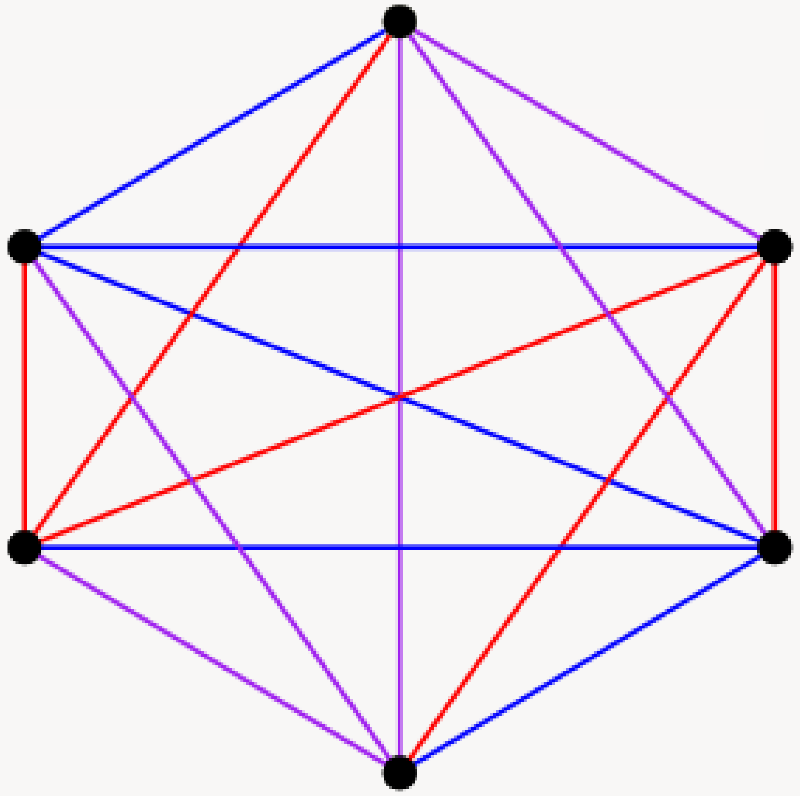
DOUBLE STARS

DEFINITION:

*A **double star** is a graph composed of two stars + an edge connecting their centers.*



DECOMPOSITION OF K_{2K} INTO DOUBLE STARS

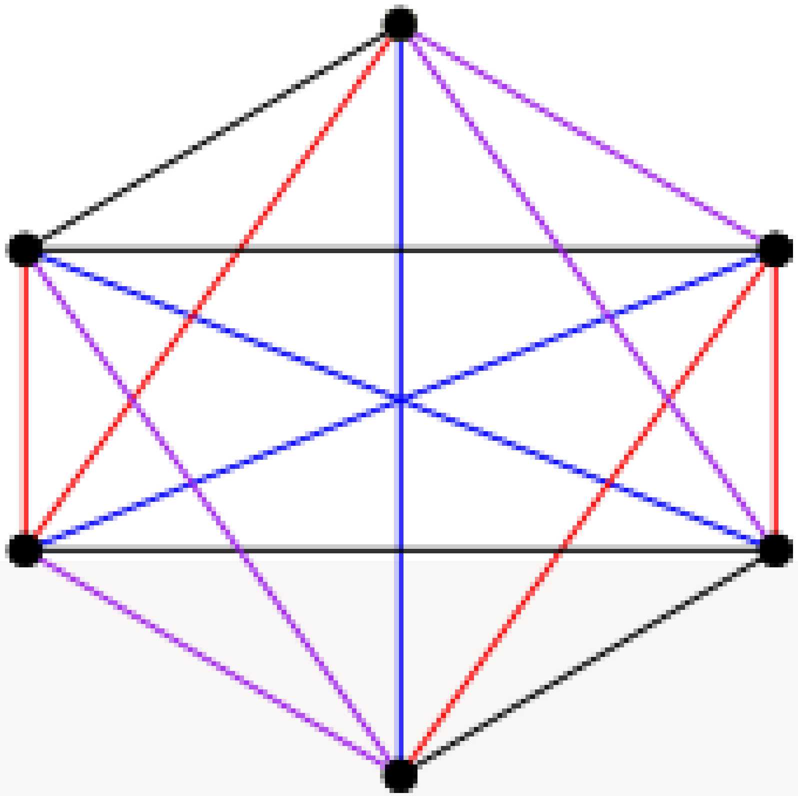


BROKEN DOUBLE STARS DECOMPOSITION

DEFINITION:

