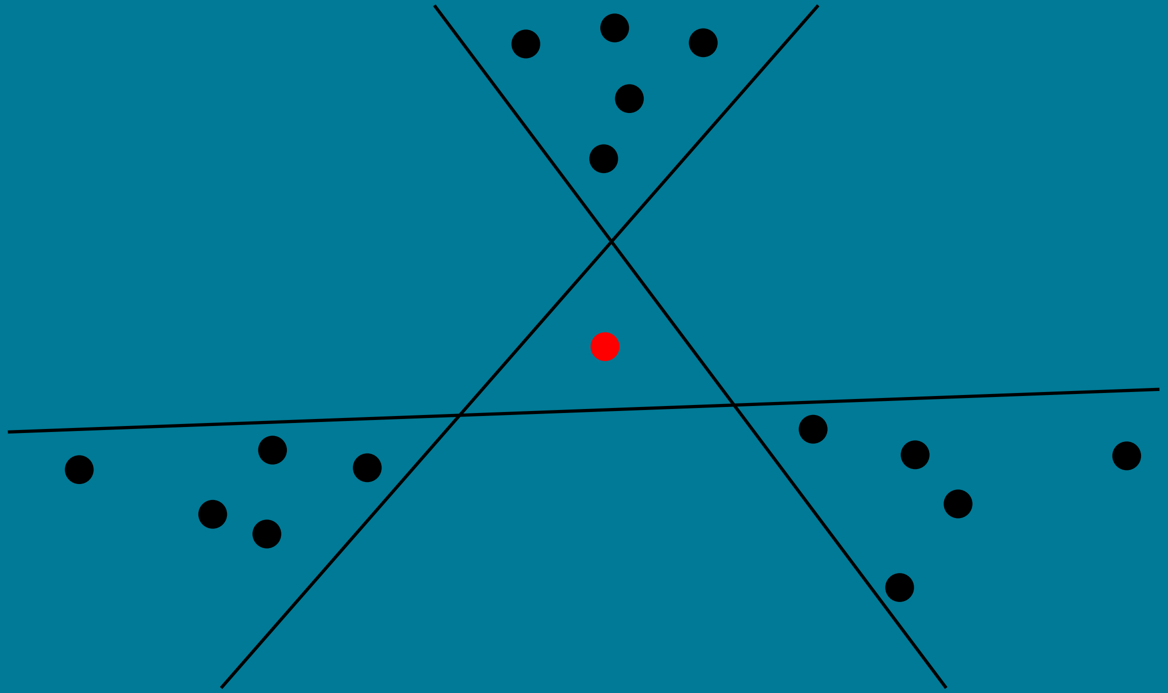
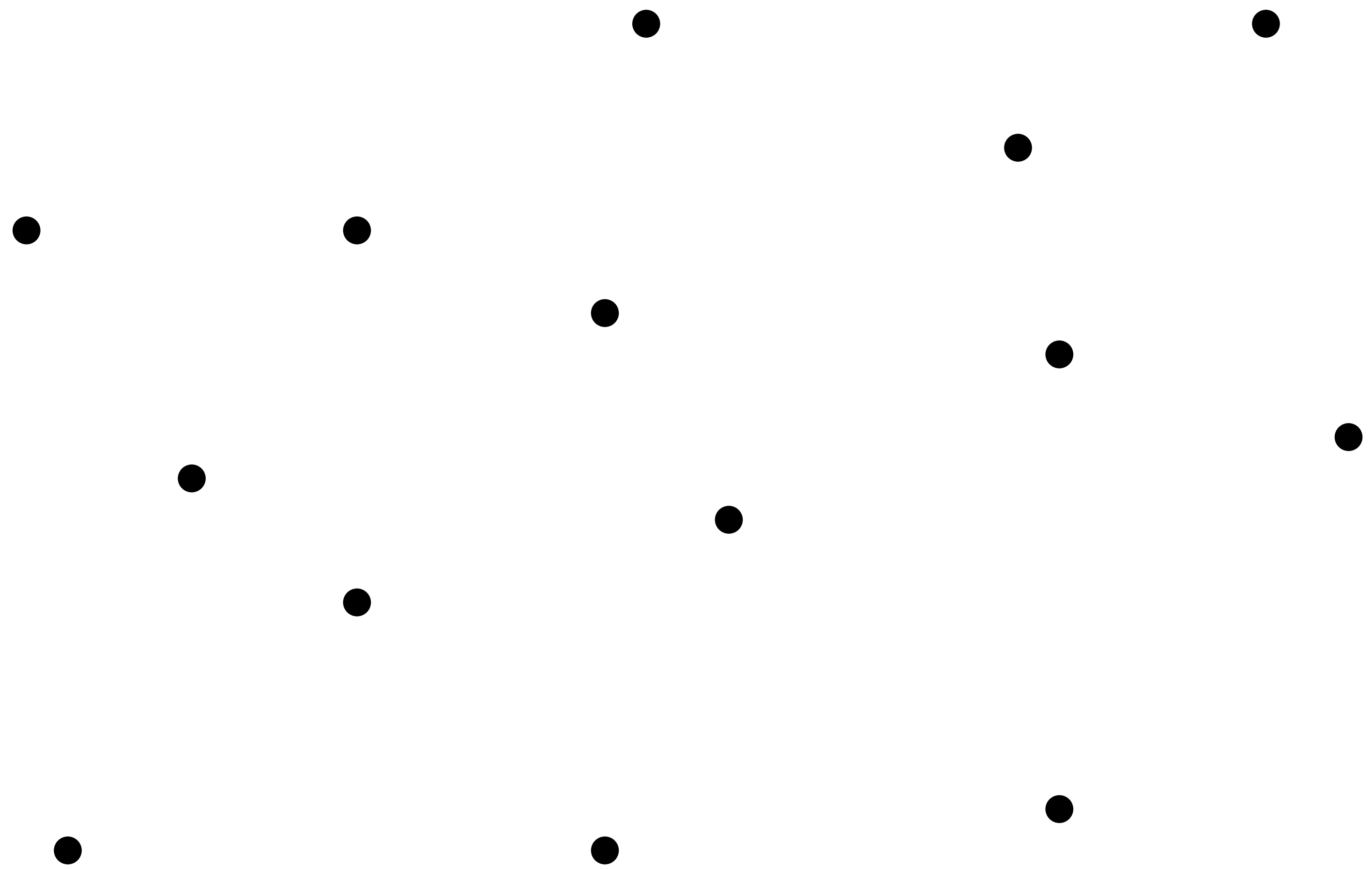
Computing Enclosing Depth

Bernd Gärtner, Fatime Rasiti, Patrick Schnider

EuroCG 2024



Introduction



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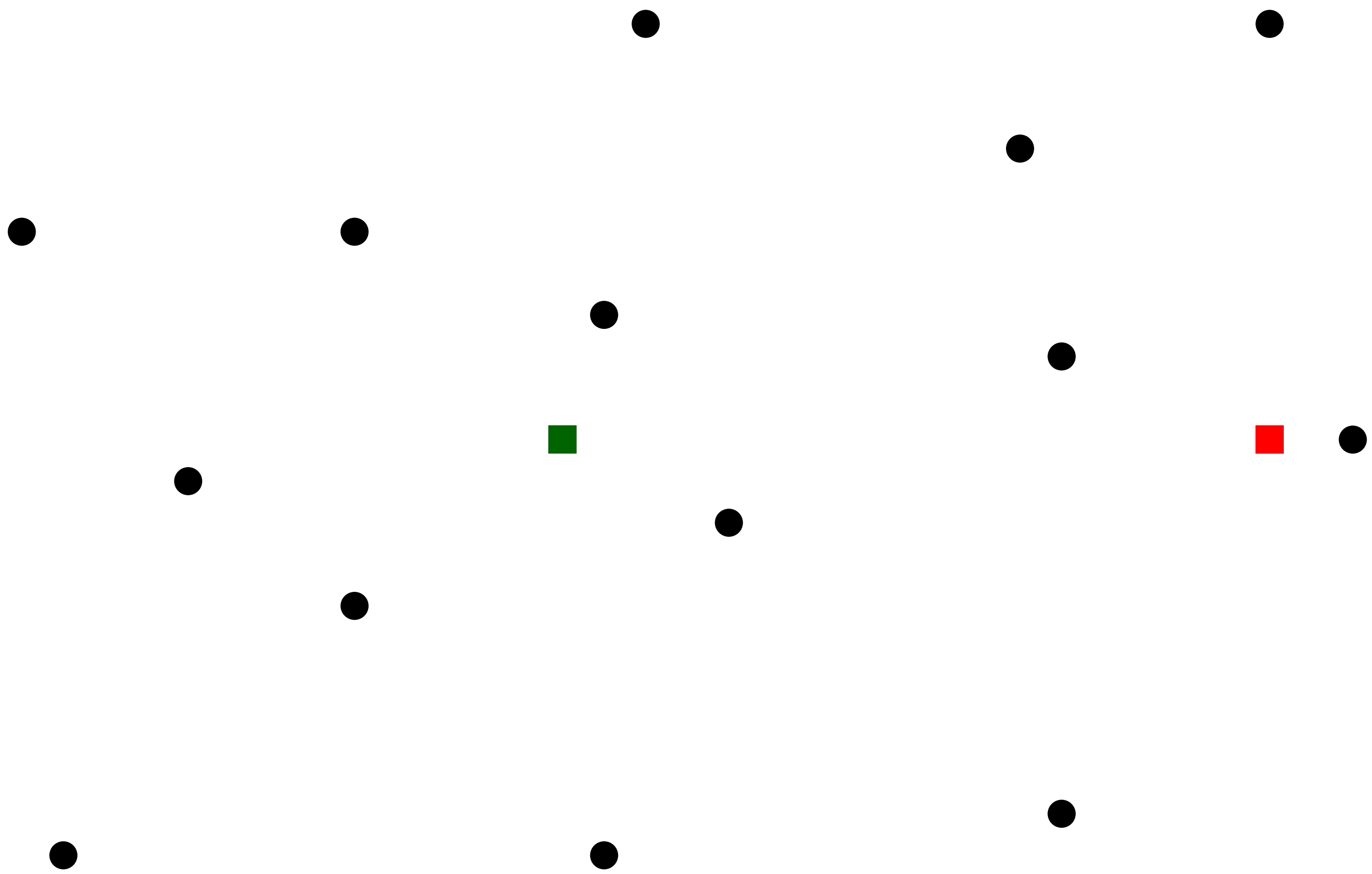

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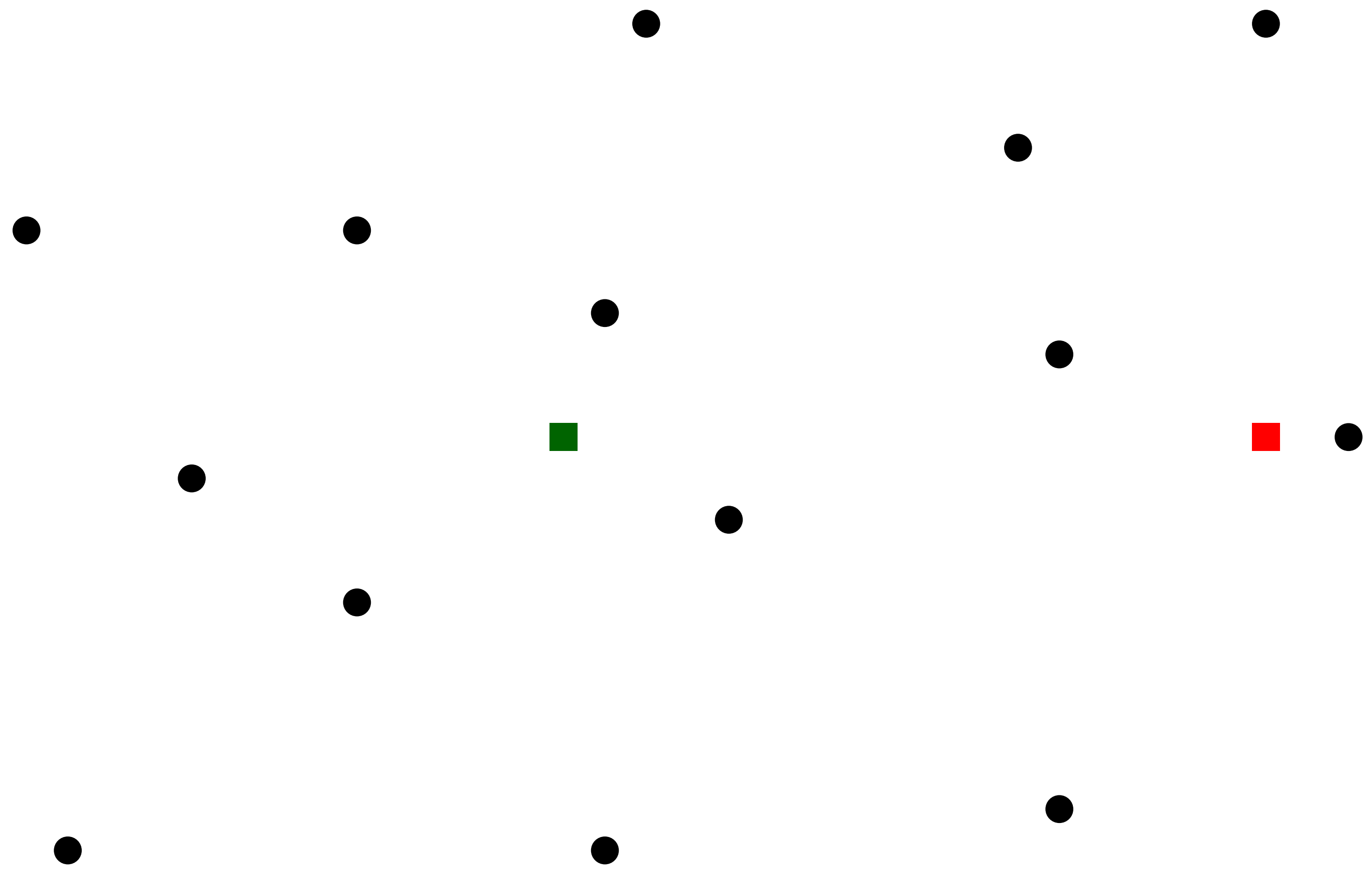
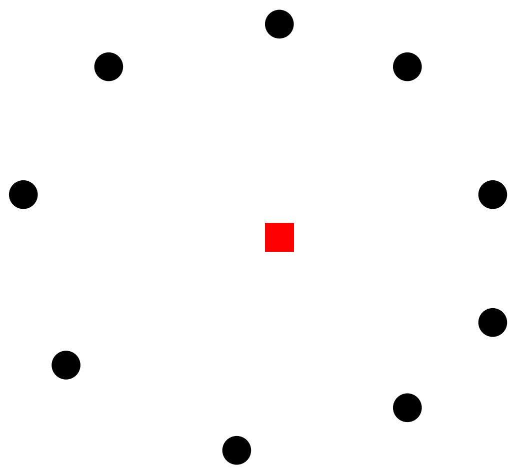
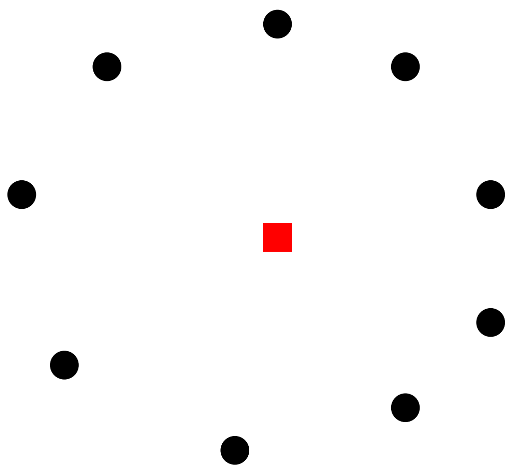


