RUB

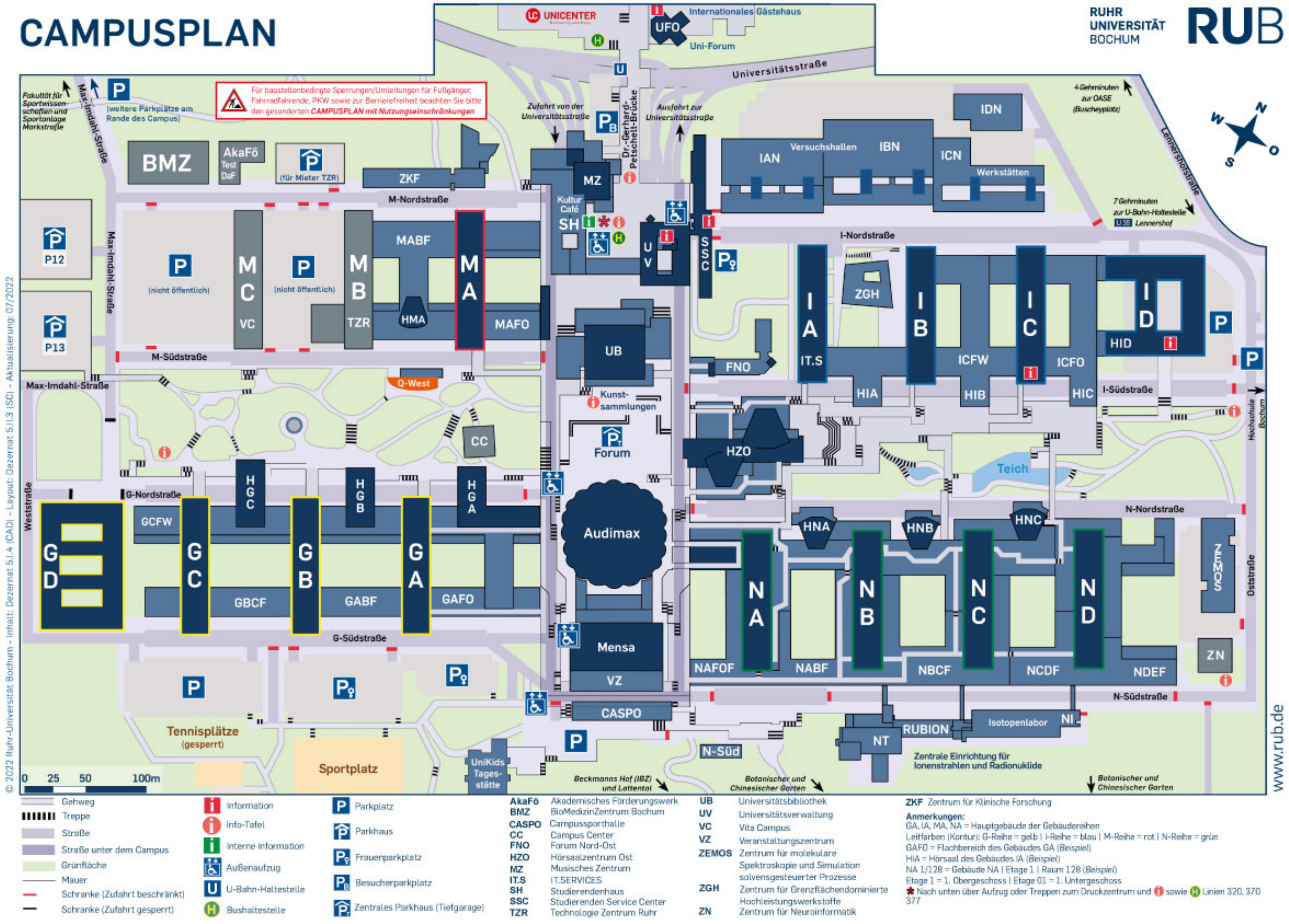
Hardness and modifications of the weak graph distance

13.03.2024

Maike Buchin, **Wolf Kißler**

Distance measures for immersed graphs

Representations of geometric networks:



Ruhr-Universität Bochum, Dezernat 5.I.4 & Dezernat 5.II.3

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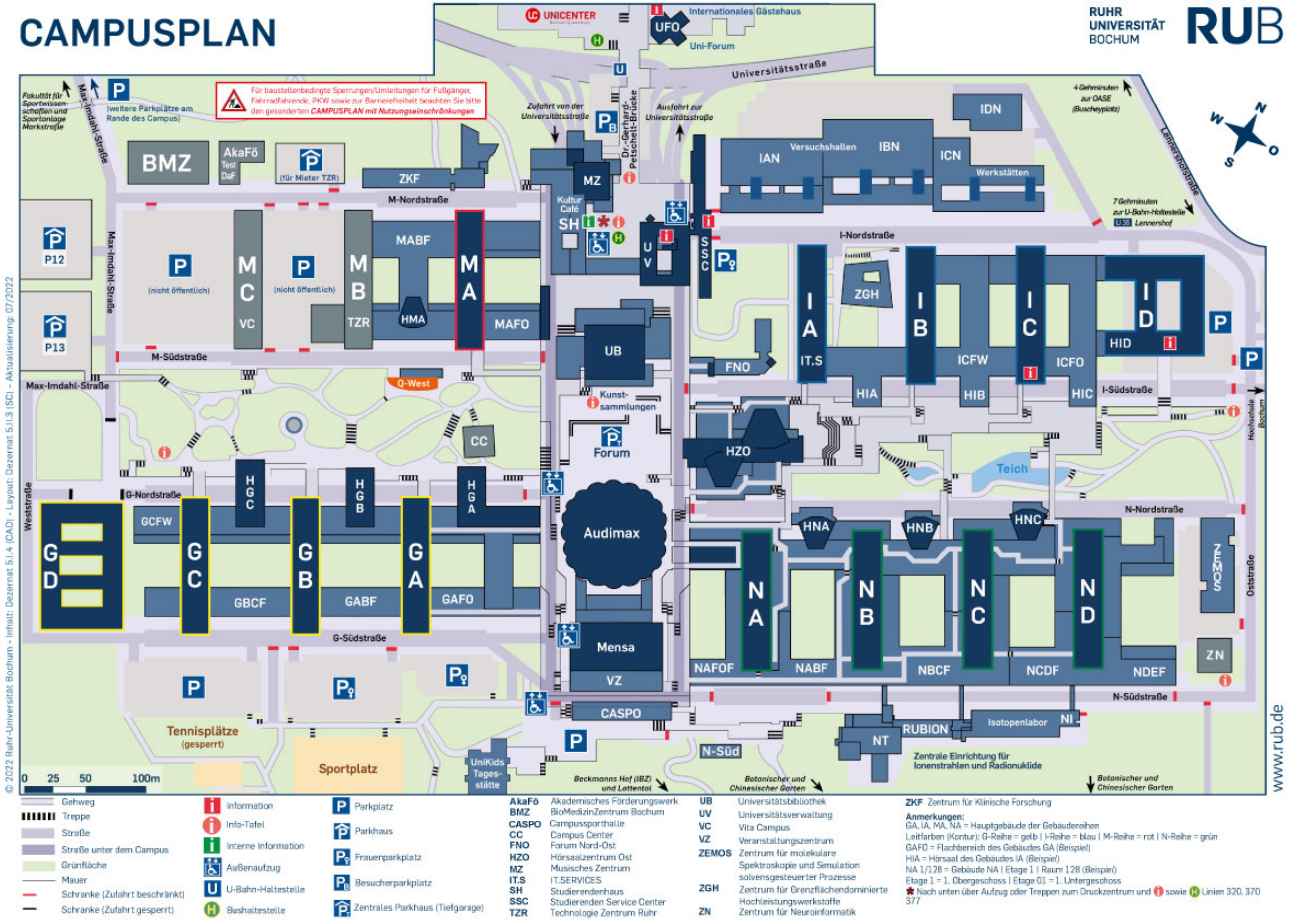

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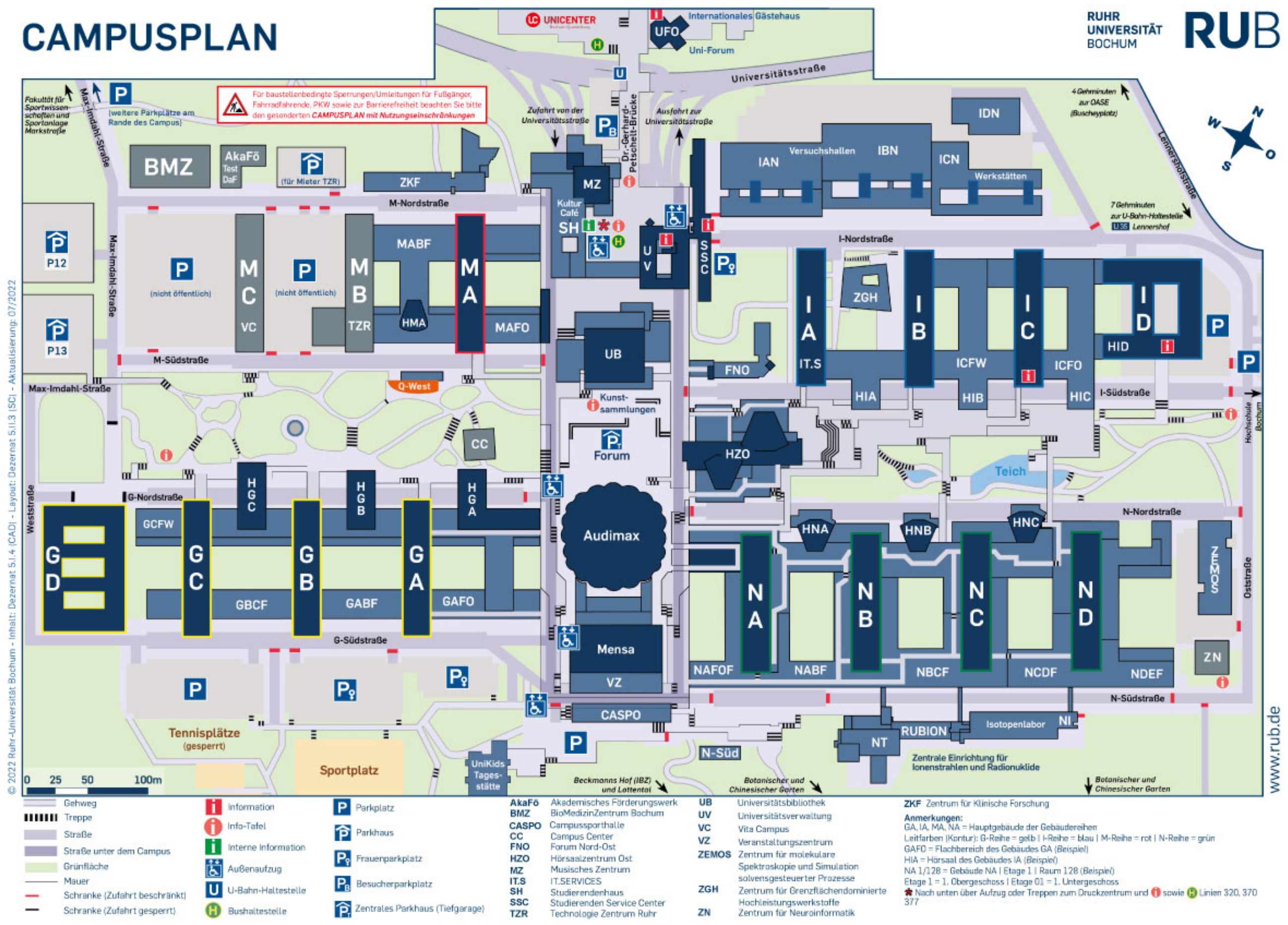