*A **broken double stars decomposition** of a complete graph on 2k vertices is a decomposition into a matching of size k and k spanning star-forests whose components are two stars with k-1 edges each, with centers at endpoints of an edge of the matching.*

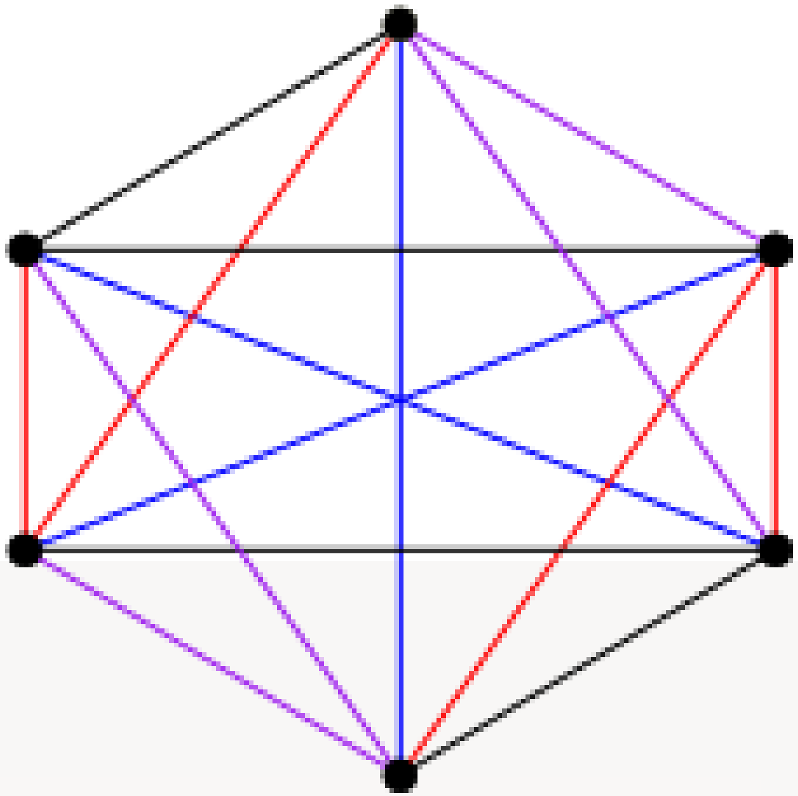
BROKEN DOUBLE STARS DECOMPOSITION OF K_{2K}



THEOREM:

Every decomposition of K_{2k} into k+1 star-forests is a broken double stars decomposition.

BROKEN DOUBLE STARS DECOMPOSITION OF K_{2K}



THEOREM:

Every decomposition of K_{2k} into k+1 star-forests is a broken double stars decomposition.

UPSHOT:

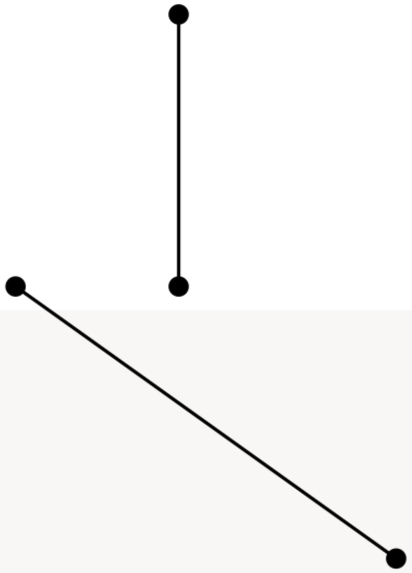
Instead of looking for pointsets we can look for arrangements of line segments.