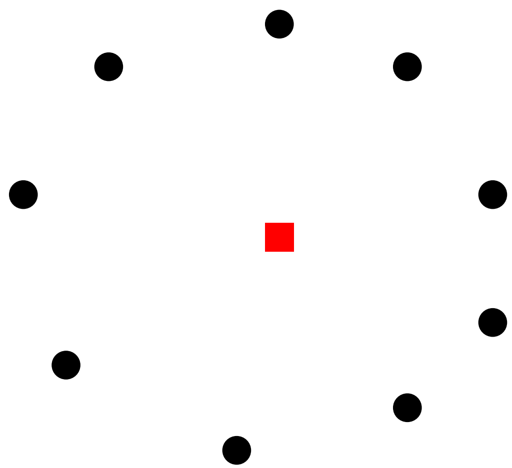
Which colored point would you rather call a ”median”?

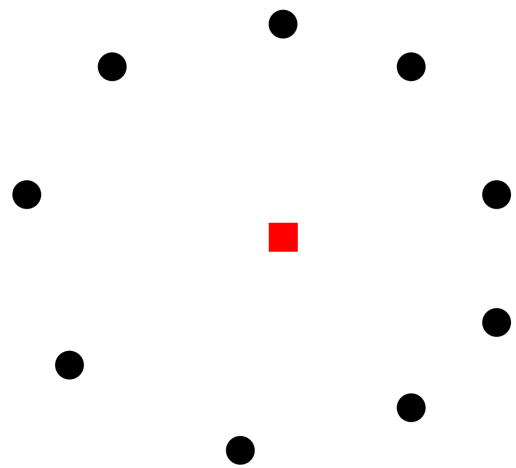
Tukey and Tverberg





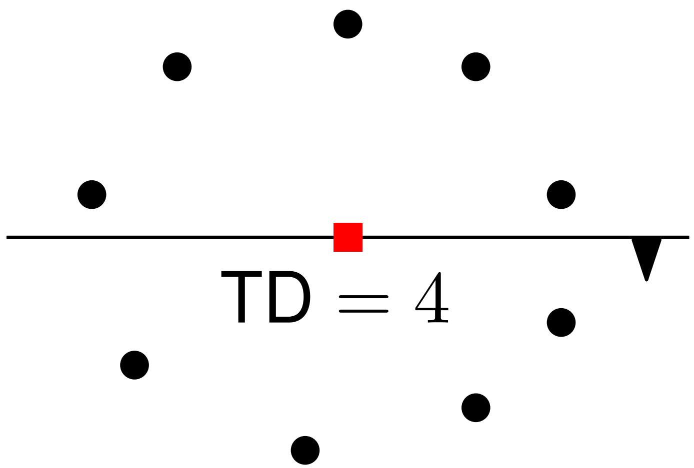
Tukey and Tverberg

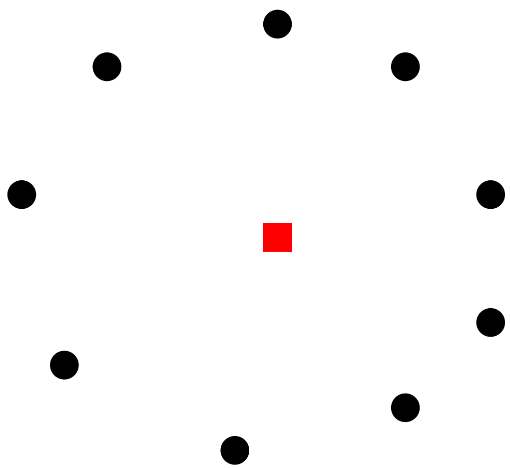




Tukey depth:
Minimum number of data points in any closed half-space containing query point q

Tukey and Tverberg





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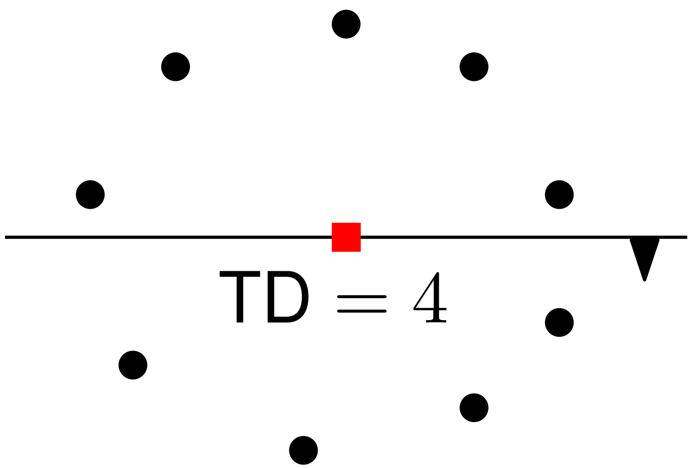

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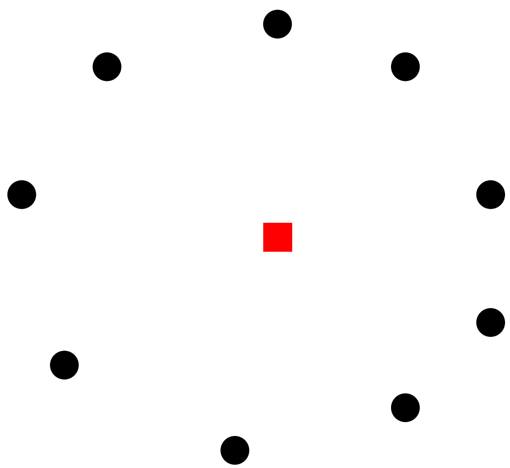
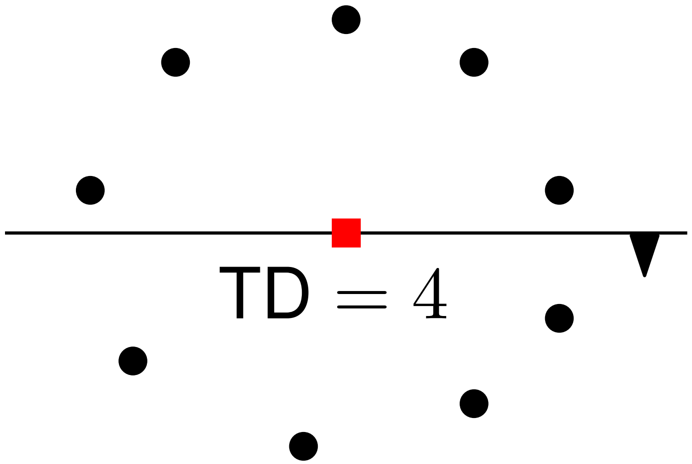
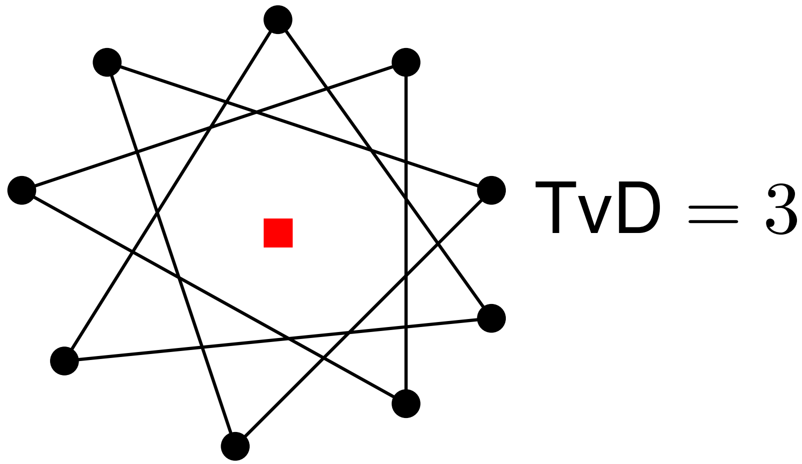


Tverberg depth:
Max. number of vertex disjoint simplices whose intersection contains q

Tukey and Tverberg





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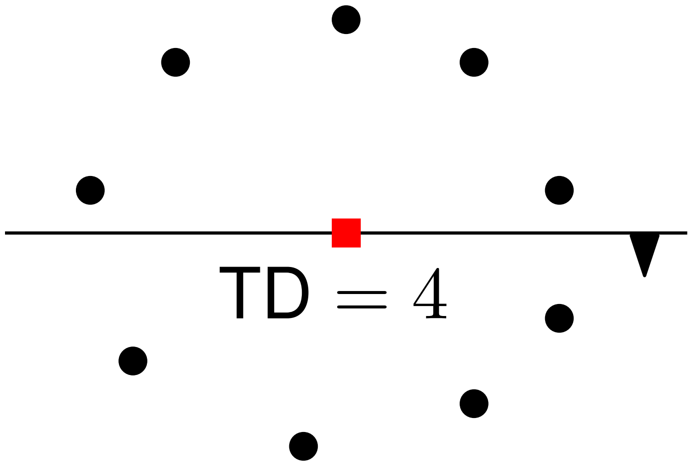

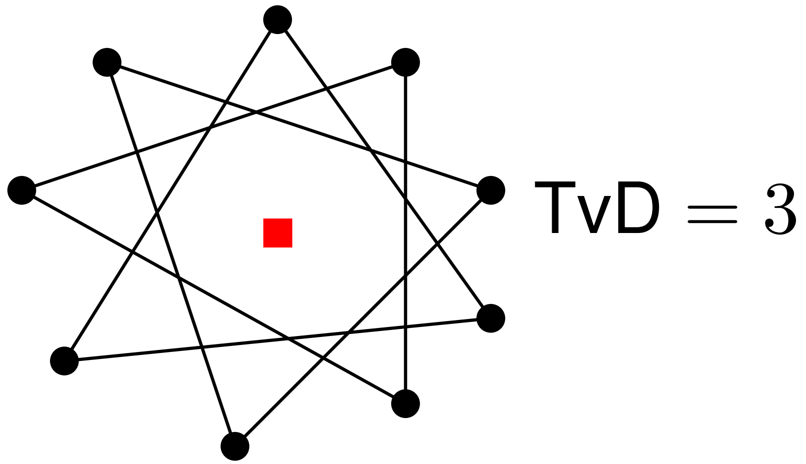

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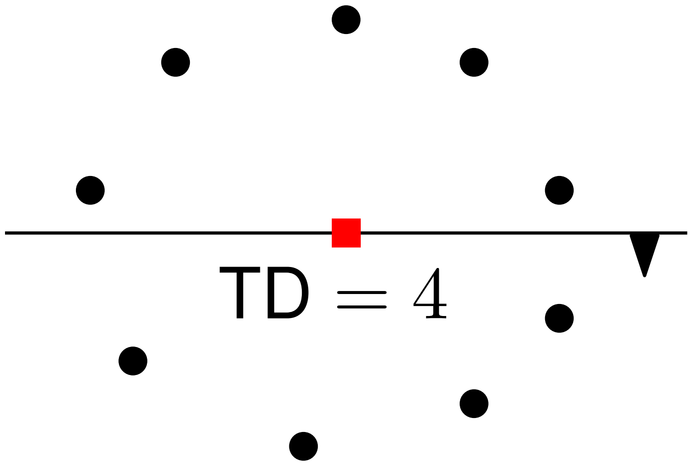
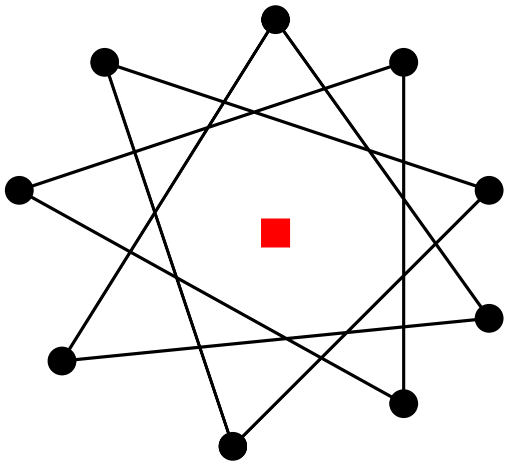
Minimum number of data points in any closed half-space containing query point q

