Constrained One-Sided Boundary Labeling

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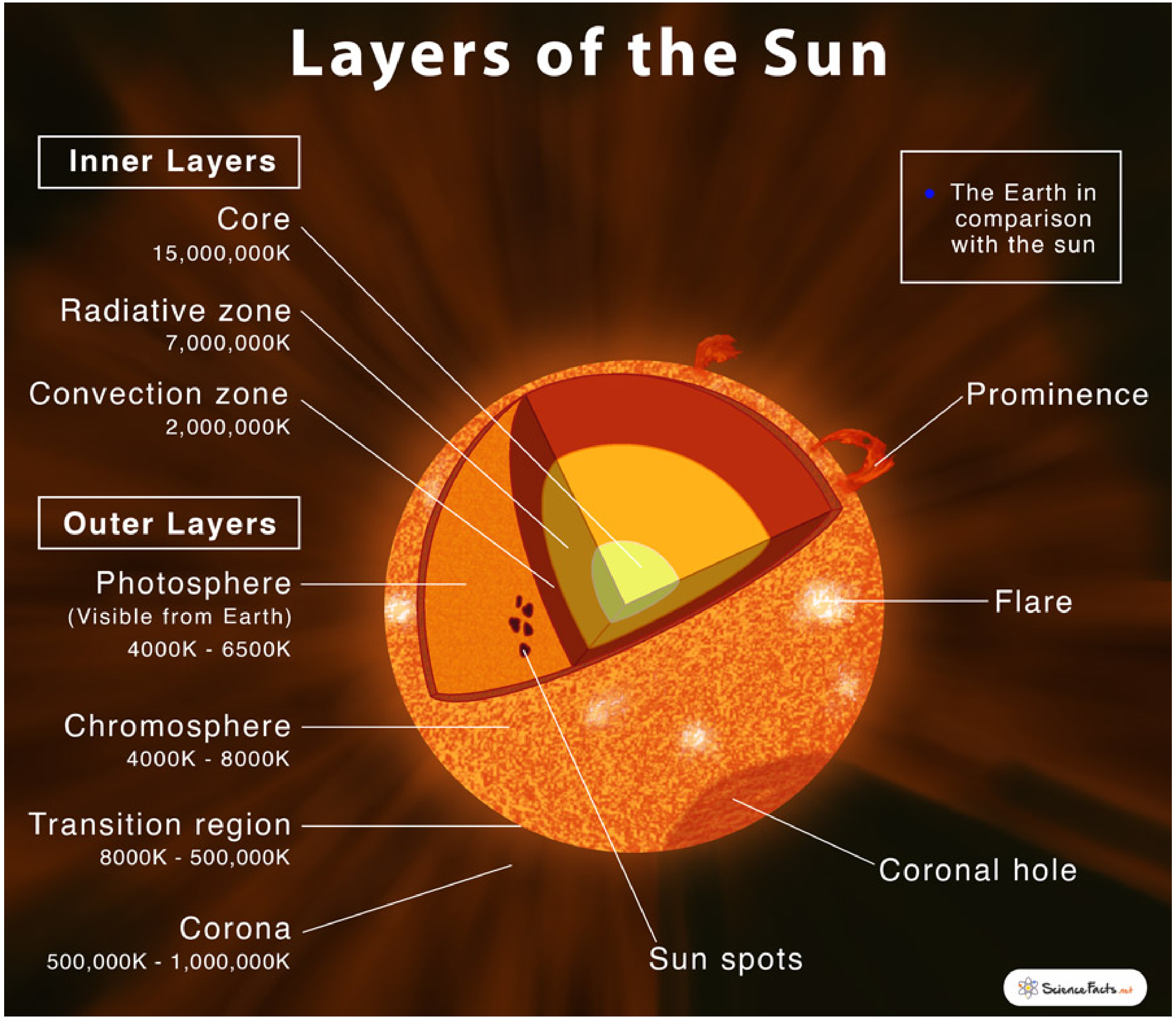
13 – 15th March · EuroCG 2024

TU WIEN
ac ALGORITHMS AND COMPLEXITY GROUP
TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

External Labeling "In-the-Wild"



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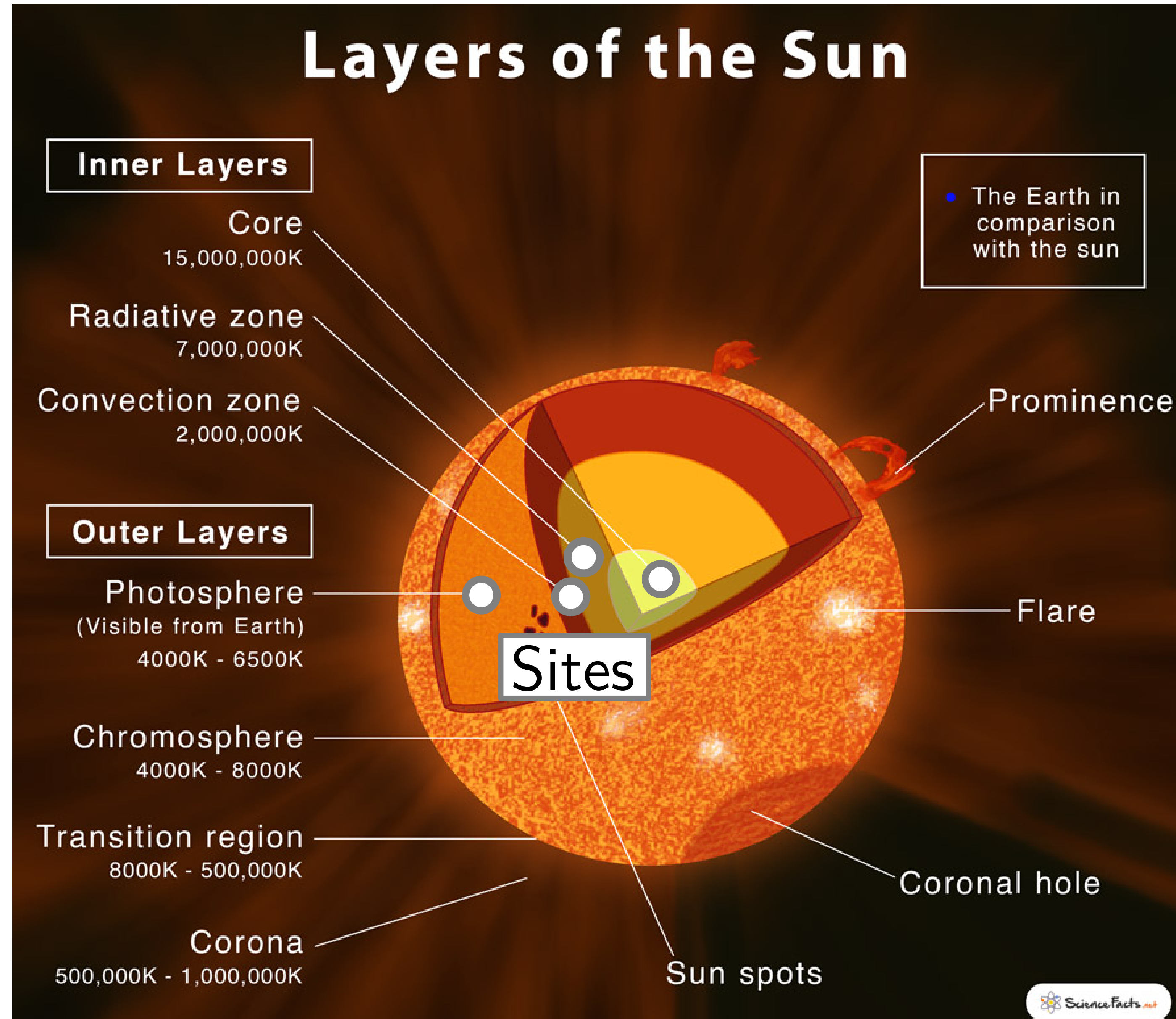

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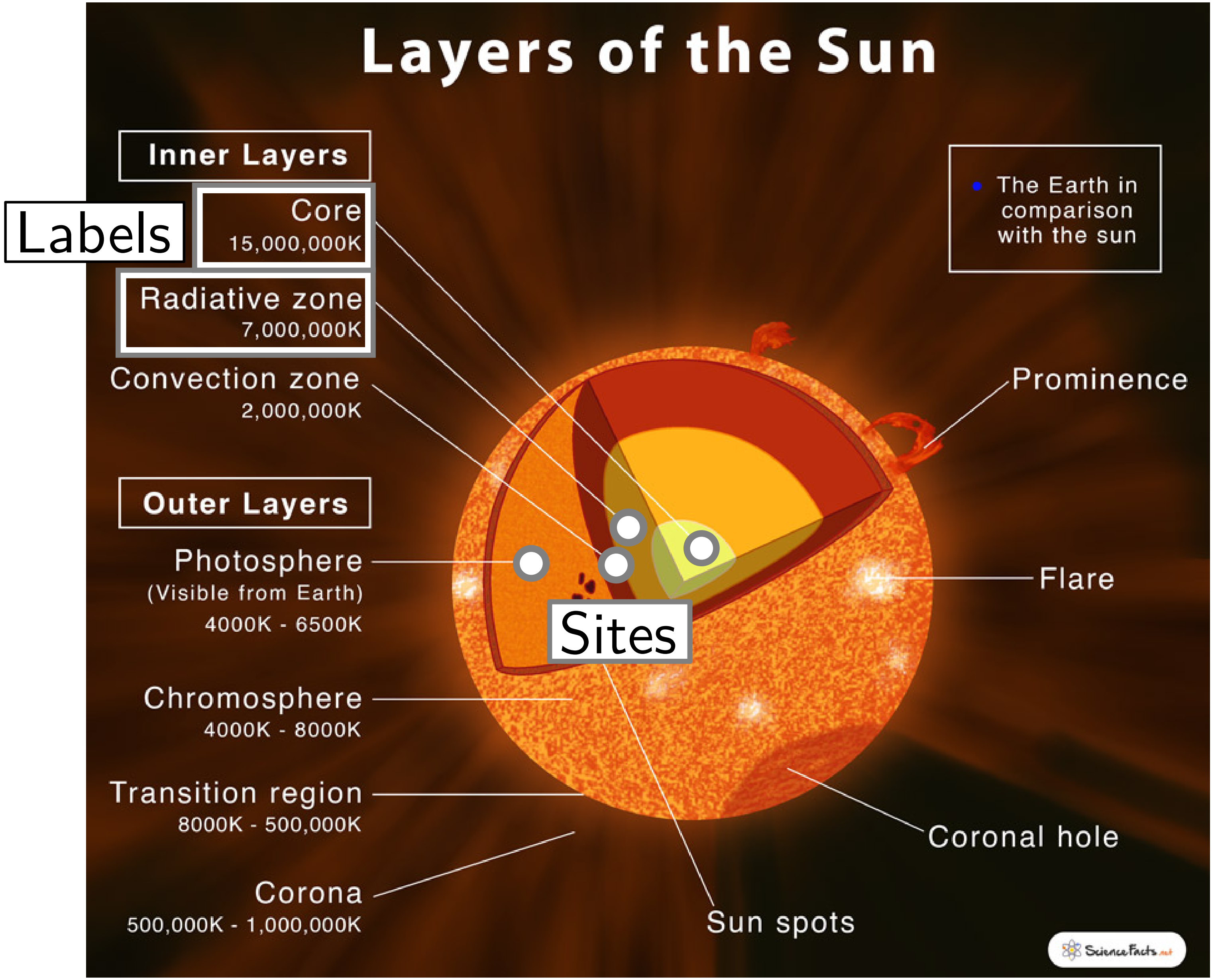

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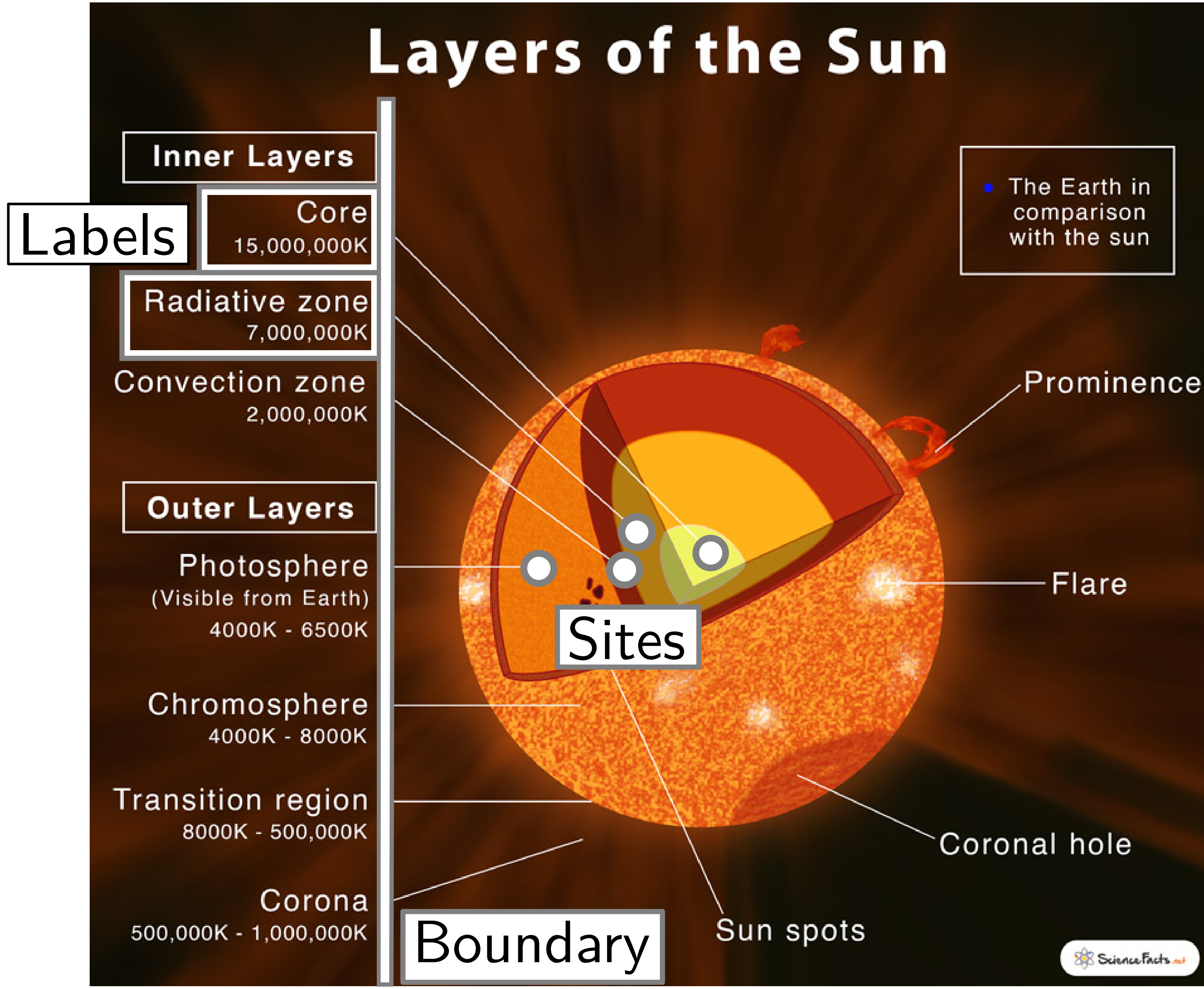

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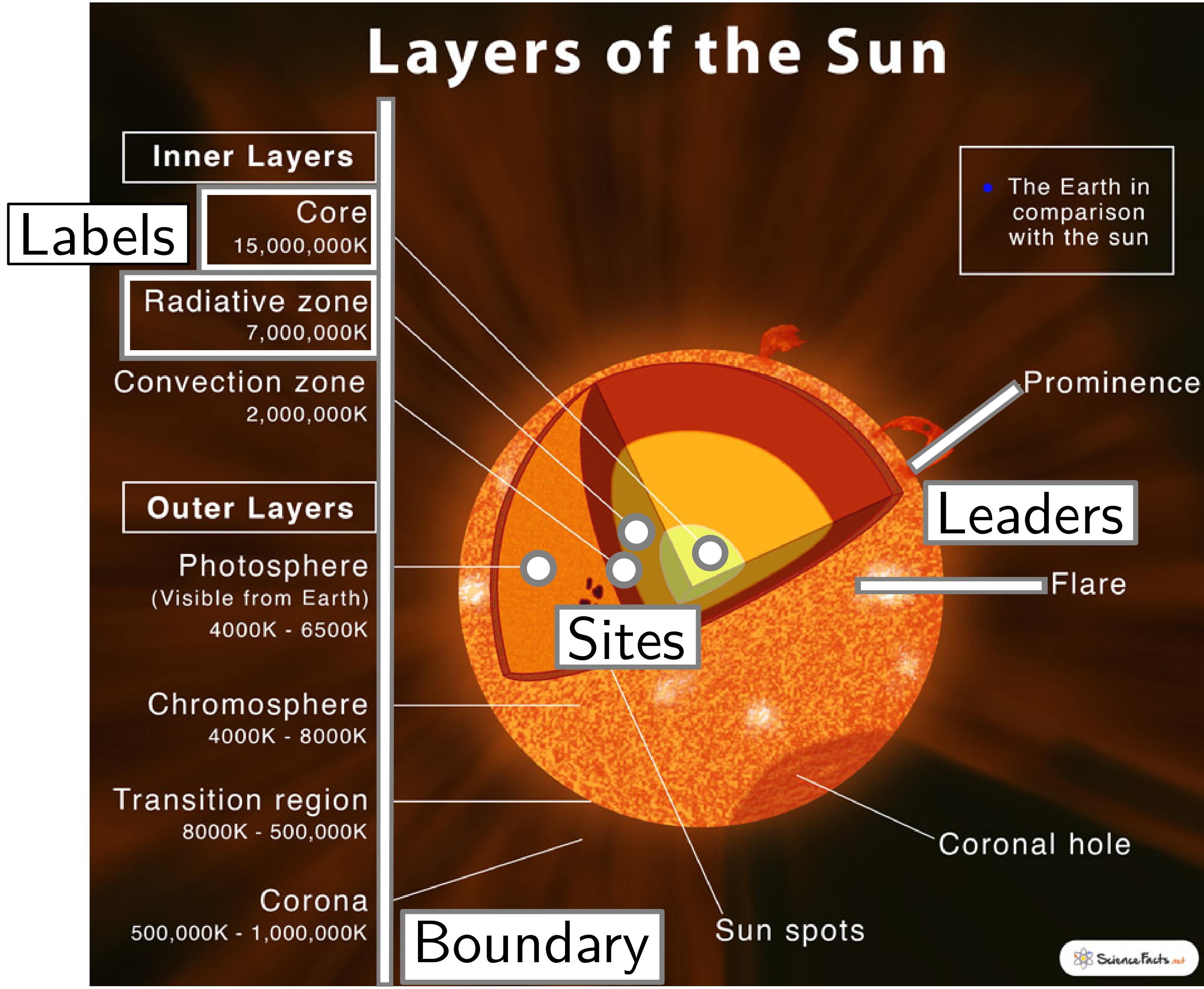

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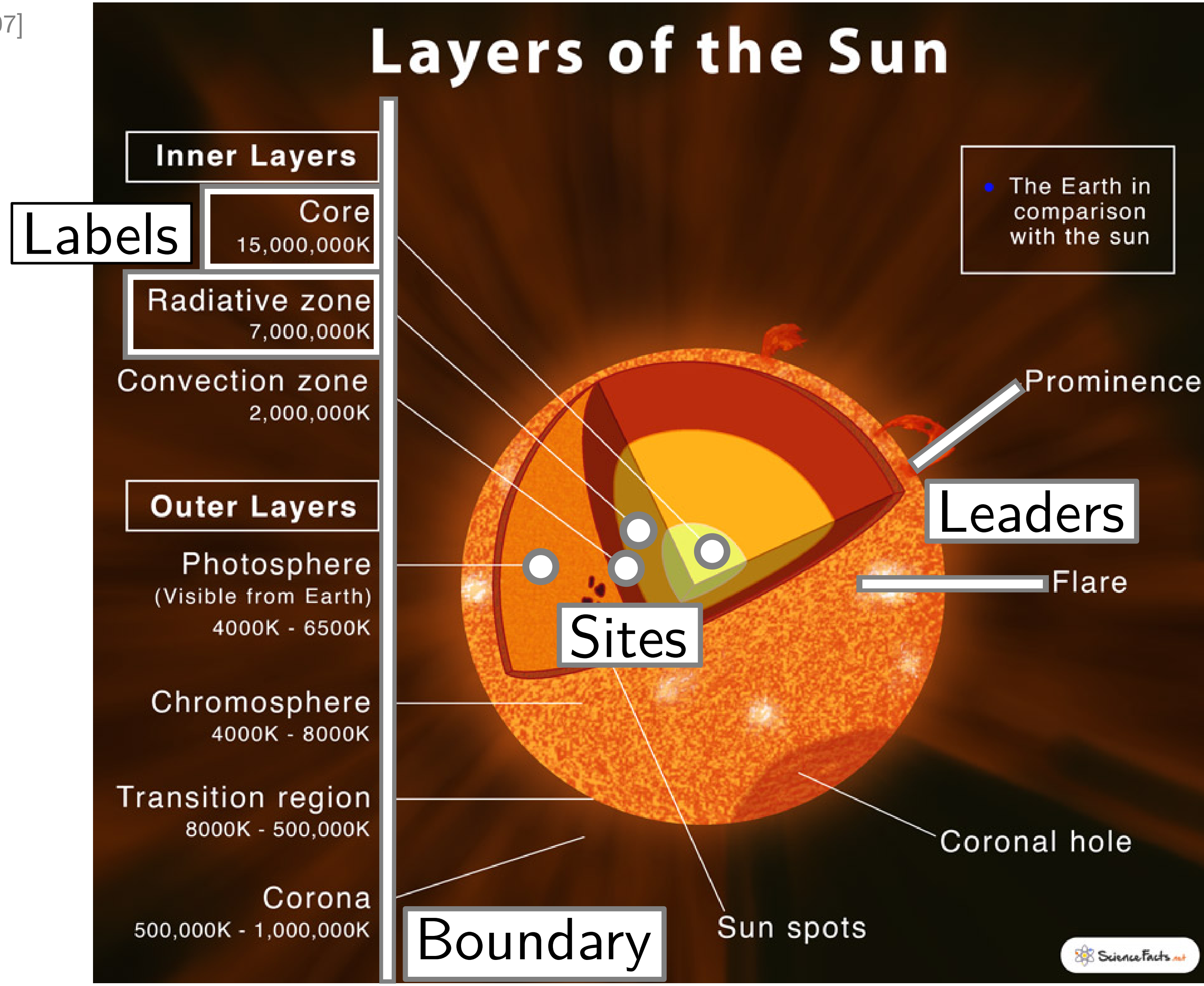

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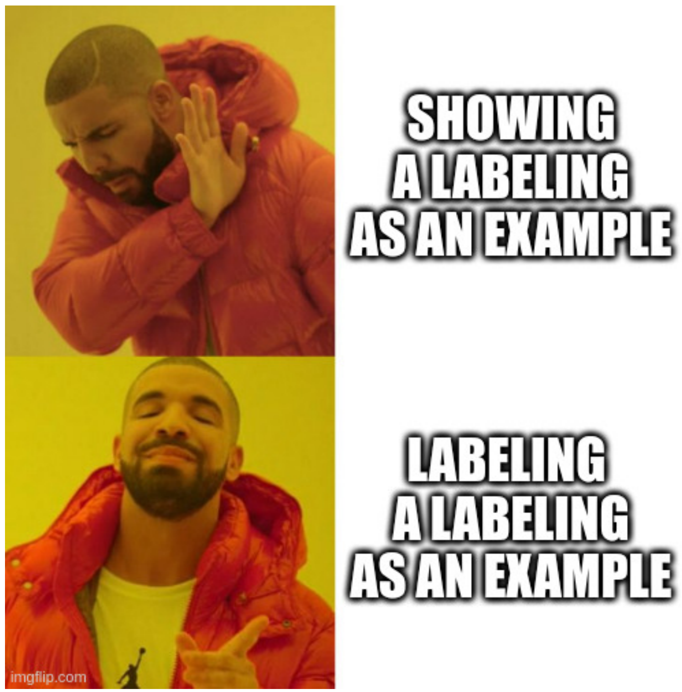