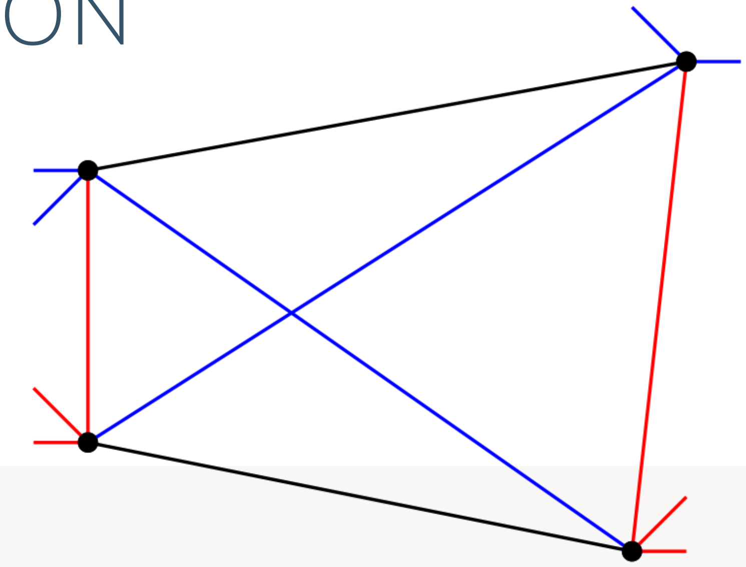
A NECESSARY CONDITION

LEMMA:

Let L be an arrangement of line segments in the plane. If L can be extended into a broken double stars decomposition of the complete geometric graph on its endpoints, then every pair of segments from L is in a stabbing position.

