Range Reporting for Time Series via Rectangle Stabbing

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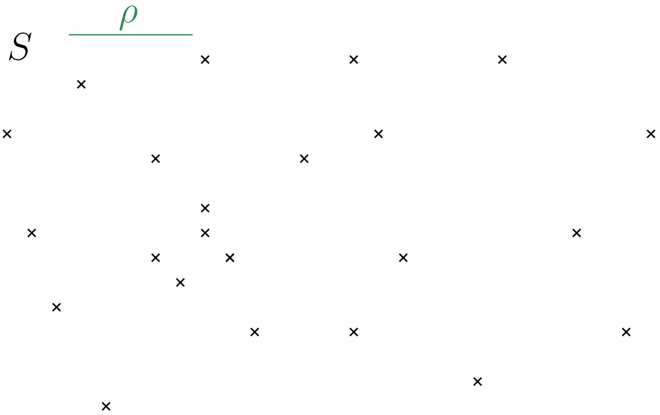
Range Reporting for Points



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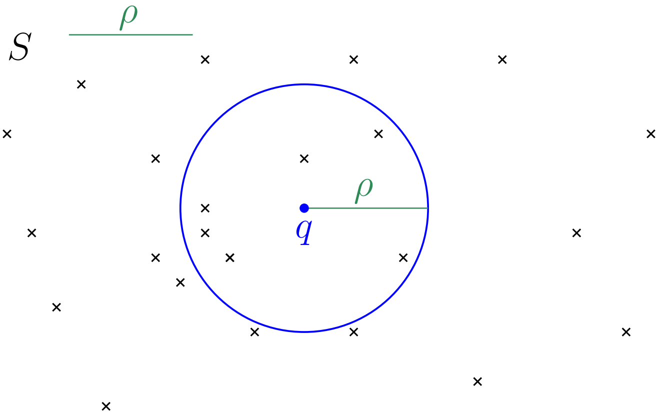

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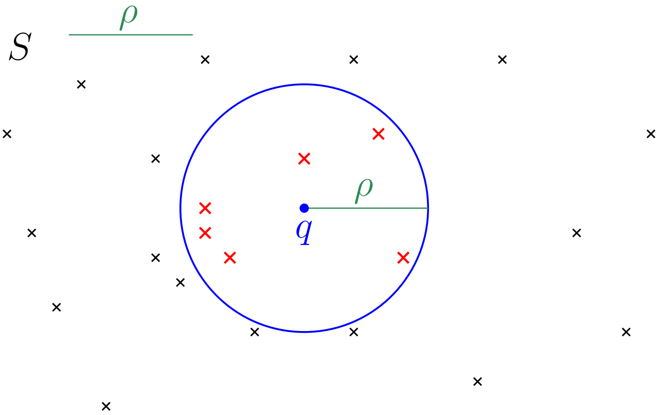
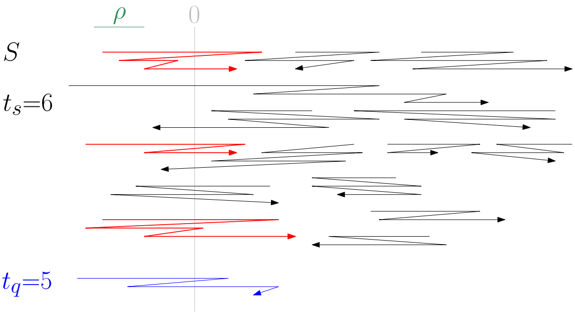


Problem (Fréchet Queries)

Given:
- set S of n time series all of complexity at most t_s,
- complexity of query time series t_q,
- distance parameter $\rho \in \mathbb{R}_{\geq 0}$

Query task: for time series q of complexity t_q
- Return $\forall s \in S$ with $d_F(s, q) \leq \rho$

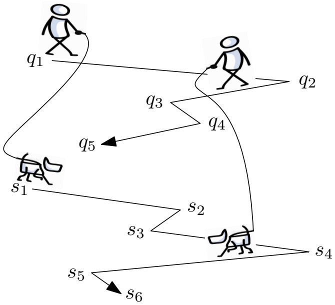


Definition: continuous Fréchet distance

Intuitive Definition:

Continuous Fréchet distance is **length of the shortest leash** such that person and dog traverse the paths

- From start to finish
- Can very speed
- Can't go backward



Rectangle Stabbing

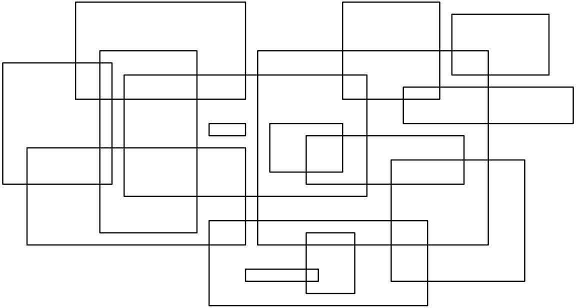
Problem (Rectangle Stabbing)

