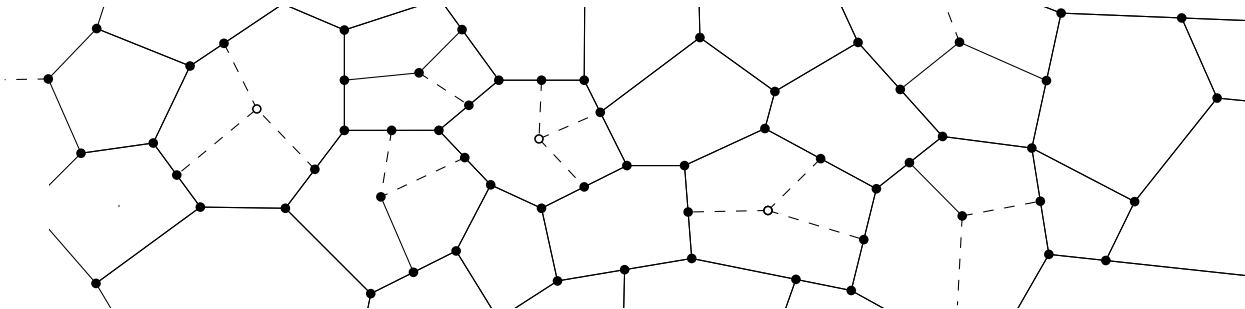


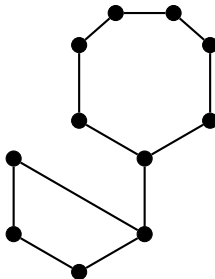
Recognition Complexity of Subgraphs of k -Connected Planar Cubic Graphs

Miriam Goetze, Paul Jungeblut, Torsten Ueckerdt | 15.03.2024



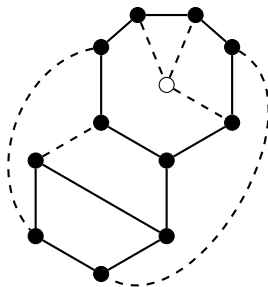
Problem: 3-AUGMENTATION

- **Input:** planar graph G with $\Delta(G) \leq 3$
- **Question:** \exists planar, cubic supergraph H ?



Problem: 3-AUGMENTATION

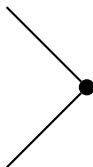
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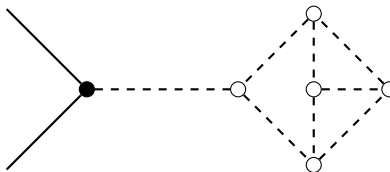
Observation



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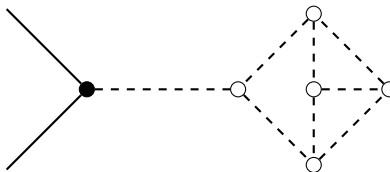
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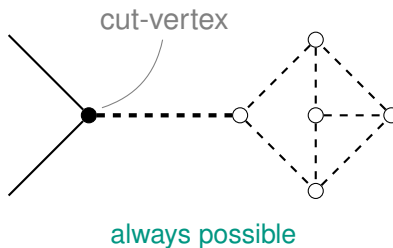


always possible

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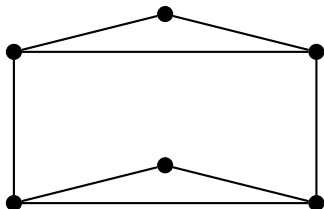
Problem: 2-CONN. 3-AUGMENTATION

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- **Question:** \exists planar, cubic, **2-connected** supergraph H ?