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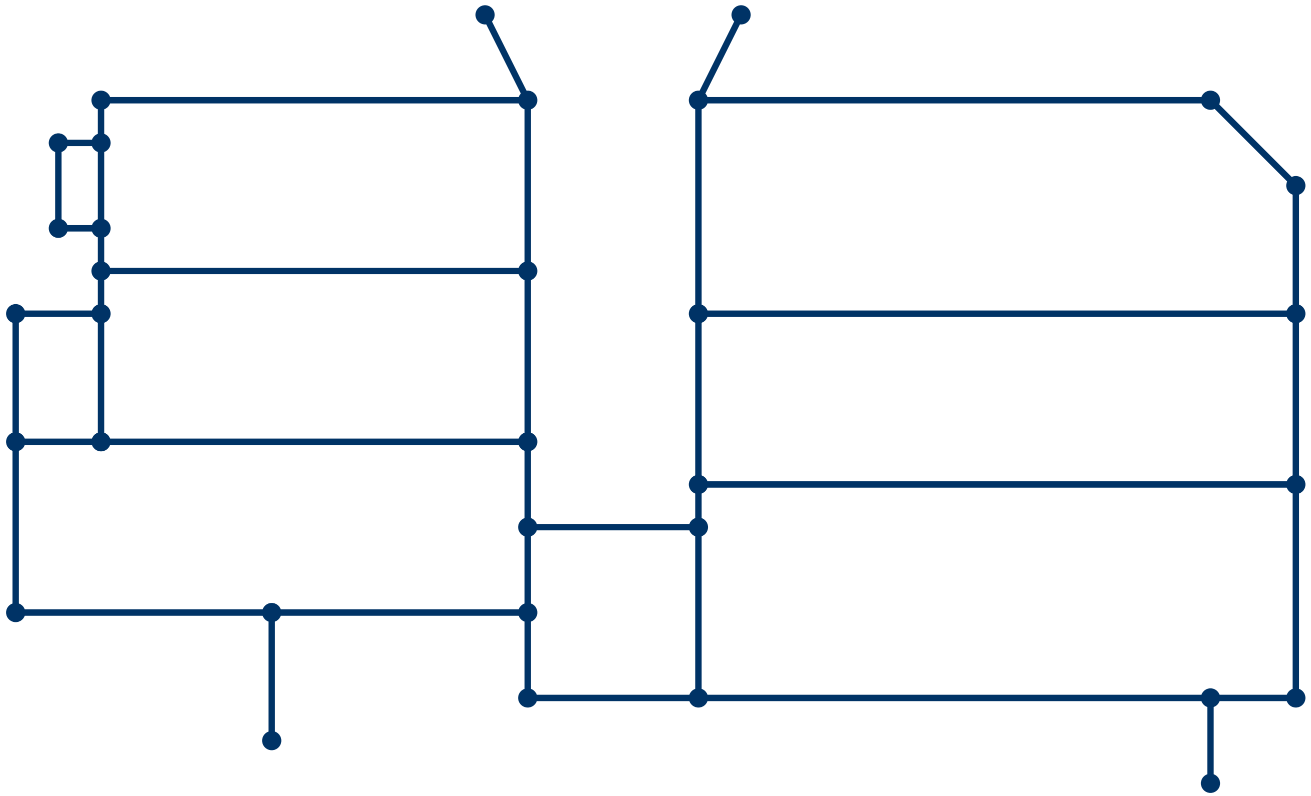

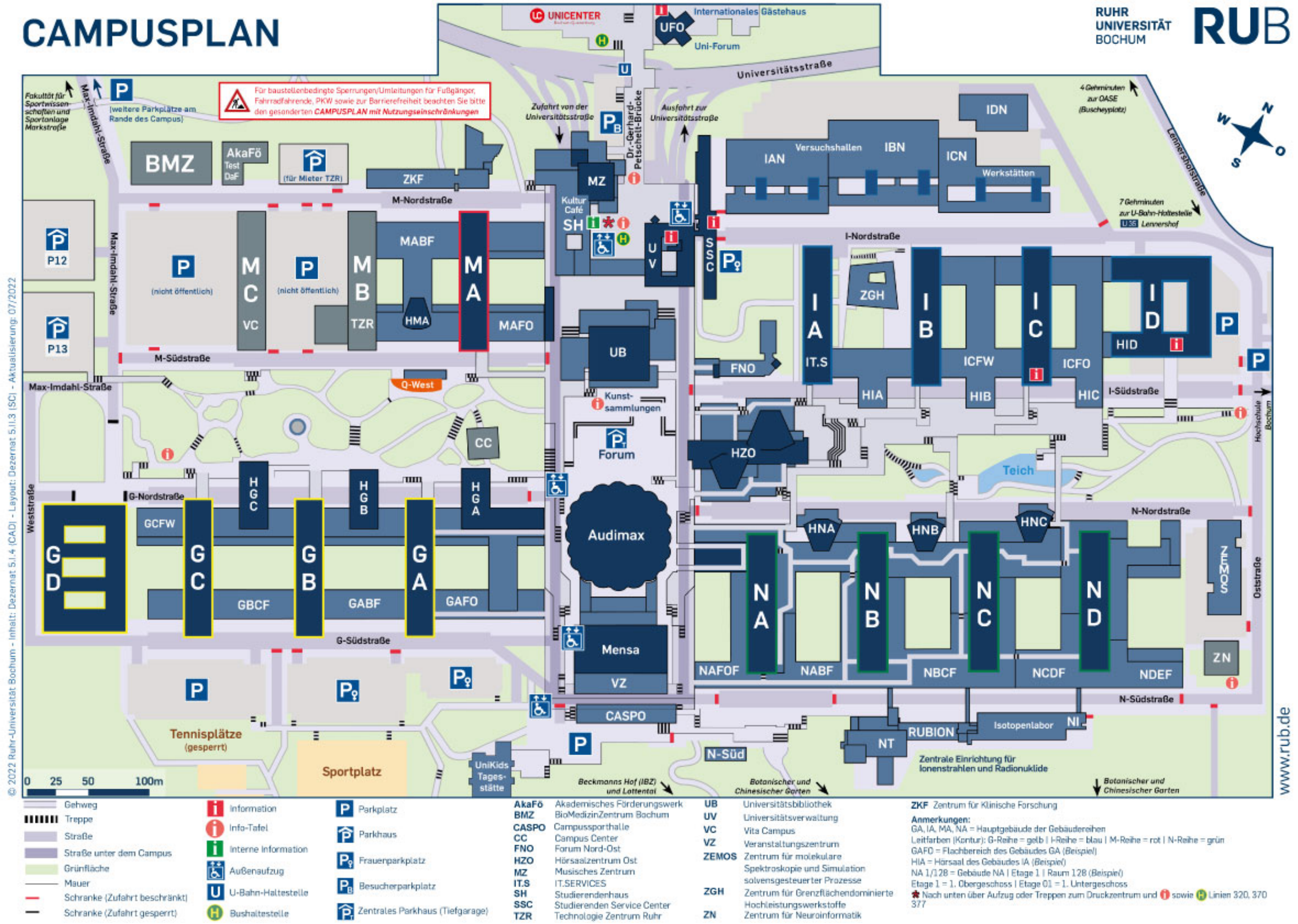

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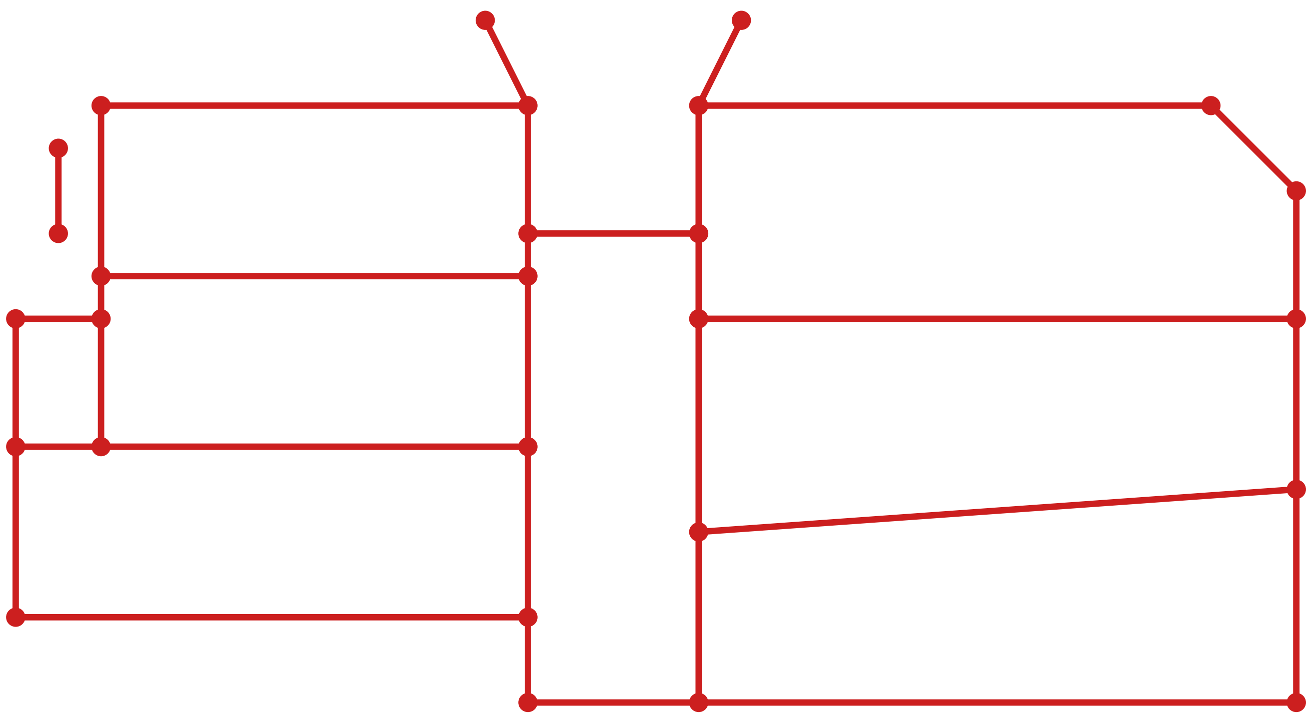