Given:

- *set S of n axis-aligned d-dimensional rectangles in $\mathbb{R}^d$*

Query task: for point $q \in \mathbb{R}^d$

- *Return $\forall R \in S$ with $q \in R$*



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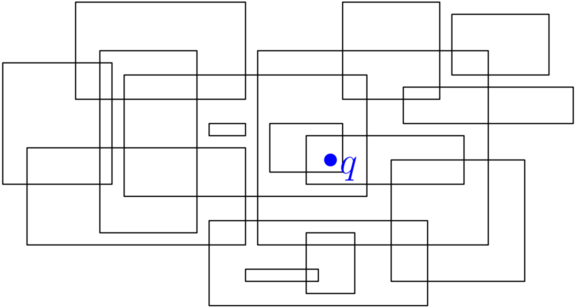

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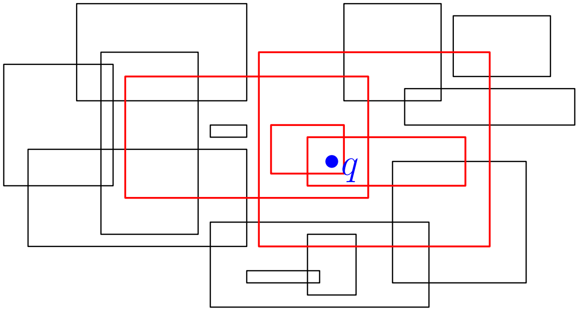
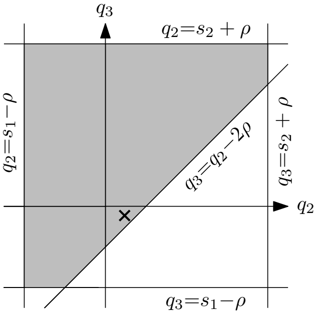
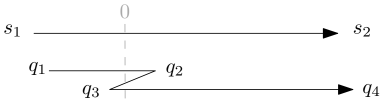


Example: Range of a Time Series s





$d_F(q,s) \leq \rho$ if and only if

- $q_1 \in [s_1-\rho, s_1+\rho]$ and $q_4 \in [s_2-\rho, s_2+\rho]$
- (q_2, q_3) in grey area

Results for the Fréchet Queries Problem

	d	Storage	Query Time
[Afshani, Driemel 18]	2	$\mathcal{O}\left(n(\log\log n)^{\mathcal{O}\left({t_s}^2\right)}\right)$	$\mathcal{O}\left(\sqrt{n}\log^{\mathcal{O}\left({t_s}^2\right)} n + k\right)$
[Cheng, Huang 23]	d	$\mathcal{O}\left(t_q t_s n\right)^{\mathcal{O}\left(d^4 {t_q}^2 \log(d t_q)\right)}$	$\mathcal{O}\left((d t_q)^{\mathcal{O}(1)} \log(t_q t_s n) + k\right)$
Our Results	1	$\mathcal{O}\left(n\left(\frac{\log n}{\log\log n}\right)^{t_s-1}\right)$	$\mathcal{O}\left(\log n\left(\frac{\log n}{\log\log n}\right)^{t_s-3} + k\right)$
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Lower bounds:[1]	Storage		Query Time
d=1	nh	$\Rightarrow$	$\Omega\left(\log n\left(\frac{\log n}{\log h}\right)^{\lfloor t_{\min}/2\rfloor-2} + k\right)$
d=1	$\Omega\left(n\left(\frac{\log n}{\log\log n}\right)^{\lfloor t_{\min}/2\rfloor-1}\right)$	$\Leftarrow$	$\mathcal{O}(\log^c n + k)$

[1] in pointer machine model

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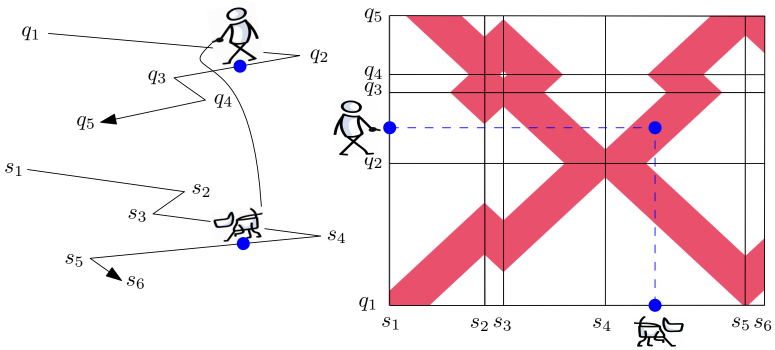
Free Space Diagram $F_\rho(q, s)$



$$F_\rho(q, s) = \{(x, y) \in [1, t_s] \times [1, t_q] \mid |q(x) - s(y)| \leq \rho\}$$