A NECESSARY CONDITION



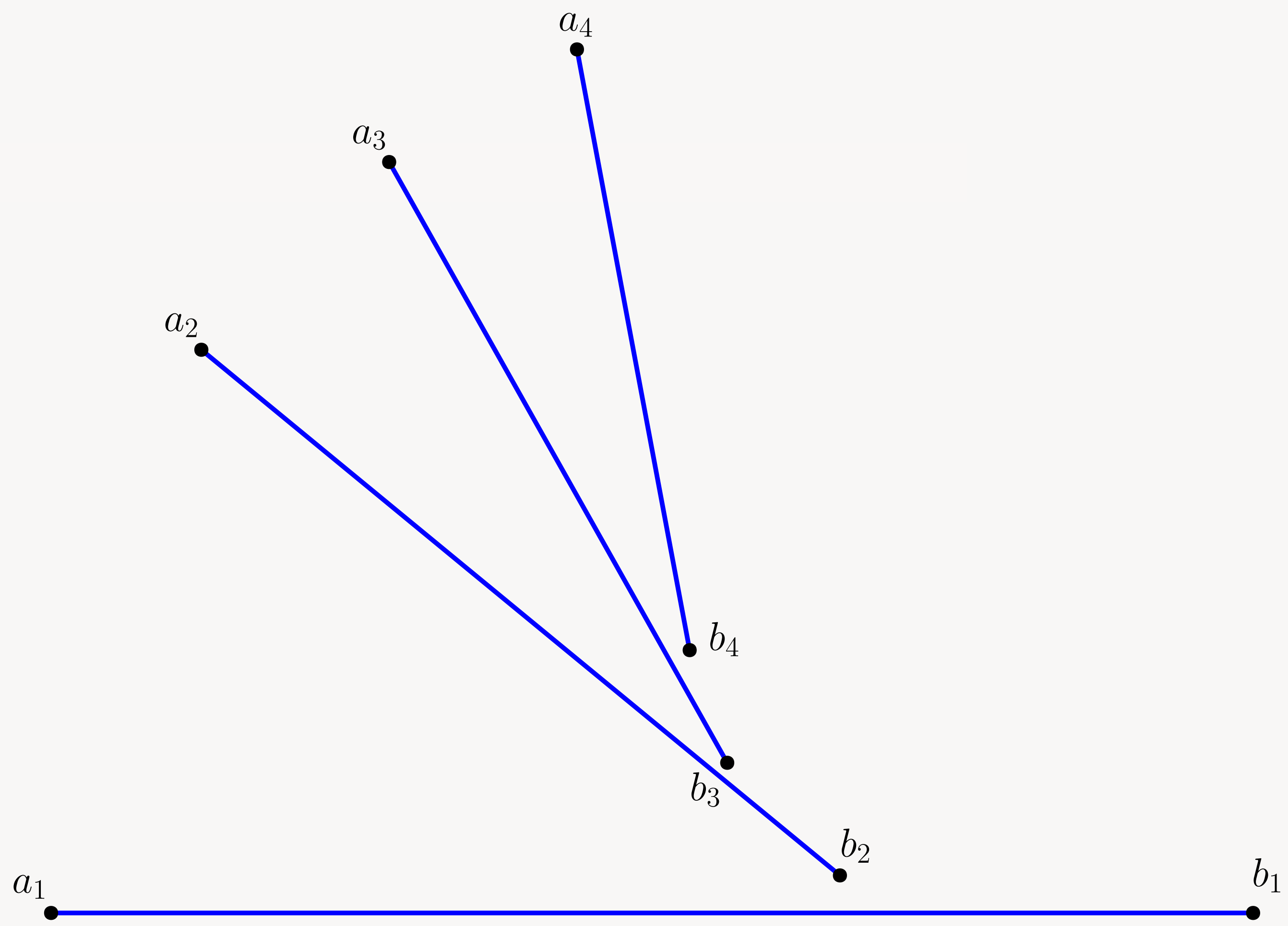


LEMMA:

Let L be an arrangement of line segments in the plane. If L can be extended into a broken double stars decomposition of the complete geometric graph on its endpoints, then every pair of segments from L is in a stabbing position.

OPTIMAL CONSTRUCTION

k- staircase



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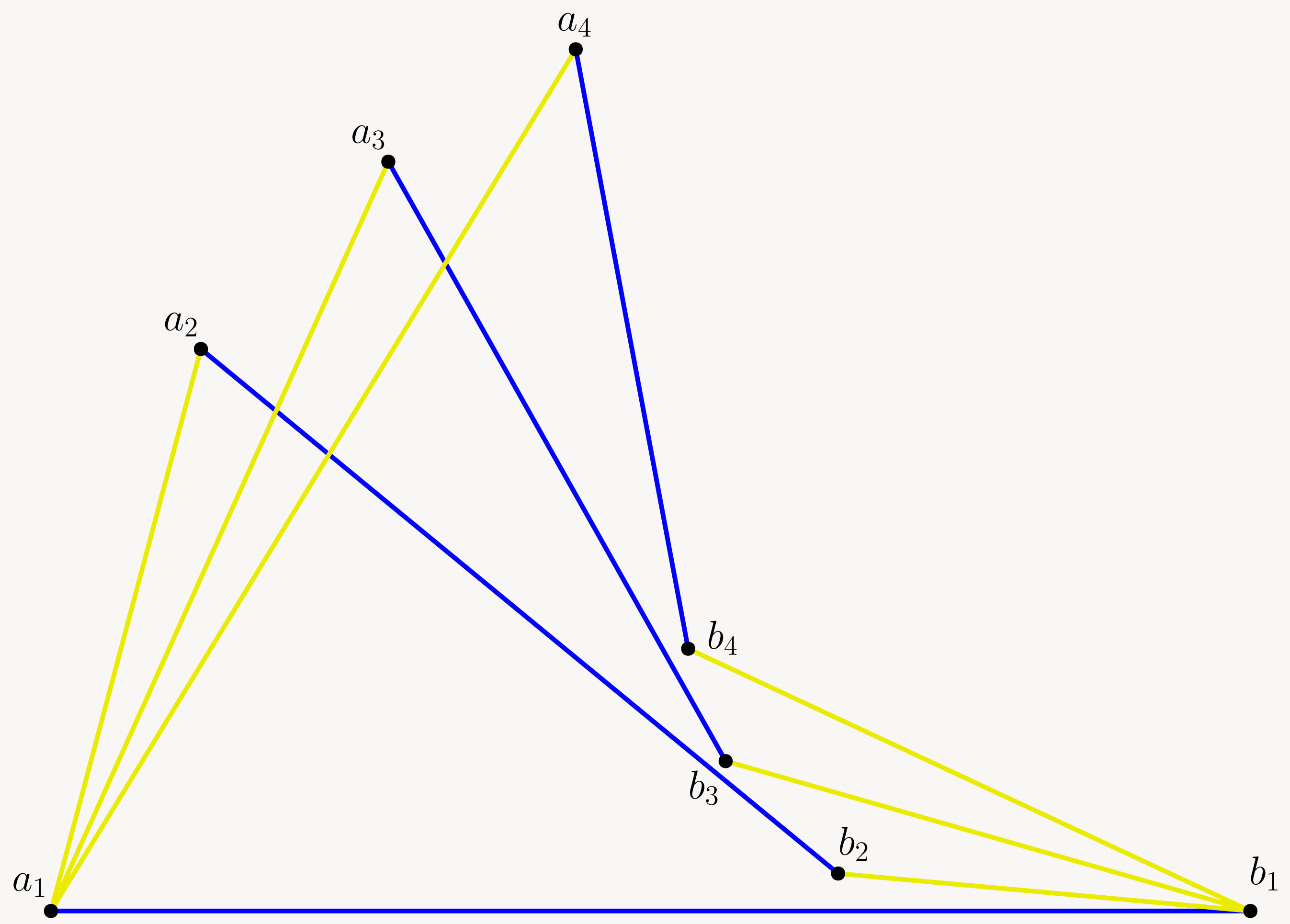