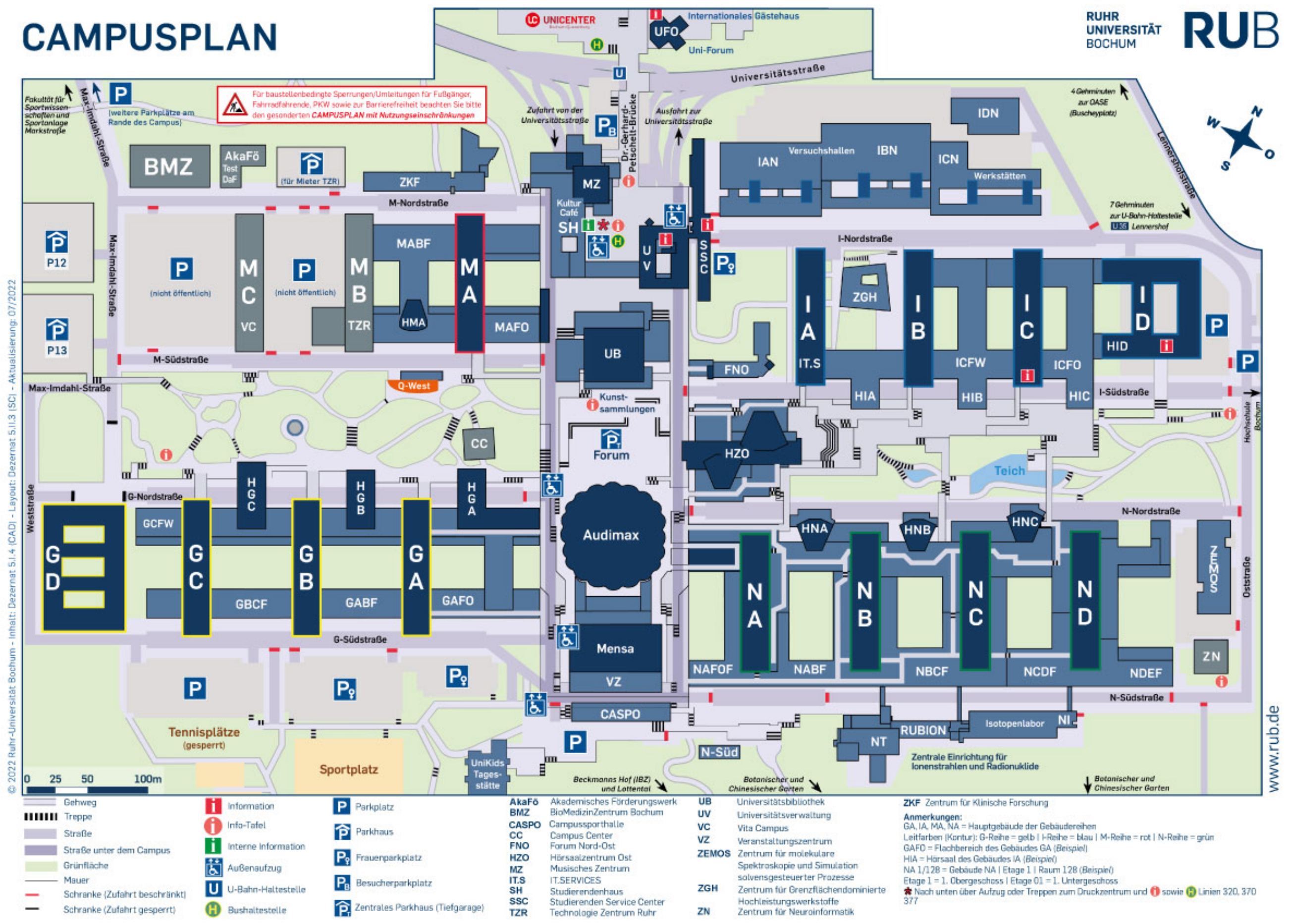

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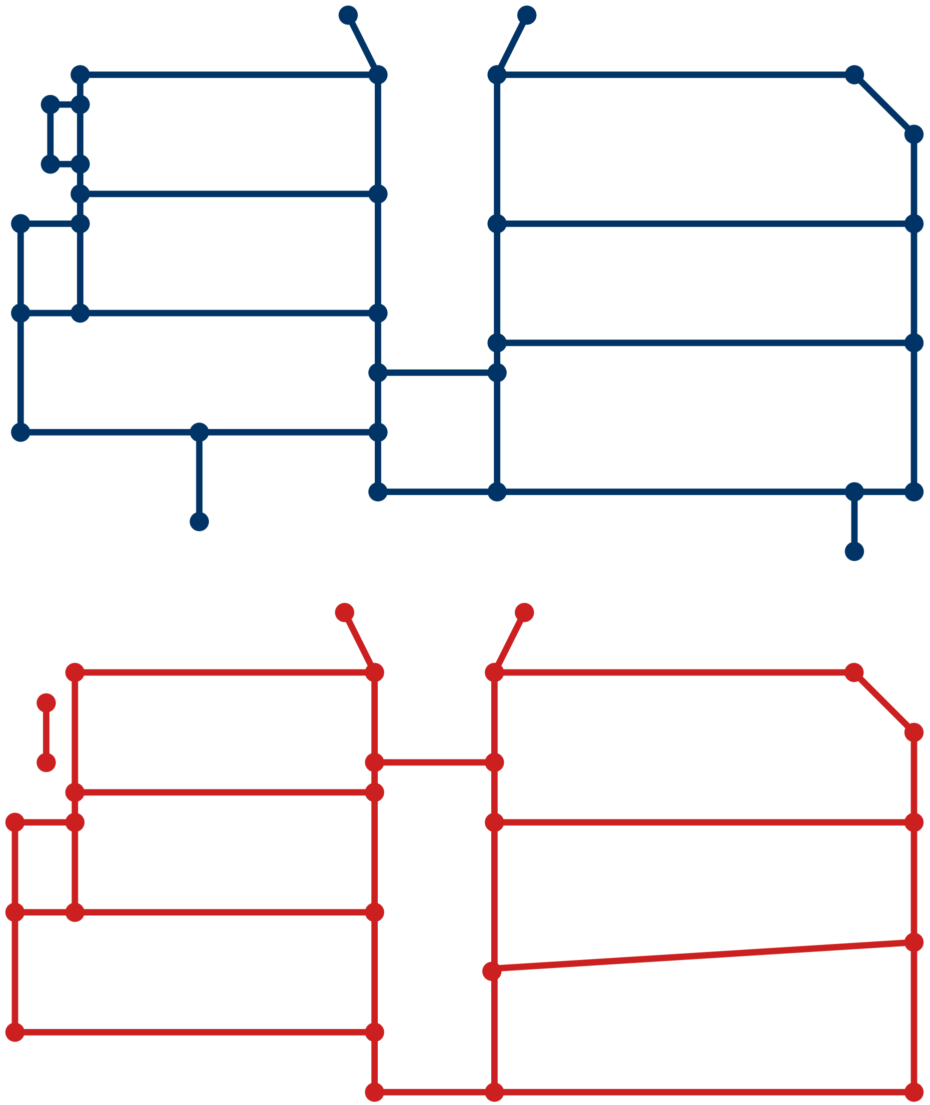

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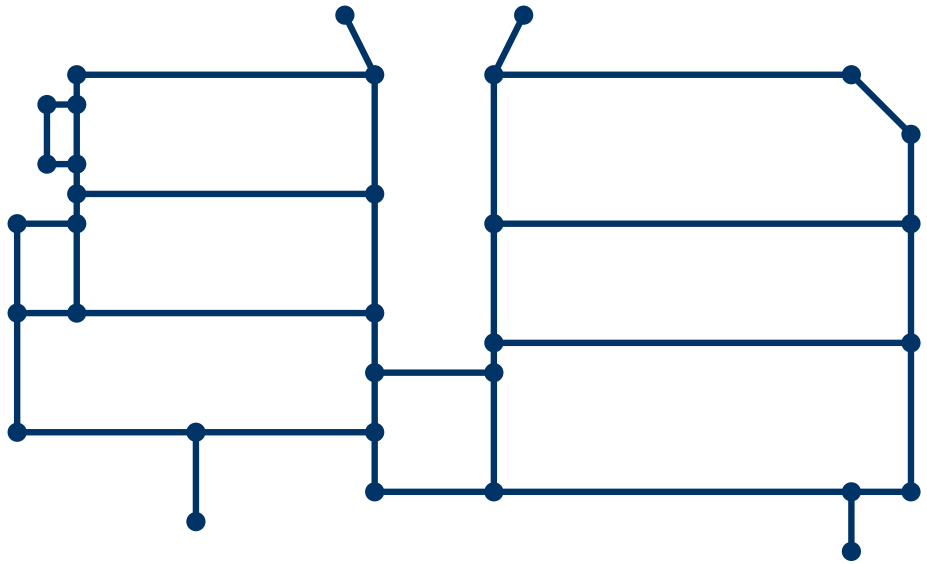
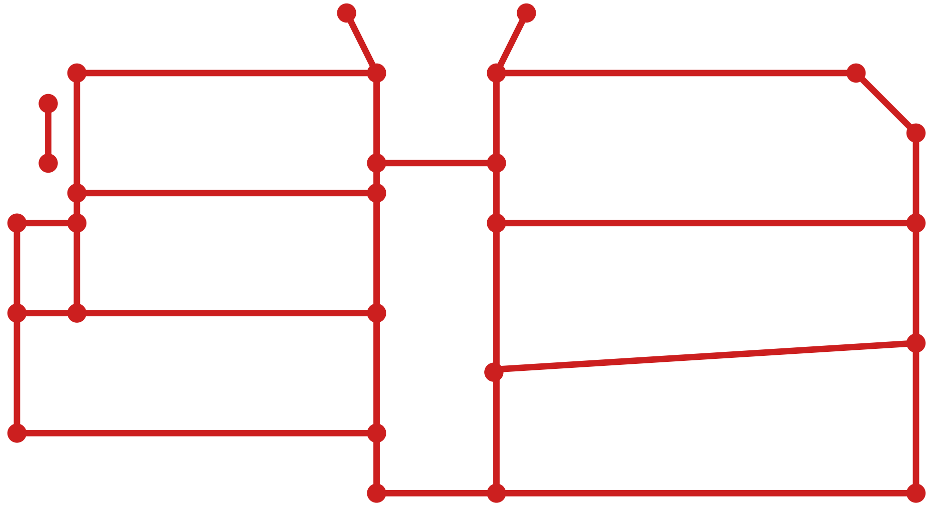


Given two representations of networks: how to compare them?

- Many approaches: edit distances, Fréchet distance, traversal based distances, LPH based distances,...
- Here: Weak Graph Distance due to Akitaya et al.



→ Akitaya et al.: Distance measures for embedded graphs, CGTA 95, 2021.