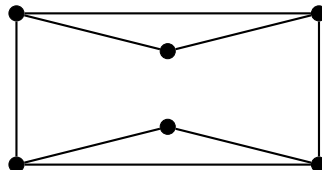
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Fixed Embedding



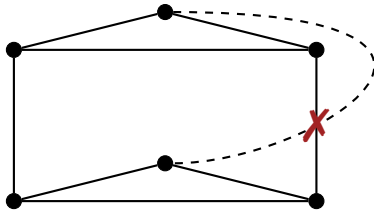
Variable Embedding



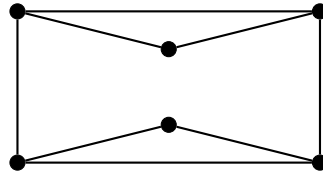
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Fixed Embedding



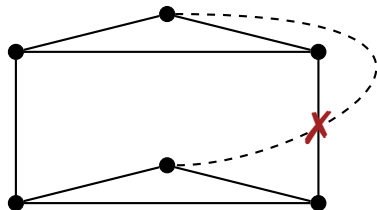
Variable Embedding



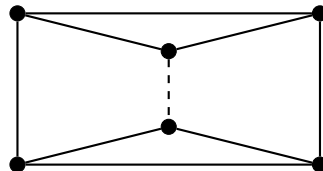
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Fixed Embedding

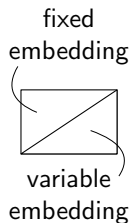


















Variable Embedding



Our Results

- **Input:** planar graph ℓ -connected graph G with $\Delta(G) \leq 3$
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| | | output H connectivity | | | |
|------------------------|--------|---|---|--|---|
| | | any | con. | 2-con. | 3-con. |
| input G connectivity | any |  |  |  |  |
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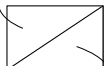
■ ESA 2022

■ EuroCG 2024

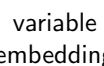
Our Results

















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fixed embedding




variable embedding



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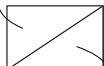
 ESA 2022

 EuroCG 2024

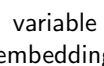
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














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 ESA 2022

 EuroCG 2024

Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

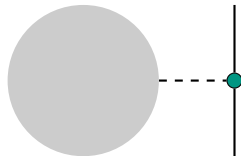
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

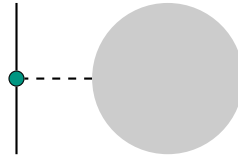
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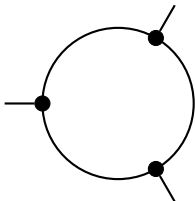
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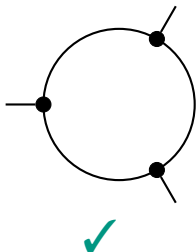
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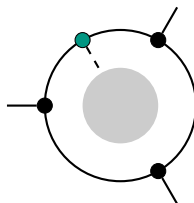
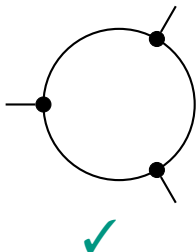
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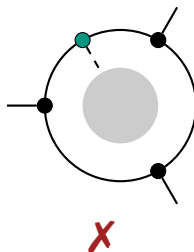
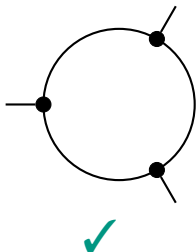
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

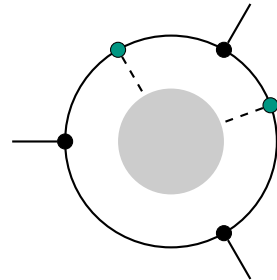
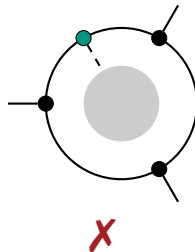
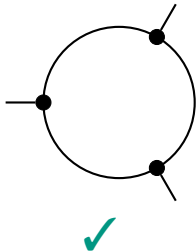
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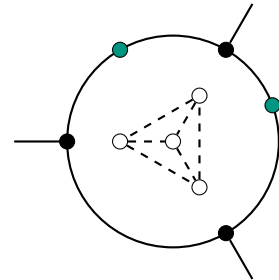
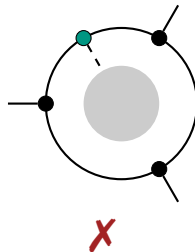
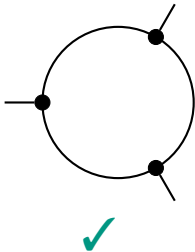
- **Input:** embedded planar **2-conn.** graph G with $\Delta(G) \leq 3$
- **Question:** \exists planar, cubic, 2-conn. supergraph H ?



Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

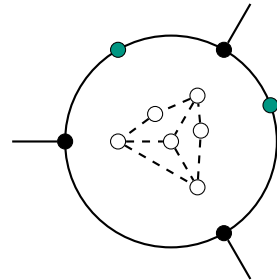
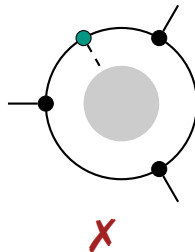
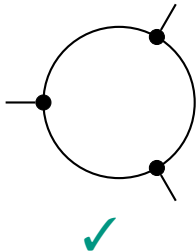
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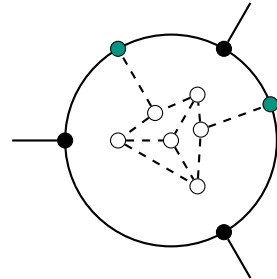
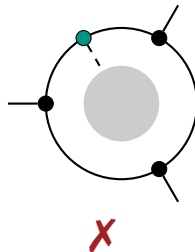
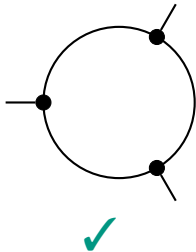
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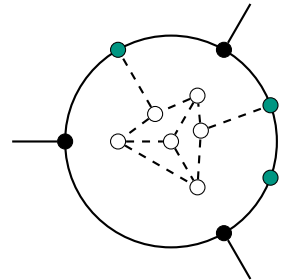
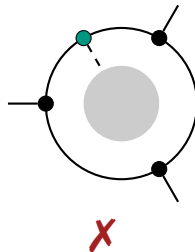
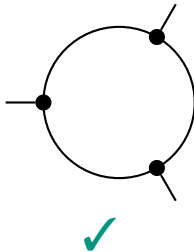
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

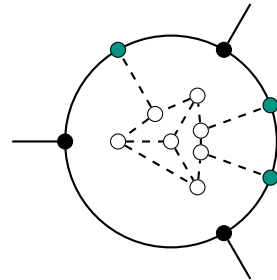
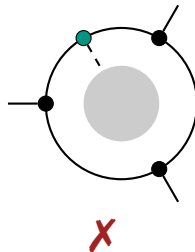
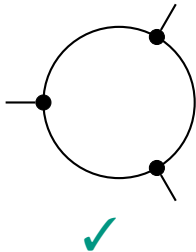
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

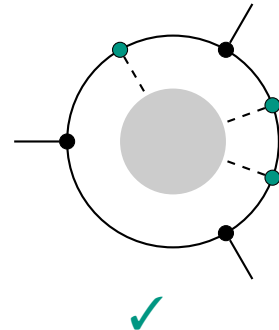
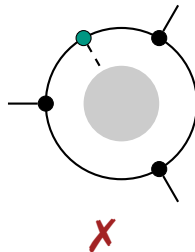
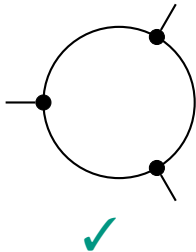
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