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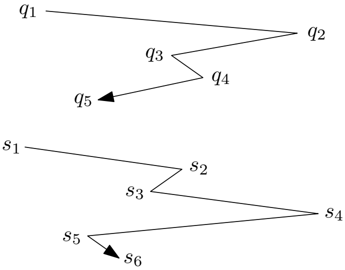

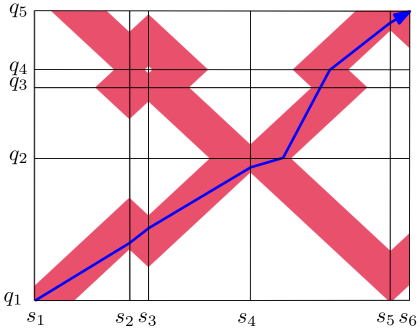

- $d_F(q, s) \leq \rho \Leftrightarrow \exists$ a path monotone in both coordinates from $(1, 1)$ to (t_s, t_q) in the free space.
- Such a path is called **feasible path**.

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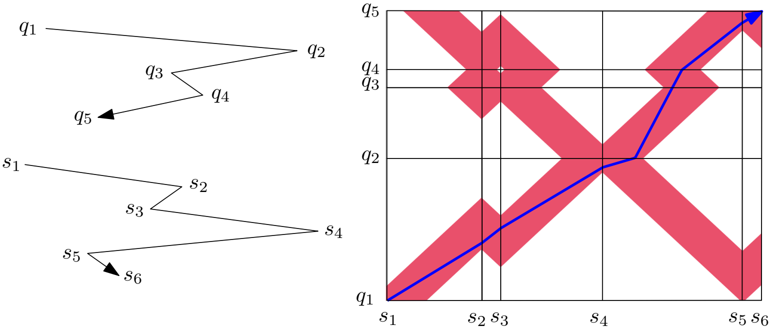

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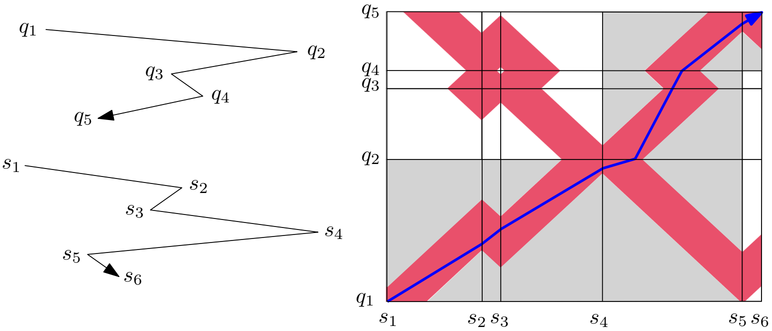

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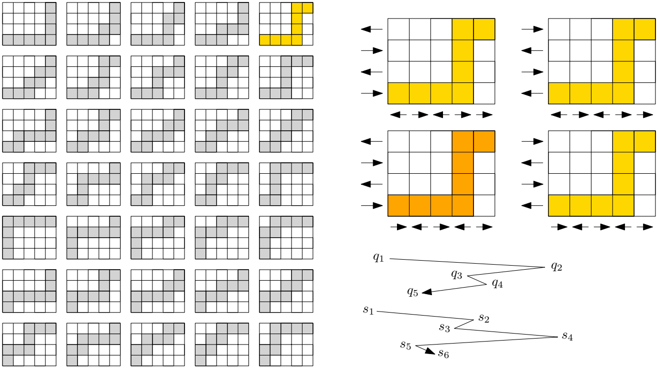
The Data Structure

For every (valid) sequence of cells: Store 4 rectangle stabbing data structures



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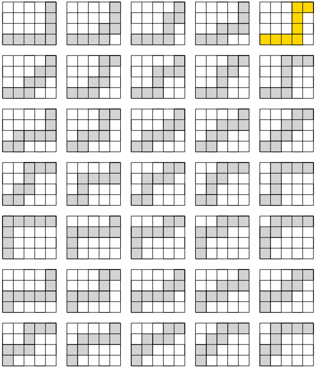
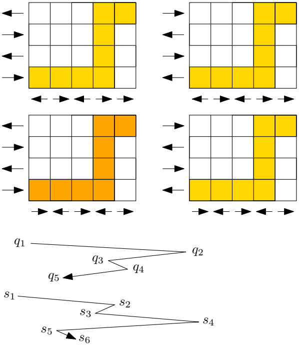


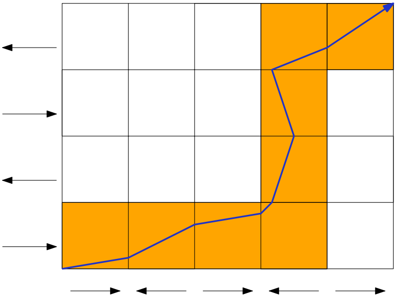


In every such data structure: Store at most one t_q-dim. rectangle per time series in S.