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Recap: Weak Fréchet distance

RUB

Let $s_1, s_2\colon [0,1] \to \mathbb{R}^d$ be curves. Their *weak Fréchet distance* is defined by

$$\delta_{wF}(s_1, s_2) = \inf_{\alpha,\beta:[0,1]\to[0,1]} \max_{t\in[0,1]} d(s_1(\alpha(t)), s_2(\beta(t)))$$

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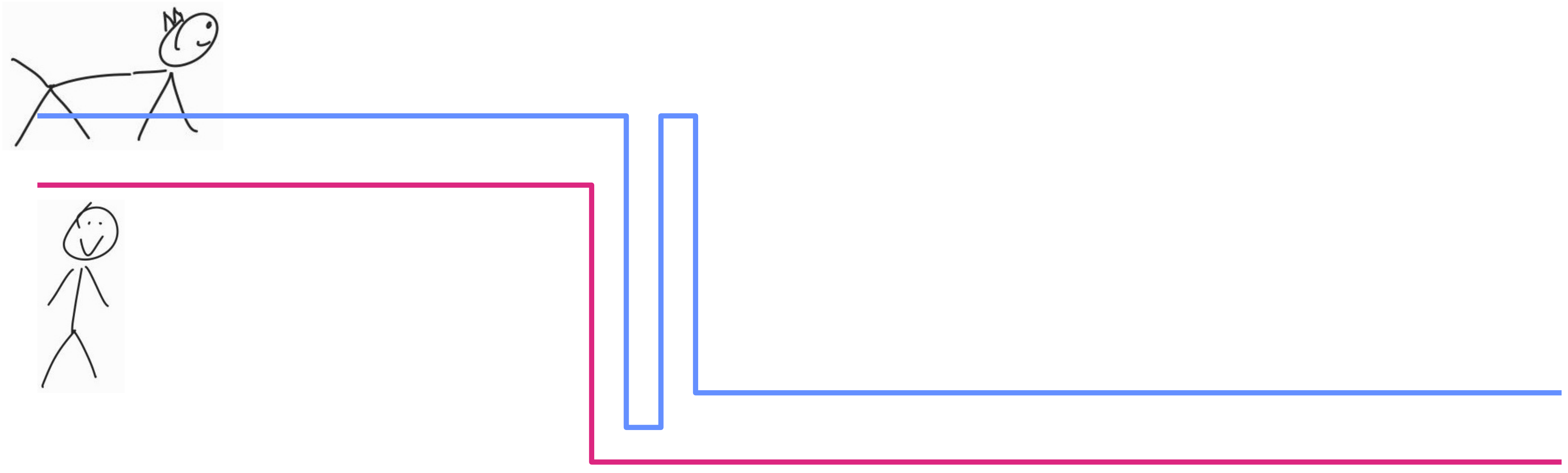
Weak Fréchet distance equals shortest leash length *when both are allowed to backtrack along their trajectory*



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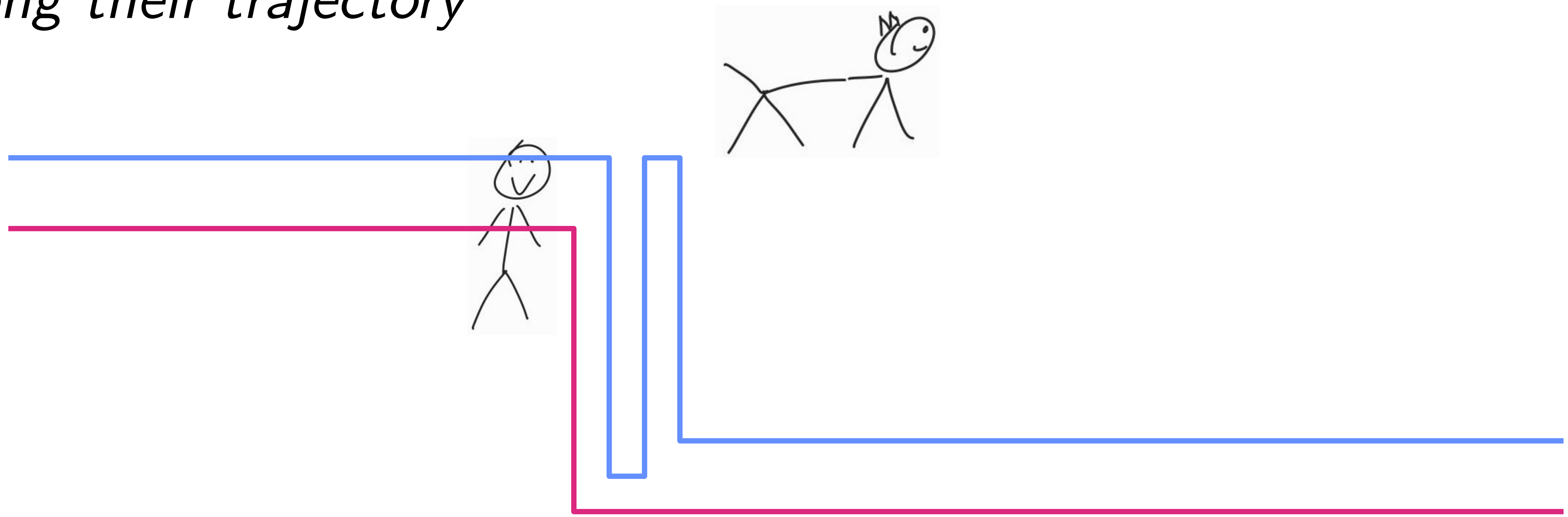

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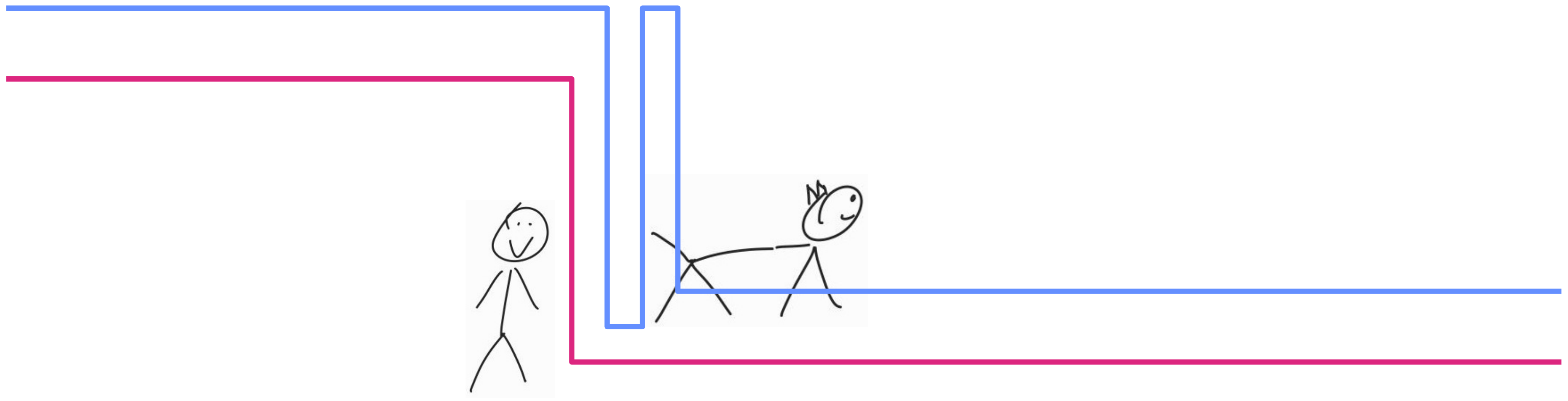

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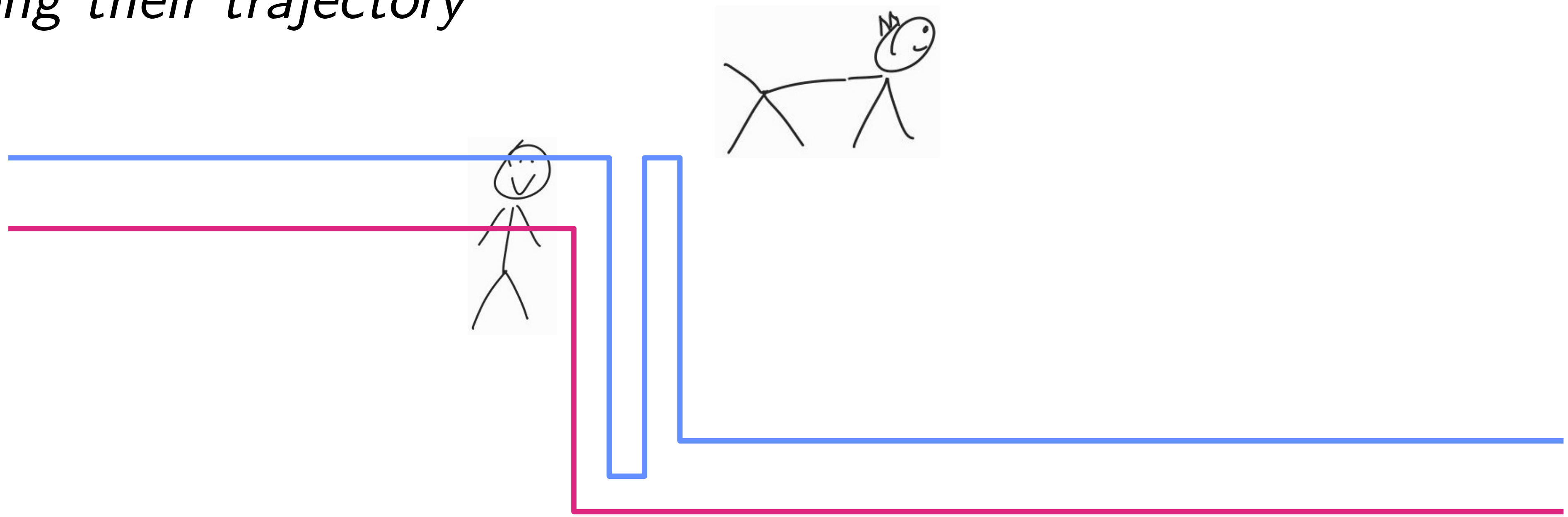

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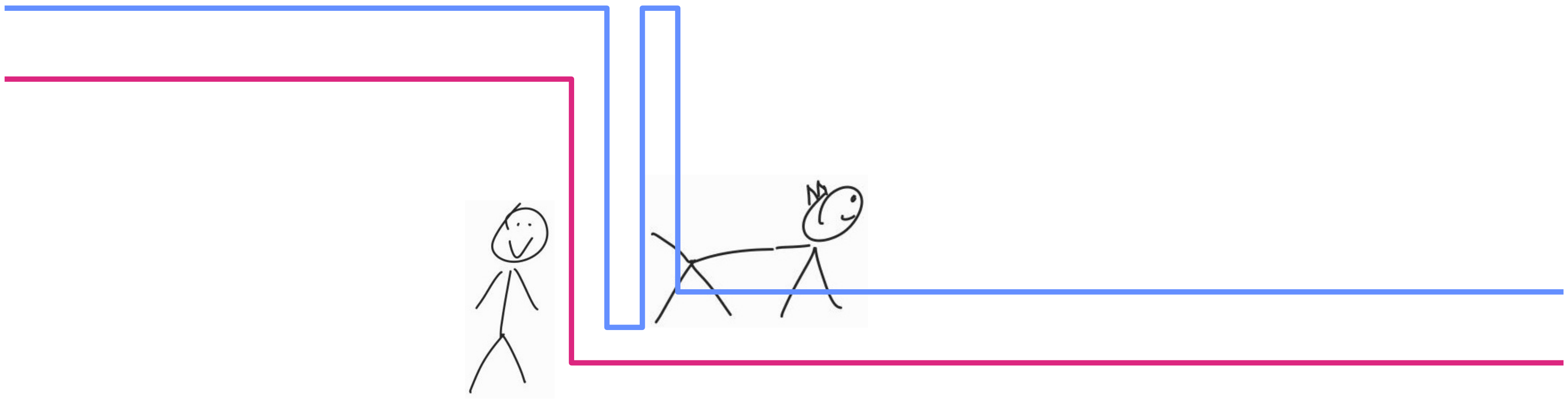