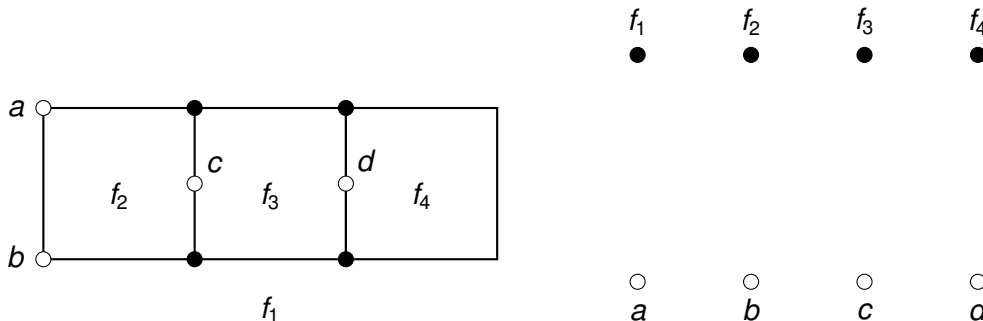
- **Input:** embedded planar **2-conn.** graph G with $\Delta(G) \leq 3$
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

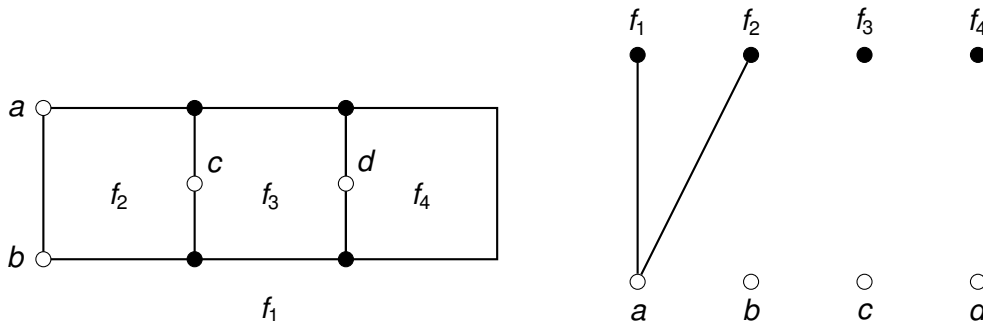
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

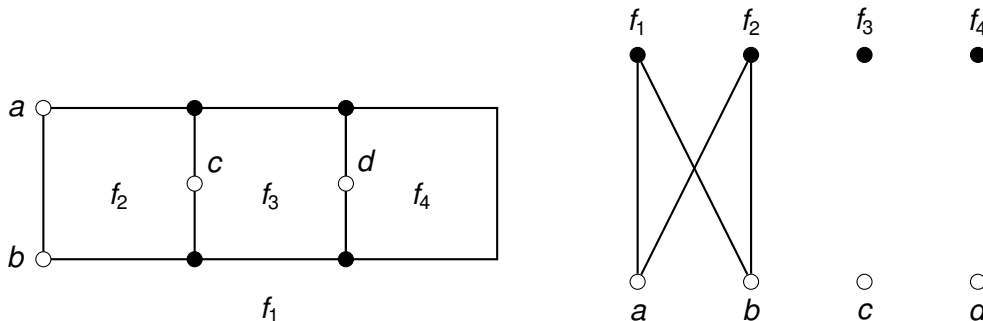
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

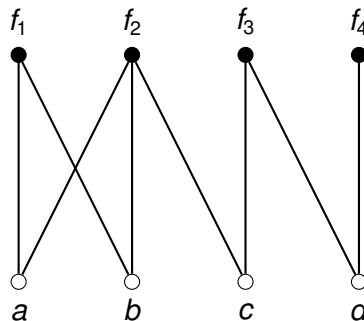
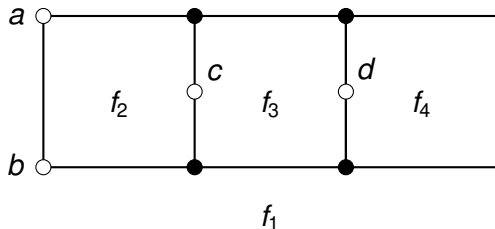
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