Tverberg depth:
Max. number of vertex disjoint simplices whose intersection contains q

Centerpoint theorem:
$\forall S \exists q : \text{TD}(S, q) \geq \frac{|S|}{d+1}$

Tukey and Tverberg





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Minimum number of data points in any closed half-space containing query point q

Tverberg depth:
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Centerpoint theorem:
$\forall S \exists q$: TD$(S, q) \geq \frac{|S|}{d+1}$

Tverbergs theorem:
$\forall S \exists q$: TvD$(S, q) \geq \frac{|S|}{d+1}$

Enclosing Depth

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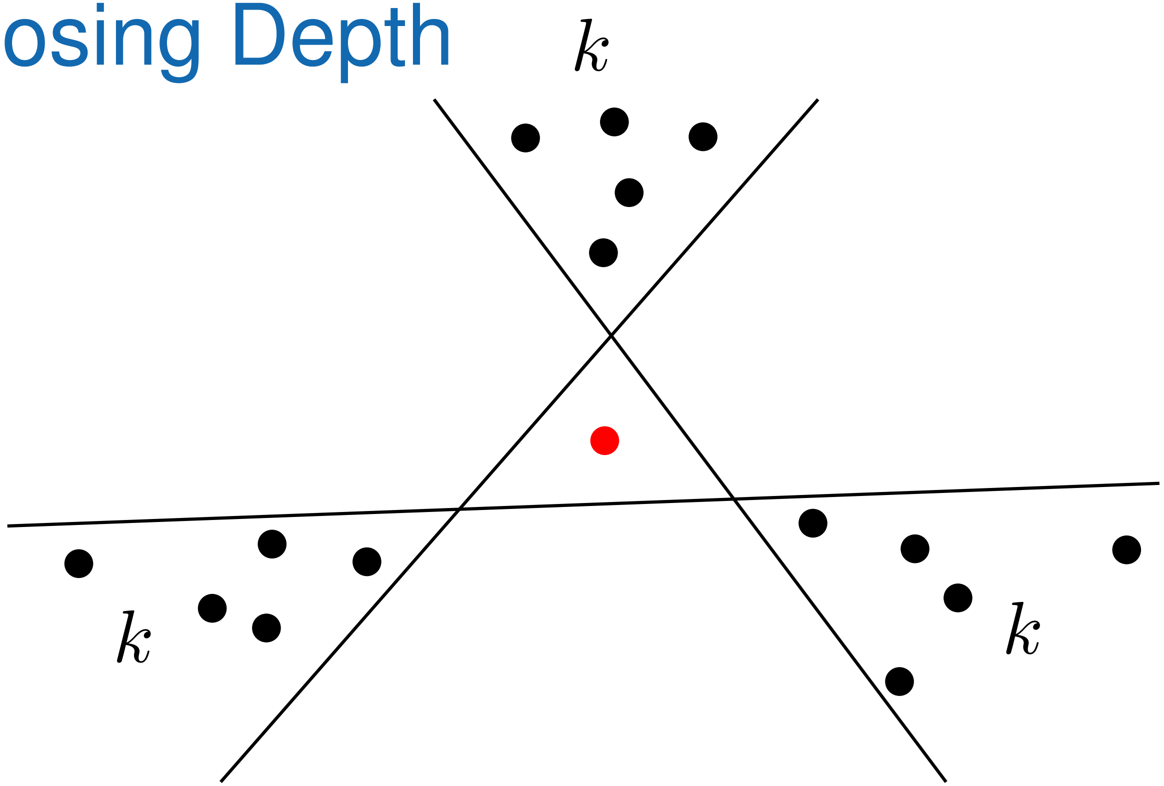

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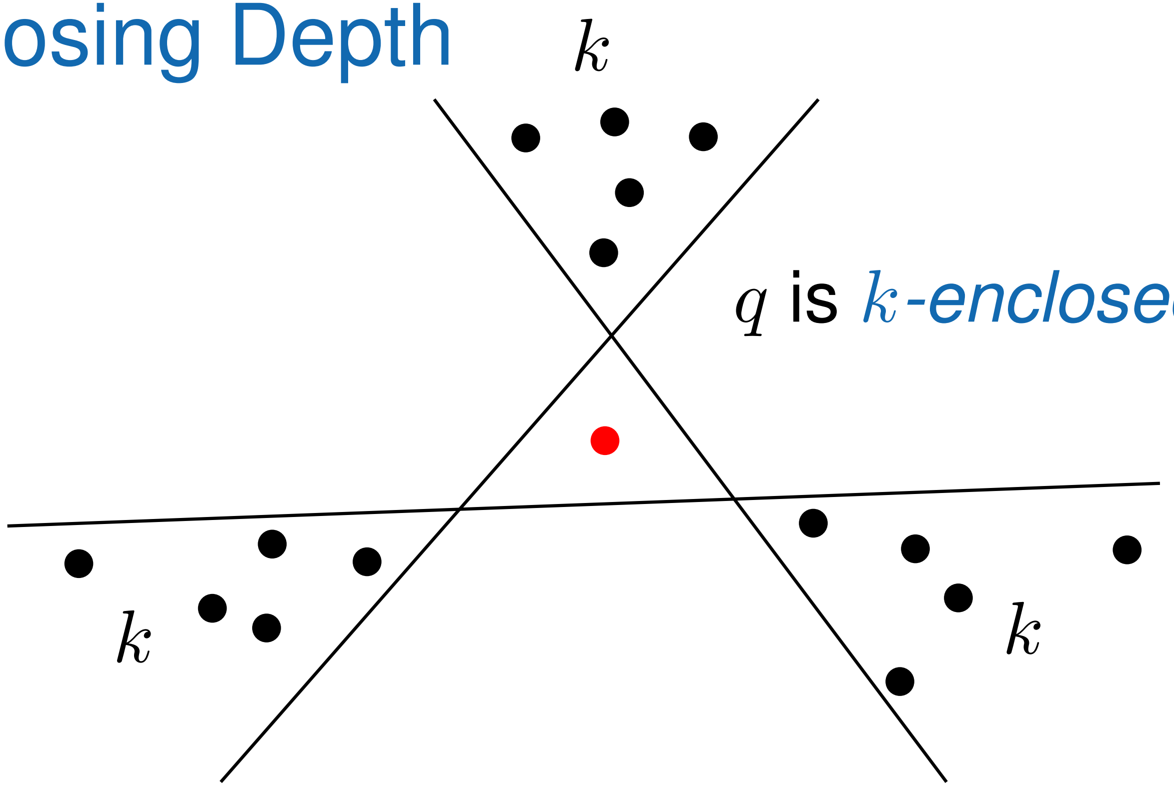
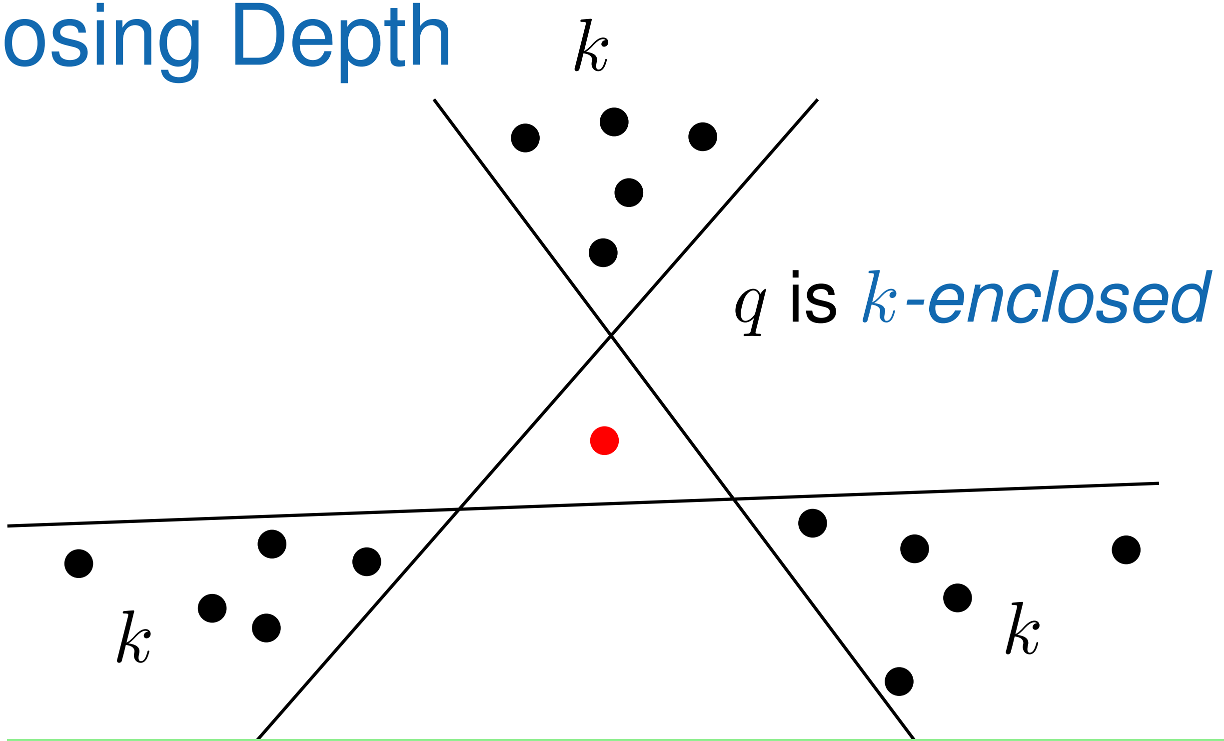


q is *k-enclosed*

Enclosing Depth



Enclosing depth:
ED$(S, q) = \max k$ s.t. q is k-enclosed

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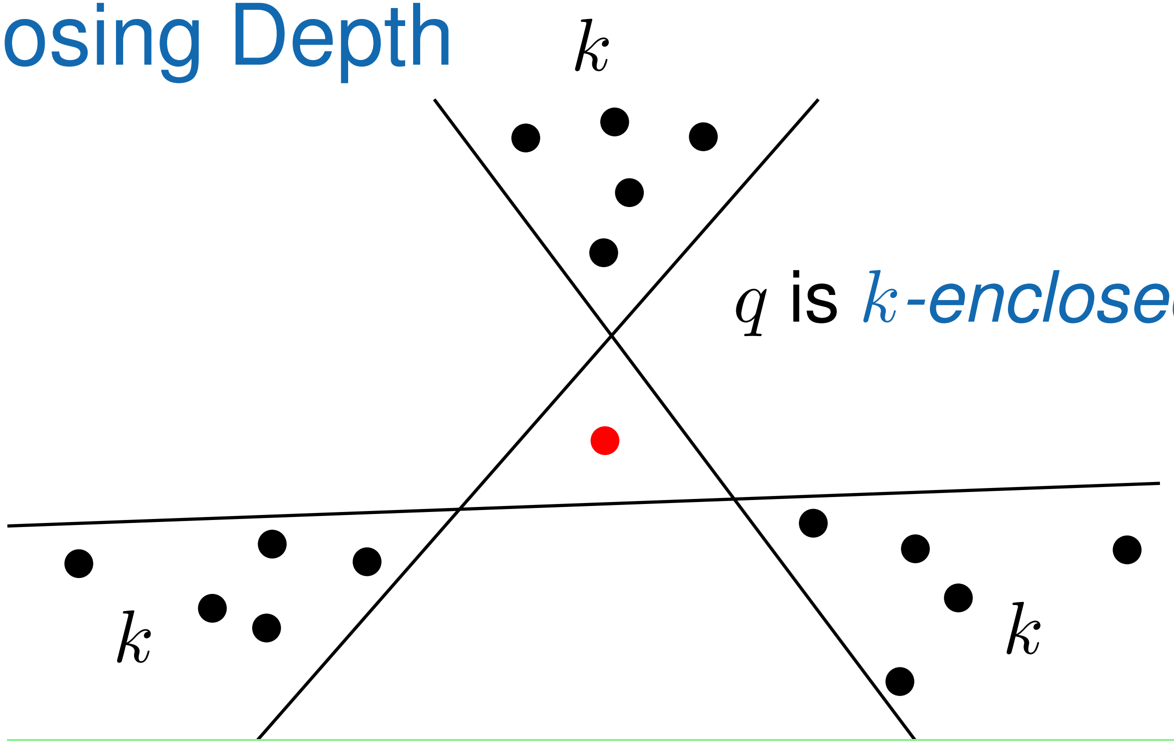