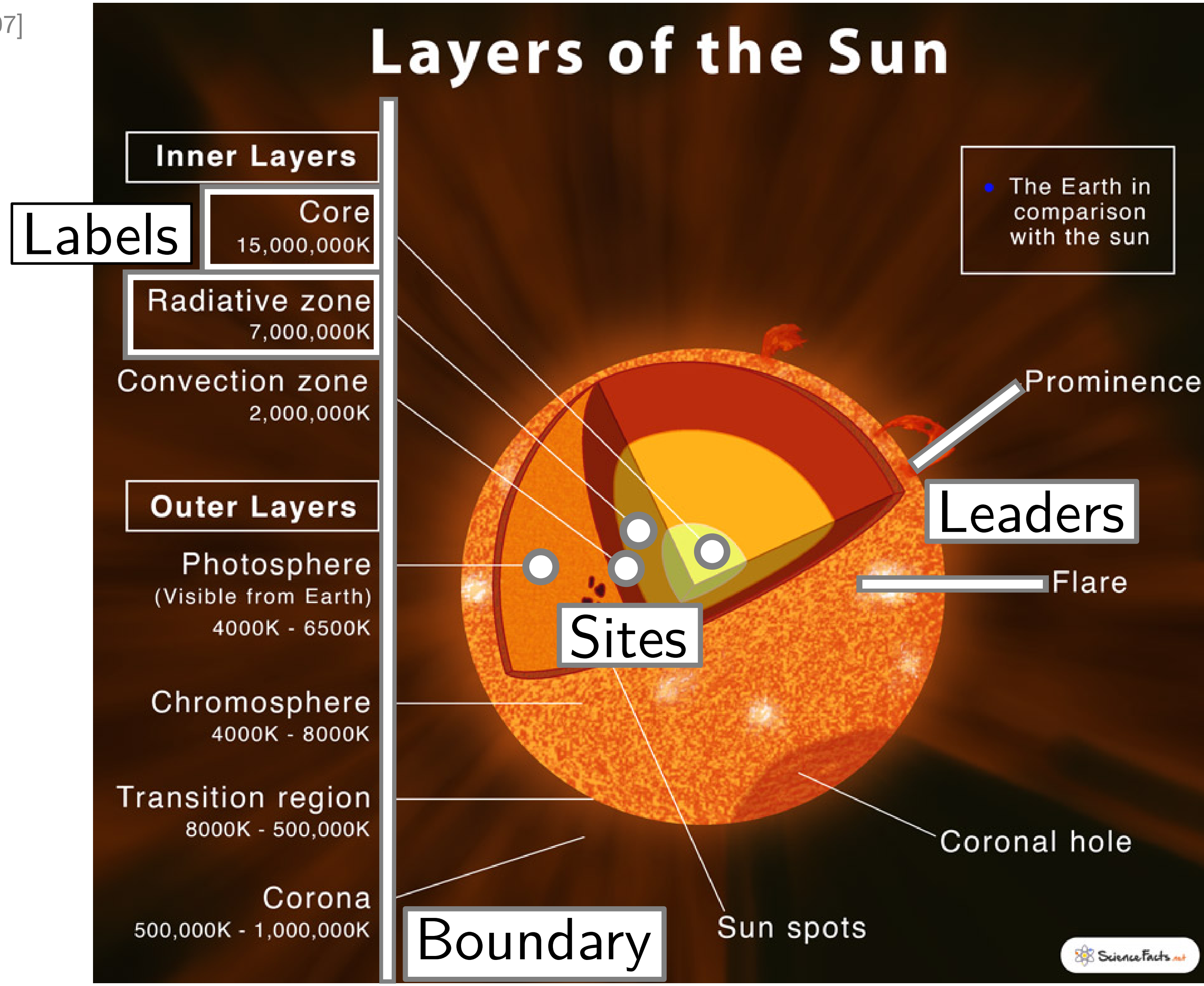

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Well-studied subject in the algorithmic and visualization field

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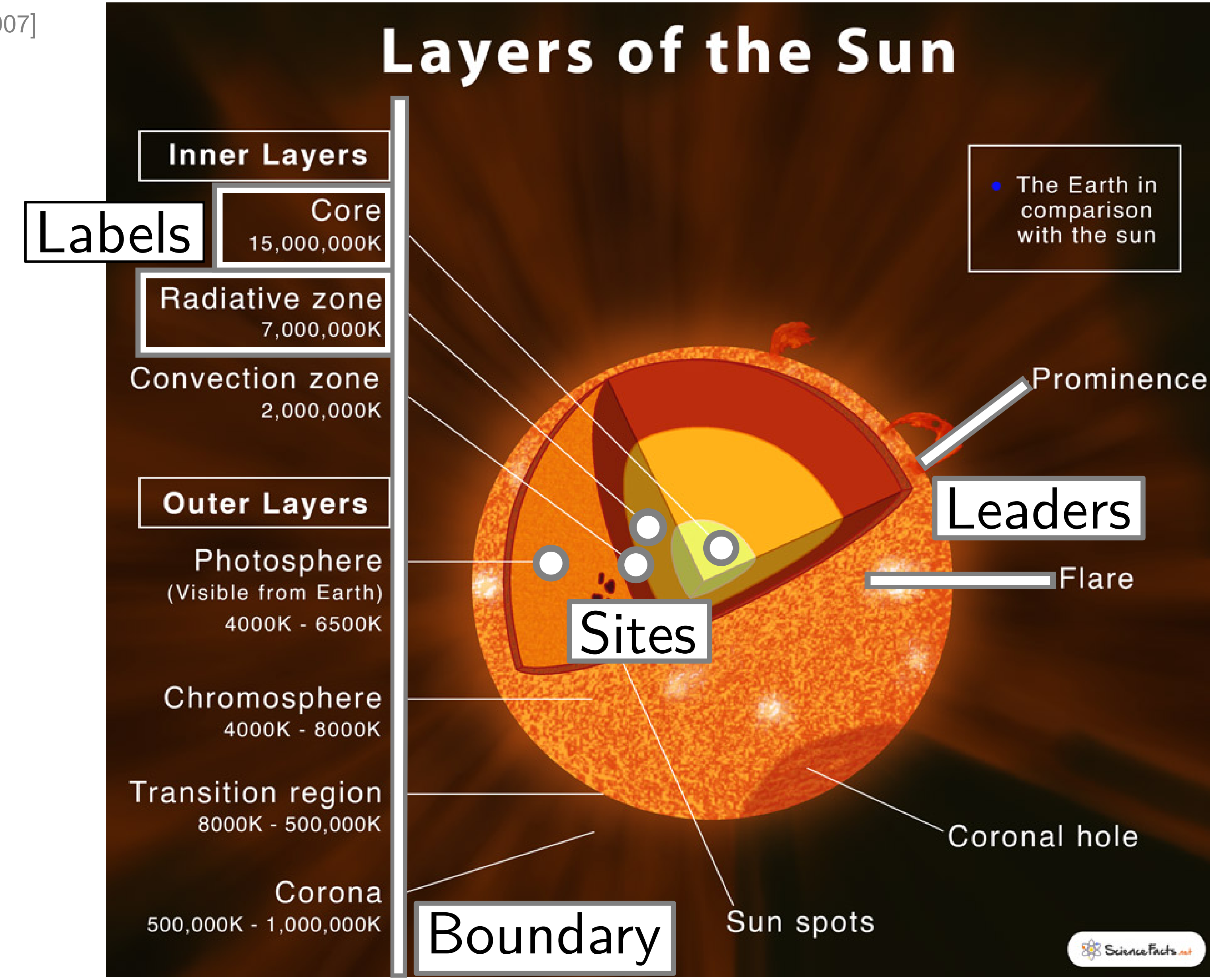

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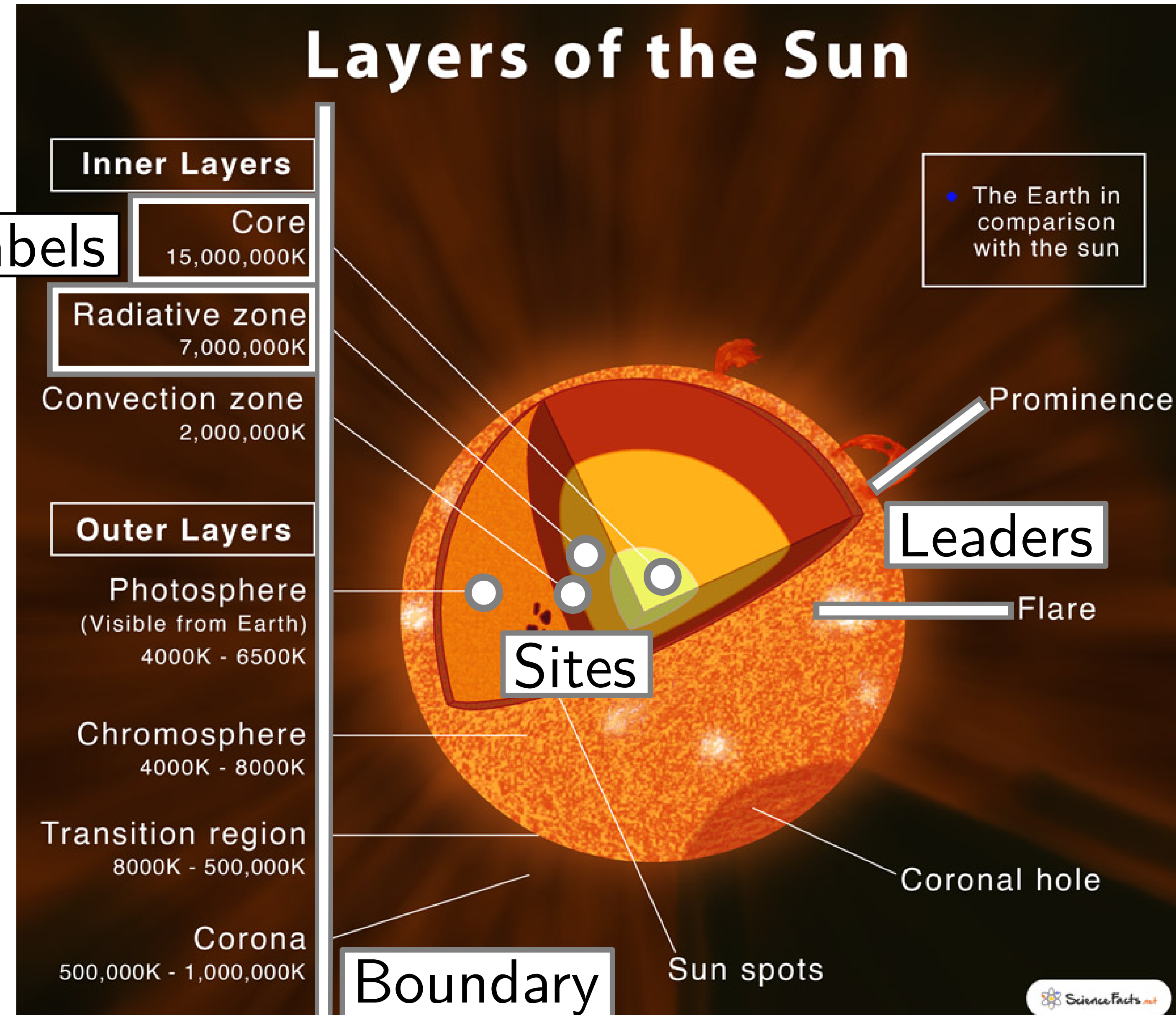

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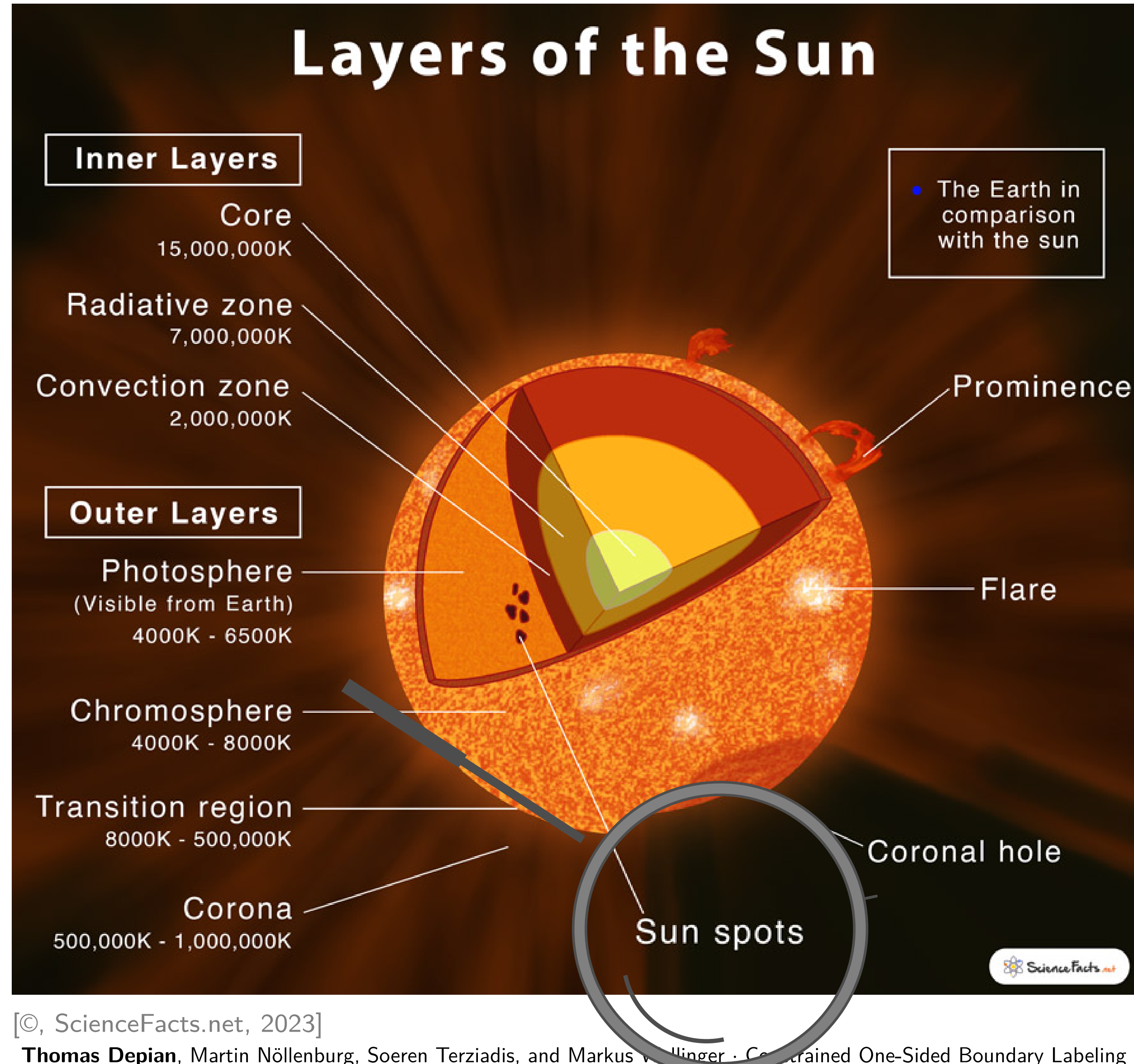

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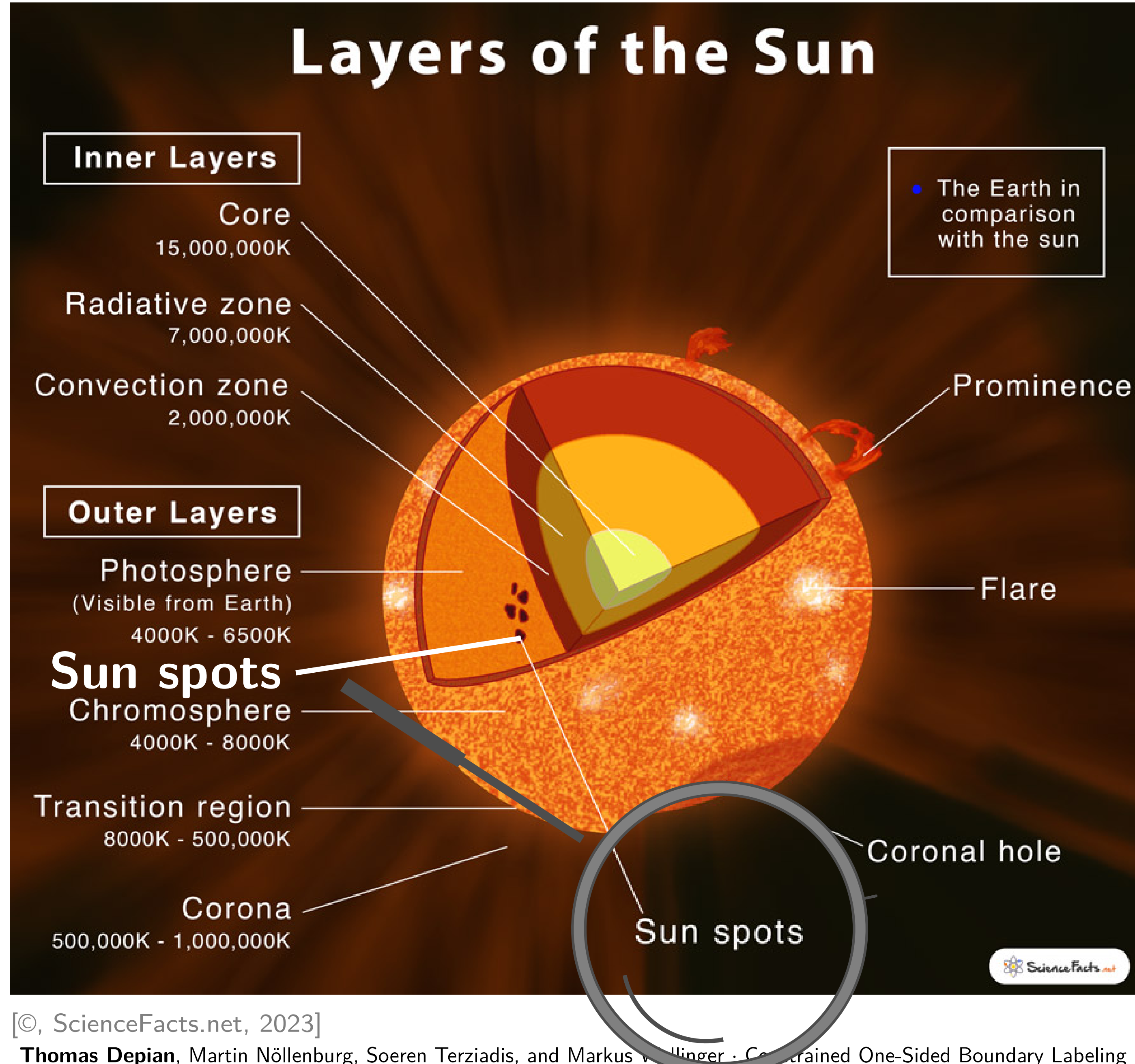

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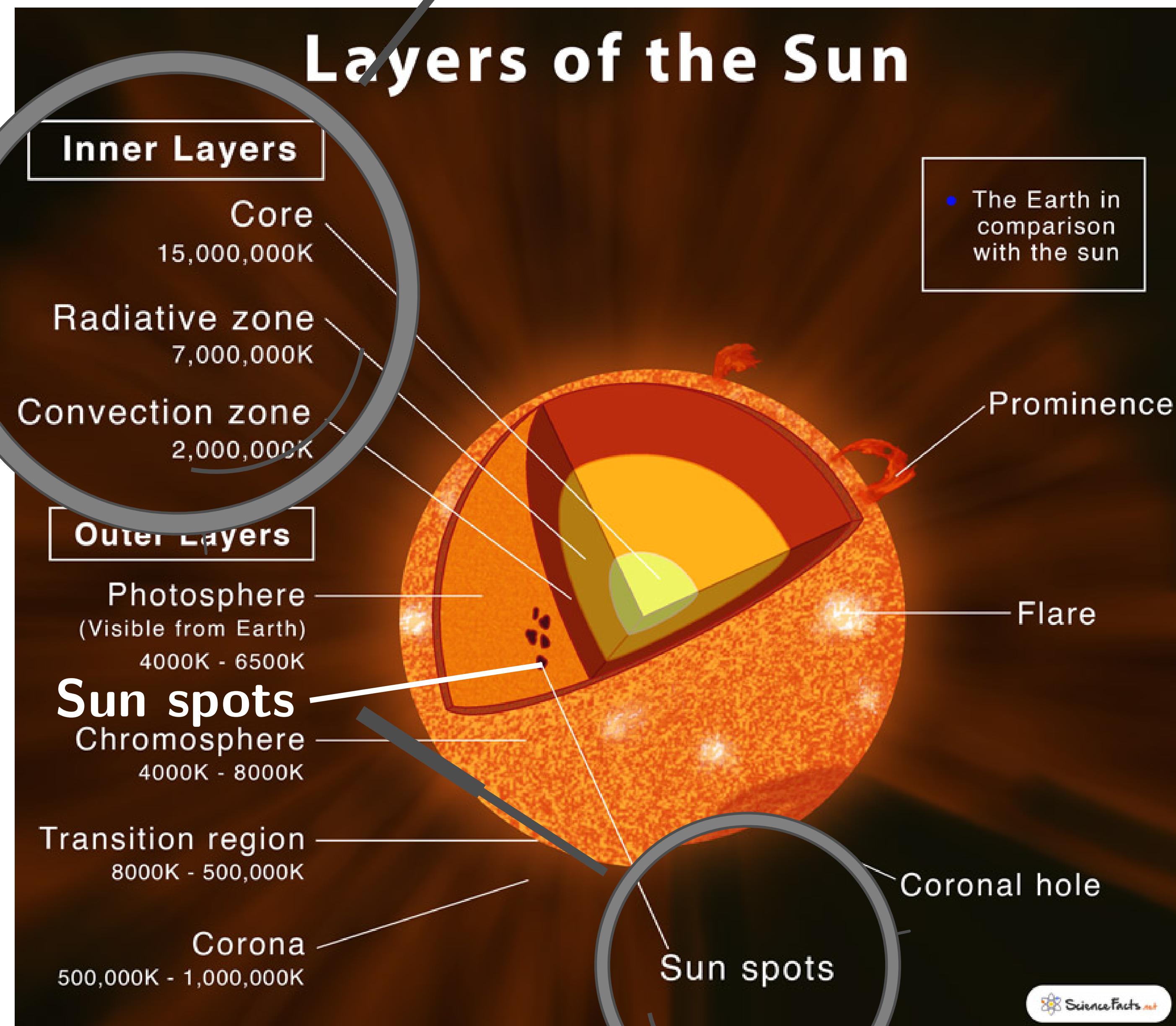

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But only limited support for semantic constraints

[Niedermann et al., PacificVis 2017] [Gedicke et al., CaGis 2023]
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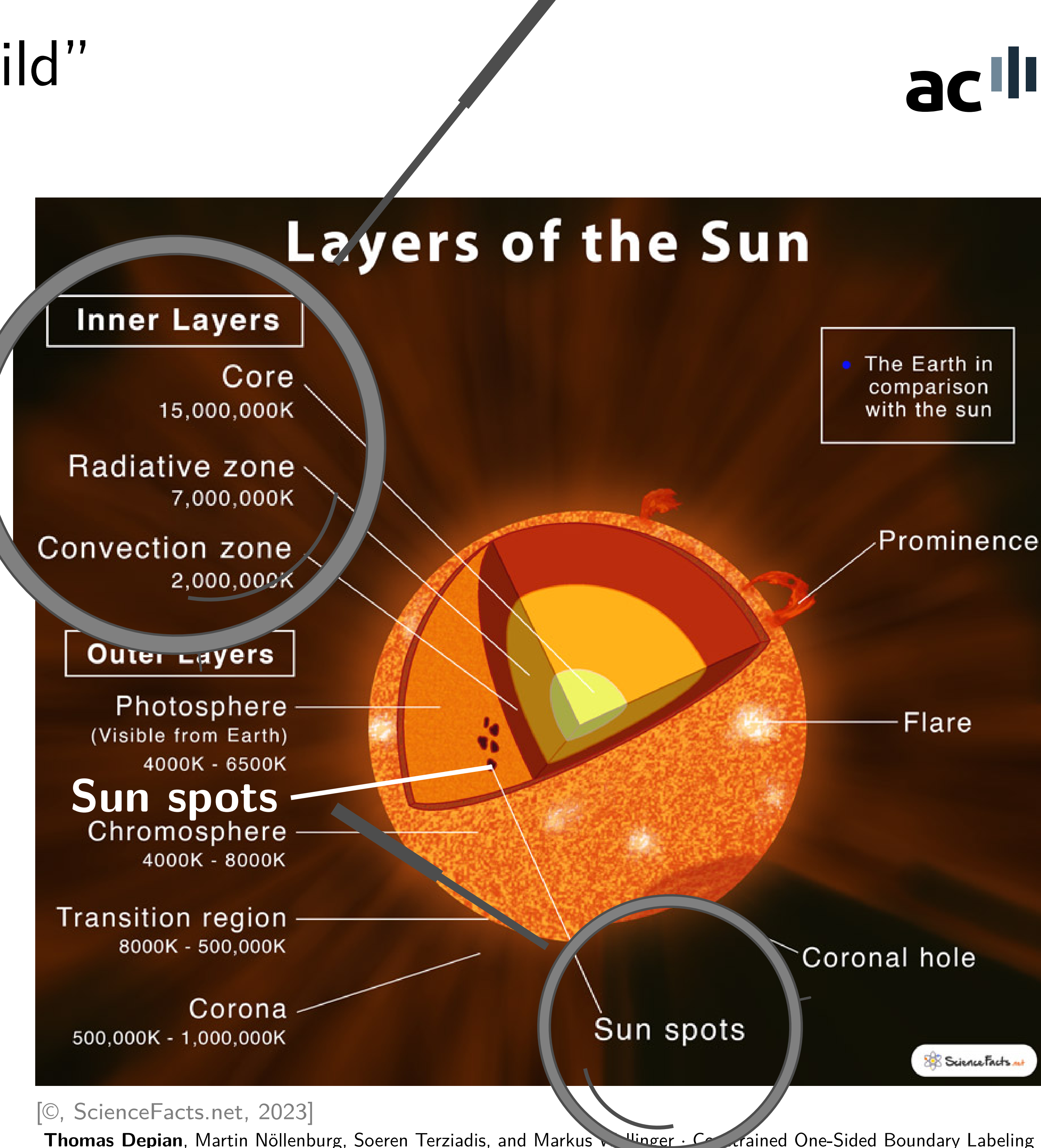