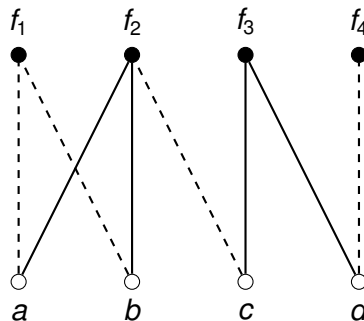
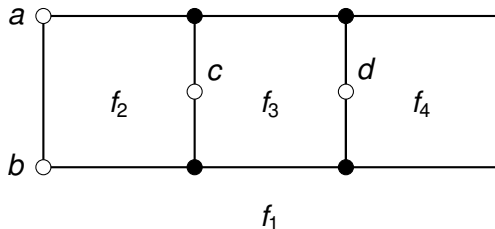
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

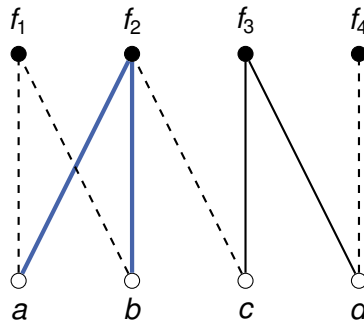
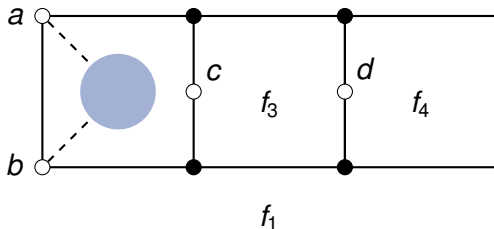
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 $O(|V(G)|^2)$


Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

- **Input:** embedded planar **2-conn.** graph G with $\Delta(G) \leq 3$
- **Question:** \exists planar, cubic, 2-conn. supergraph H ?

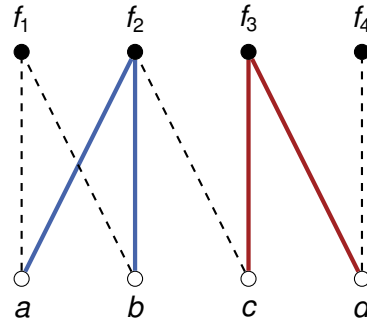
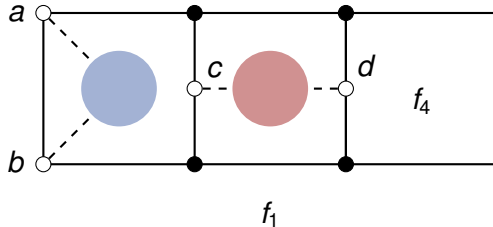
 $O(|V(G)|^2)$


Proof Idea

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- **Input:** embedded planar **2-conn.** graph G with $\Delta(G) \leq 3$
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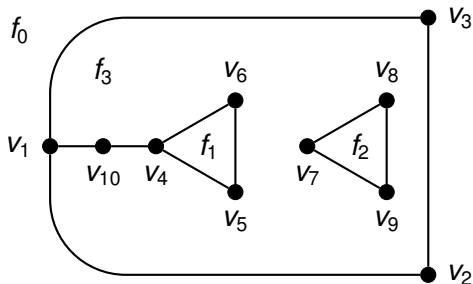
$$O(|V(G)|^2)$$



Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

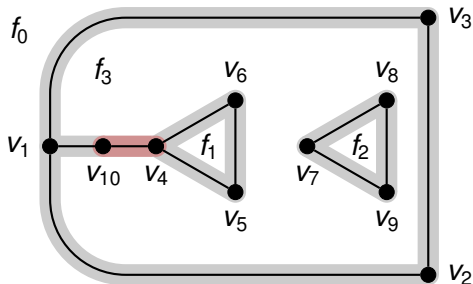
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