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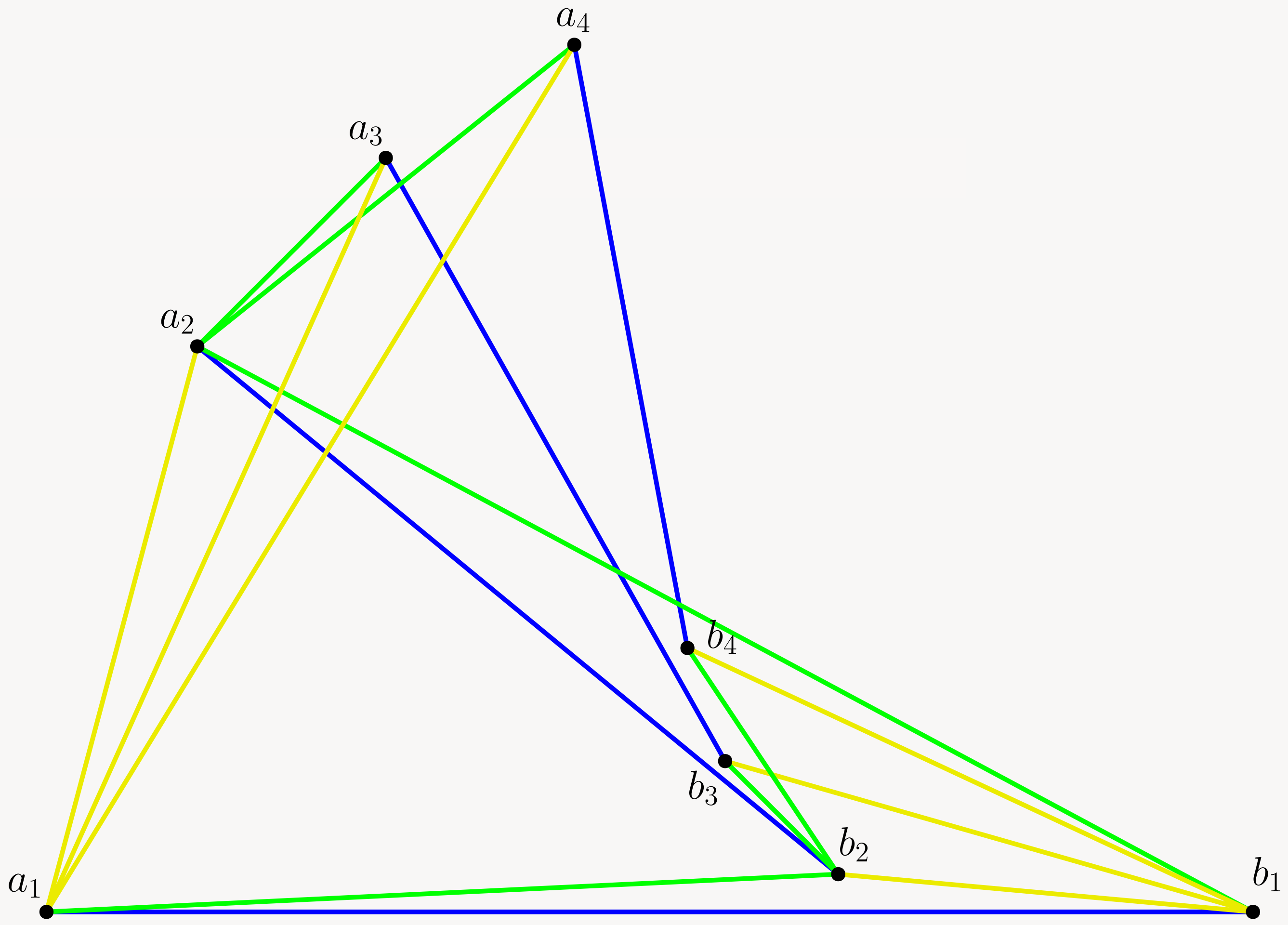