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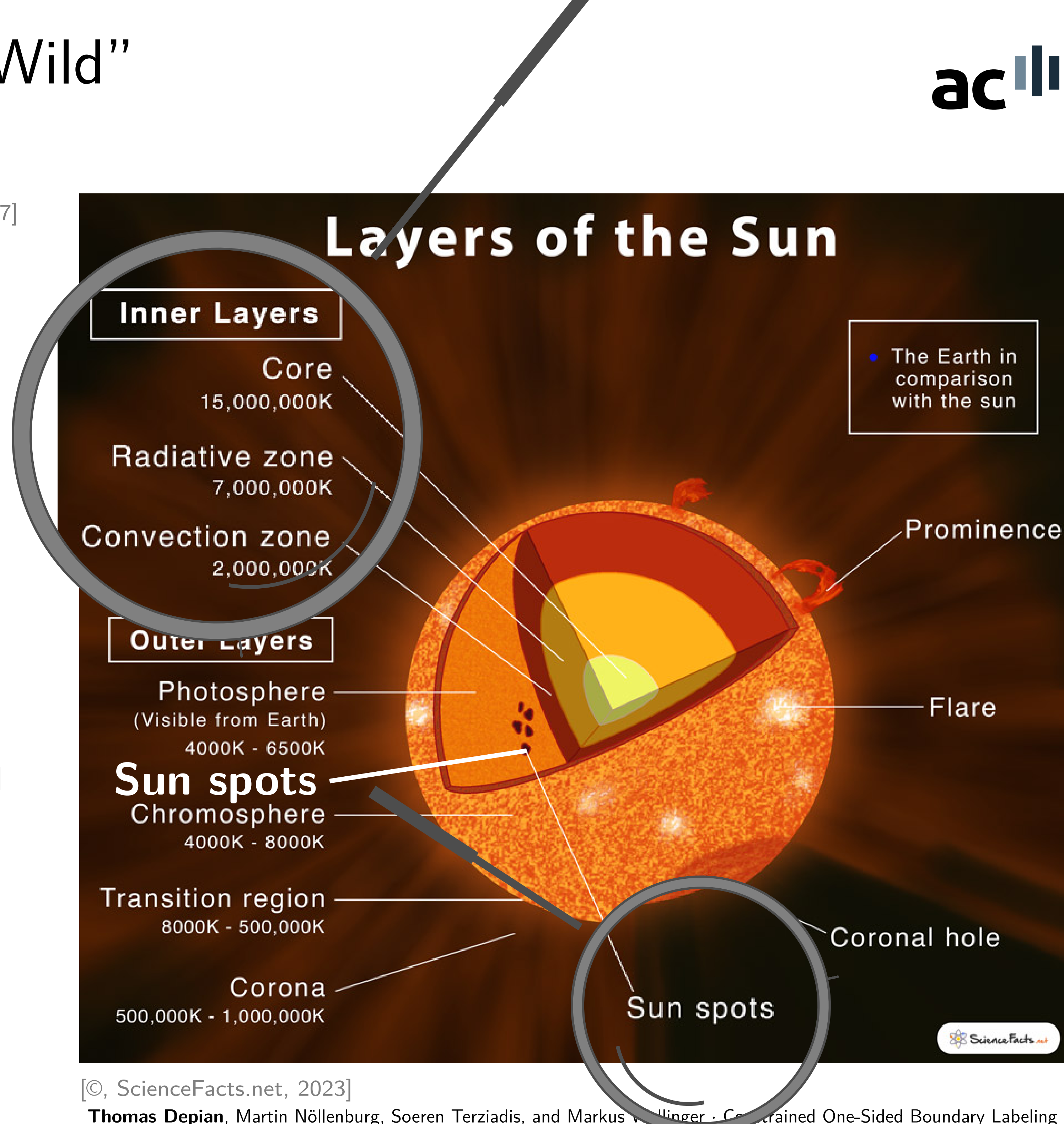

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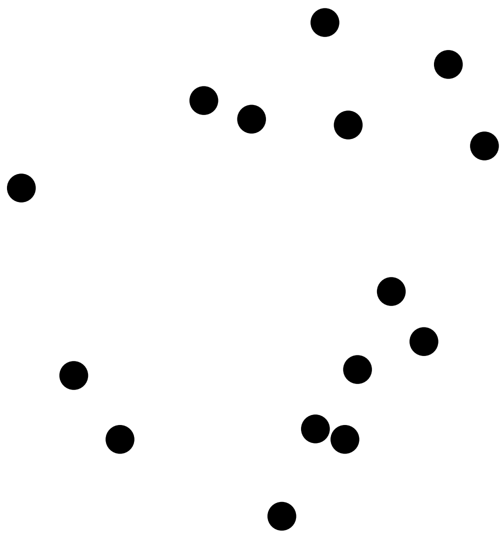
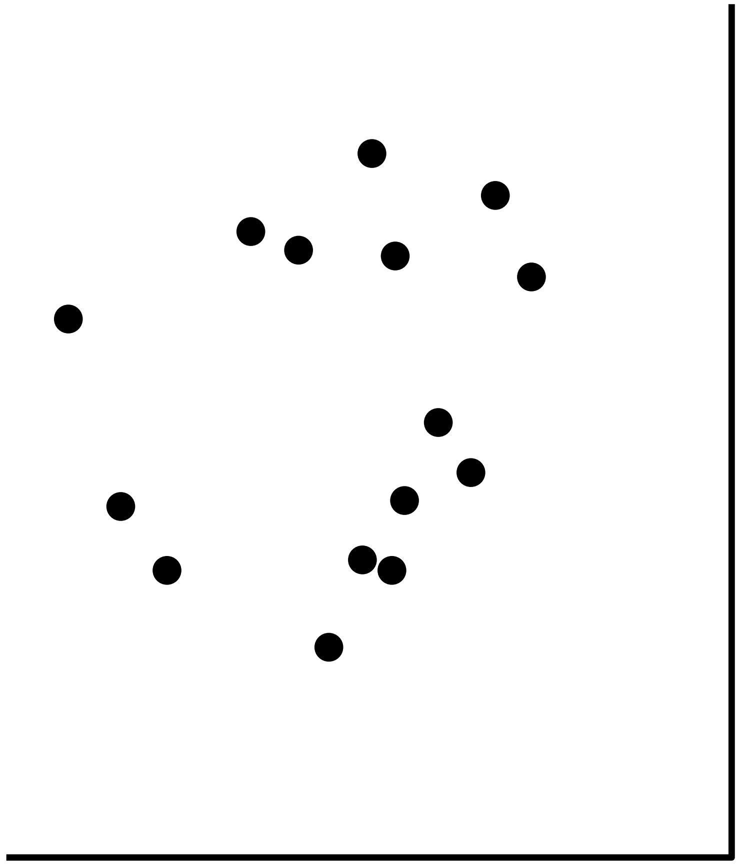
One-Sided Boundary Labeling



Given:

Sites $\mathcal{S} = \{s_1, \ldots, s_n\}$

One-Sided Boundary Labeling



Given:

Sites $\mathcal{S} = \{s_1, \ldots, s_n\}$
enclosed in a bounding rectangle $\mathcal{B}$

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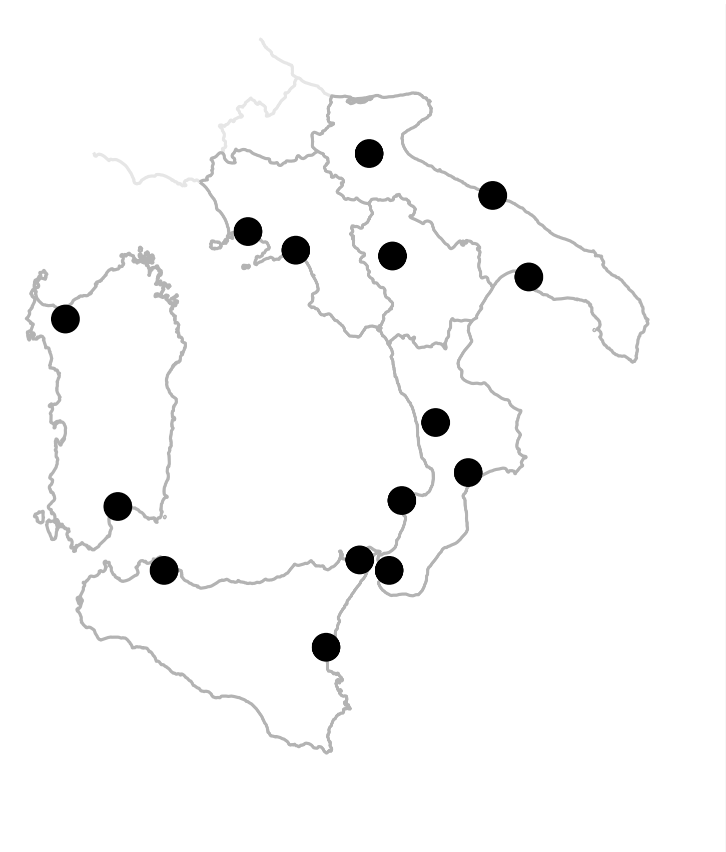

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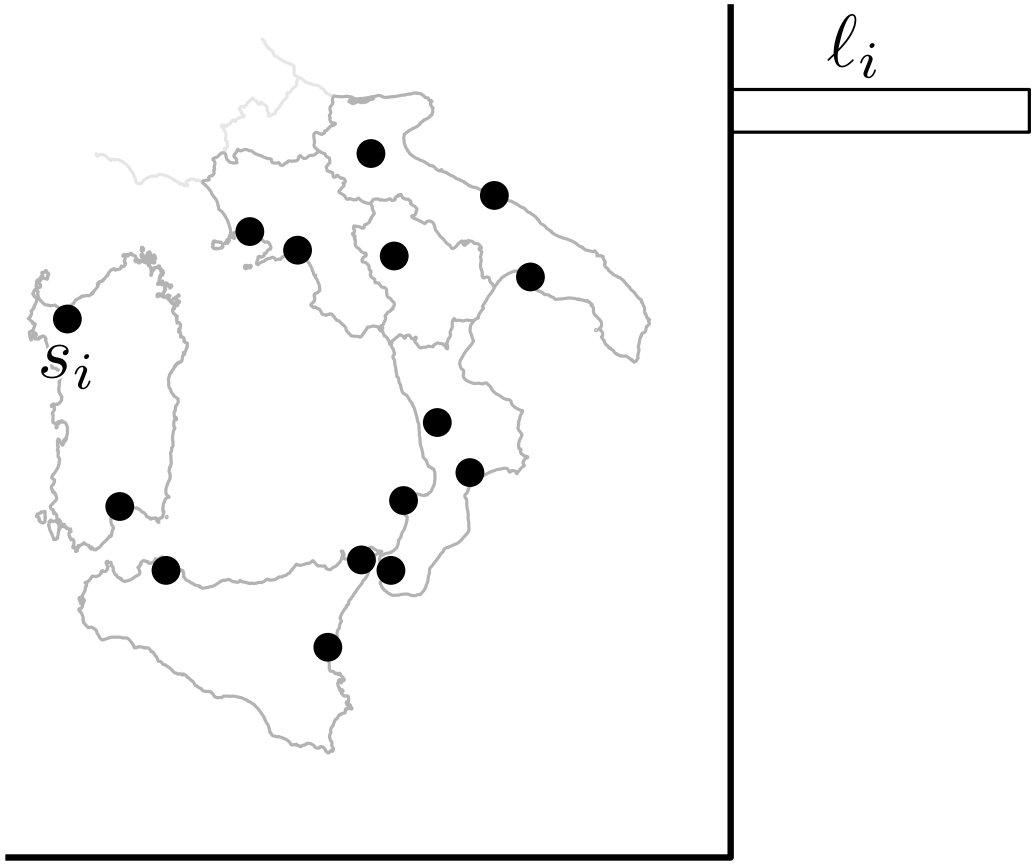
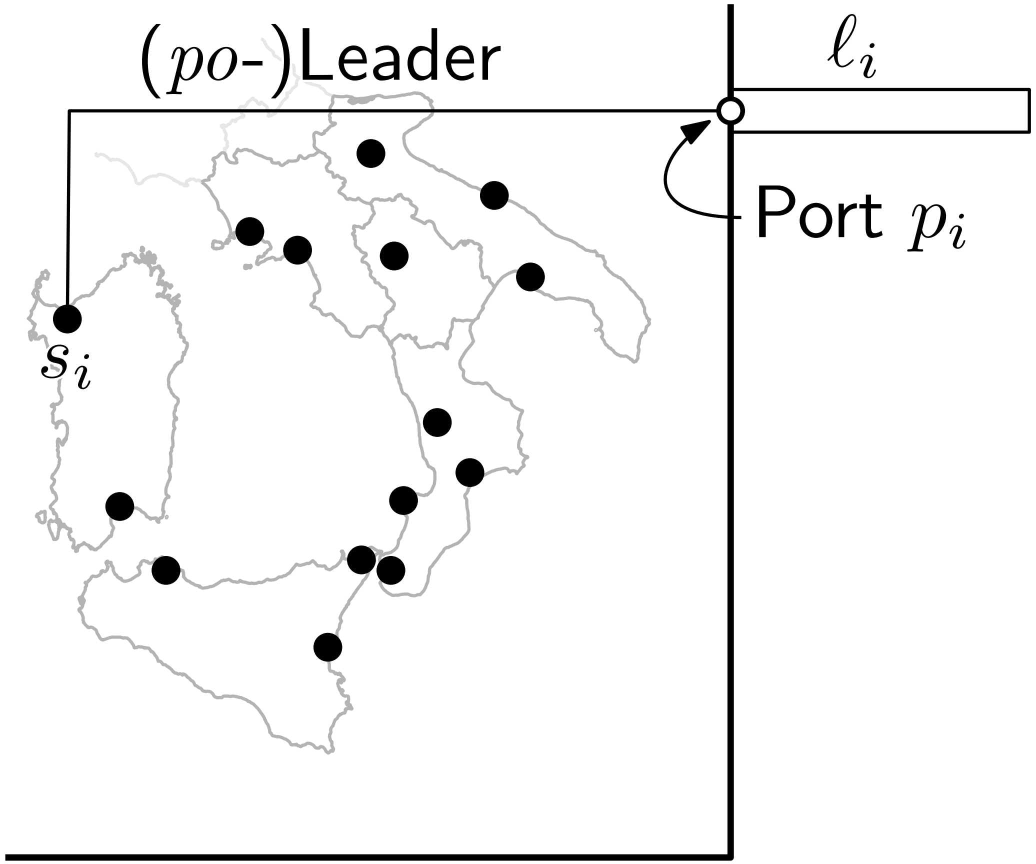


Given:

Sites $\mathcal{S} = \{s_1, \ldots, s_n\}$
enclosed in a bounding rectangle $\mathcal{B}$

Rectangular label ℓ_i of height $h(\ell_i)$
that can be placed on the right side of $\mathcal{B}$



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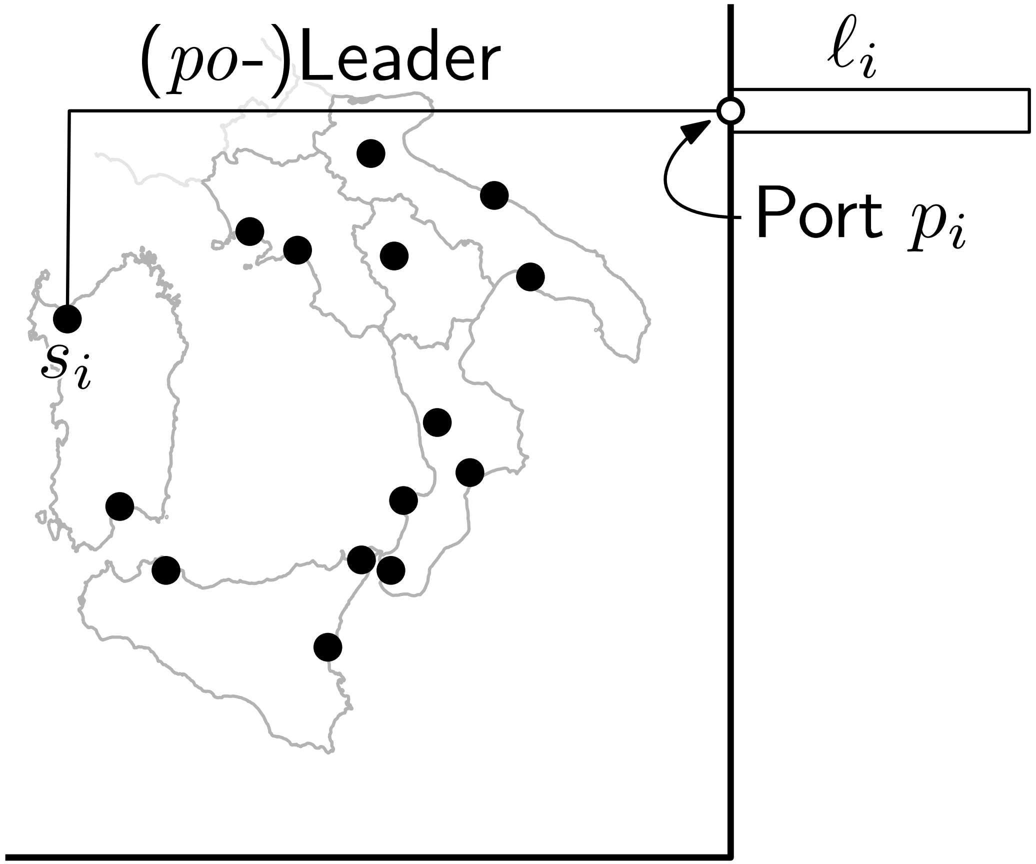

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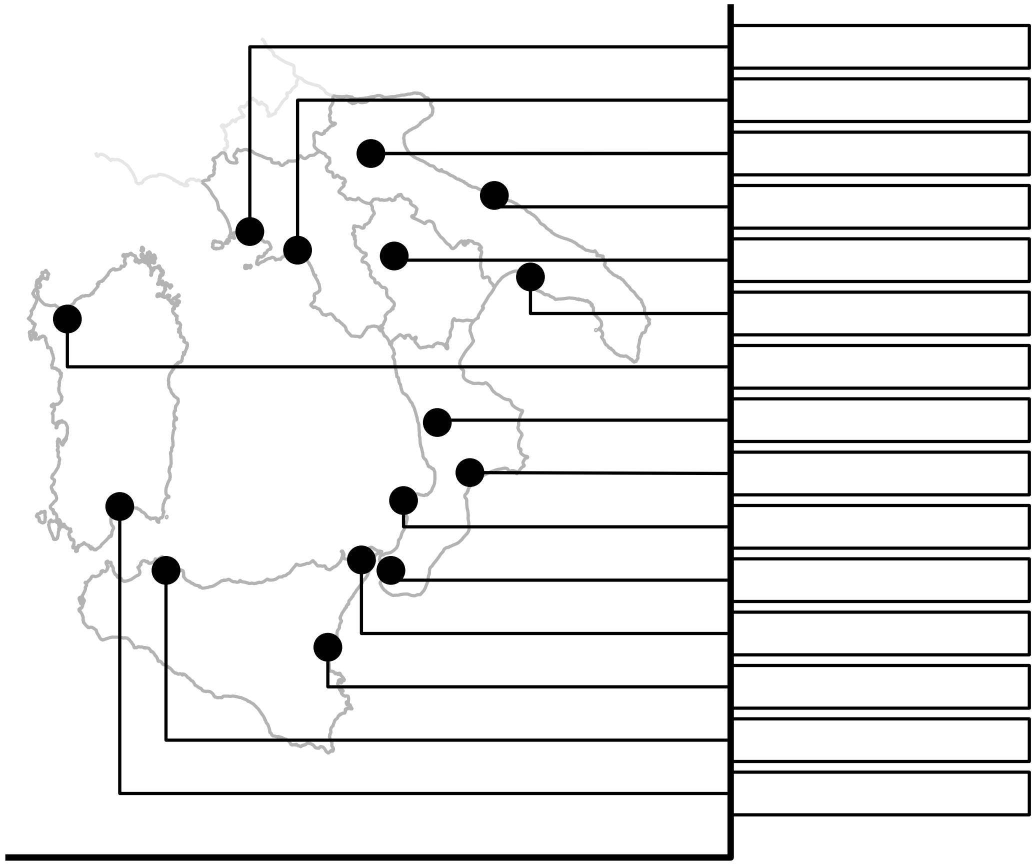
Optimization function $f\colon \Lambda \to \mathbb{R}_0^+$

Set of all possible leaders

One-Sided Boundary Labeling



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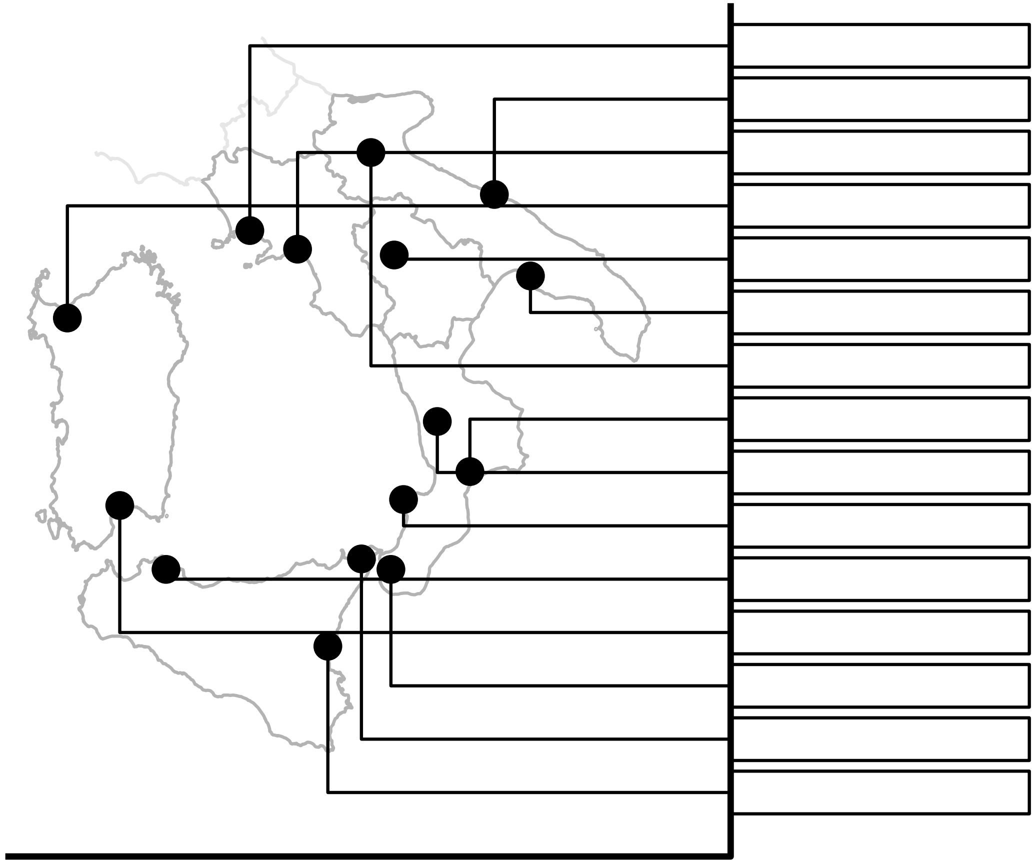
Optimization function $f \colon \Lambda \to \mathbb{R}_0^+$

Set of all possible leaders

Task:

Find a planar one-sided *po*-labeling $\mathcal{L}^*$ that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.

One-Sided Boundary Labeling



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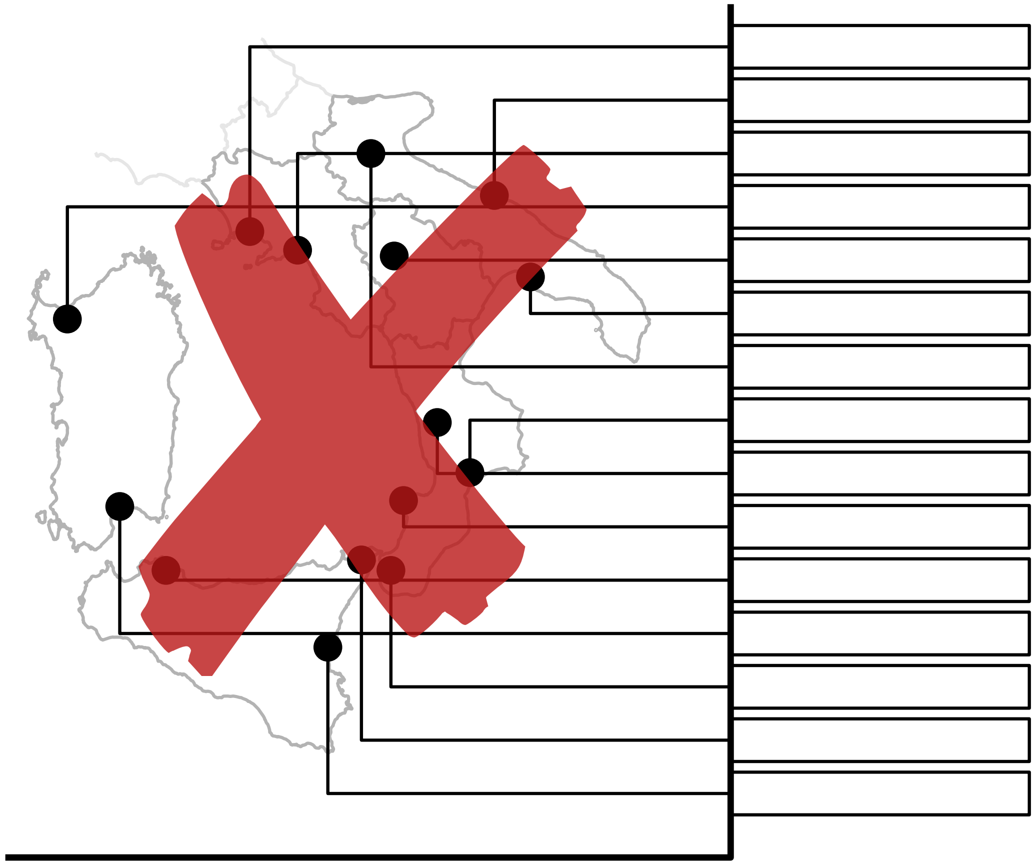
Optimization function $f \colon \Lambda \to \mathbb{R}_0^+$

Set of all possible leaders

Task:

Find a **planar** one-sided po-labeling $\mathcal{L}^*$ that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.