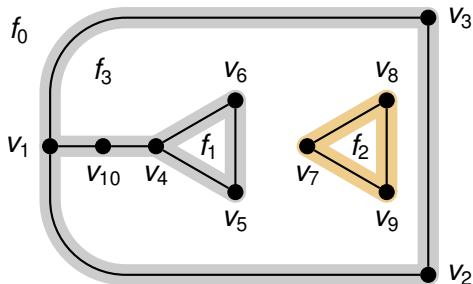
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Proof Idea

Problem: fixed 2-conn. 3-AUGMENTATION

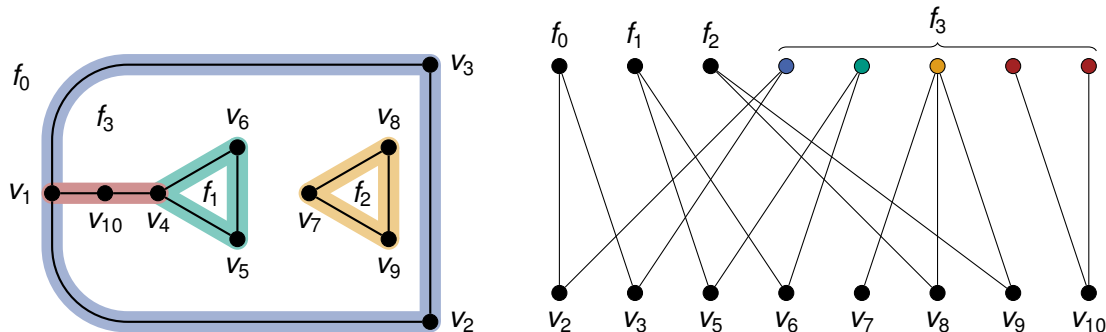
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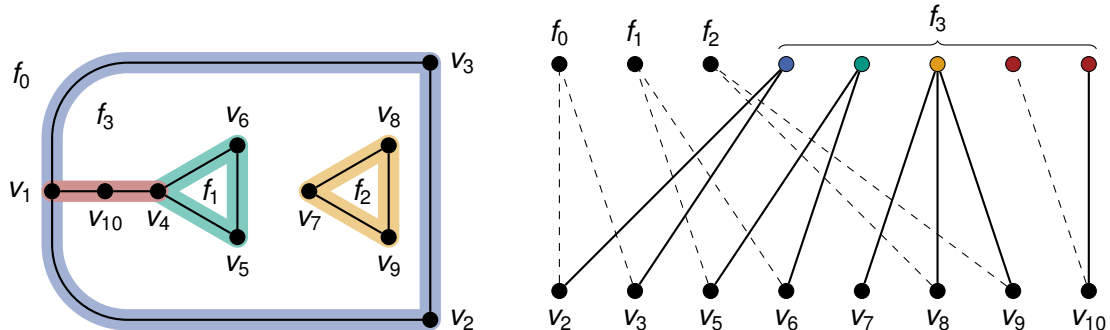
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Proof Idea

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- **Question:** \exists planar, cubic, 2-conn. supergraph H ?

 $O(|V(G)|^4)$


Open Questions

Problem: 3-AUGMENTATION

| | | output H connectivity | | | |
|------------------------|--------|-------------------------|-------|--------|---------|
| | | any | con. | 2-con. | 3-con. |
| input G connectivity | any | ✓ / ✓ | P / P | P / P | ? / NPC |
| | con. | ✓ / ✓ | ✓ / ✓ | P / P | ? / NPC |
| | 2-con. | ✓ / ✓ | ✓ / ✓ | P / P | ? / ? |
| | 3-con. | ✓ / ✓ | ✓ / ✓ | ✓ / ✓ | ✓ / ✓ |

Problem: r -AUGMENTATION

- **Input:** planar graph G with $\Delta(G) \leq r$
- **Question:** \exists planar, r -regular supergraph H ?

Values: $r \in \{4, 5\}$