Defining the Intervals

Lemma

Given: sequence of cells $\mathscr{C}$*, time series* s*, distance* ρ*, edge directions of* q
Then: Exists intervals $I_1, \ldots, I_{t_q}$ *such that* $(q_1, \ldots, q_{t_q}) \in I_1 \times \ldots \times I_{t_q} \Leftrightarrow \exists$ *feasible path traversing* $\mathscr{C}$ *except for monotonicity x-coordinate*



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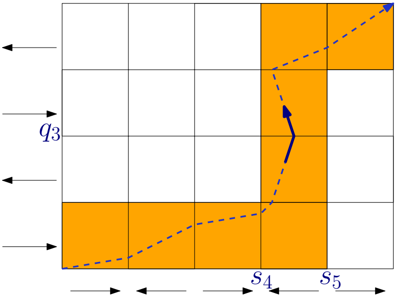
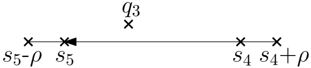
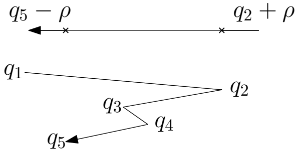


For example,
$q_3 \in [s_5 - \rho, s_4 + \rho]$.



Key Concept: Backward and Forward Numbers



Here, $b_2(q) = 5$.

Backward number $b_i(q)$: highest index $k \in \{i, \ldots, t_q\}$ s.t.

- $d_F(\langle q_i, \ldots, q_k \rangle, \longleftarrow) \leq \rho$

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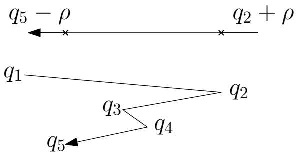

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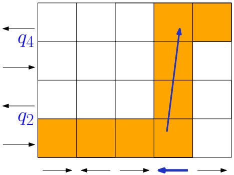
Backward number $b_i(\mathscr{C})$: highest index $k \in \{i, \ldots, t_q\}$ s.t.

- $(i-1, j), \ldots, (k, j) \in \mathscr{C}$
- $\overline{s_j s_{j+1}}$ oriented backward



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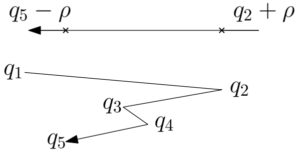

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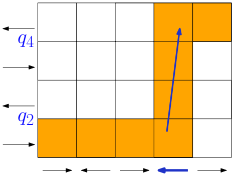

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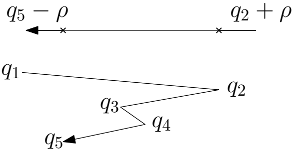
Similarly, define forward numbers $f_i(q)$ and $f_i(\mathscr{C})$.



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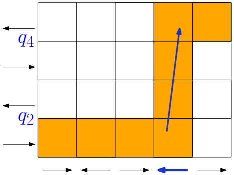

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Lemma (Equivalent statements)

- *$\exists$ feasible path in $F_\rho(q, s)$ traversing exactly cells in $\mathscr{C}$*
- *$b_i(q) \geq b_i(\mathscr{C})$, $f_i(q) \geq f_i(\mathscr{C})$ and $q_i \in I_i$ for all i*

The Query Algorithm

The Query Algorithm.

Compute $f_1(q), \ldots, f_{t_q}(q), b_1(q), \ldots, b_{t_q}(q)$

for *all valid sequences of cells* $\mathscr{C}$ **do**

if $f_i(\mathscr{C}) \leq f_i(q)$ *and* $b_i(\mathscr{C}) \leq b_i(q)$ *for all* i **then**

Query search in associated rectangle stabbing data structure with $(q_1, q_2, \ldots, q_{t_q})$

Output all time series associated with rectangle containing this point

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Result

Theorem

For constant t_s and t_q, there exists a data structure solving the Fréchet Queries problem

- *of size in $\mathcal{O}(n \log^{t_q - 2} n)$ and*
- *query time in $\mathcal{O}(\log^{t_q - 1} n + k)$.*

Proof.

- Correctness by discussion before
- Rectangle stabbing data structure by Chazelle (1986) with size in $\mathcal{O}(n \log^{d-2} n)$ and query time in $\mathcal{O}(\log^{d-1} n + k)$

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Thank you for your attention!
Questions?

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Lower bounds:[1]	Storage		Query Time
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[1] in pointer machine model