No leader-leader crossings
No leader-site crossings
...



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Optimization function $f \colon \Lambda \to \mathbb{R}_0^+$

Set of all possible leaders

Task:

Find a **planar** one-sided *po*-labeling $\mathcal{L}^*$ that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.

No leader-leader crossings
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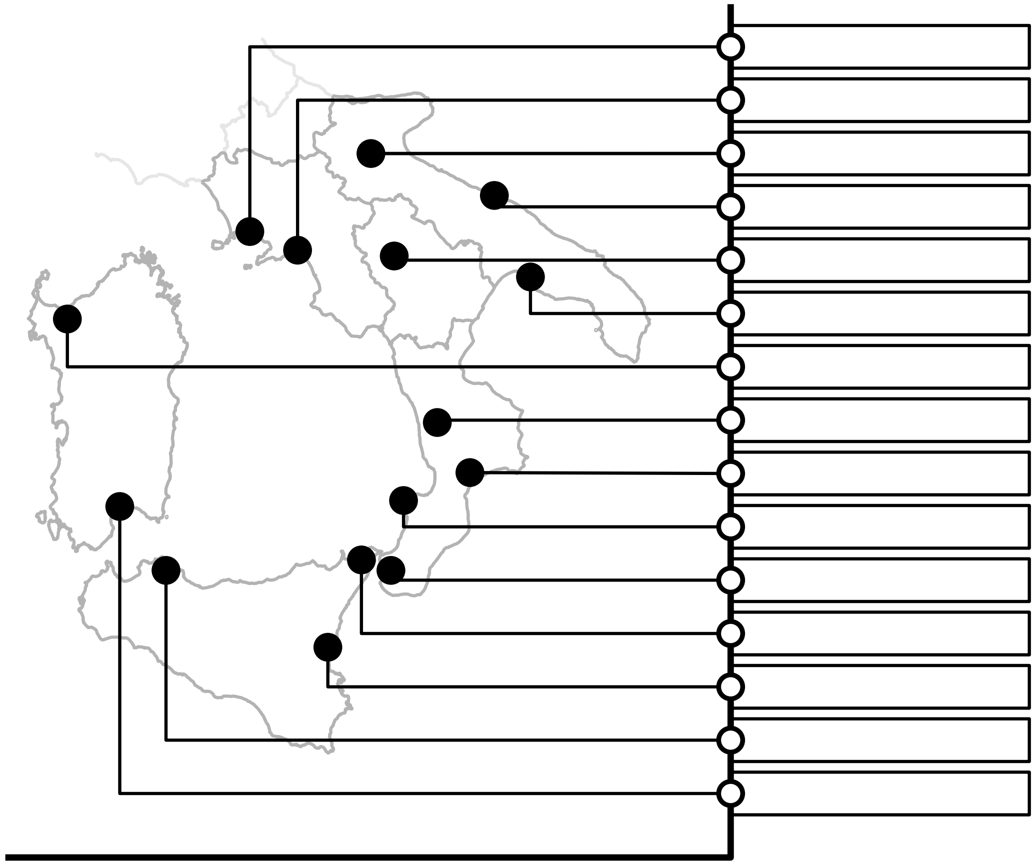
One-Sided Boundary Labeling



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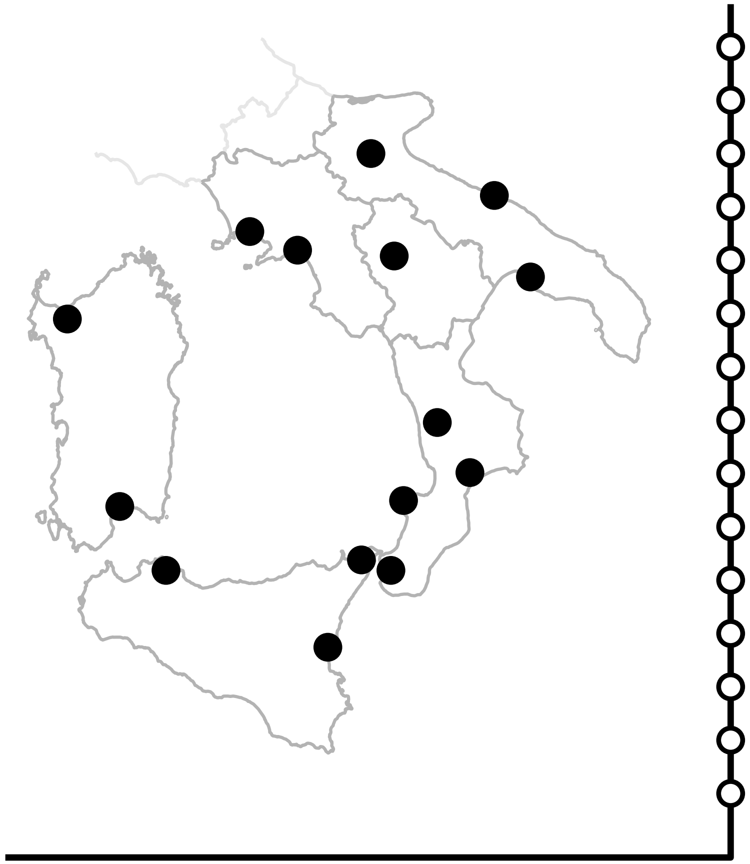
Optimization function $f\colon \Lambda \to \mathbb{R}_0^+$

(Ports $\mathcal{P} = \{p_1, \ldots, p_m\}$, $n \leq m$)

Task:

Find a planar one-sided po-labeling $\mathcal{L}^*$ (on a set of m **candidate ports $\mathcal{P}$**) that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.

Constrained One-Sided Boundary Labeling



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Constraints $\mathcal{C} = (\mathscr{G}, \preceq)$

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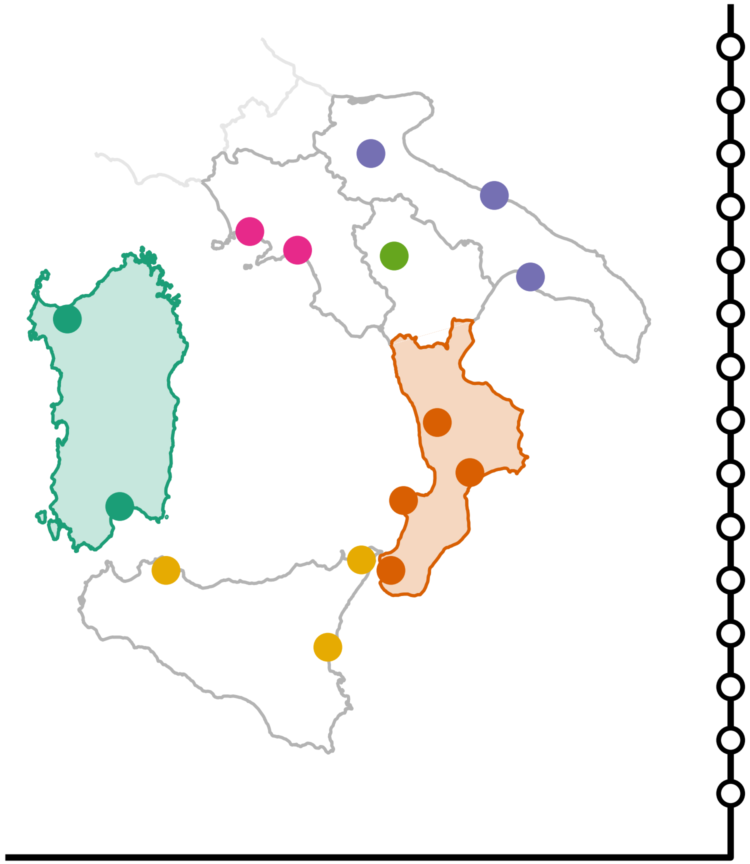

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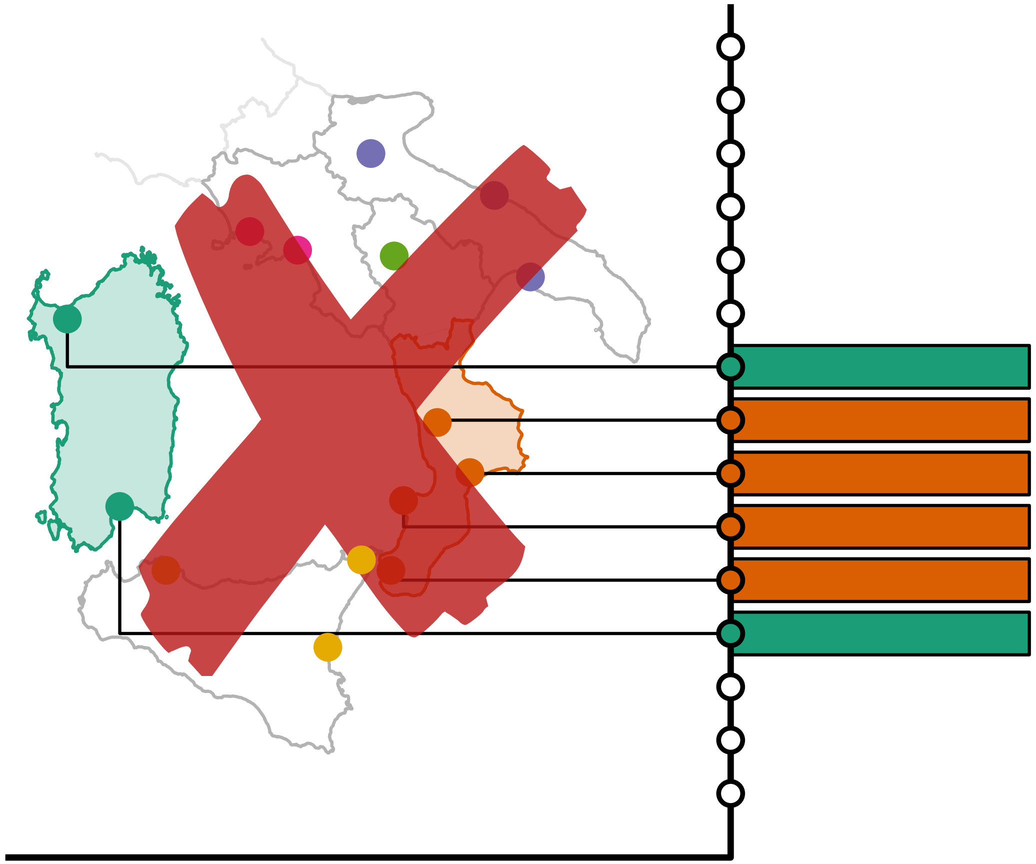
Constraints $\mathcal{C} = (\mathscr{G}, \preceq)$

Grouping Constraints ($\mathscr{G}$): Labels for sites in group $\mathcal{G} \in \mathscr{G}$, $\emptyset \subset \mathcal{G} \subseteq \mathcal{S}$, must appear **consecutively** on $\mathcal{B}$.



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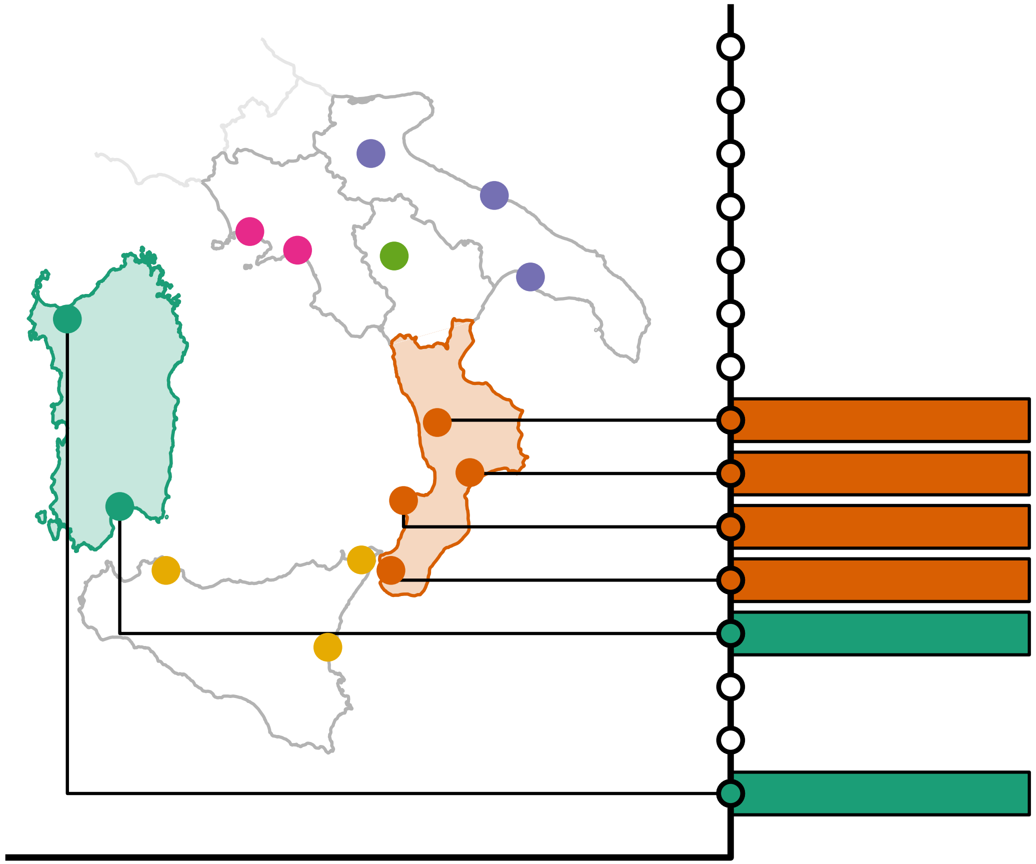

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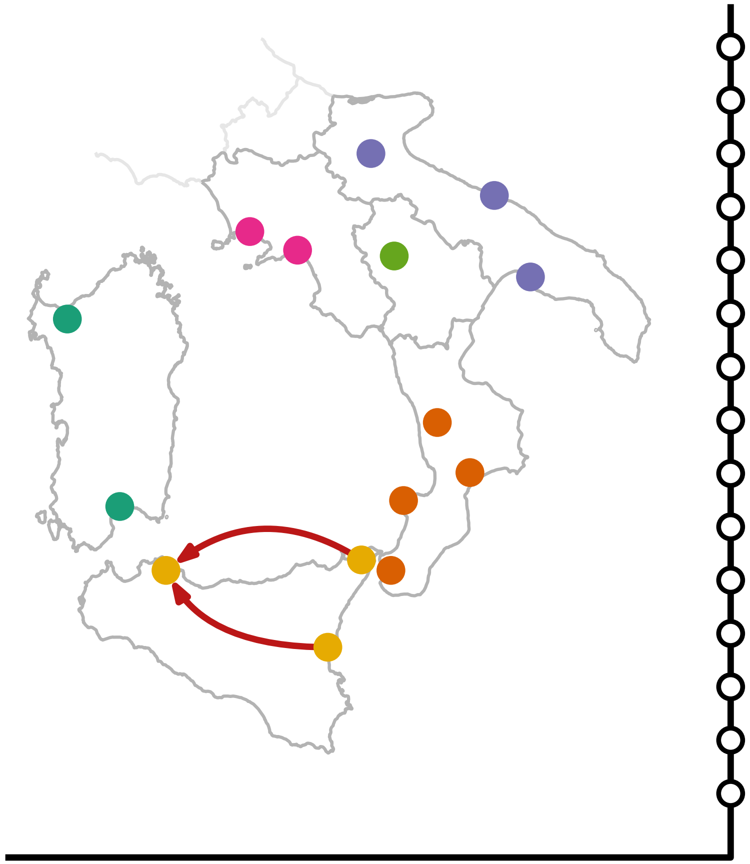

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Ordering Constraints ($\preceq$): The **partial** order $\preceq$ on the sites must be **extended** to a **total** order on the labels.

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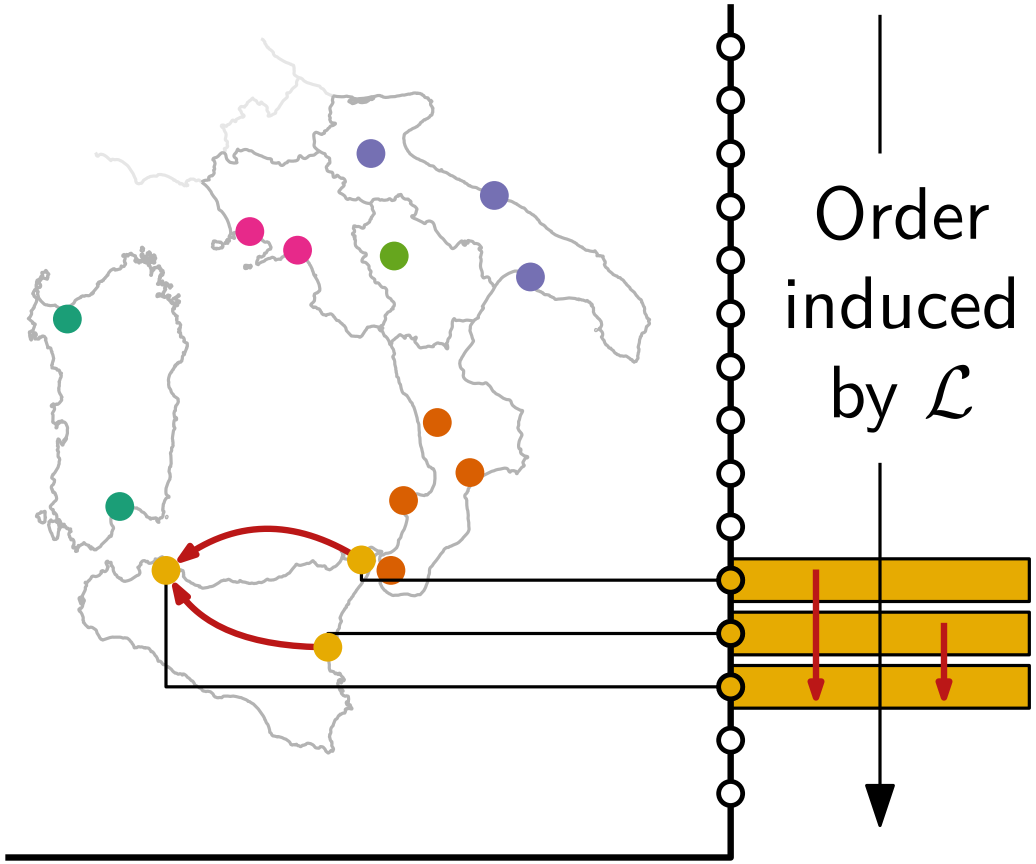

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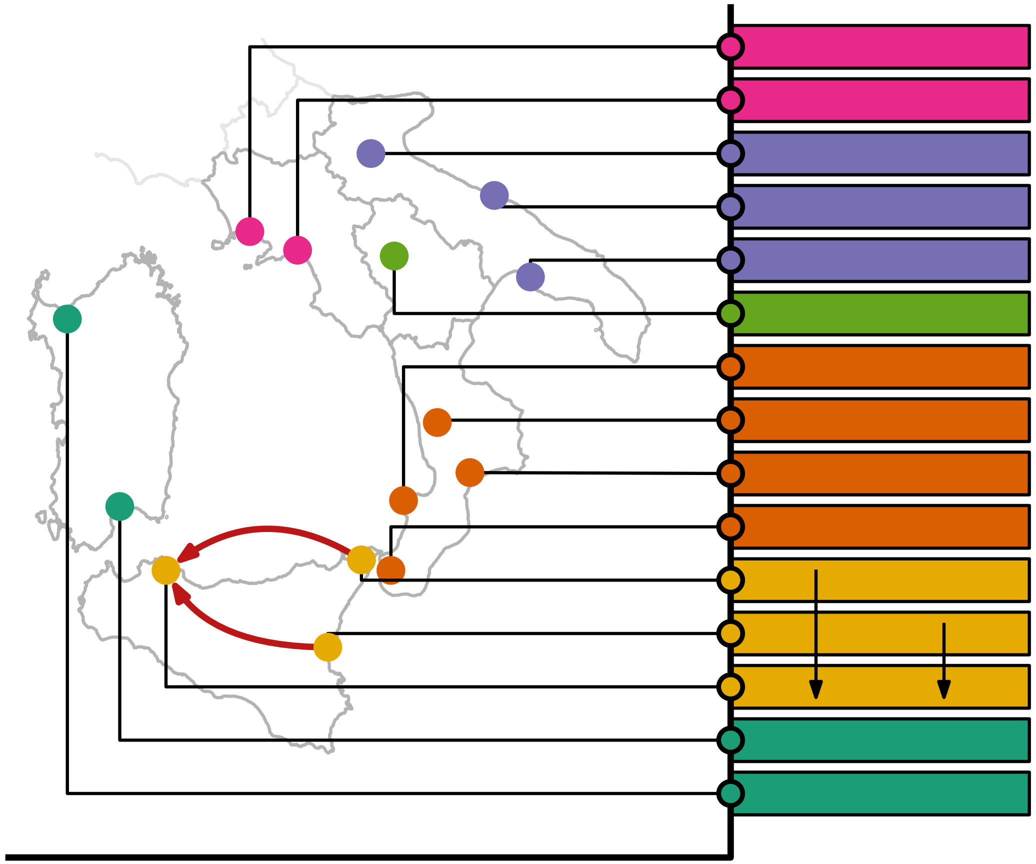

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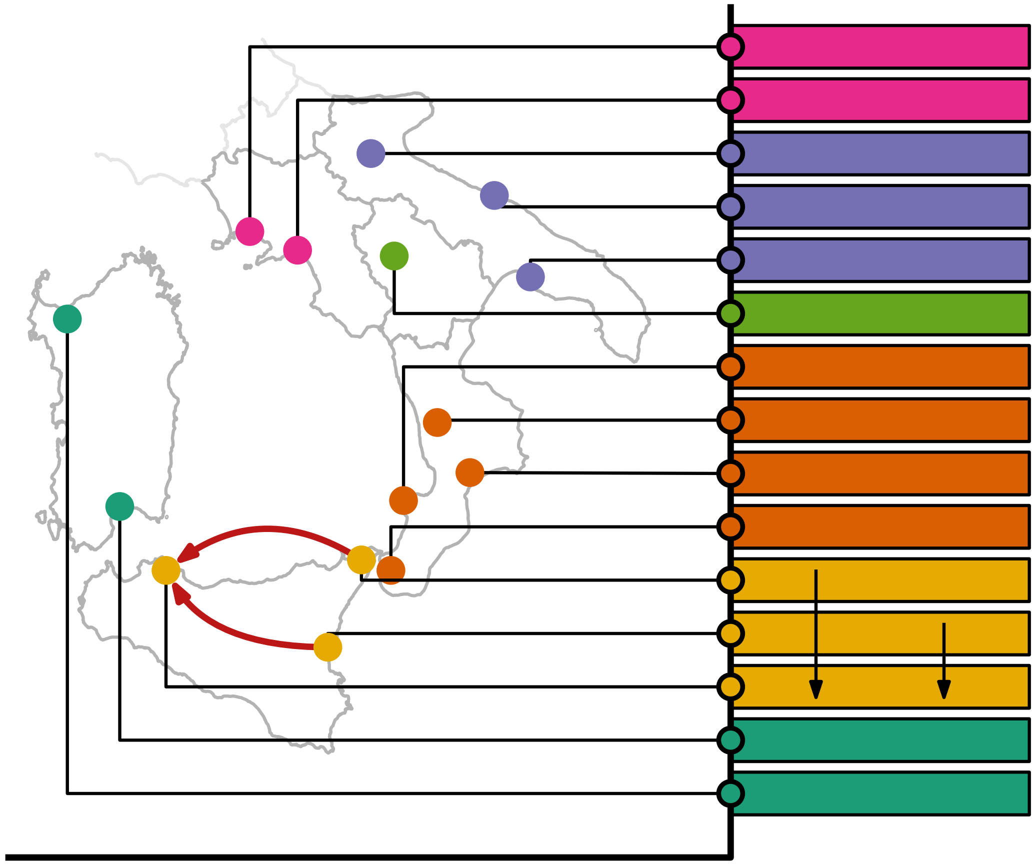
Constraints $\mathcal{C} = (\mathcal{G}, \preceq)$

Task:

Find an admissible one-sided *po*-labeling $\mathcal{L}^*$ (on a set of m candidate ports $\mathcal{P}$) that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.