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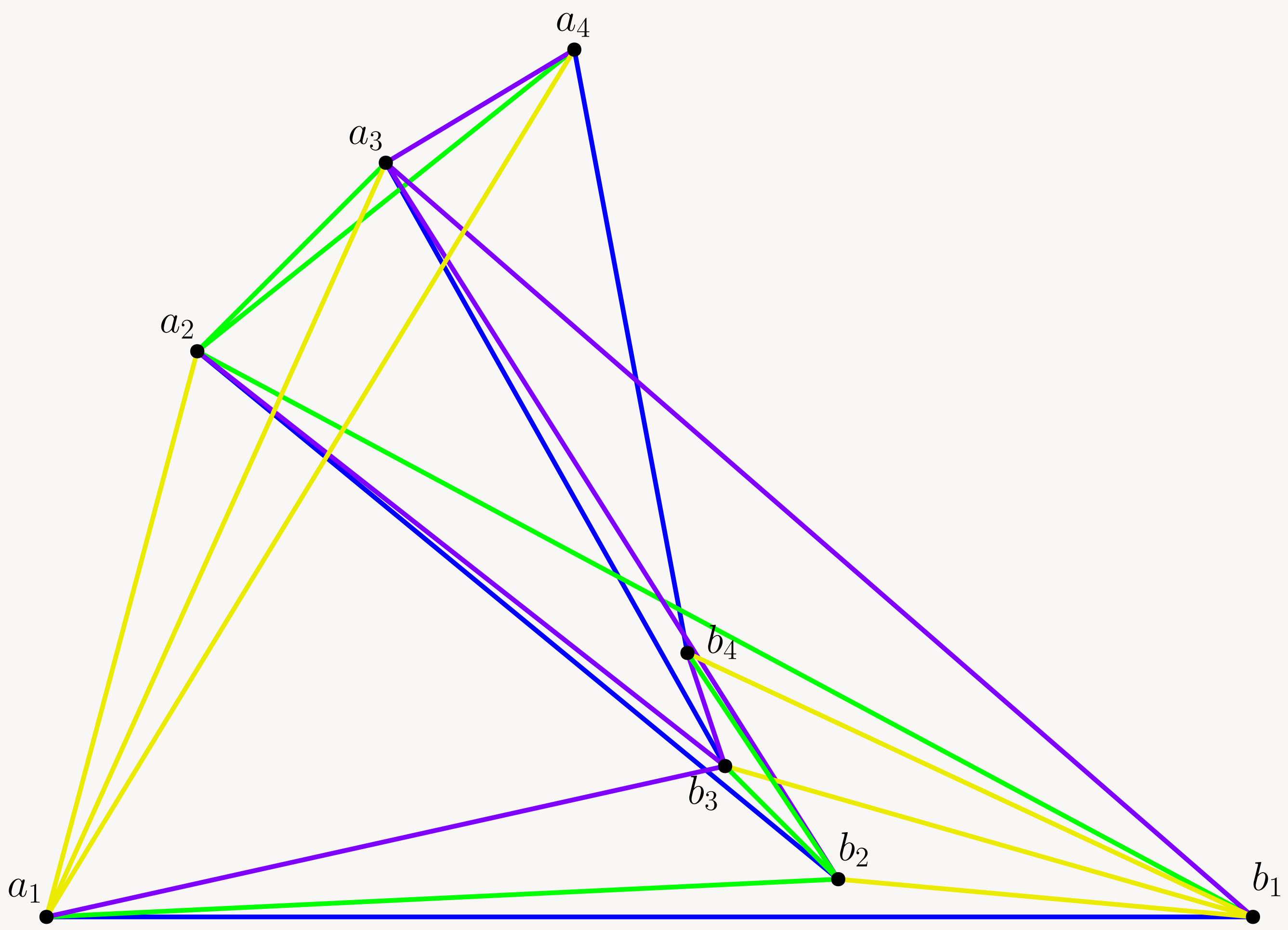