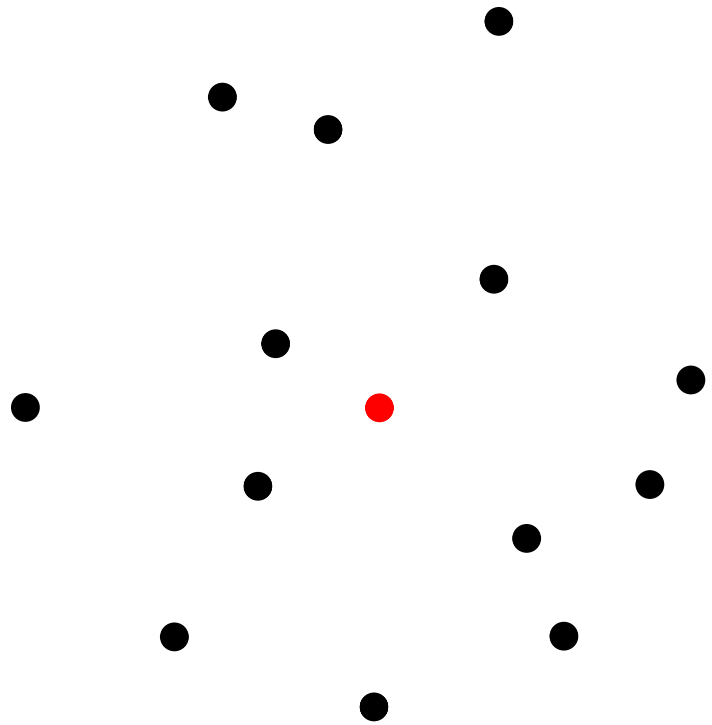
q is *k-enclosed*

Enclosing depth:
ED$(S, q) = \max k$ s.t. q is k-enclosed

Theorem [S', '23]:
For a large family of depth measures, we have
TD$(S, q) \geq \rho(S, q) \geq$ ED$(S, q) \geq c \cdot$ TD(S, q).

The algorithm in the plane

The algorithm in the plane



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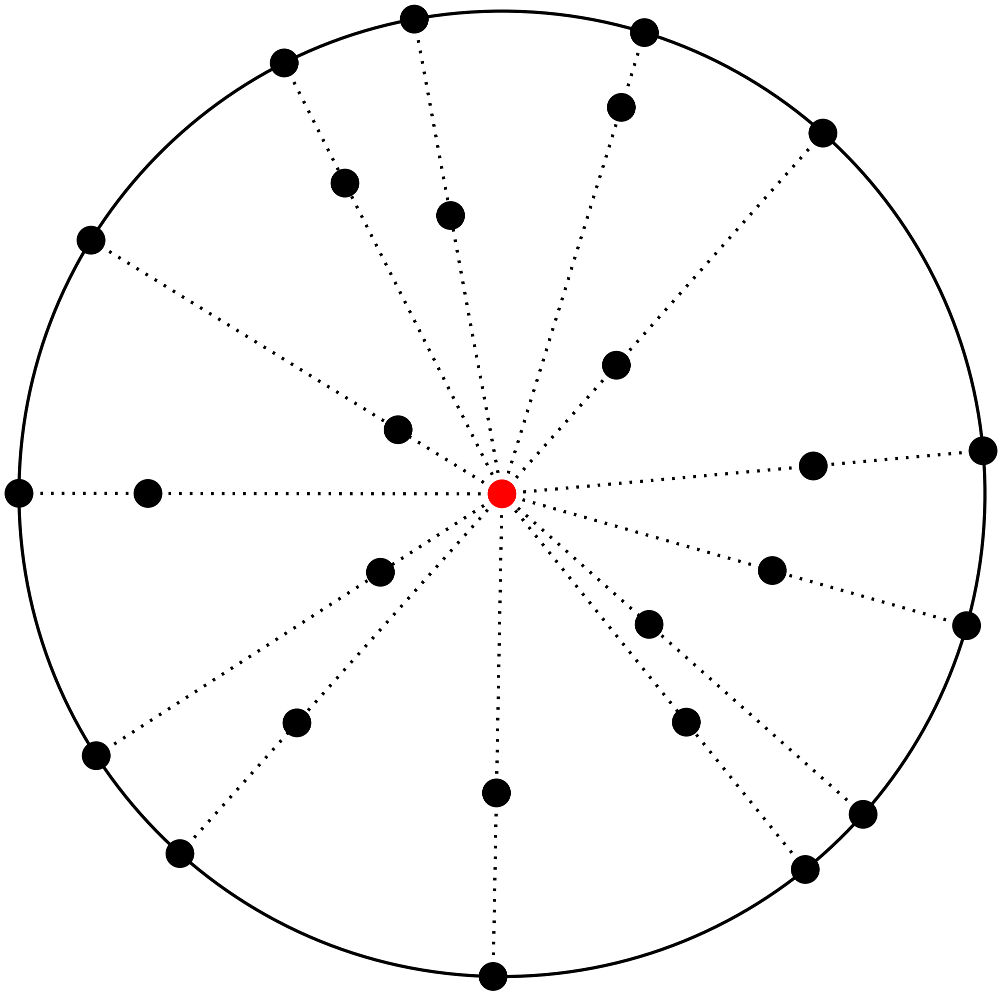

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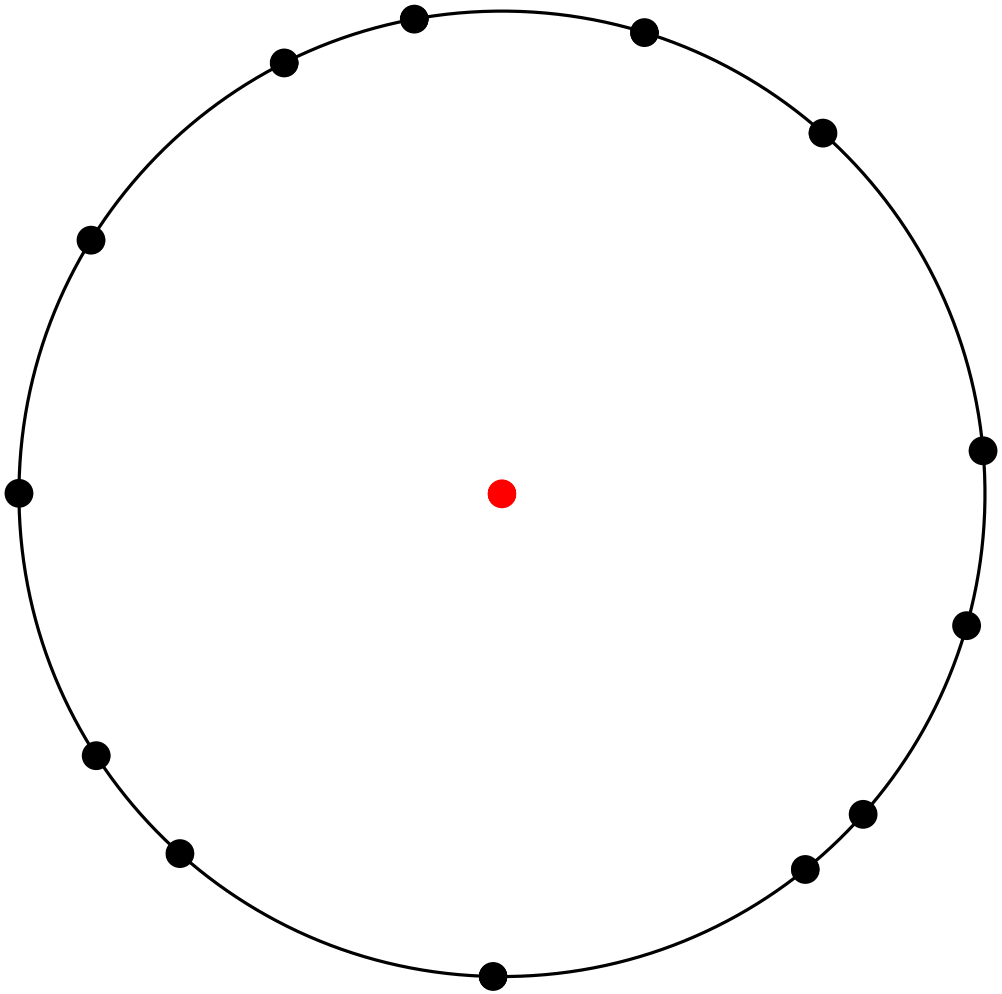

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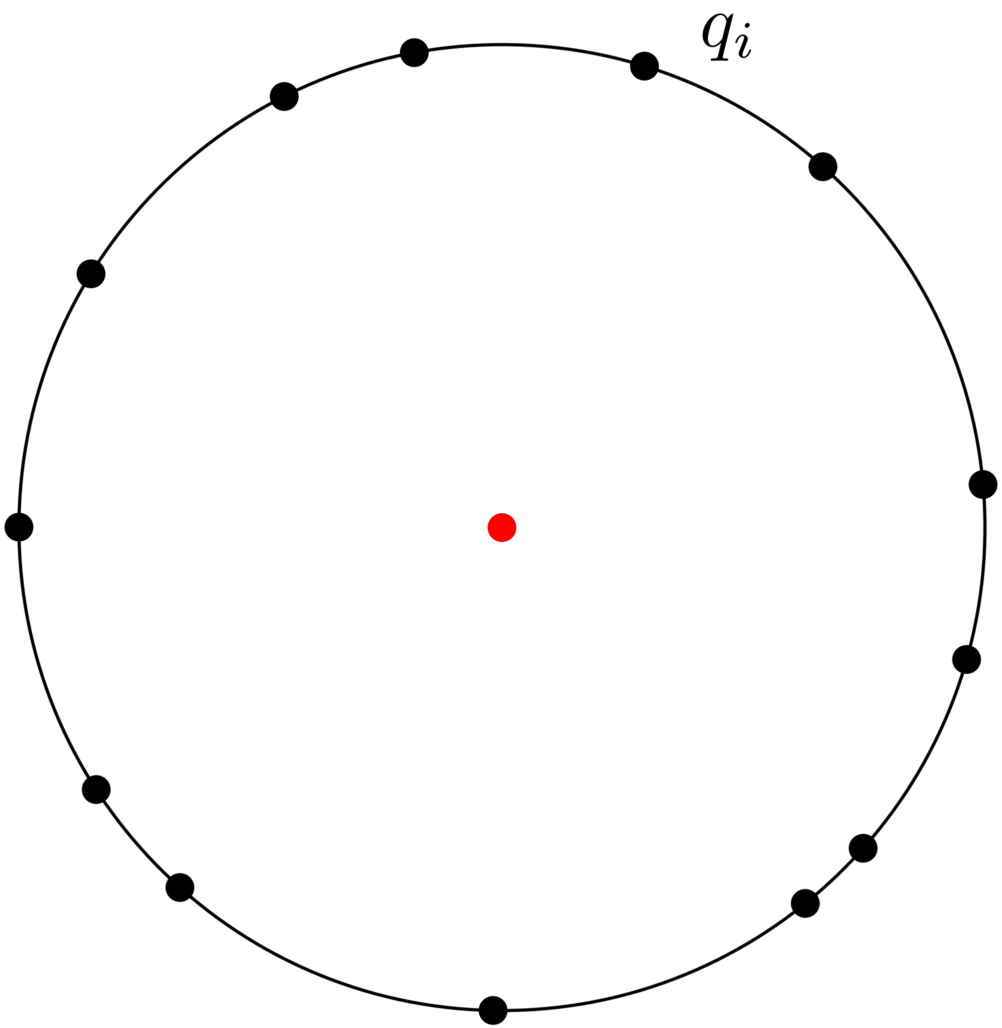