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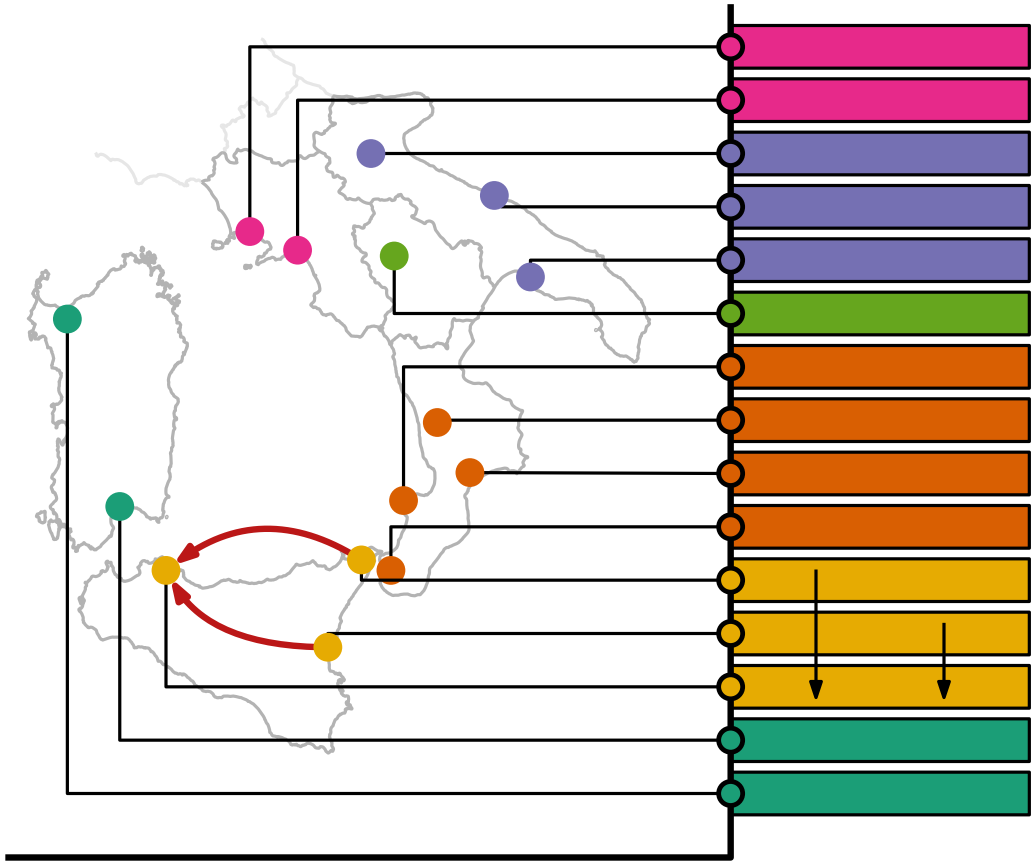
Task:

planar & respects constraints $\mathcal{C}$

Find an **admissible** one-sided po-labeling $\mathcal{L}^*$ (on a set of m candidate ports $\mathcal{P}$) that minimizes $\sum_{\lambda \in \mathcal{L}^*} f(\lambda)$.

1-CBL



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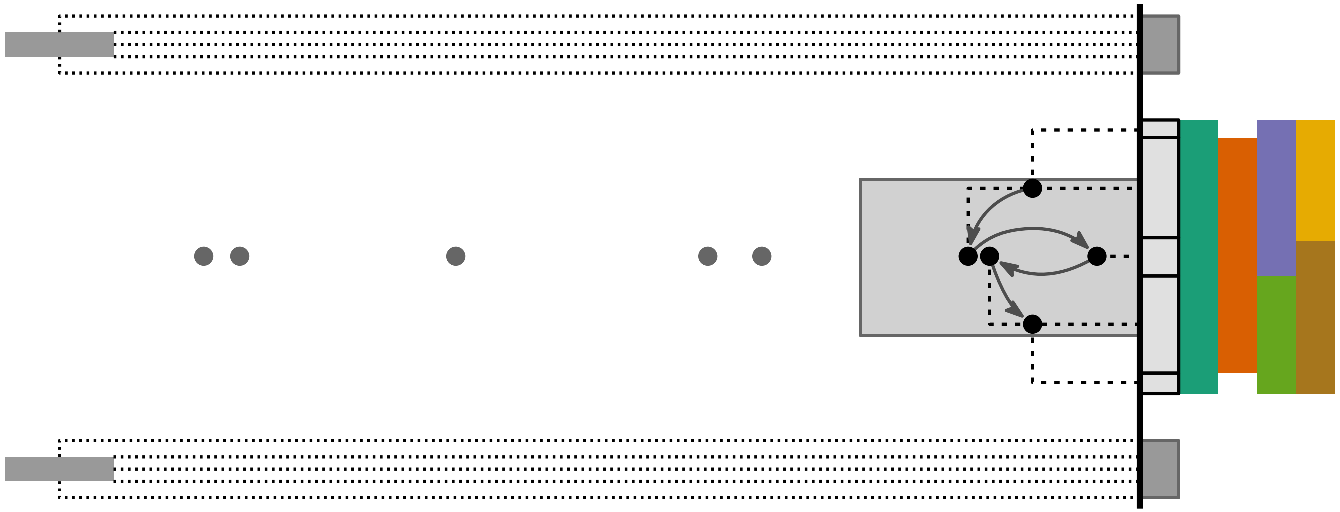
Our Results

1-CBL has

no candidate ports

non-uniform-height labels

Our Results



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[Fink and Suri, CCCG 2016]

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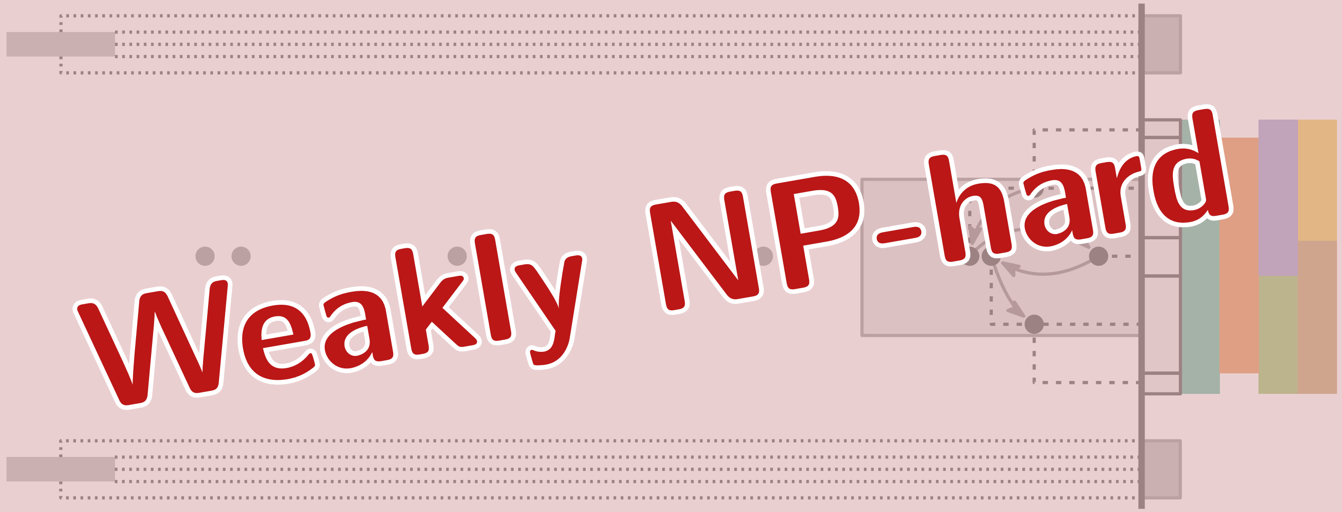

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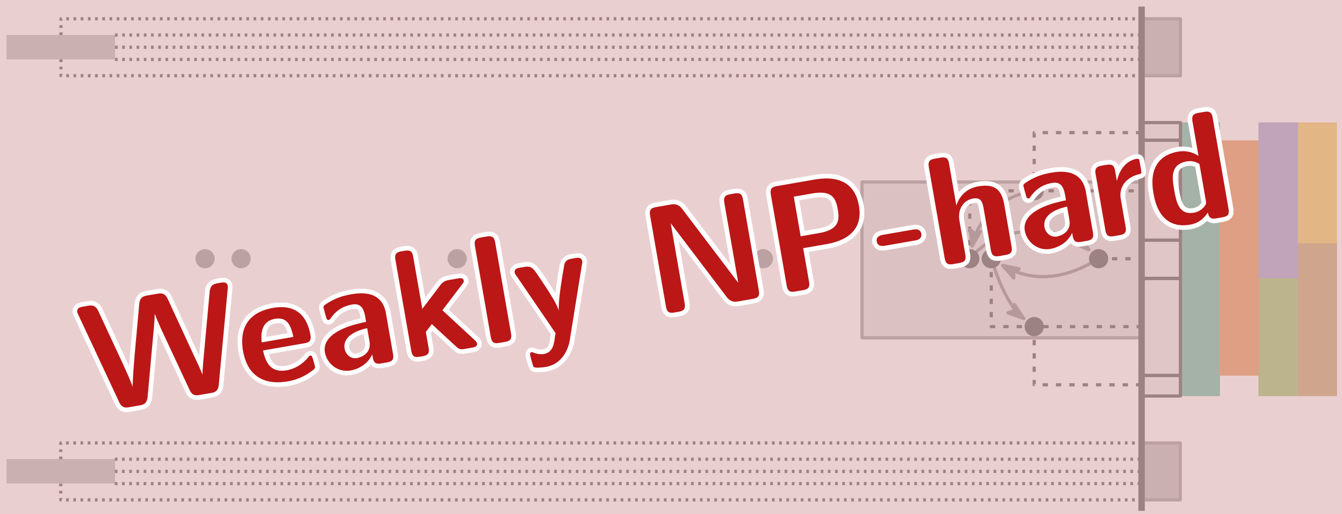

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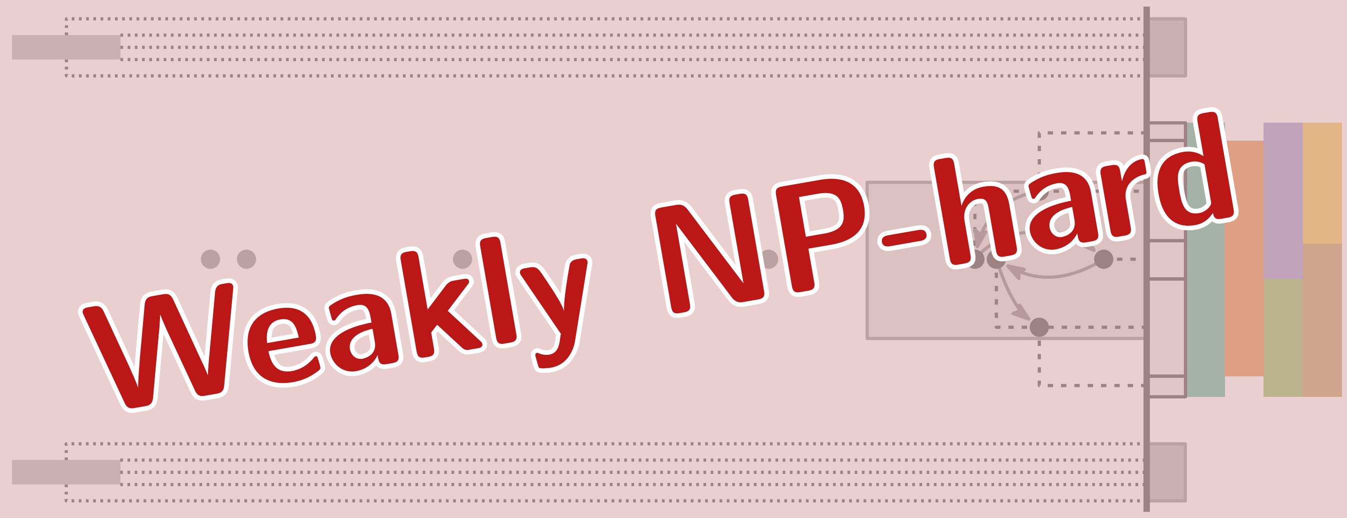

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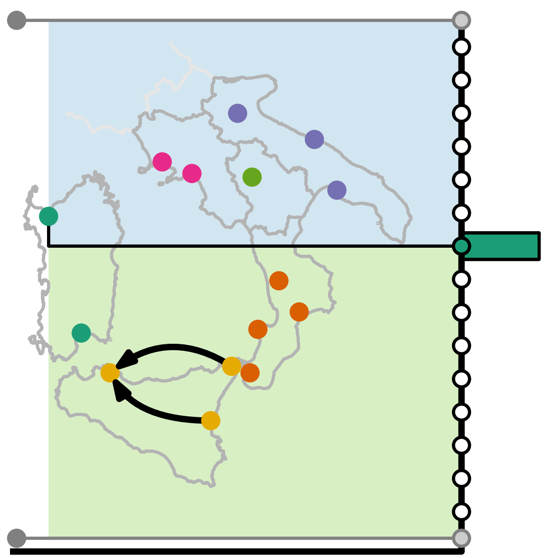

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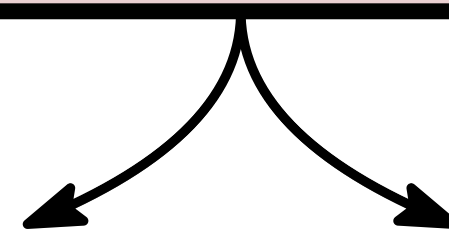

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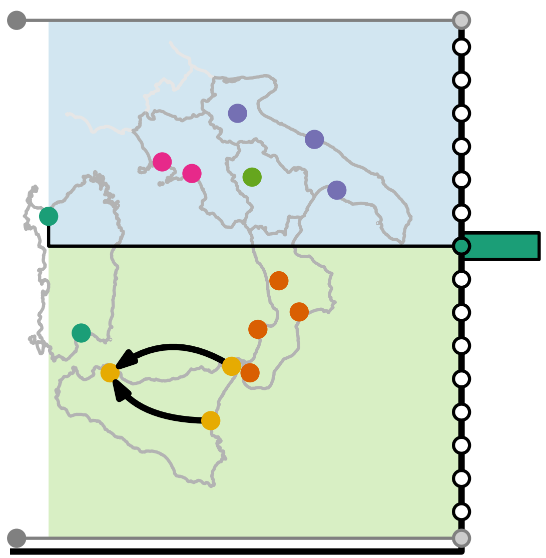

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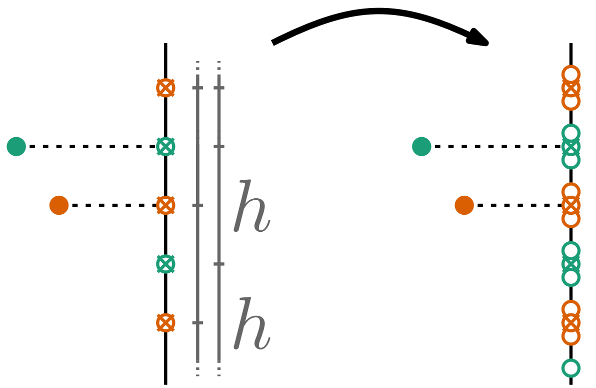

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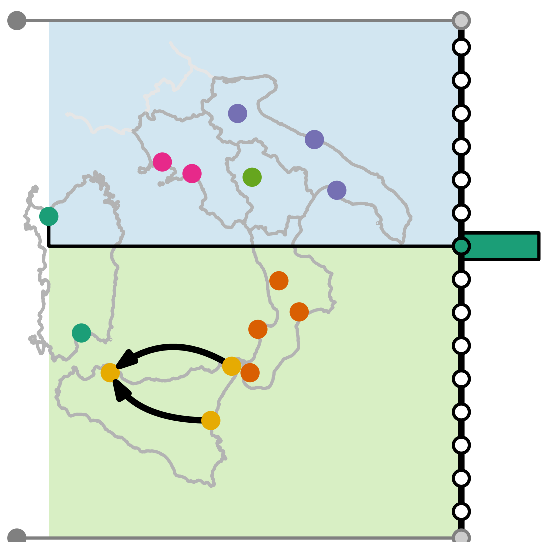

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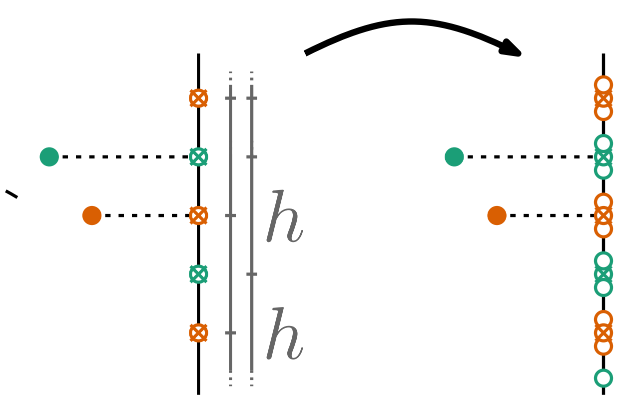

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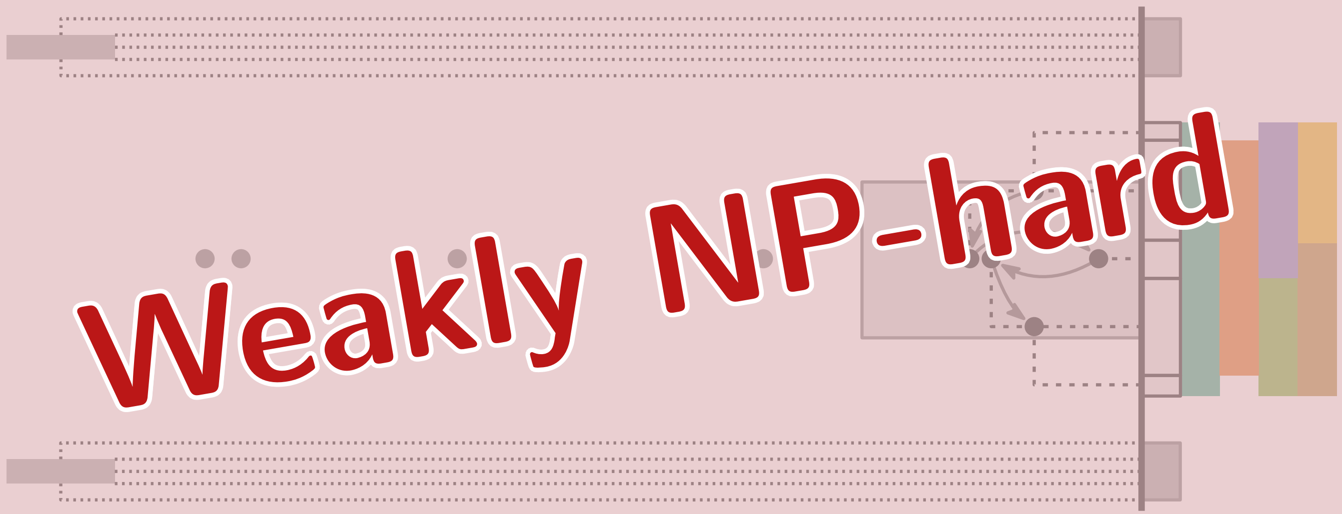

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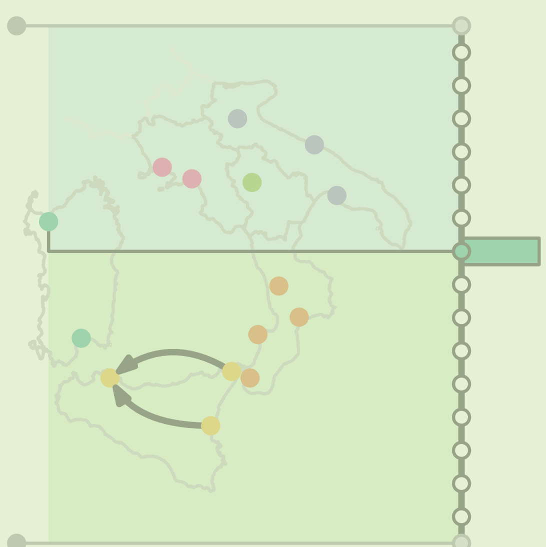
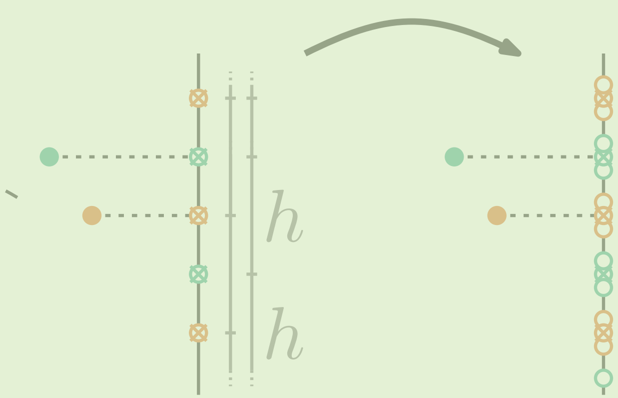


Uniform-height labels

Poly-Time



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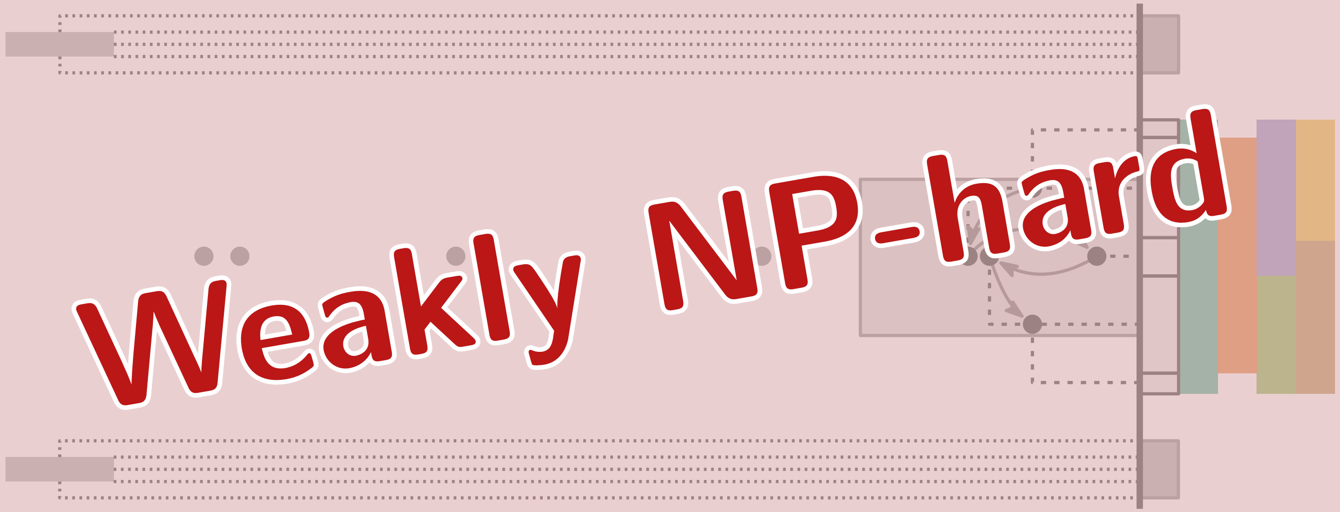

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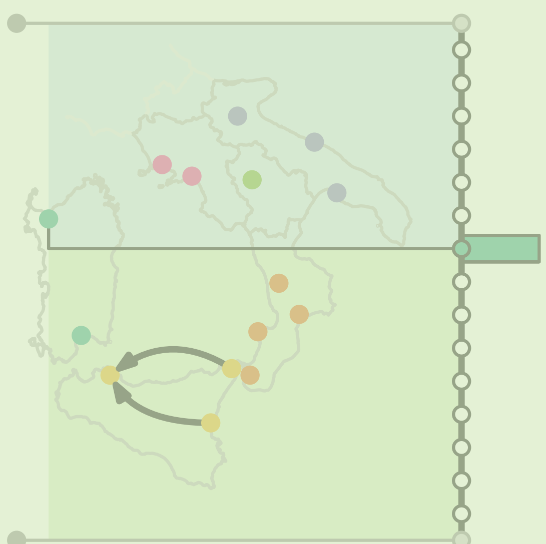
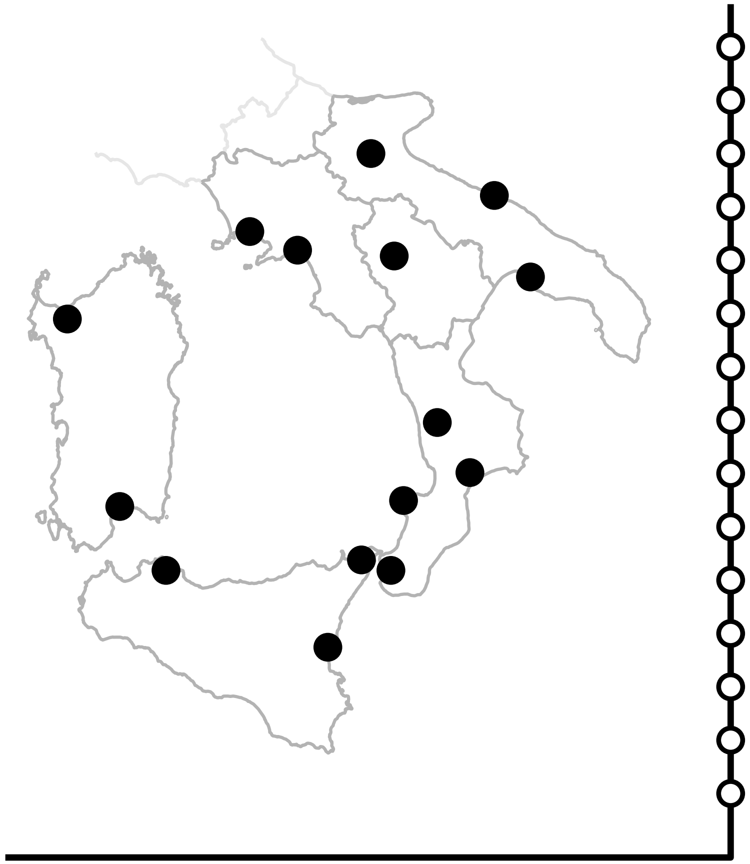