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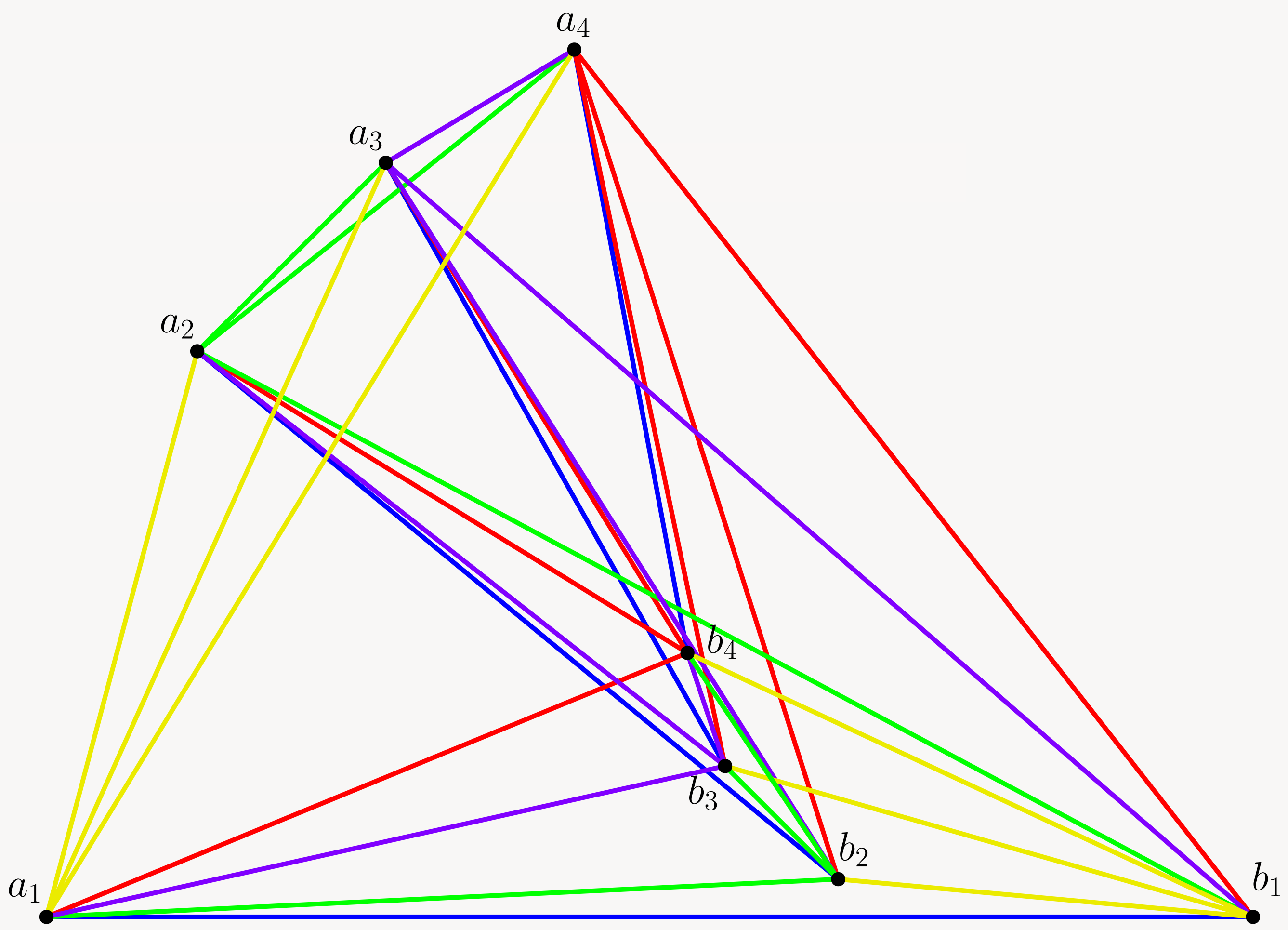
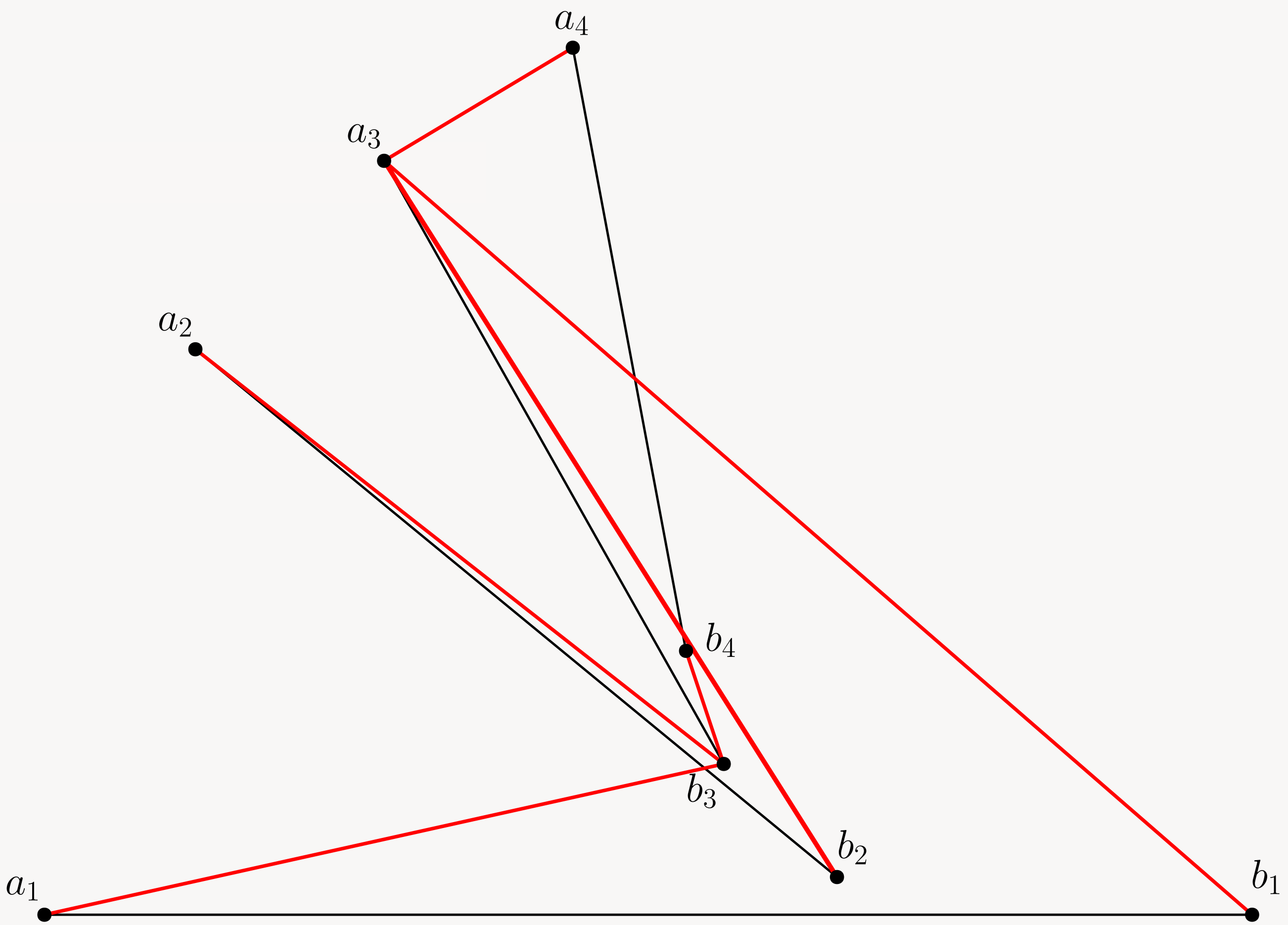


MORE FORMALLY:

- Convex (k+1)-gon P with vertices $a_{1,\dots,}a_k, b_1$
- $\forall i > 1, a_i$ in top left quadrant of the plane
- $\overline{a_1 b_1} = \overline{(-1,0)(1,0)}$
- $\forall i > 1$ place b_i in the intersection of top right quadrant of the plane with triangles (a_l, b_l, a_j) where $l < j \leq i$.

STAR-FOREST WITH CENTERS a_i, b_i CONTAINS EDGES:

$$\{\{a_i, a_j\}: j > i\} \cup \{\{a_i, b_k\}: k < i\}$$
$$\cup \{\{b_i, b_j\}: j > i\} \cup \{\{b_i, a_k\}: k < i\}$$



OPEN QUESTIONS

CONJECTURE [PACH, SAGHAFIAN AND SCHNIDER]:

The number of plane k-star-forests needed to decompose a complete geometric graph is at least $\frac{(k+1)n}{2k}$.

THANK YOU!

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