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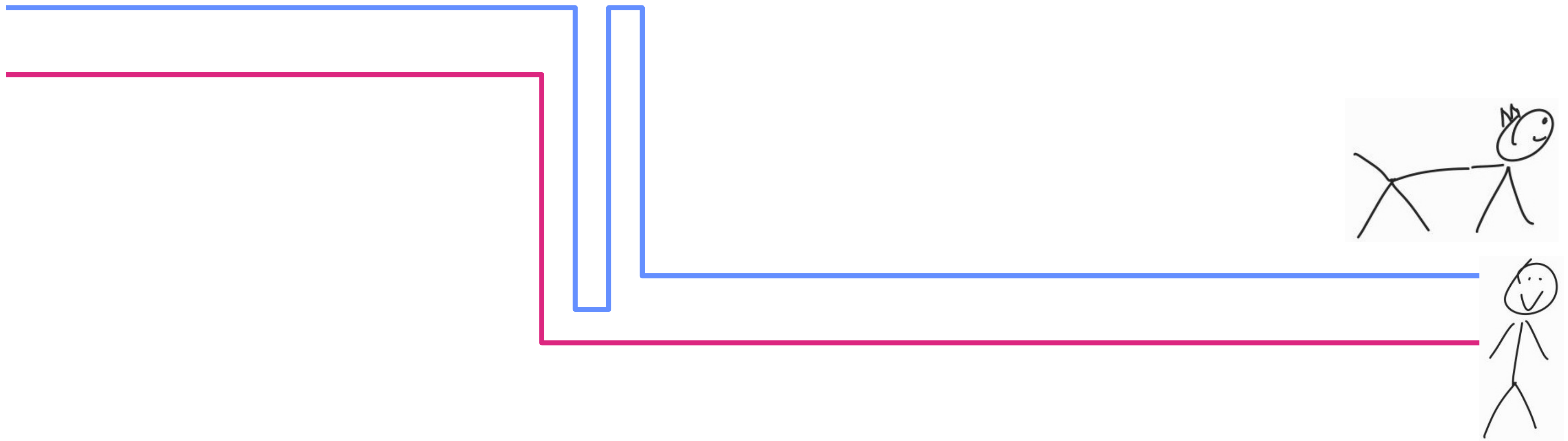

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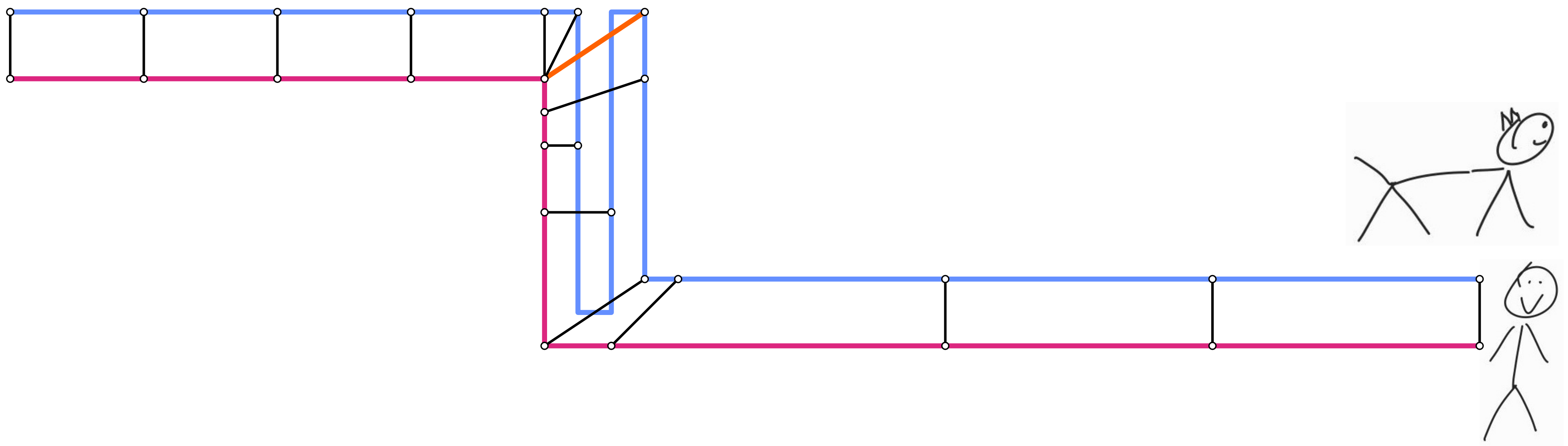

Weak graph distance

Let G_1, G_2 be immersed graphs. A *graph mapping* $s\colon G_1 \to G_2$ maps

- each vertex v of G_1 to a point $s(v)$ on an edge of G_2
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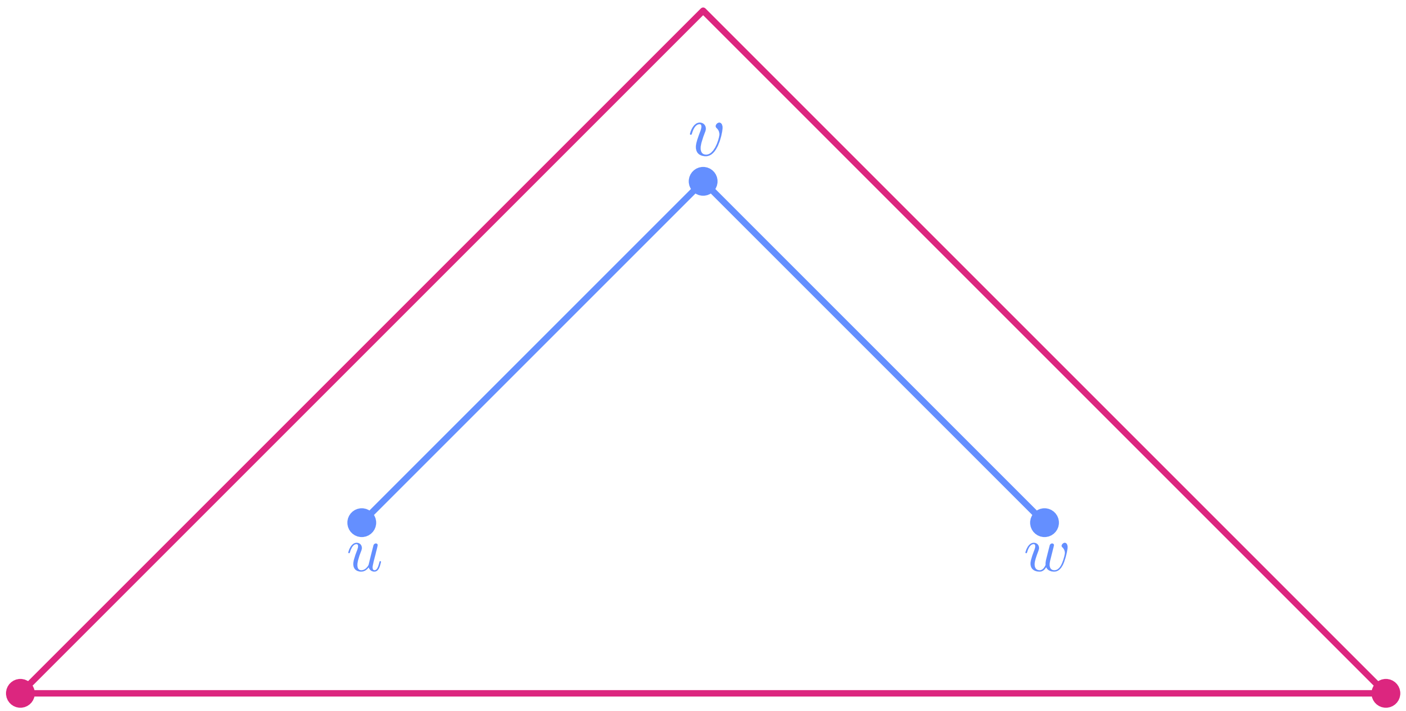

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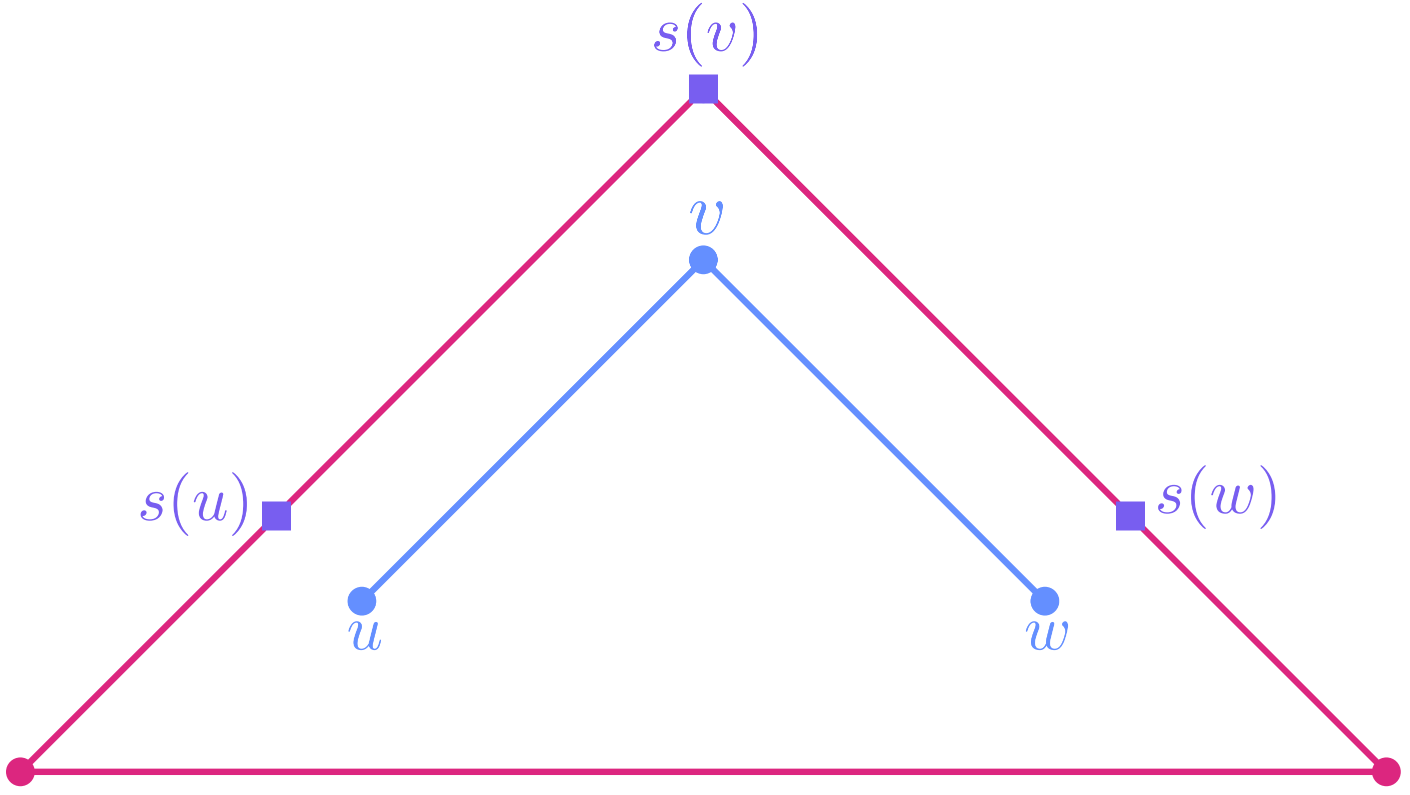