Poly-Time

A Useful Observation



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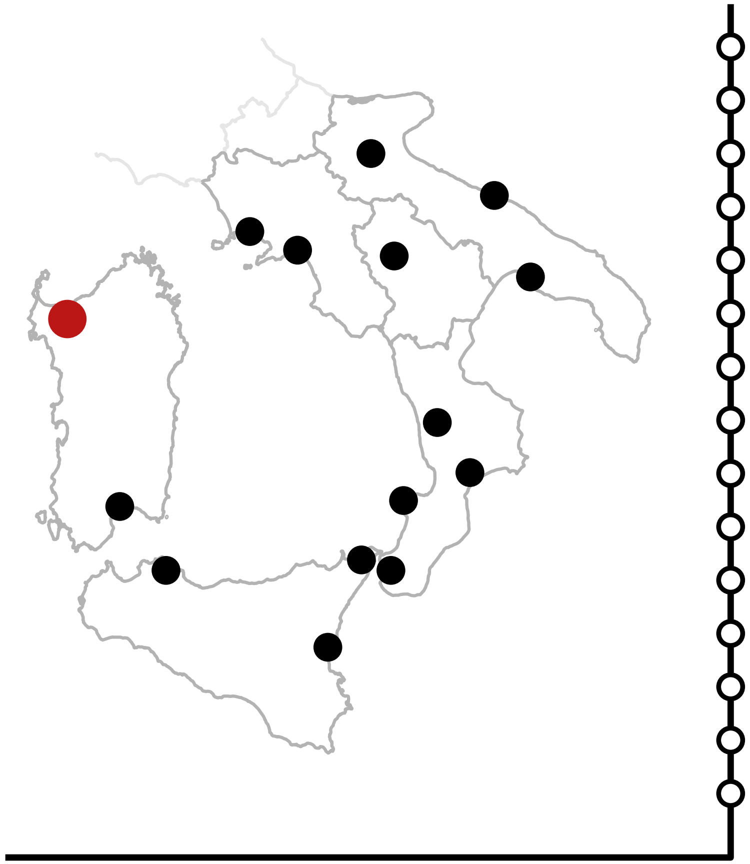

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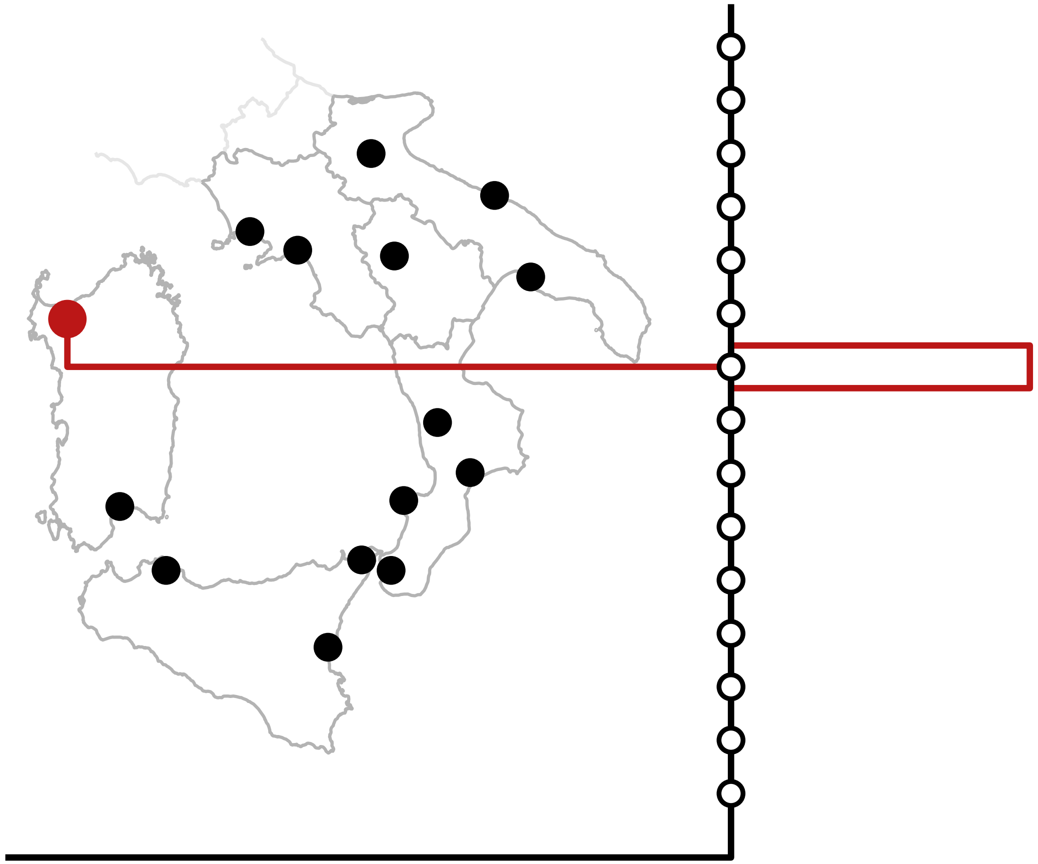

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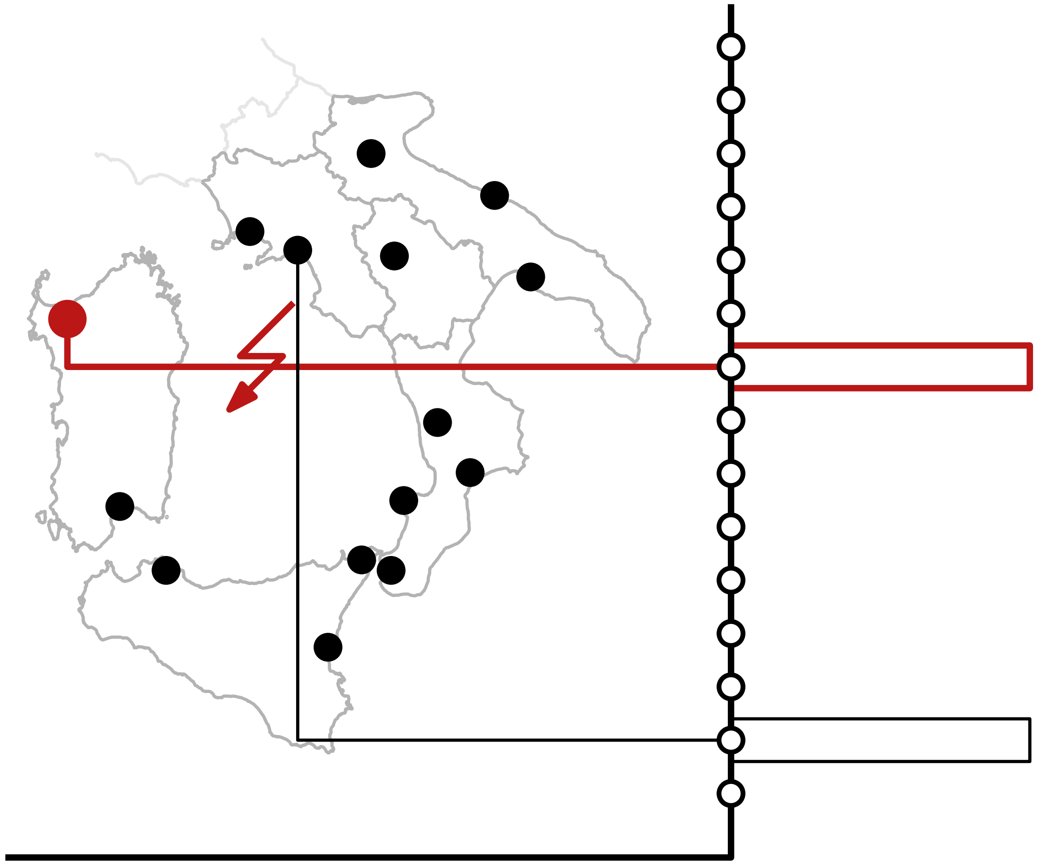

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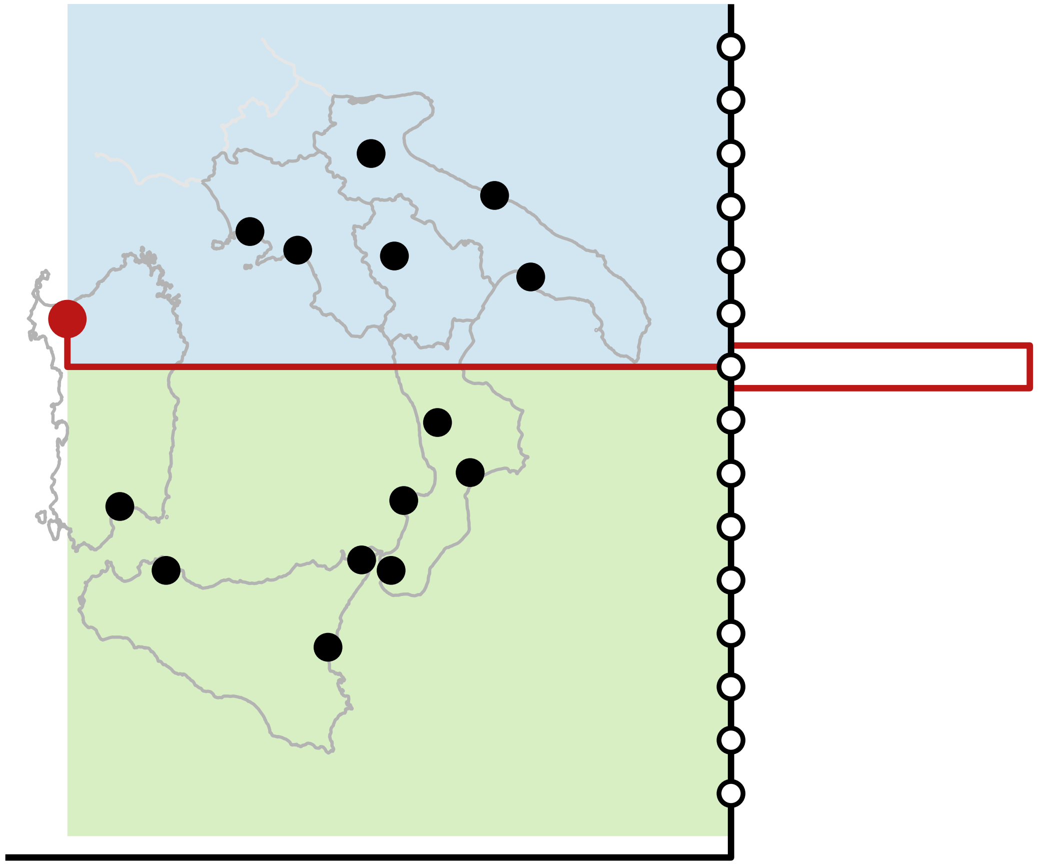

Observation:
The leader of the leftmost site splits an instance into two **independent** sub-instances.

[Benkert et al., JGAA 2009]

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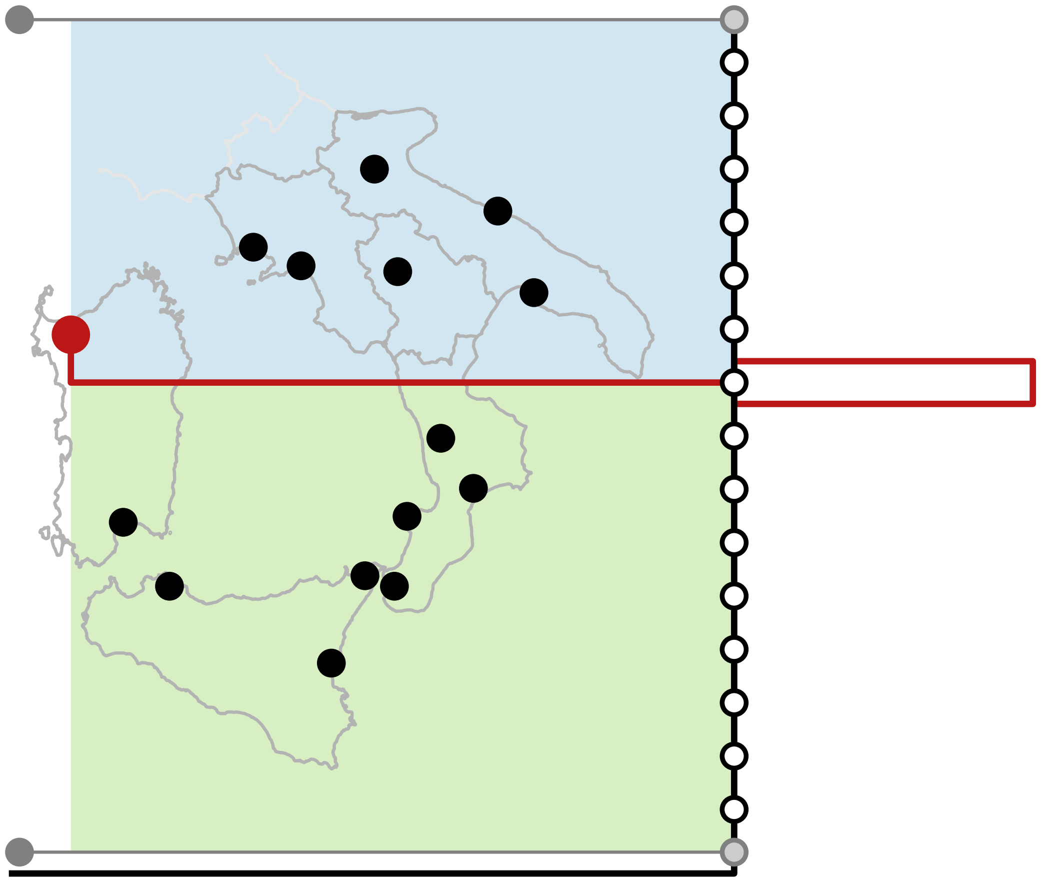

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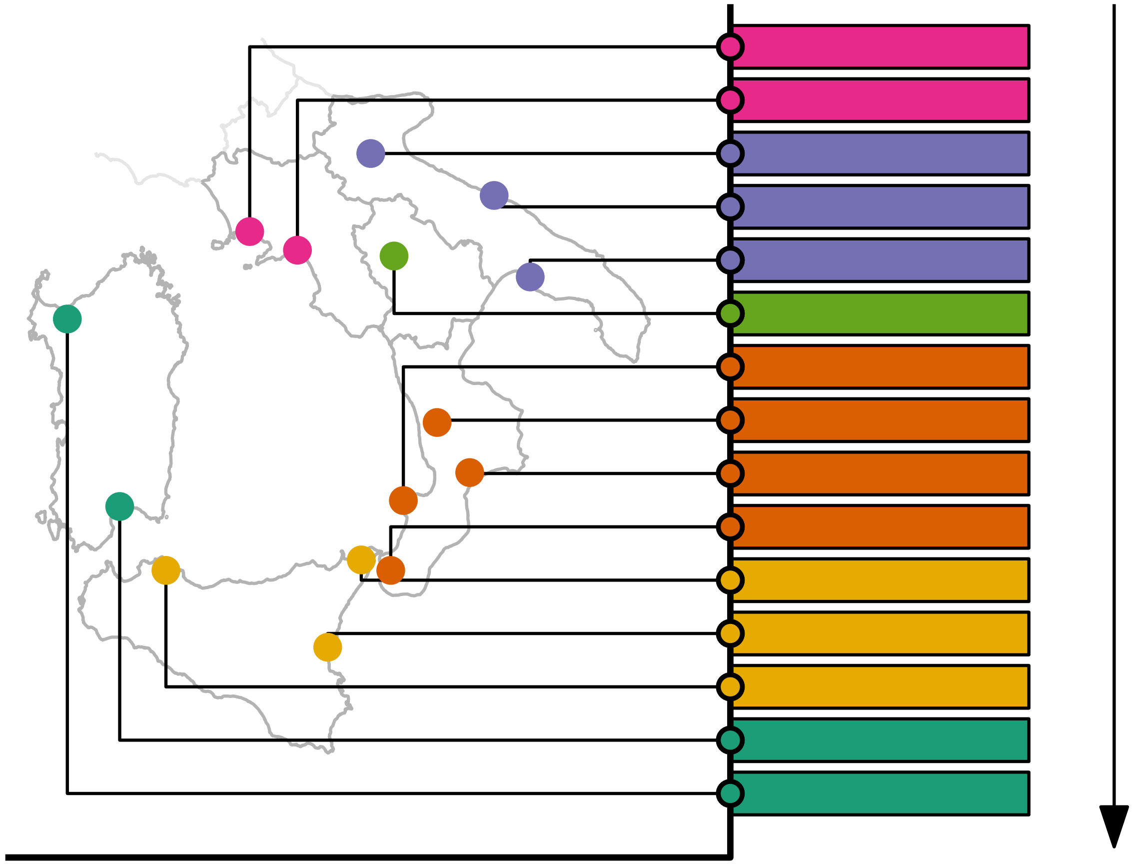
Encoding the Constraints: PQ-A-Graphs



π:

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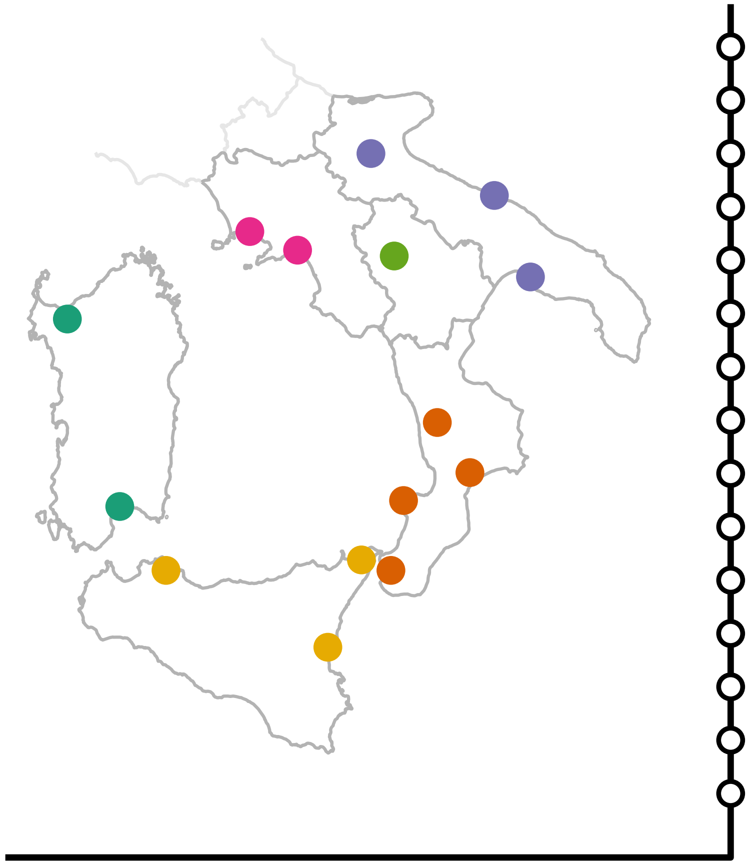
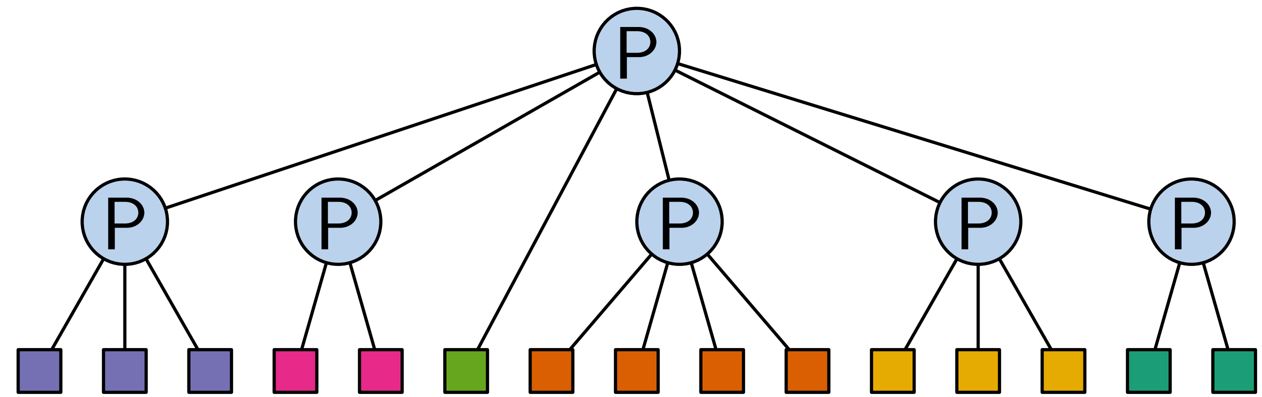


PQ-A-Graph: **PQ**-Tree



[Booth and Lueker, JCSS 1976]

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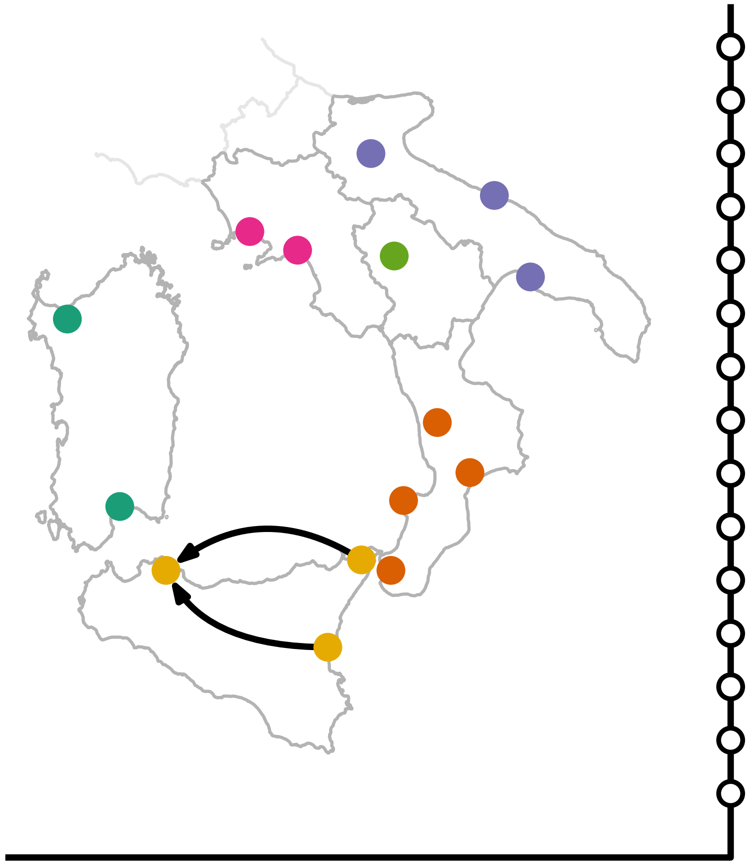
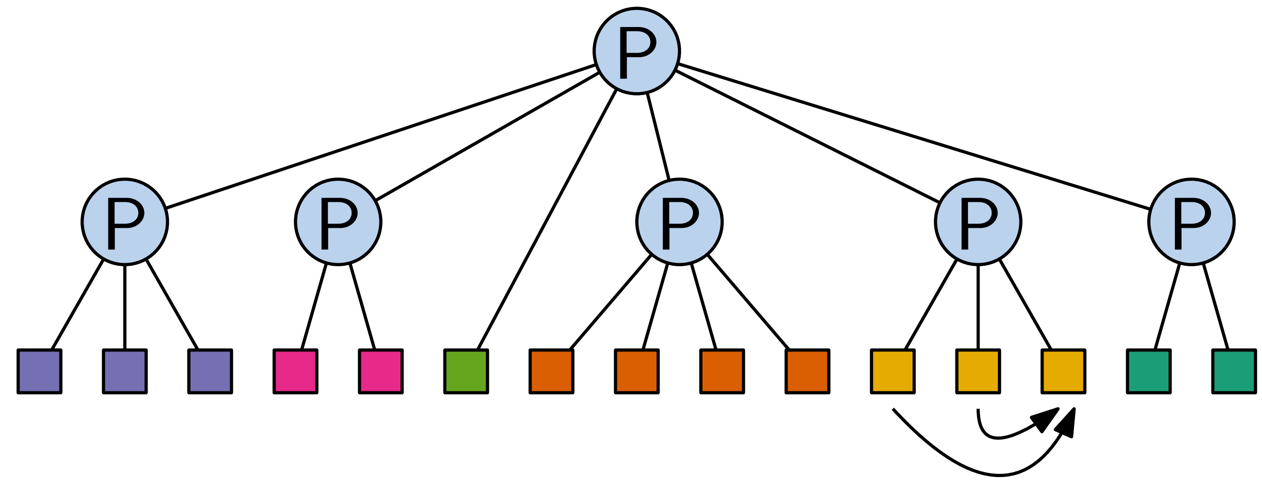