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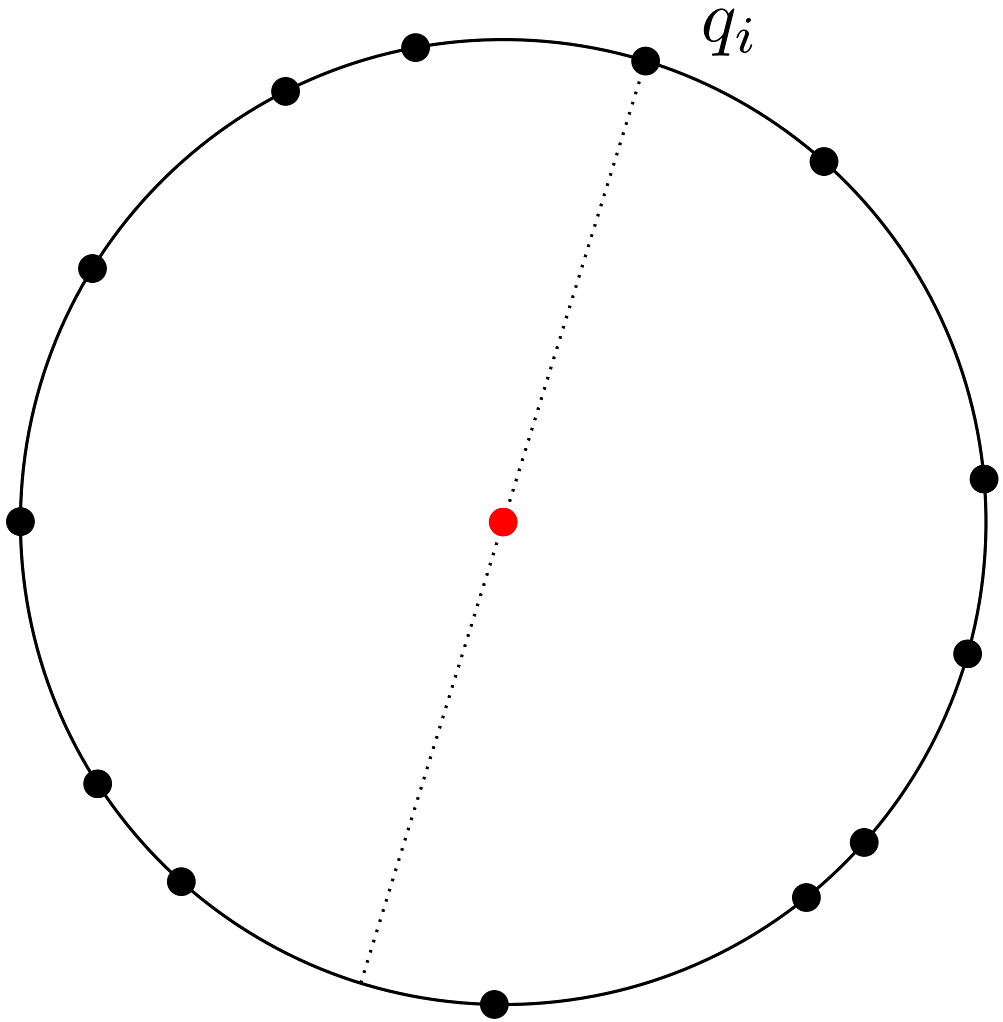

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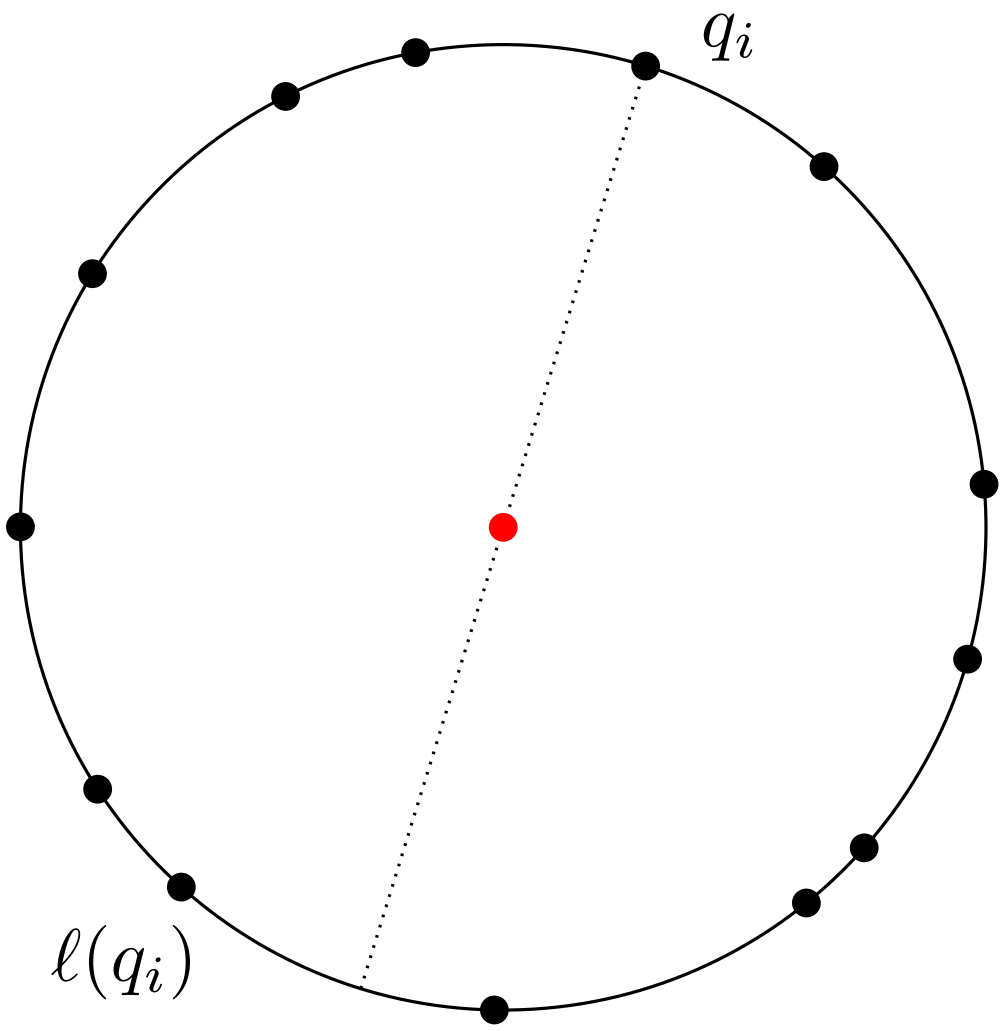

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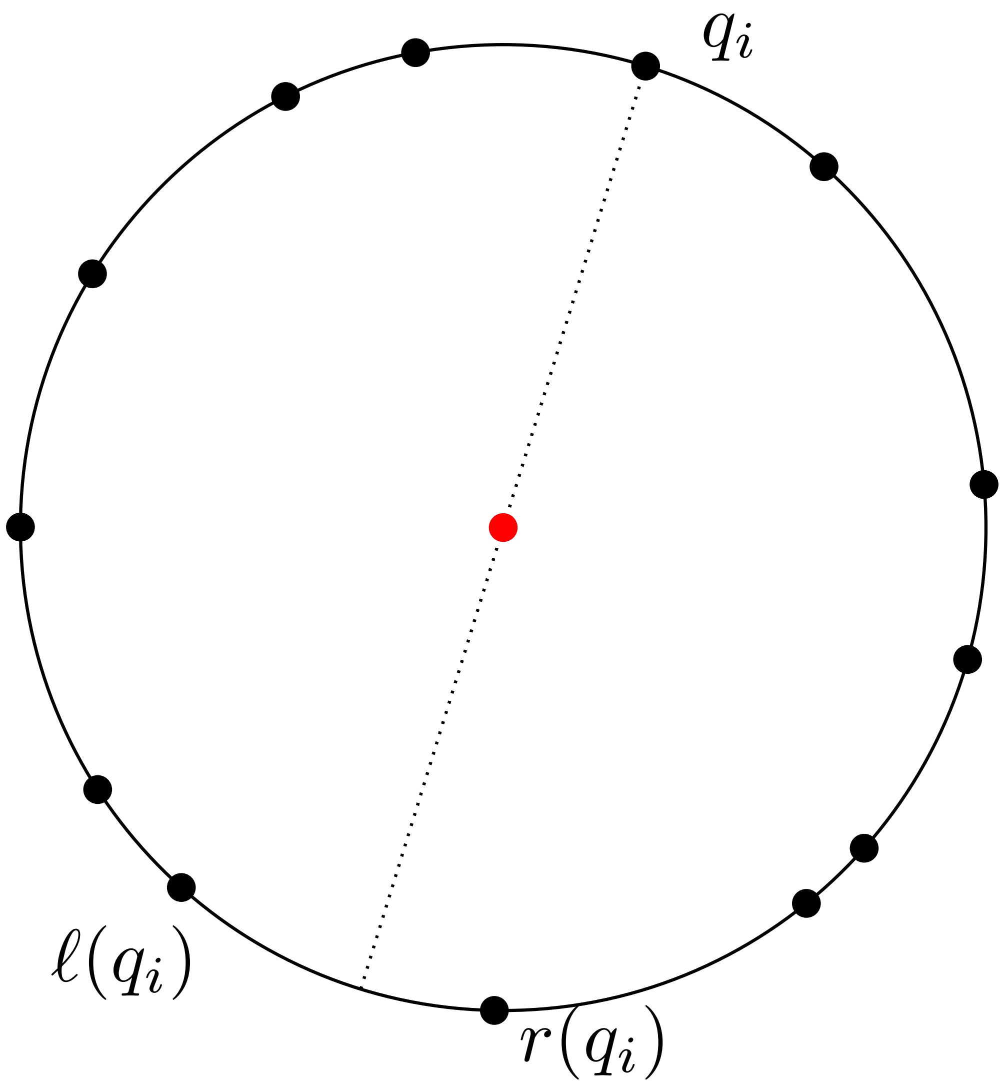

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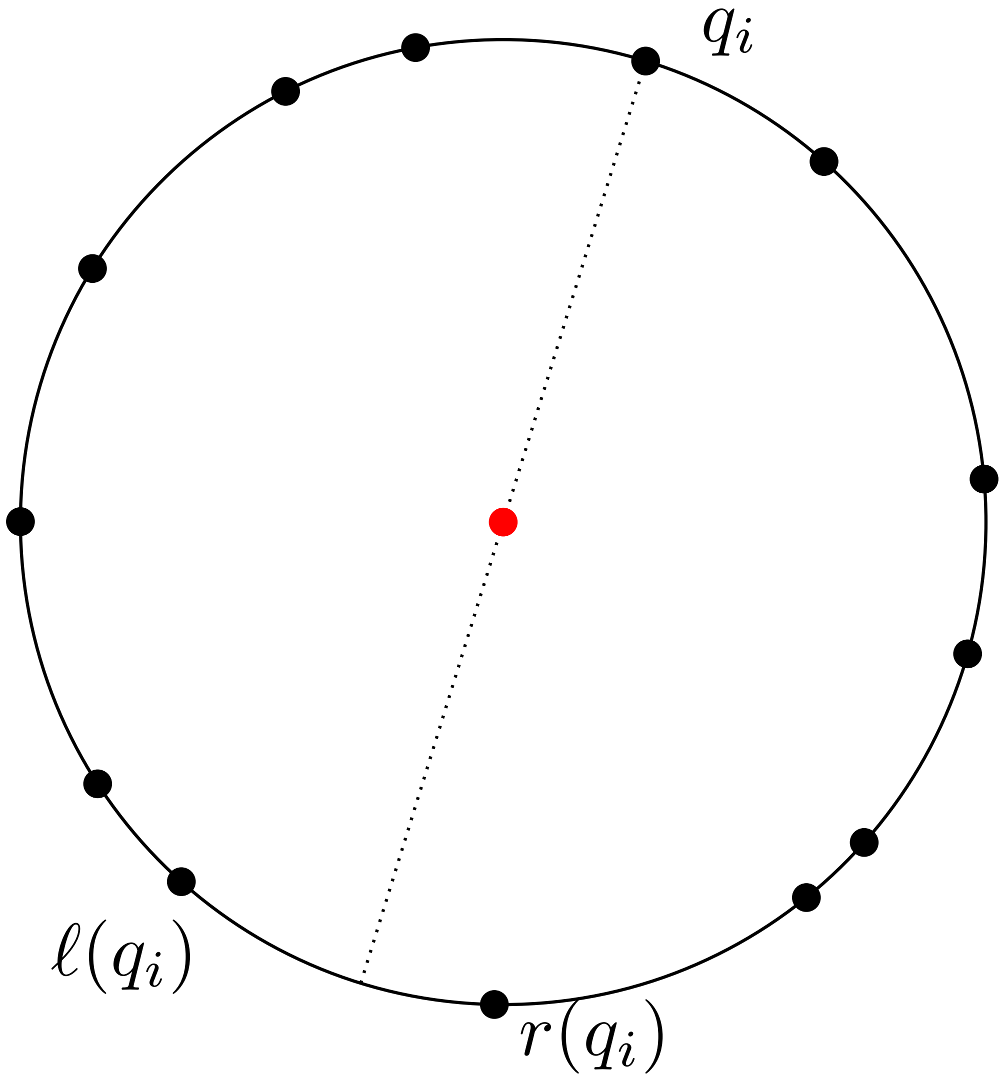
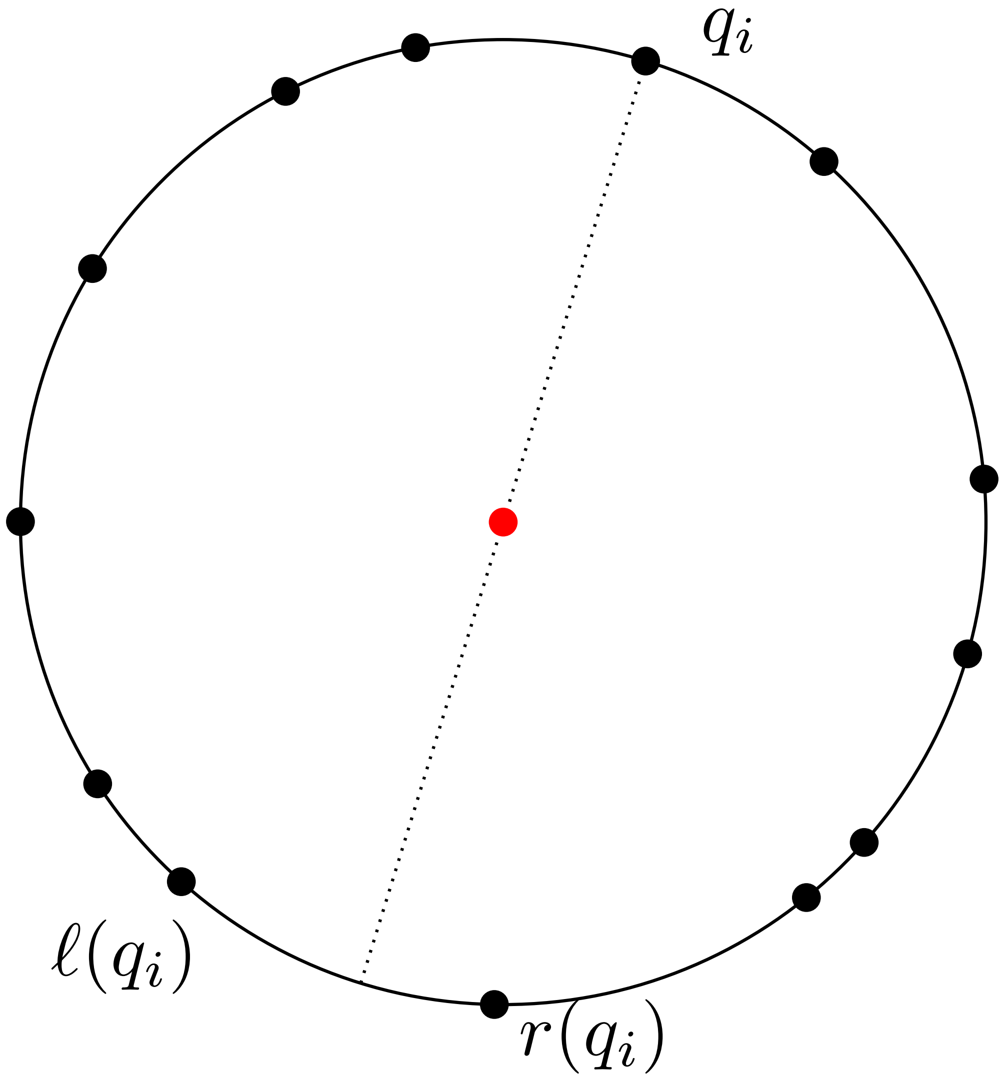