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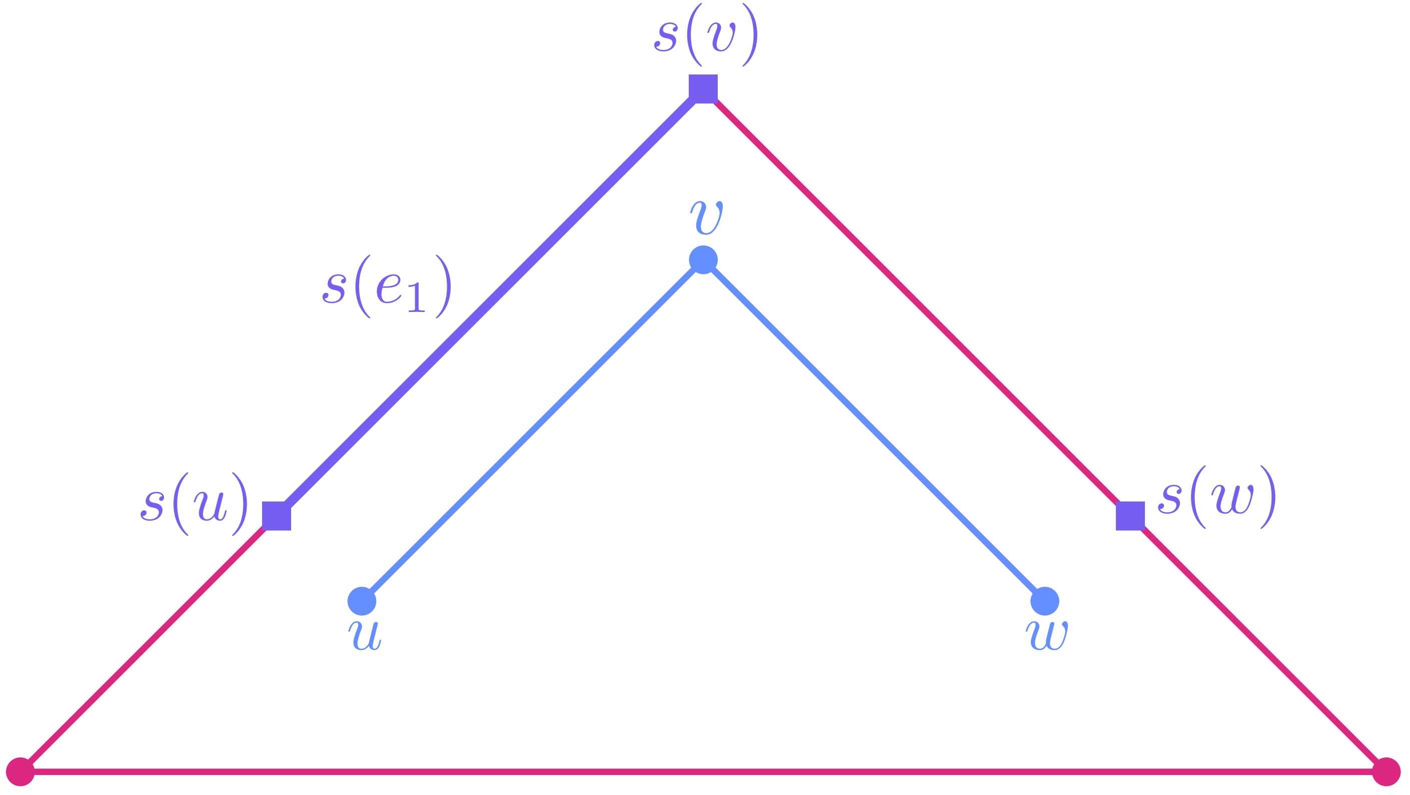

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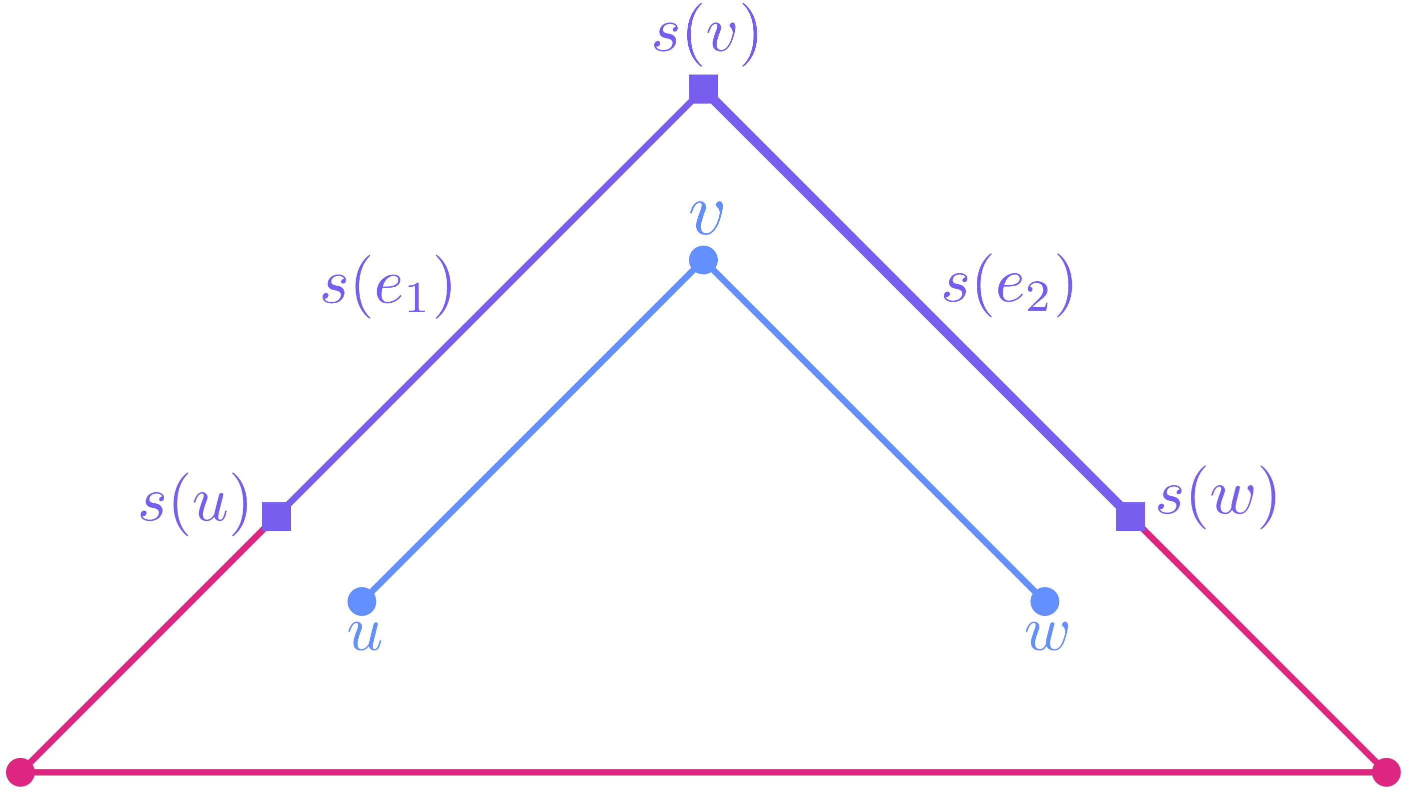

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The *directed weak graph distance* from G_1 to G_2 is defined as

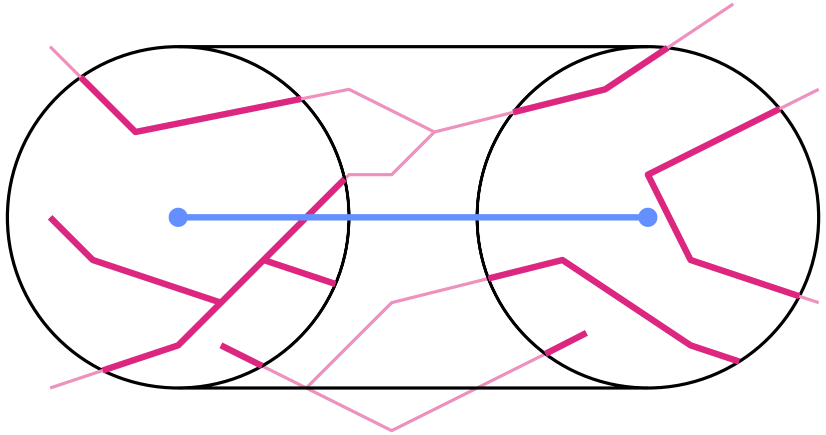
$$\vec{\delta}_{wG} = \min_{s:G_1 \to G_2} \max_{e \in E(G_1)} \delta_{wF}(e, s(e))$$

graph mapping   interpreted as curves

Undirected version: $\delta_{wG}(G_1, G_2) = \max\{\vec{\delta}_{wG}(G_1, G_2), \vec{\delta}_{wG}(G_2, G_1)\}$

General decision algorithm due to Akitaya et al.

Vertex placement of $v \in V(G_1)$: connected component of $G_2 \cap B_\varepsilon(v)$



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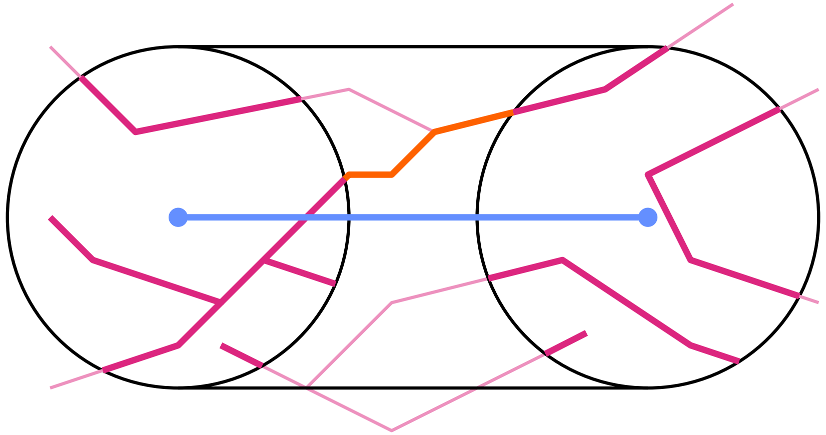

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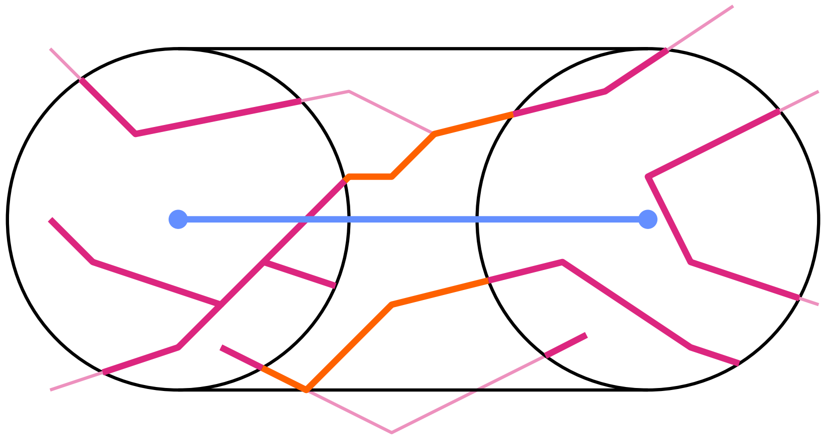