PQ-A-Graph: **PQ**-Tree + **A**rcs of a graph



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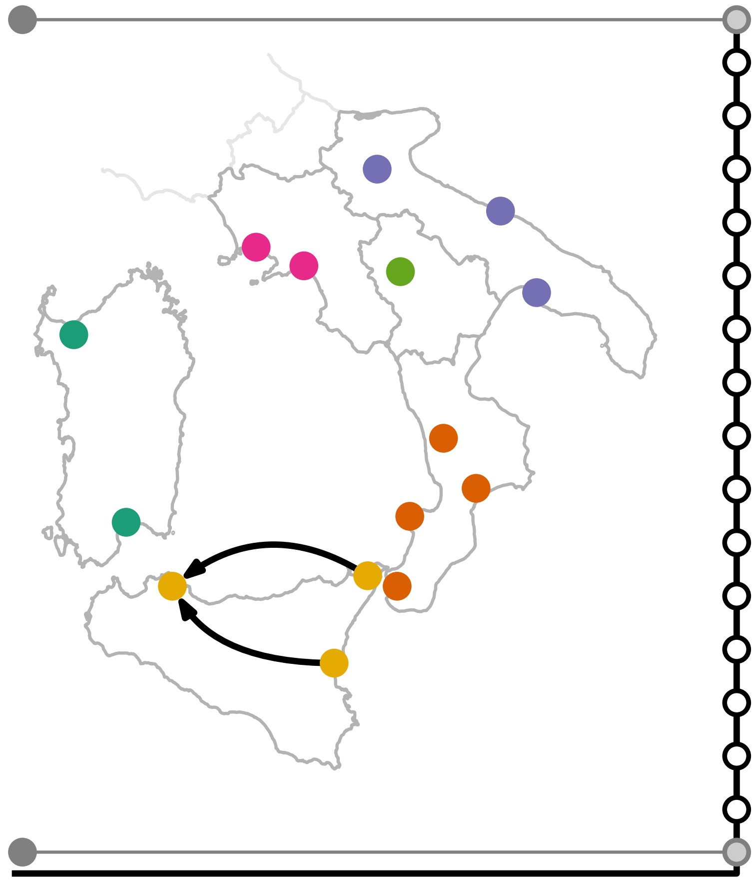
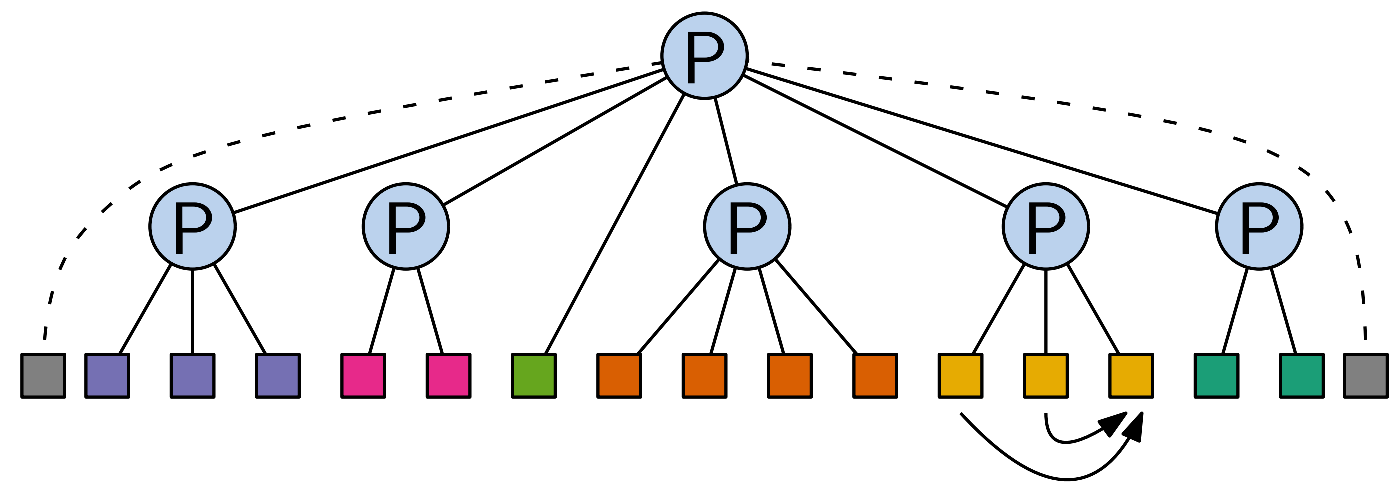
Dynamic Programming Algorithm for Fixed Ports





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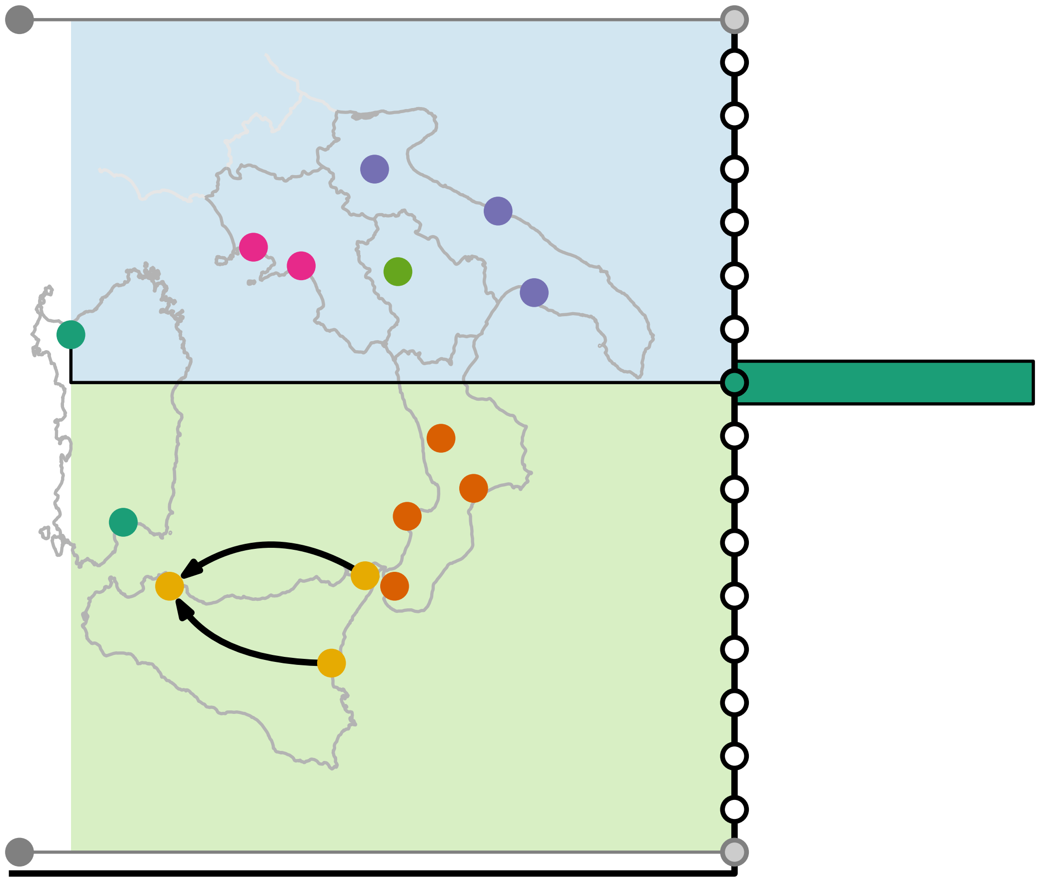

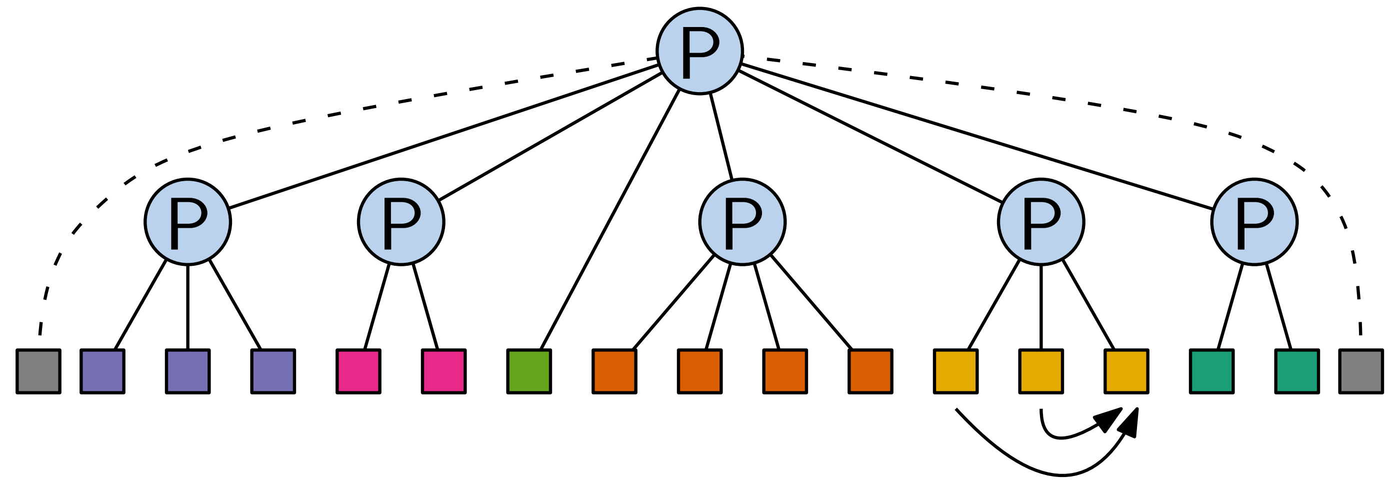

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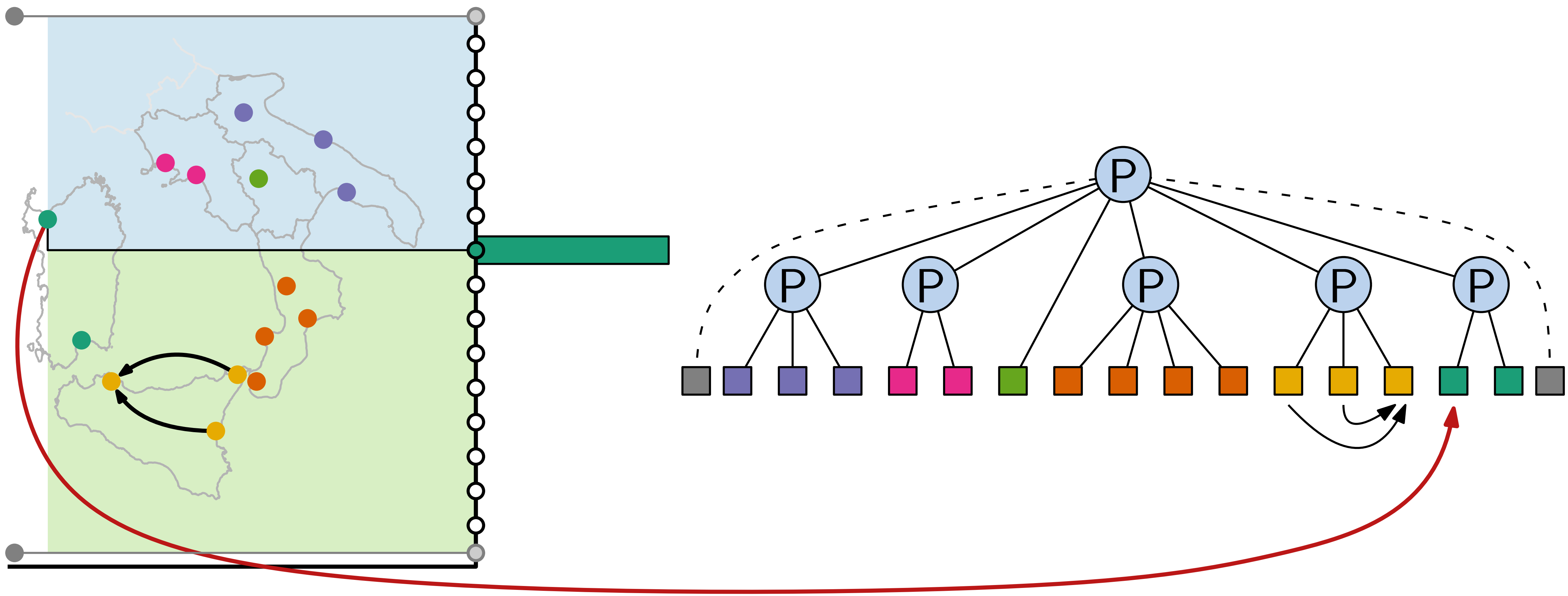

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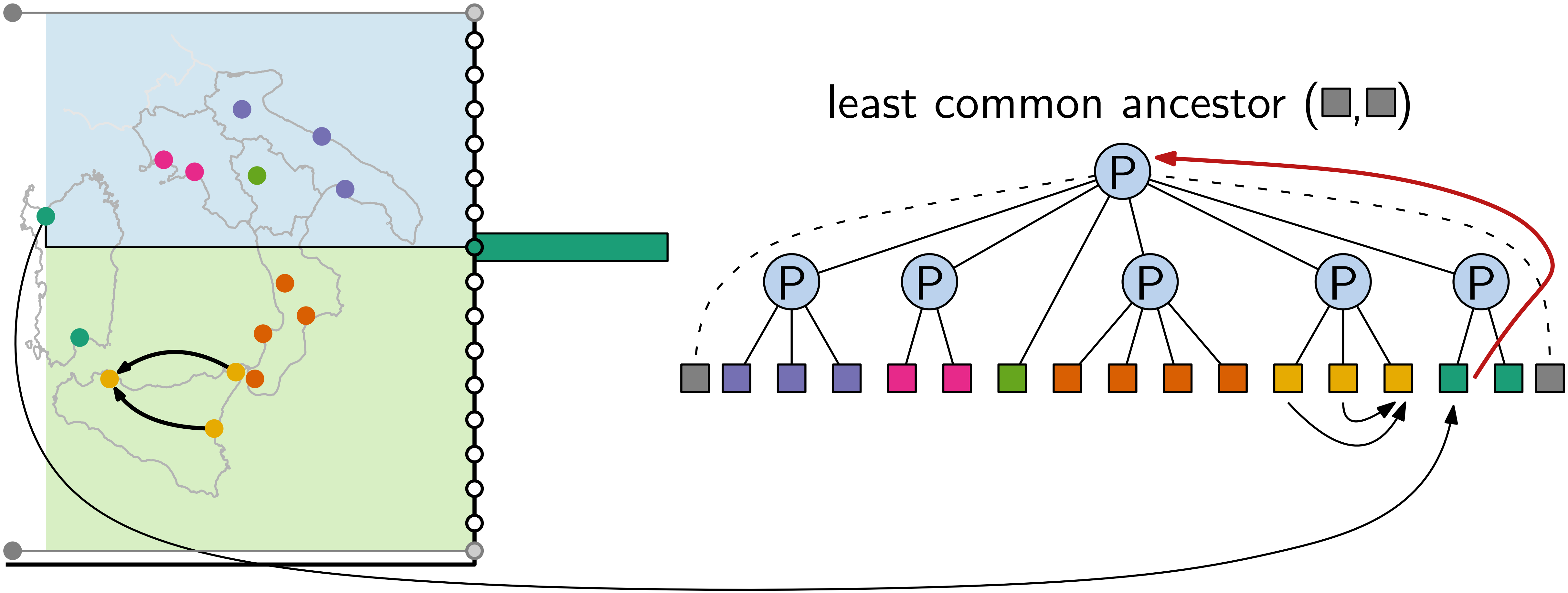

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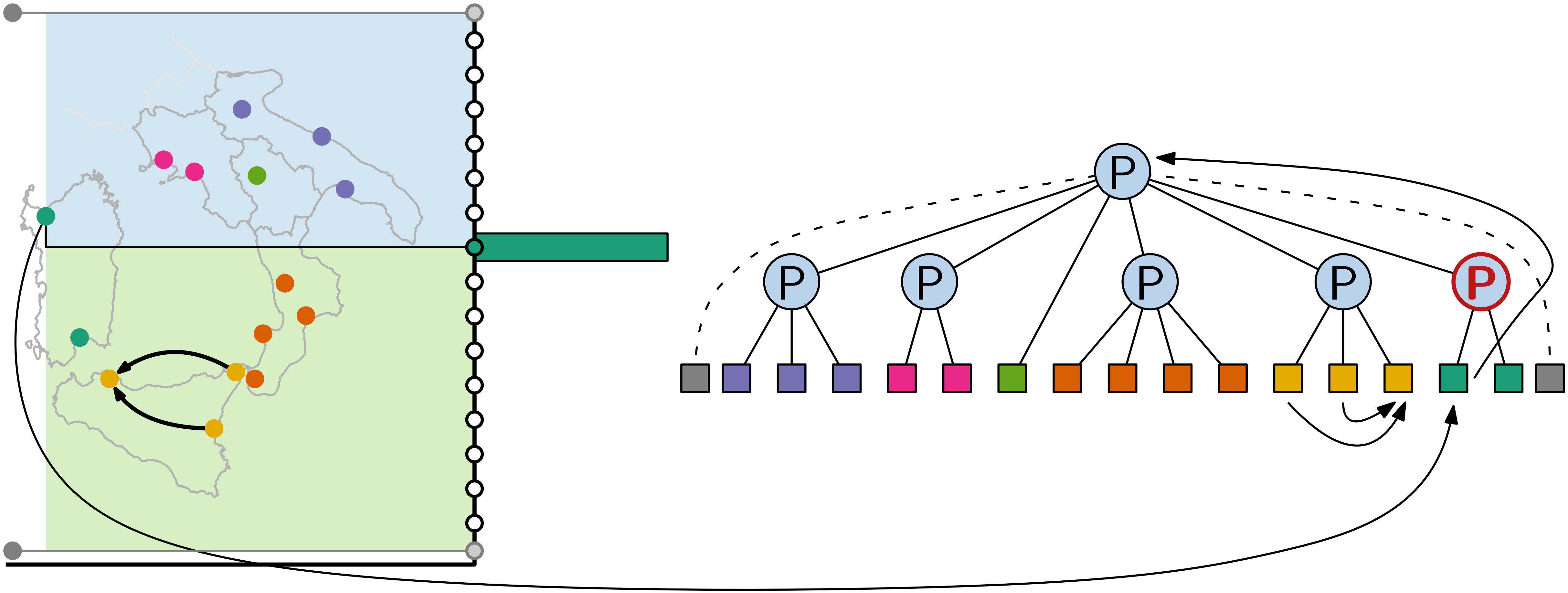

Does the candidate leader respect the constraints at the node?

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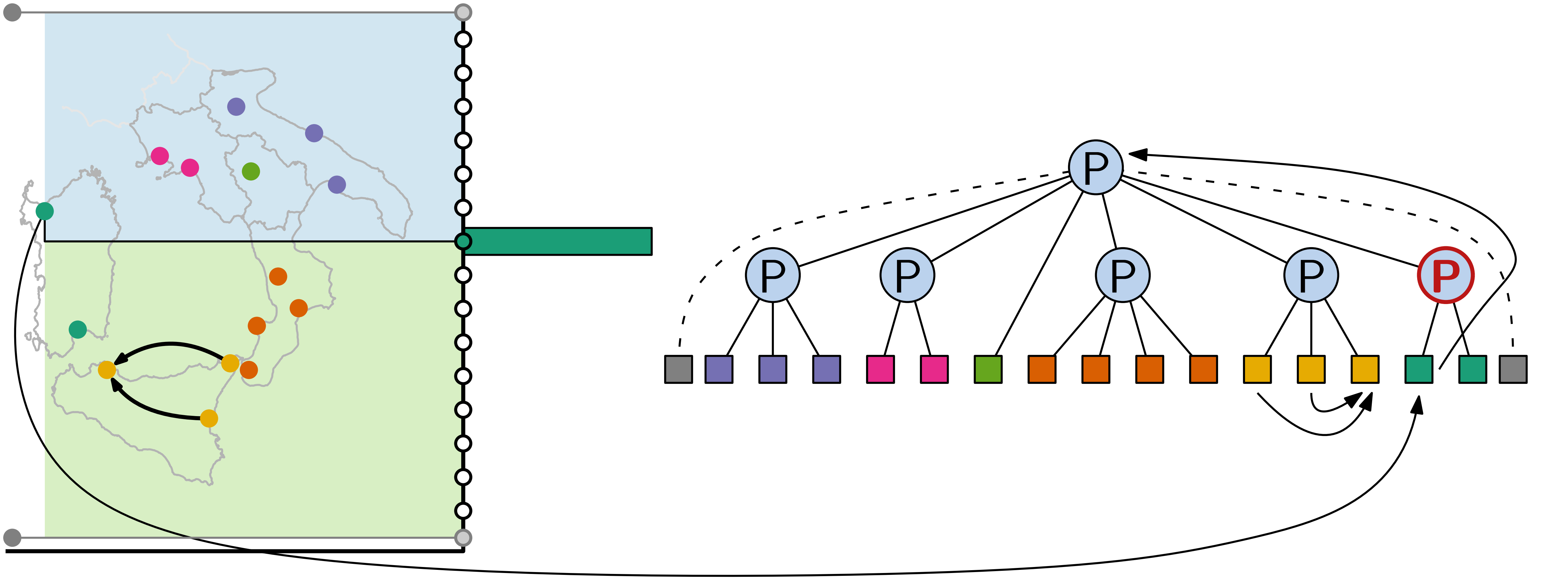

Does the candidate leader respect the constraints at the node?

(Can we reorder the children of the node so that they are on the correct side of the leader and the arcs run from left to right?)