Preprocessing:

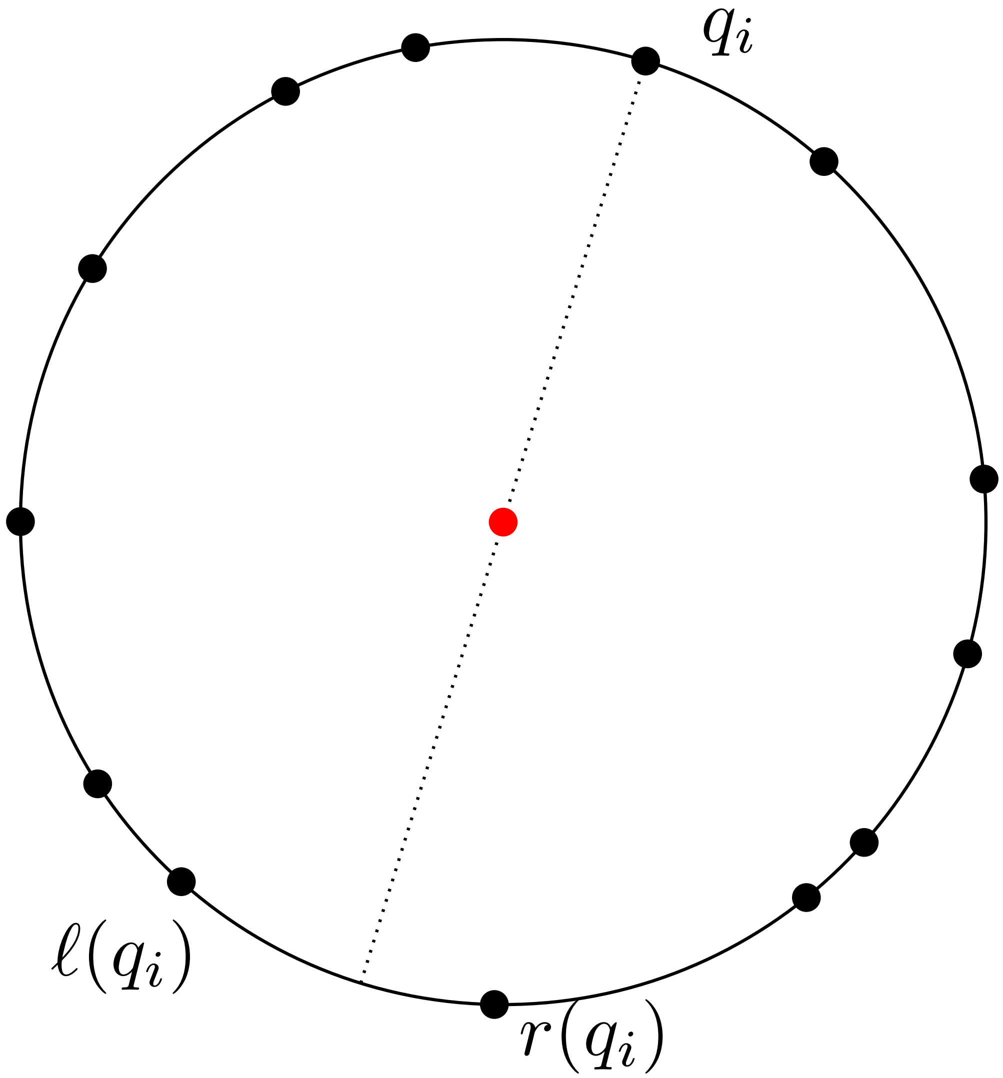
The algorithm in the plane



Preprocessing:

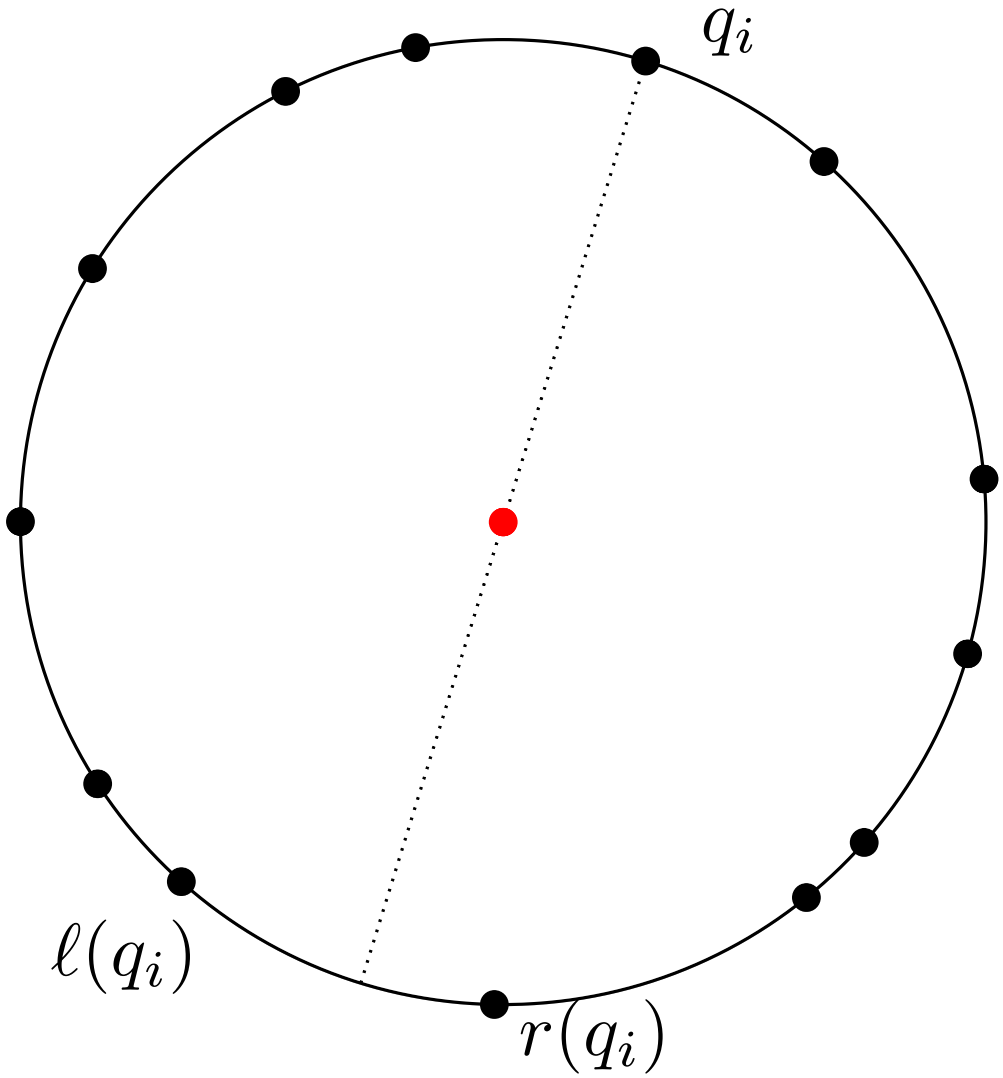
1. radial sort

The algorithm in the plane



Preprocessing:
1. radial sort
2. compute $\ell(q_i)$'s, $r(q_i)$'s

The algorithm in the plane

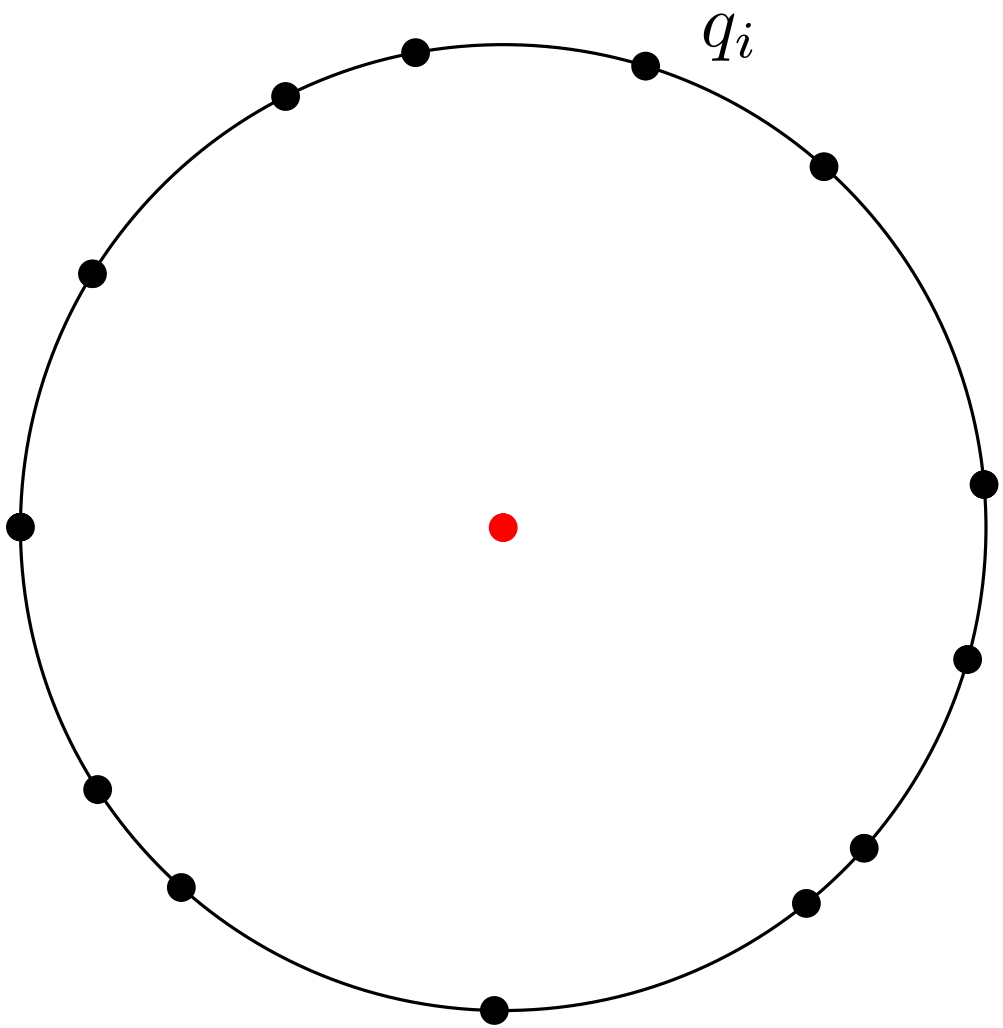


Preprocessing:

1. radial sort
2. compute $\ell(q_i)$'s, $r(q_i)$'s

time $O(n \log n)$

The algorithm in the plane



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Decide ED $\geq k$:

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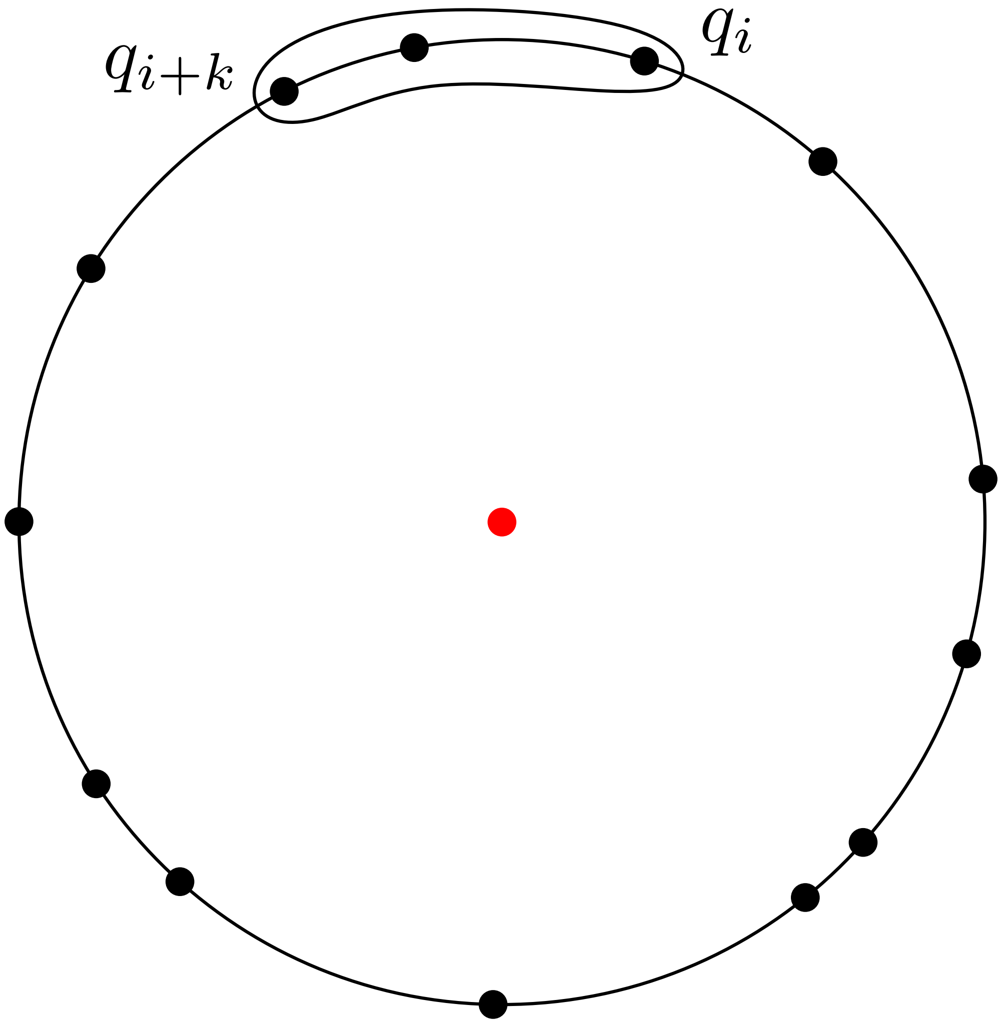

Preprocessing:

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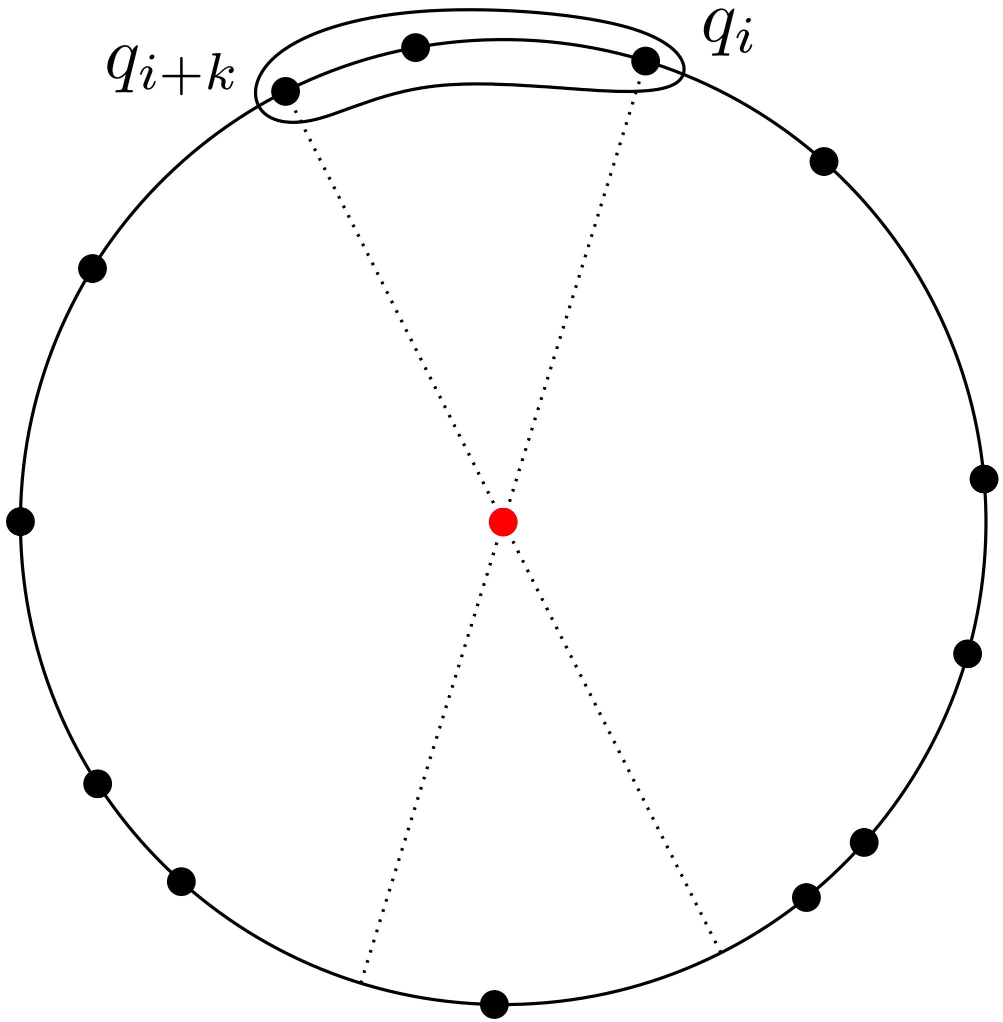
Decide ED $\geq k$:

for each $q_i, \ldots, q_{i+k}$:

The algorithm in the plane



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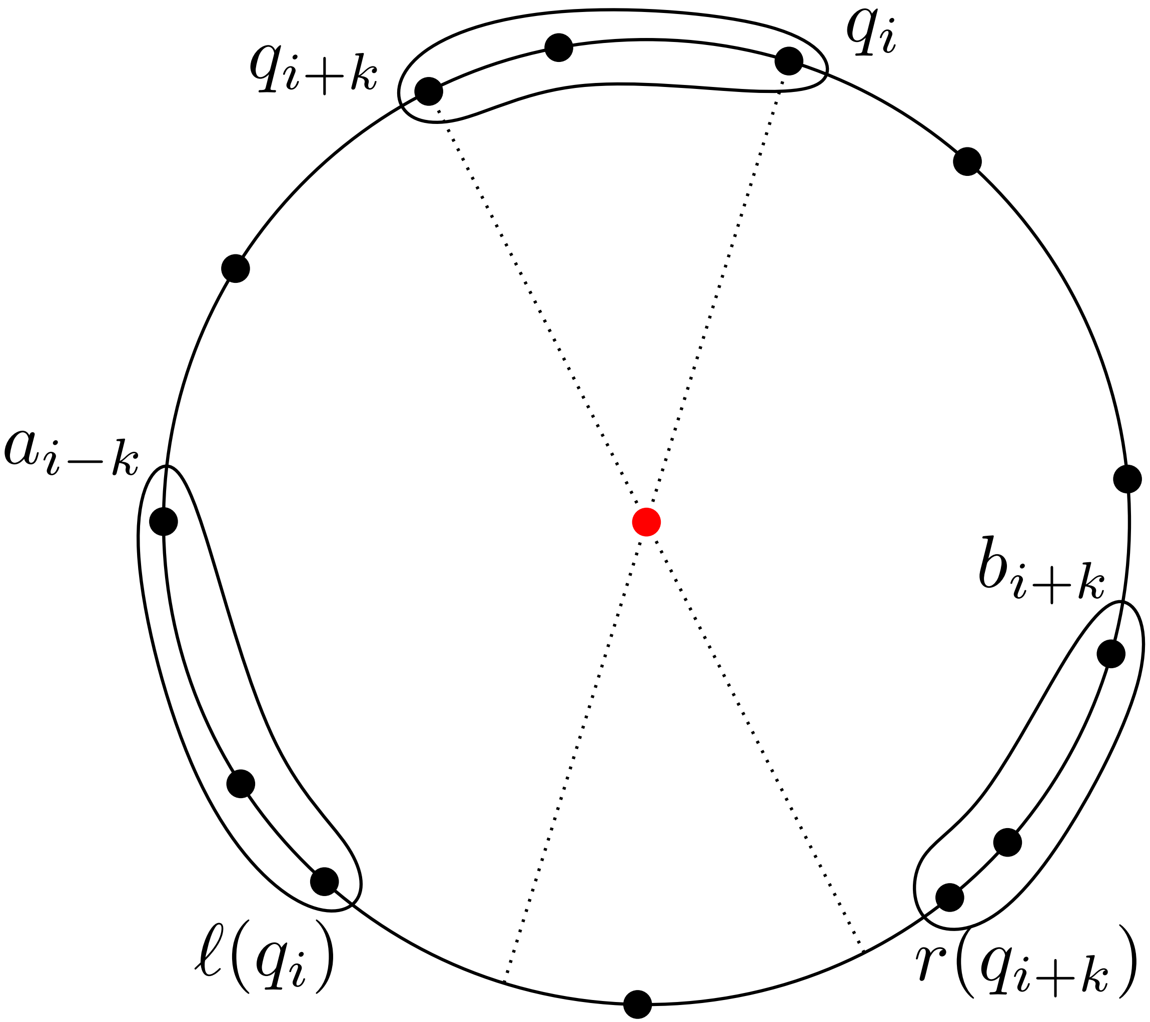

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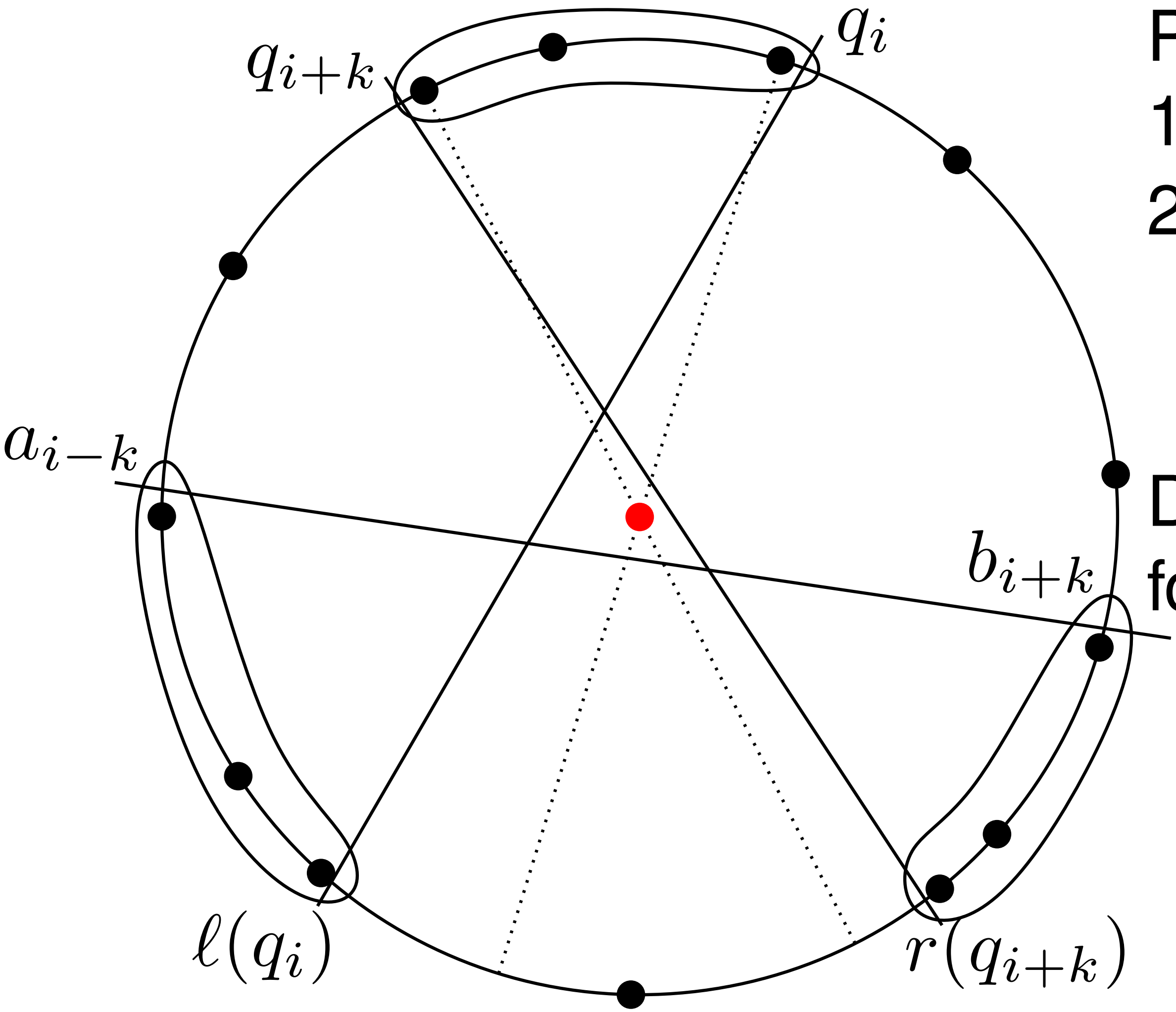