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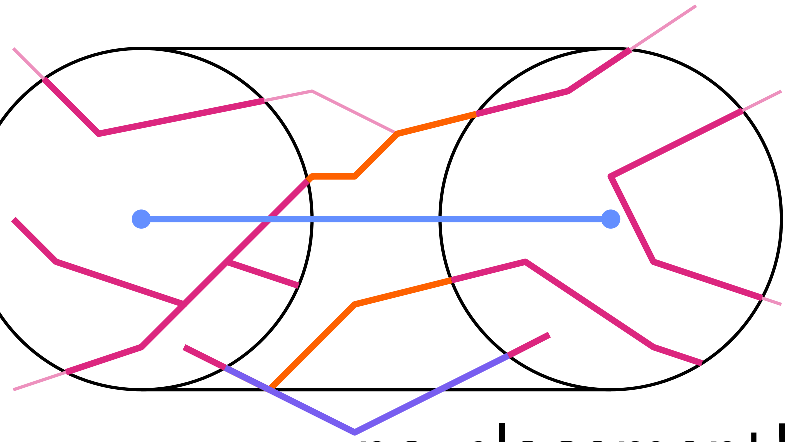

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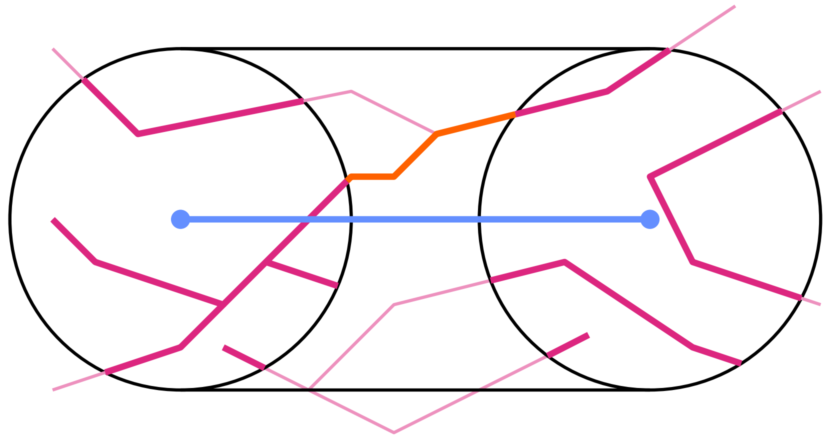

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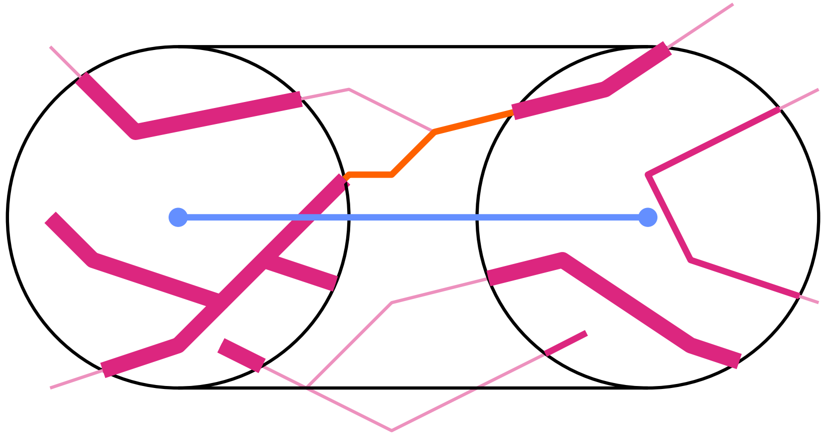

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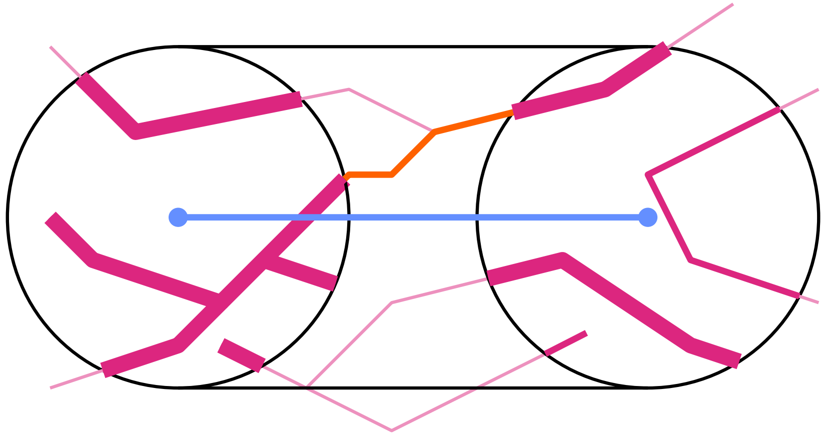

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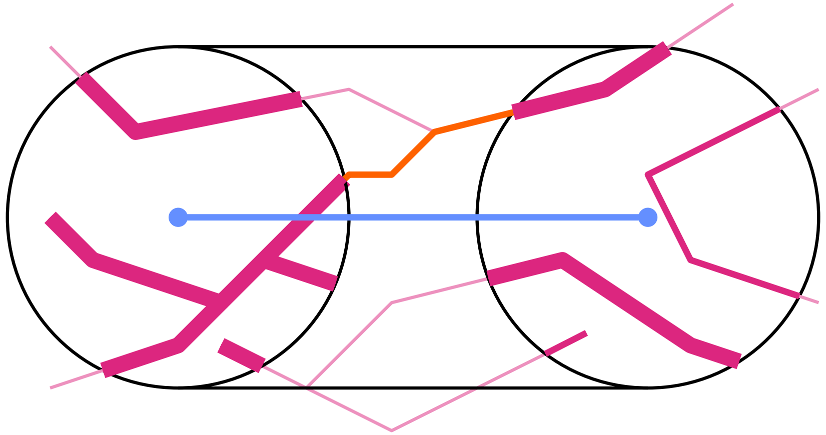

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1. Compute vertex placements.	
2. Compute reachability information.	quadratic time
3. Delete invalid placements.	
4. Decide whether a placement of G_1 exists.	NP-complete in general

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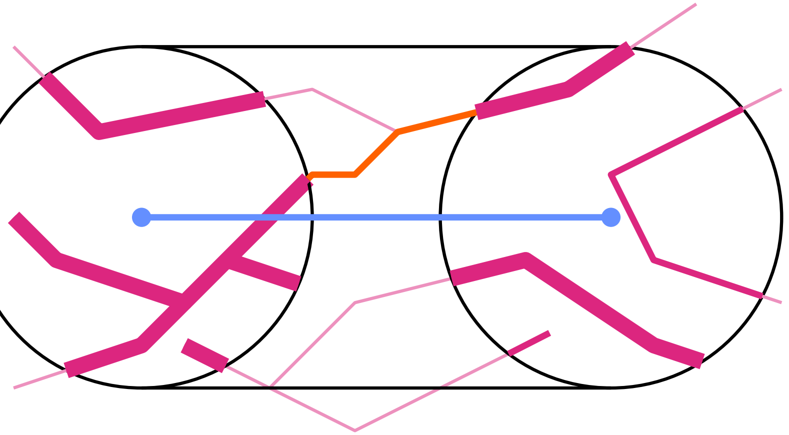

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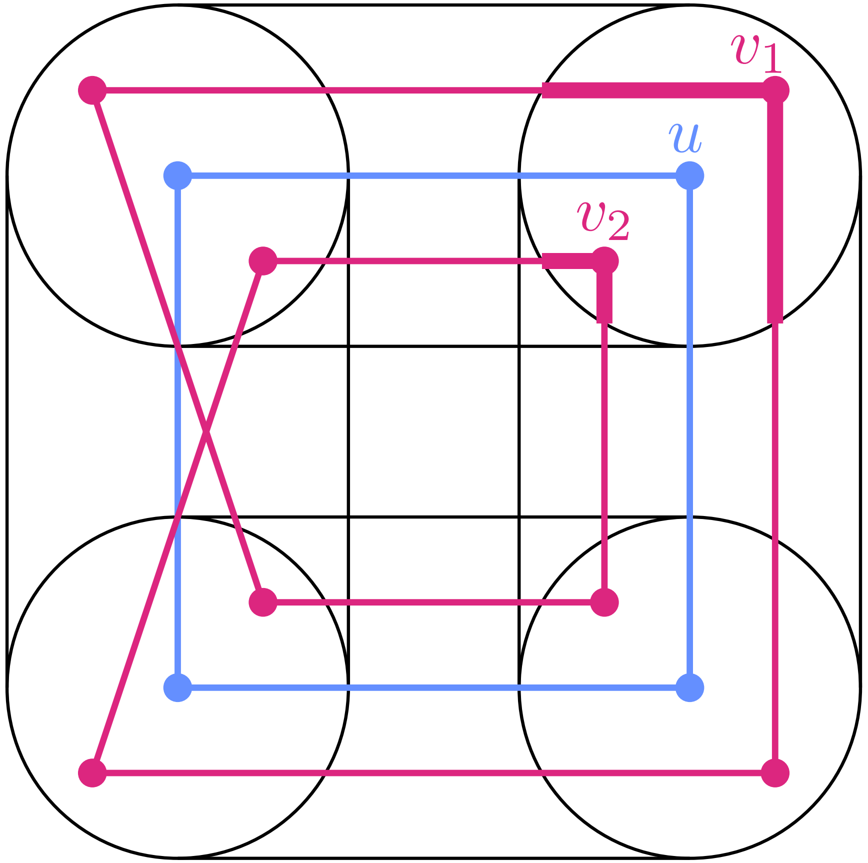

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Hardness of deciding $\vec{\delta}_{wG}(G_1, G_2) \leq \varepsilon$ if G_1 is plane

RUB

Theorem: Deciding whether $\vec{\delta}_{wG}(G_1, G_2) \leq \varepsilon$ is NP-complete even if G_1 is plane and G_2 is immersed in $\mathbb{R}^2$.