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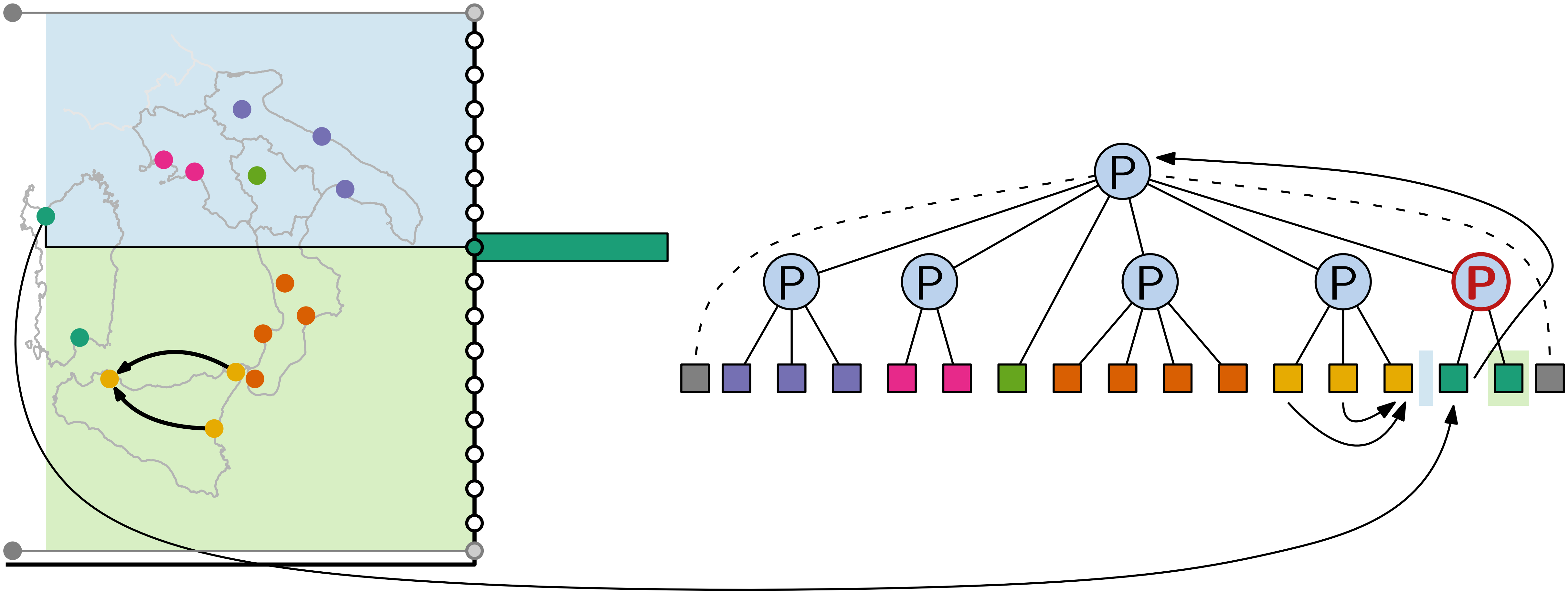

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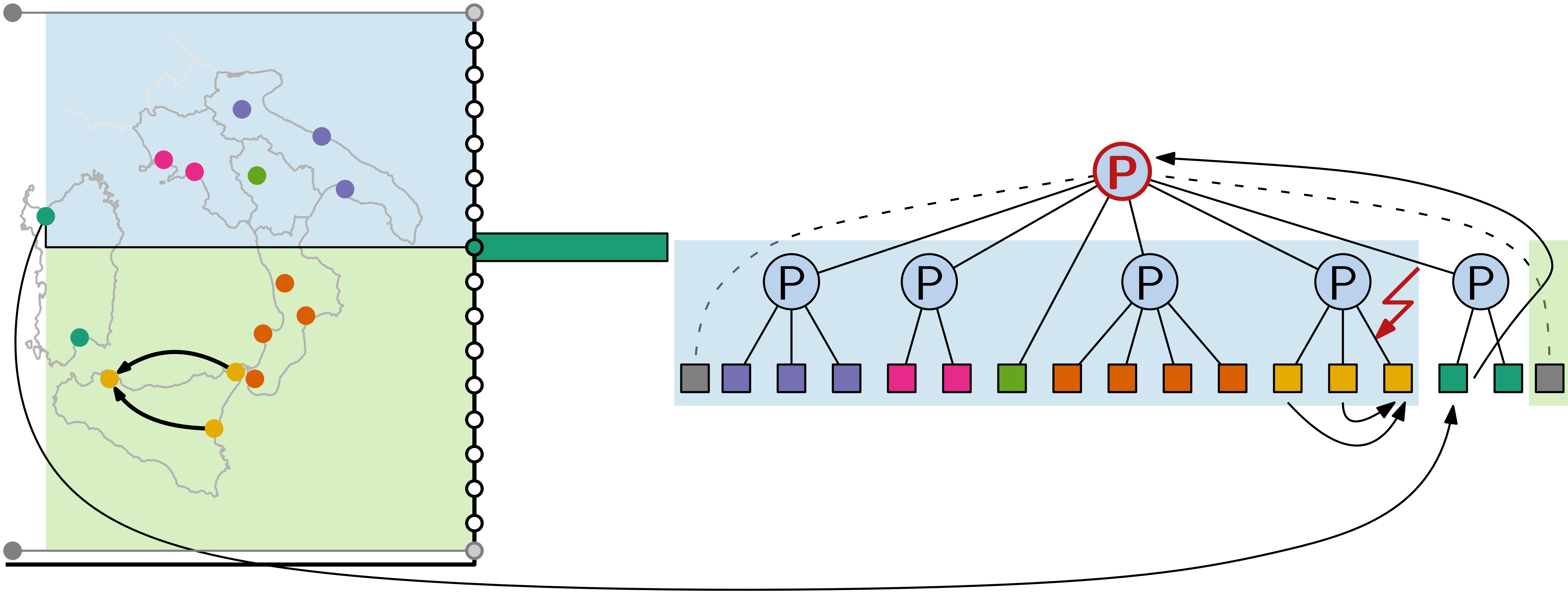

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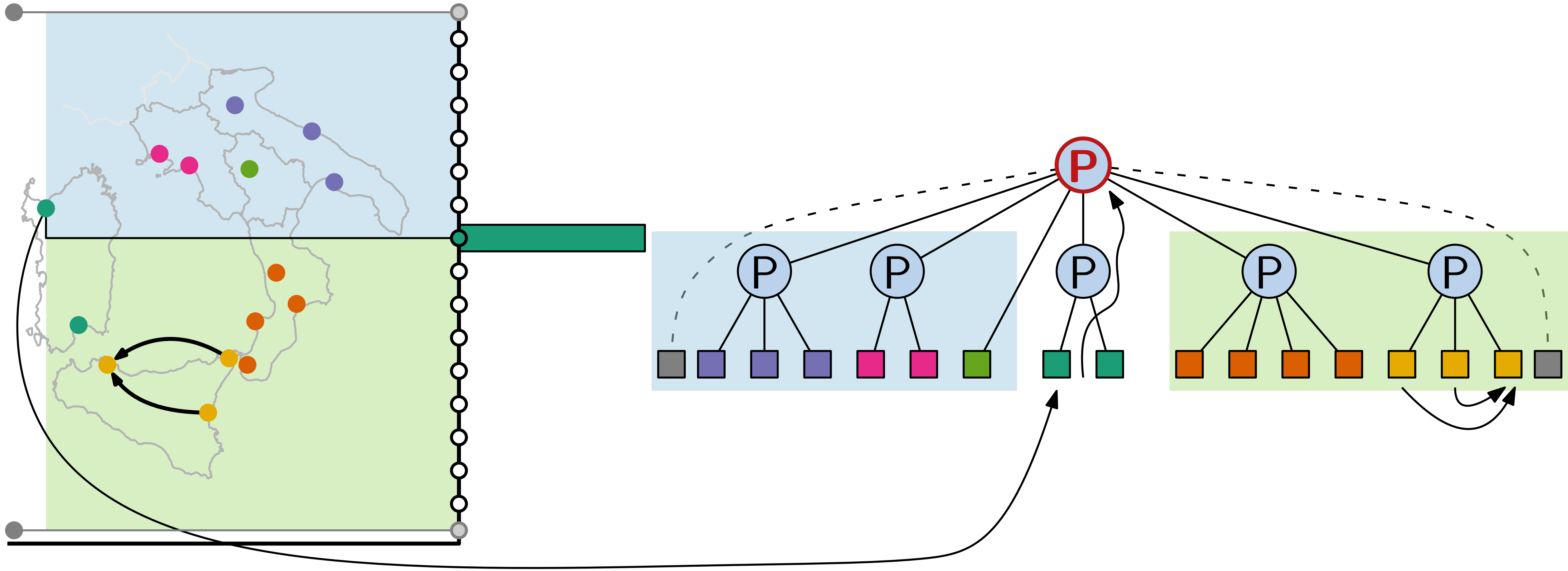

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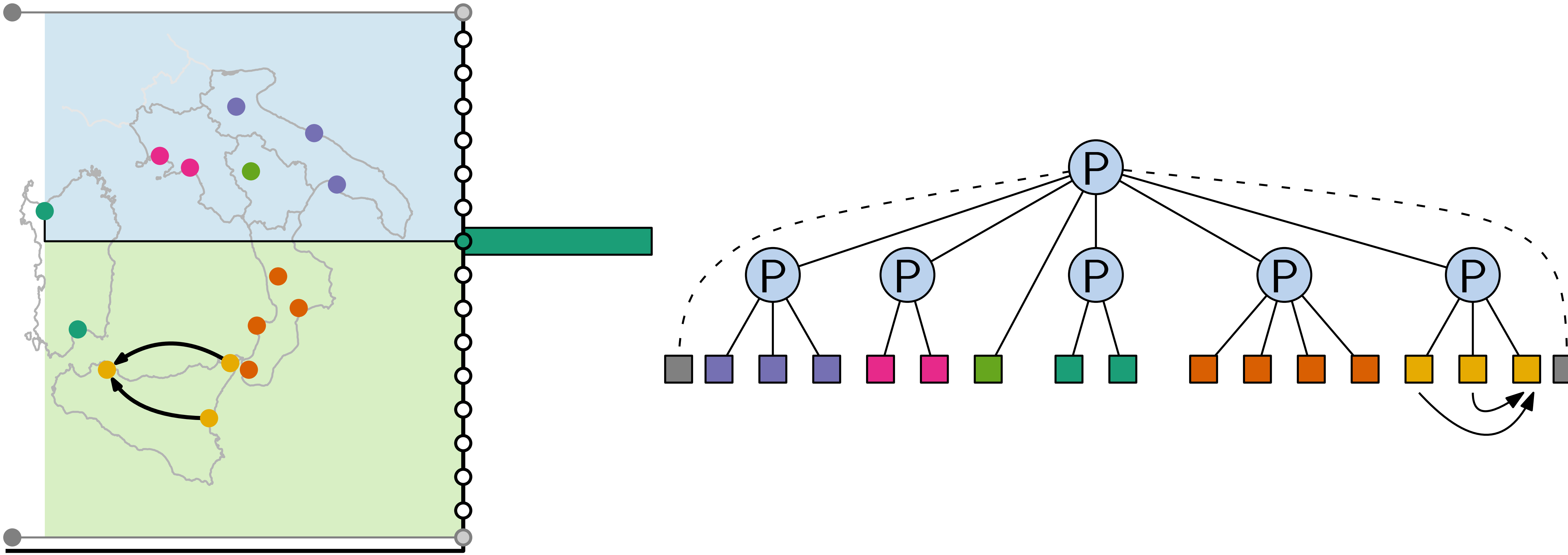

. . . evaluate the created sub-instances . . .

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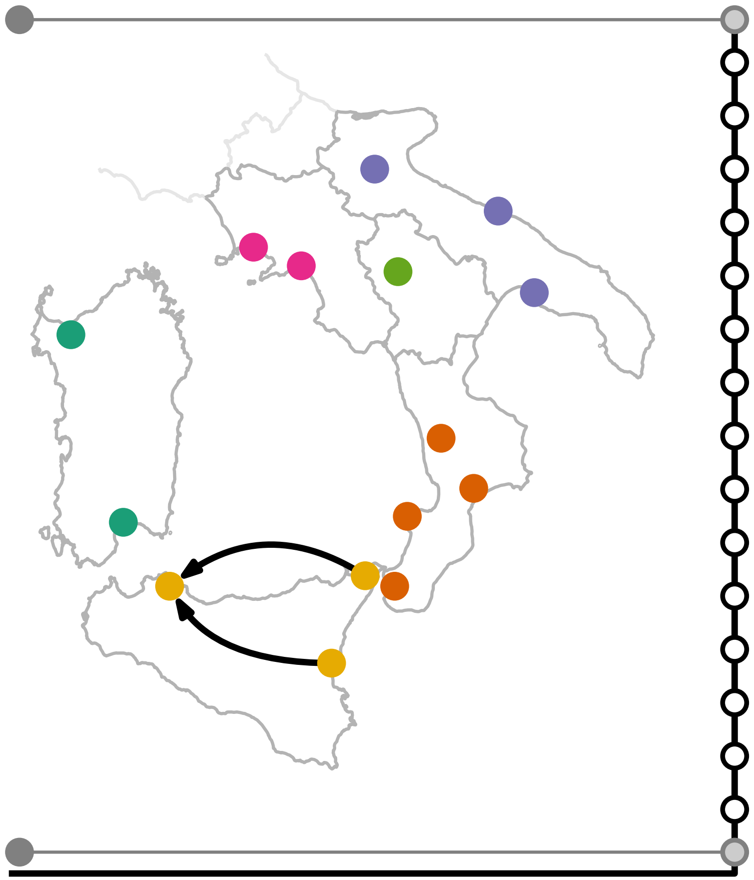

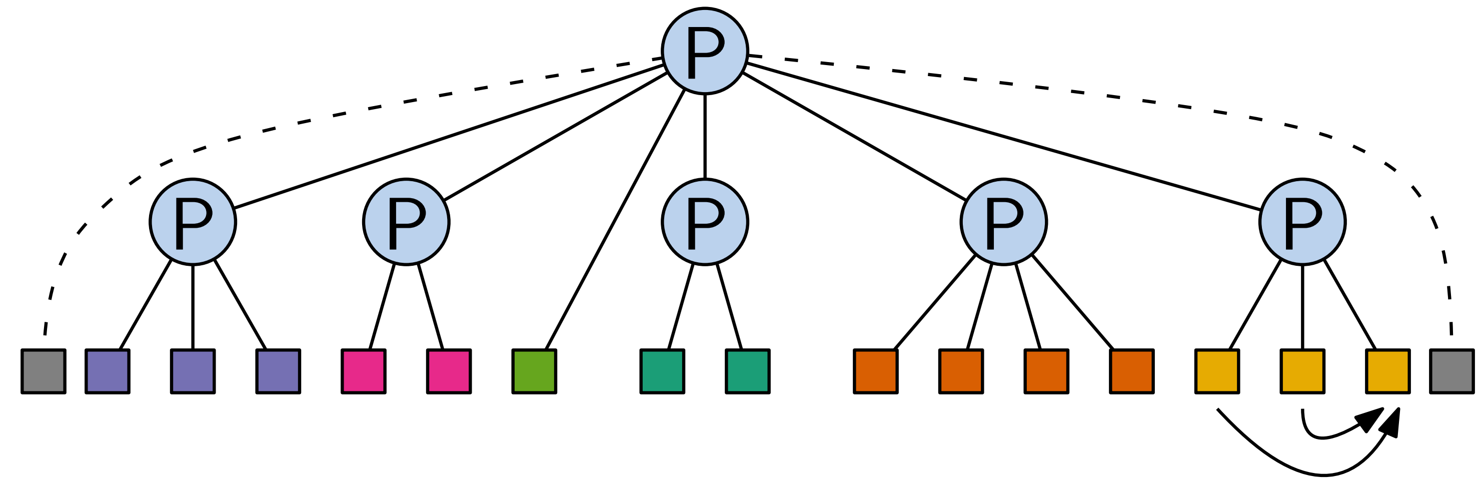

Theorem:
1-CBL, with fixed ports, can be solved in
$\mathcal{O}\left(|\mathcal{S}|^5|\mathcal{P}|^3\log|\mathcal{P}| + |\mathscr{G}| + \sum_{\mathcal{G}\in\mathscr{G}}|\mathcal{G}|\right)$ time and $\mathcal{O}\left(|\mathcal{S}|^2|\mathcal{P}|^2\right)$ space.

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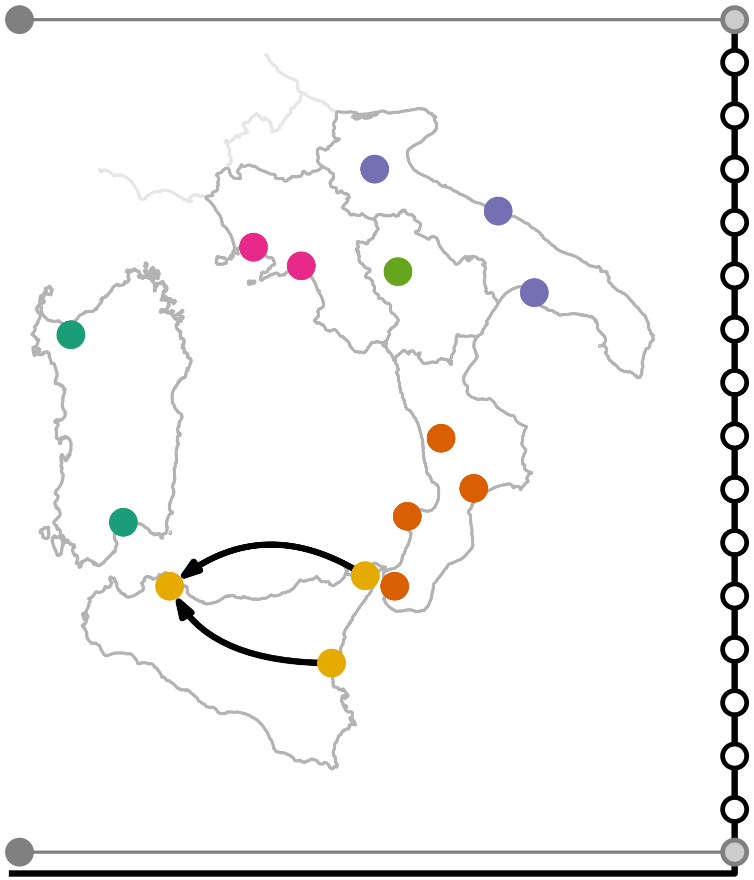

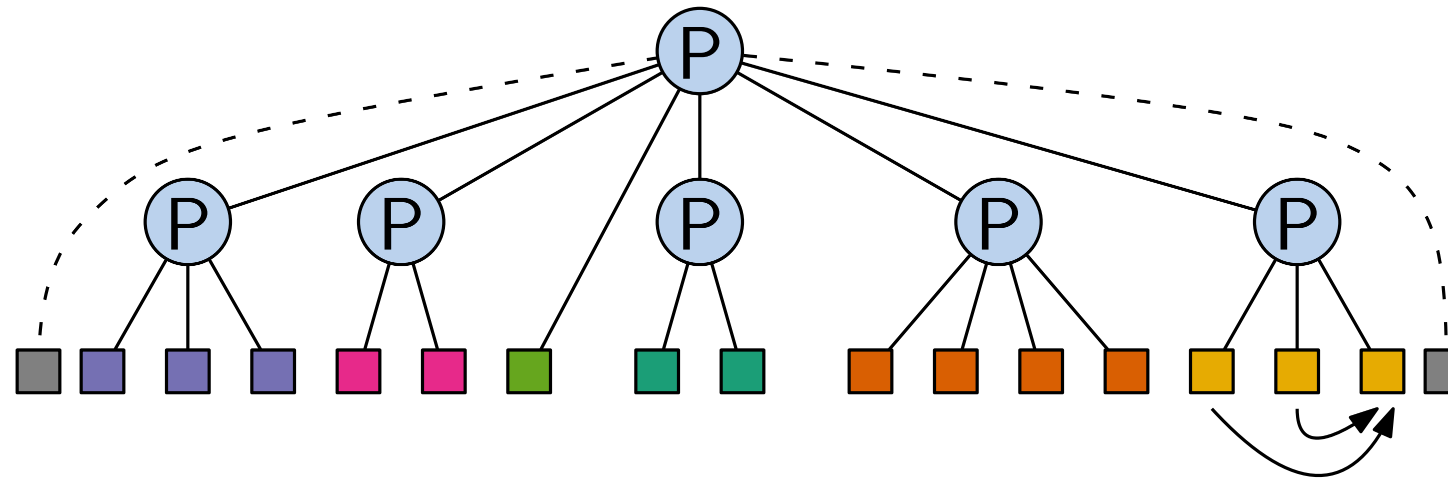

Theorem:
1-CBL, with fixed ports, can be solved in
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Conclusion

Grouping and ordering constraints occur in real-world labelings

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. . . but have barely been considered so far.

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Constrained One-Sided Boundary Labeling

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CONSTRAINED ONE-SIDED BOUNDARY LABELING

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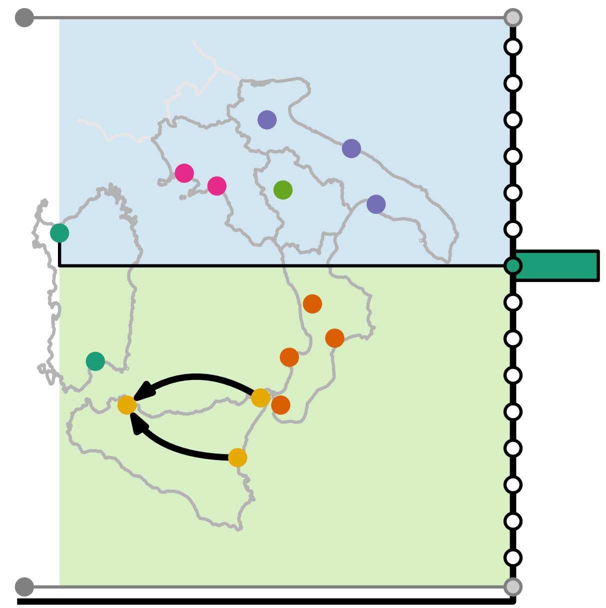
... can be solved in polynomial-time for fixed ports

... can be solved in polynomial-time for uniform-height labels



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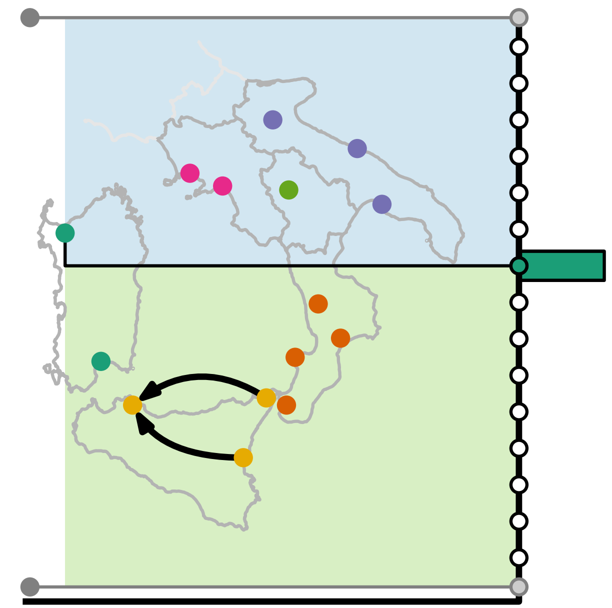

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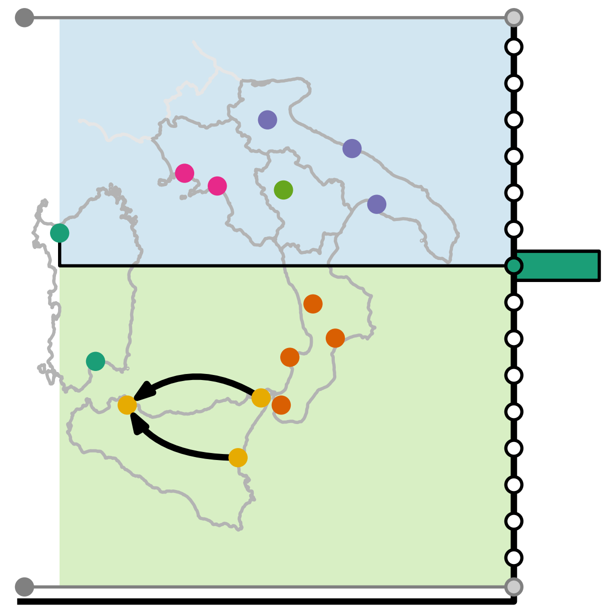

Many more interesting open questions

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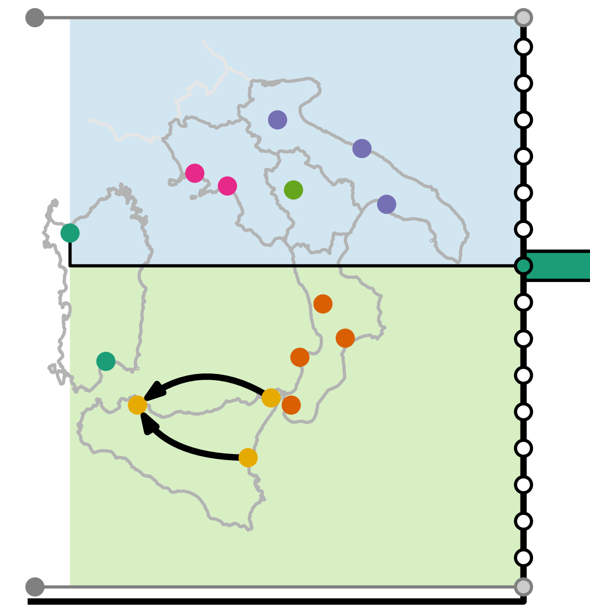

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Reduce the running time of our algorithm (heuristics, . . .)

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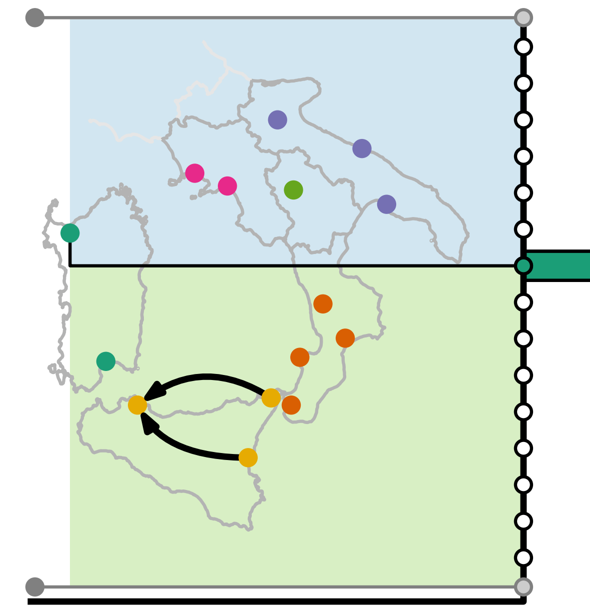

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Semantic soft constraints

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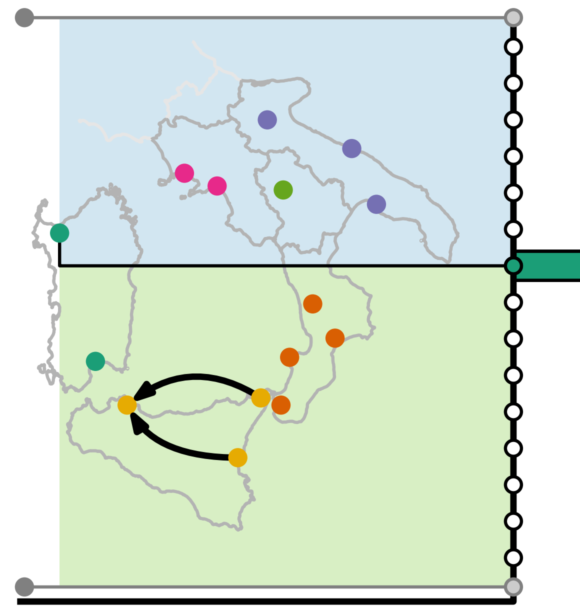

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Semantic soft constraints

Thank you for your attention!

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