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for each $q_i, \ldots, q_{i+k}$:

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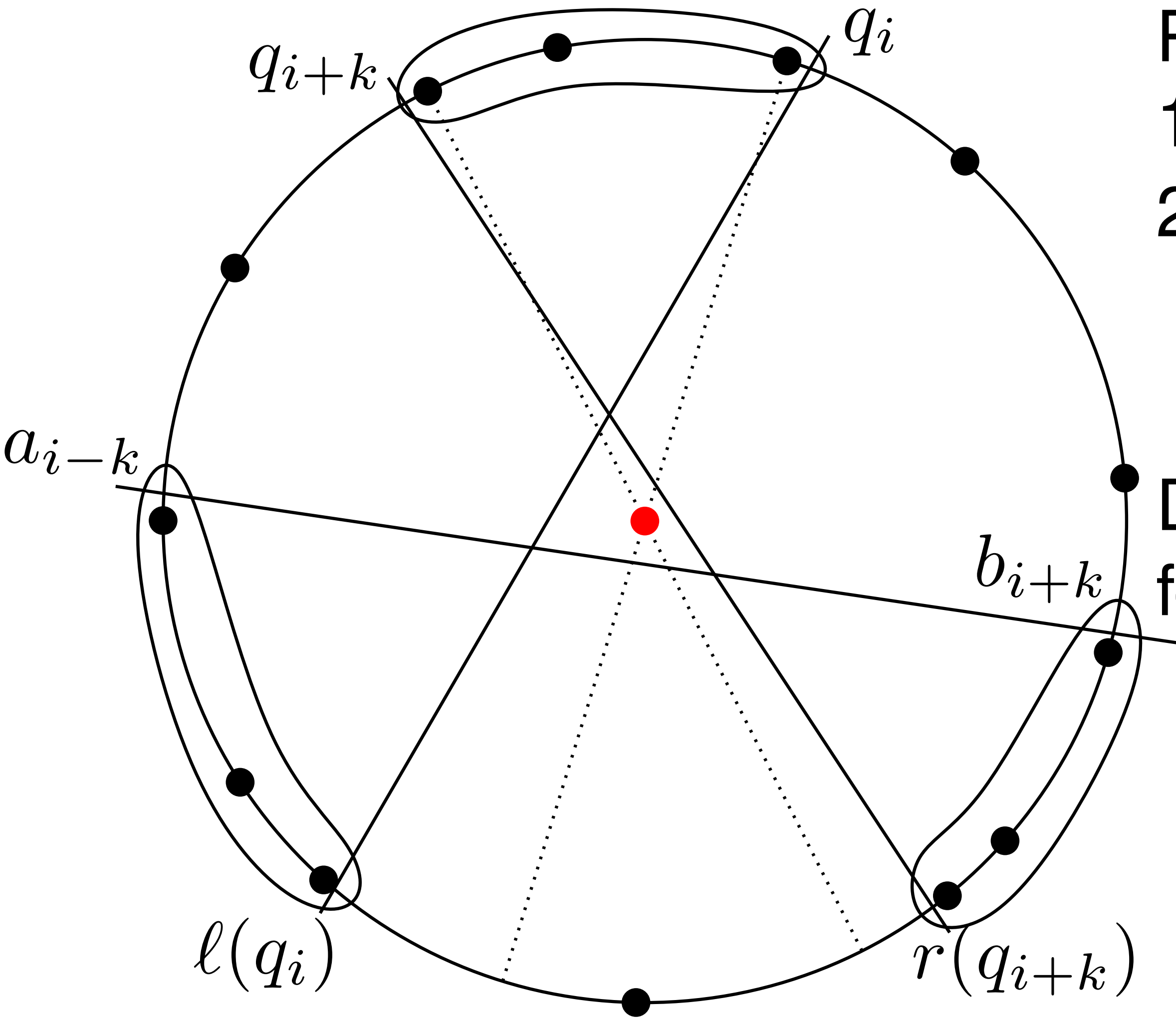
check if $\{q_i, q_{i+k}\}$,

$\{a_i, a_{i-k}\}$, $\{b_i, b_i + k\}$

enclose origin

The algorithm in the plane



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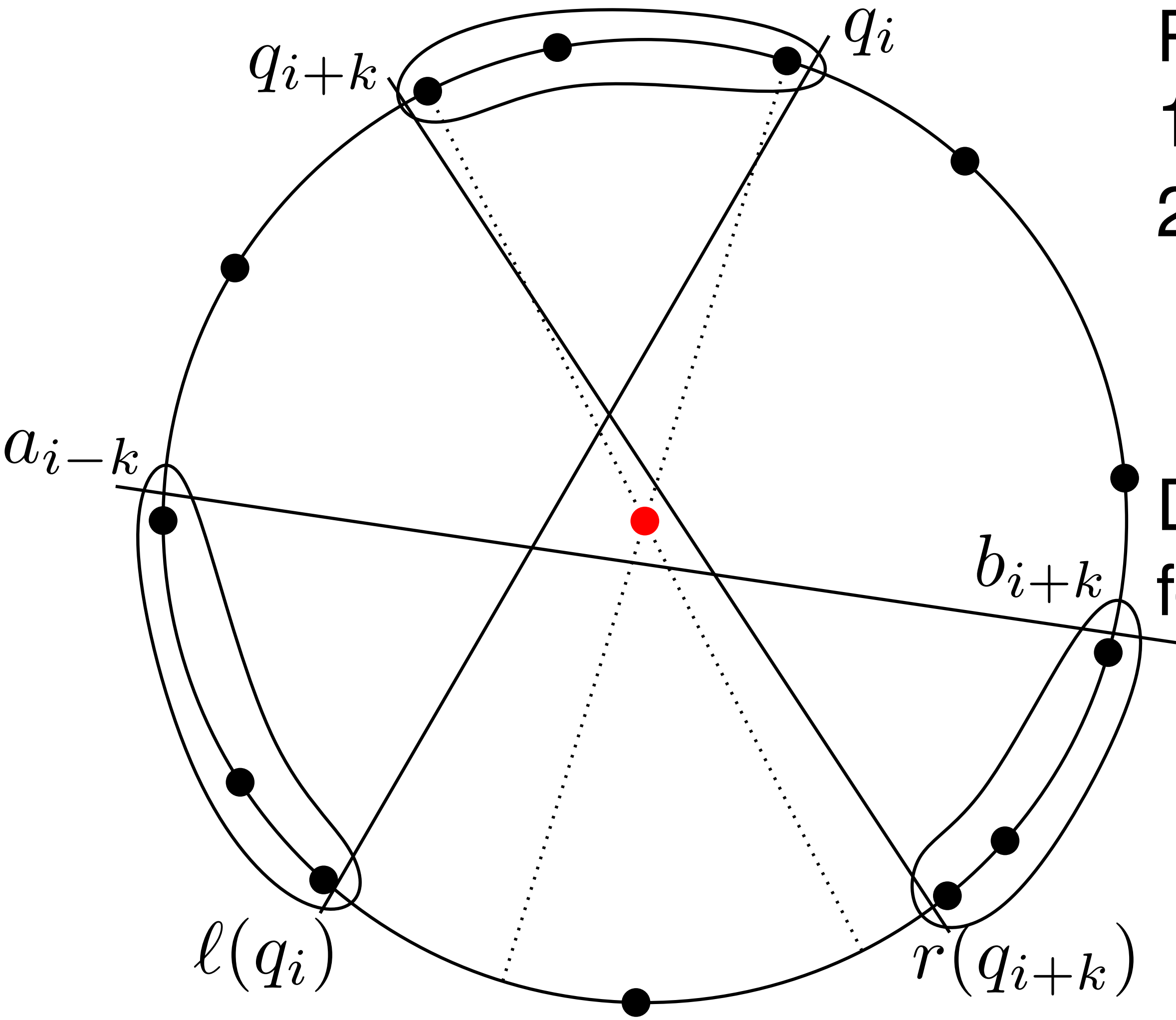
time $O(n)$

The algorithm in the plane



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time $O(n)$

binary search over k

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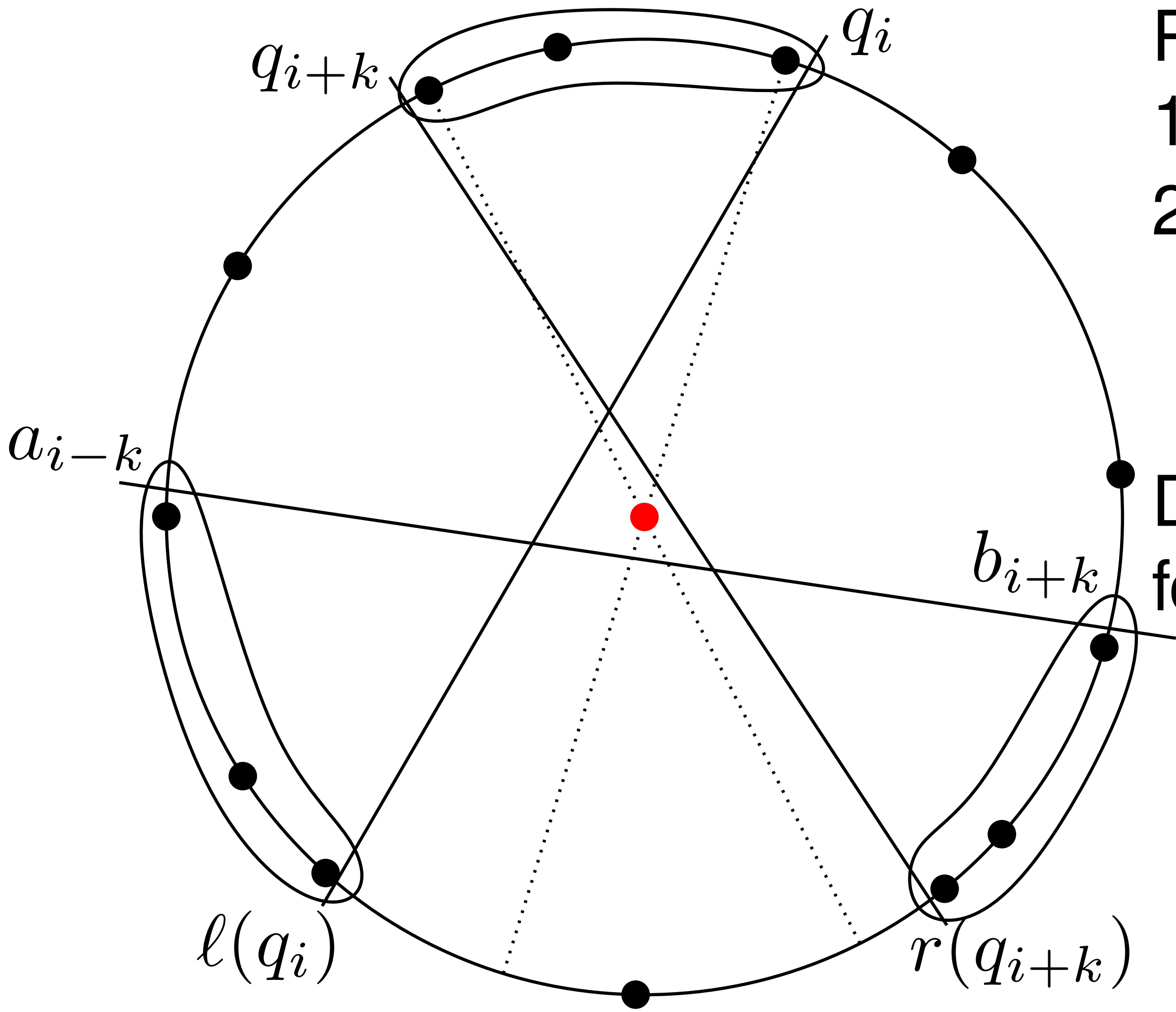

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time $O(n)$

binary search over k

Total runtime: $O(n \log n)$

Conclusion

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- $O(n \log n)$ in the plane
- $O(n^{d^2})$ in dimension d

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Thank you!

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Thank you!