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Sketch of the proof by reduction from $\textsc{Planar 3Col}$:

Let $G = (V, E)$ be the (planar) input graph

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Let $G = (V, E)$ be the (planar) input graph

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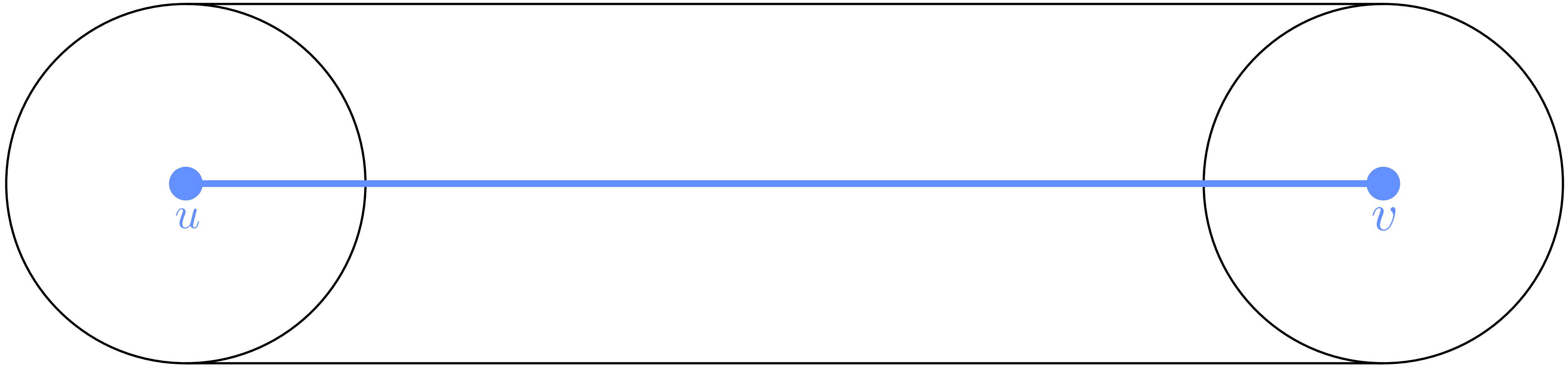
2. Choose ε s.t. all ε-balls and tubes are separated



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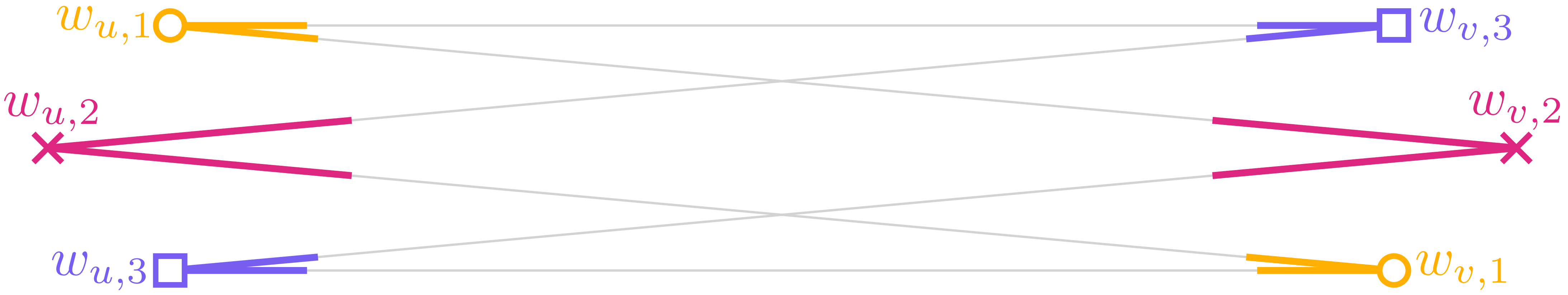
Sketch of the proof by reduction from PLANAR 3COL:

Let $G = (V, E)$ be the (planar) input graph

3. Construct G_c with
   - vertices $w_{u,i}$ for $u \in V$, $i \in [3]$
   - edges $\{w_{u,i}, w_{v,j}\}$ for $\{u, v\} \in E$, $i \neq j$



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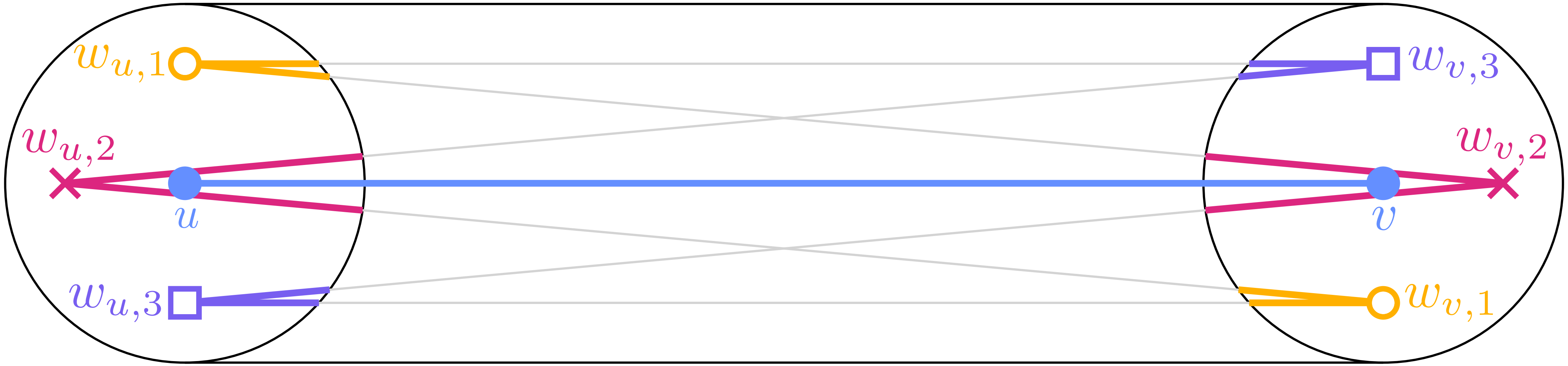
Sketch of the proof by reduction from Planar 3Col:

Let $G = (V, E)$ be the (planar) input graph

4. Immerse G_c s.t. $w_{u,i}$ lies in $B_\varepsilon(u)$



Hardness of deciding $\vec{\delta}_{wG}(G_1, G_2) \leq \varepsilon$ if G_1 is plane

RUB

Theorem: Deciding whether $\vec{\delta}_{wG}(G_1, G_2) \leq \varepsilon$ is NP-complete even if G_1 is plane and G_2 is immersed in $\mathbb{R}^2$.

Sketch of the proof by reduction from PLANAR 3COL:

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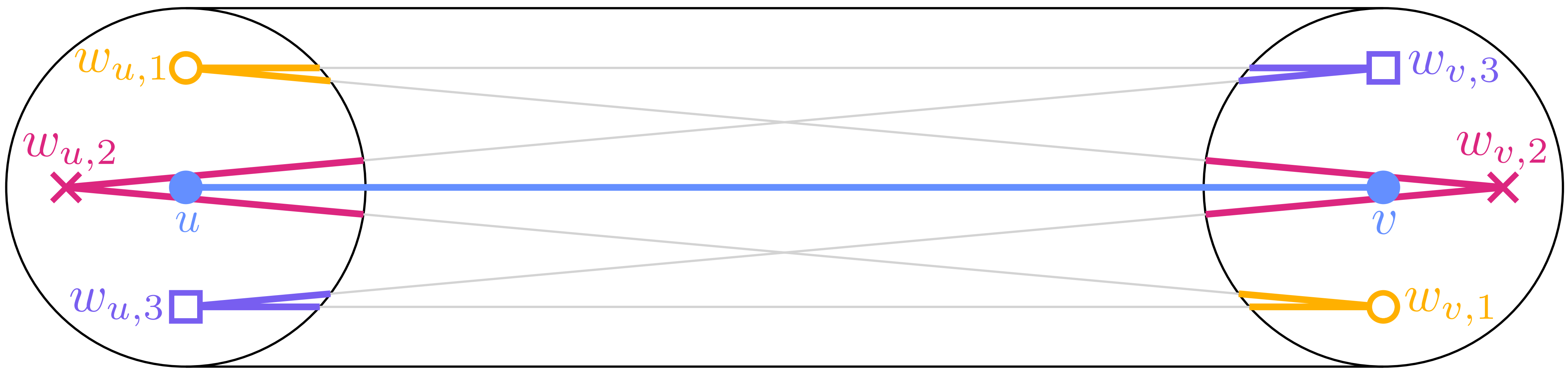
Idea: Vertex placements $\leftrightarrow$ colors



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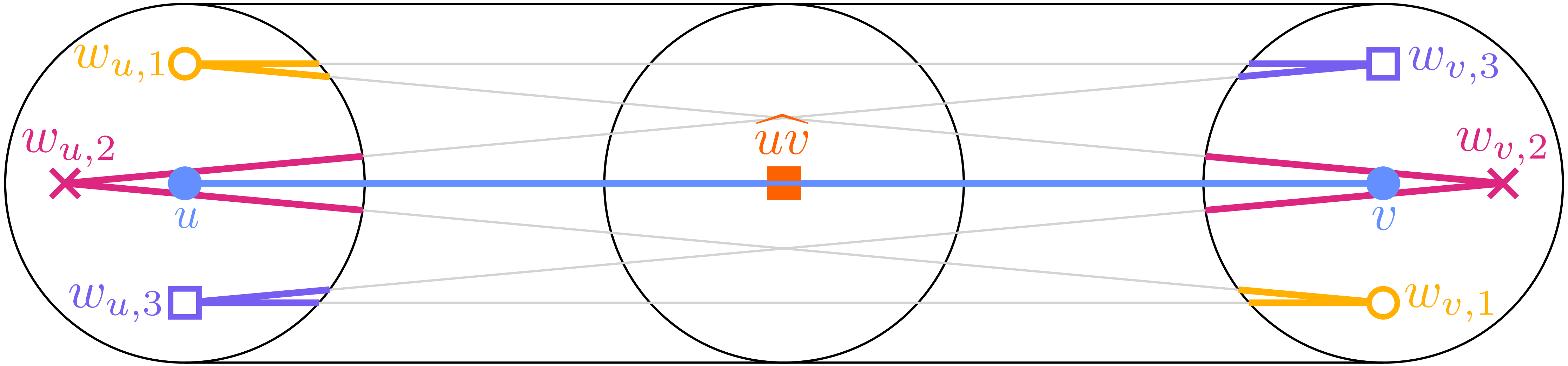
$\tilde{1}$. Construct a crossing-free embedding of G and insert a vertex $\widehat{uv}$ in the middle of each edge $\{u, v\}$ $\quad \rightarrow G_p$



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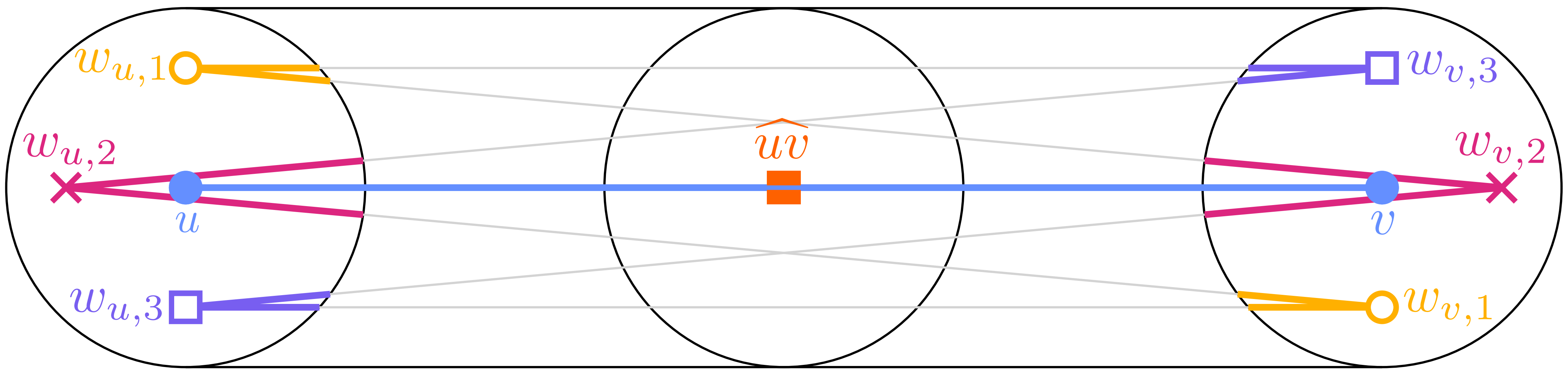

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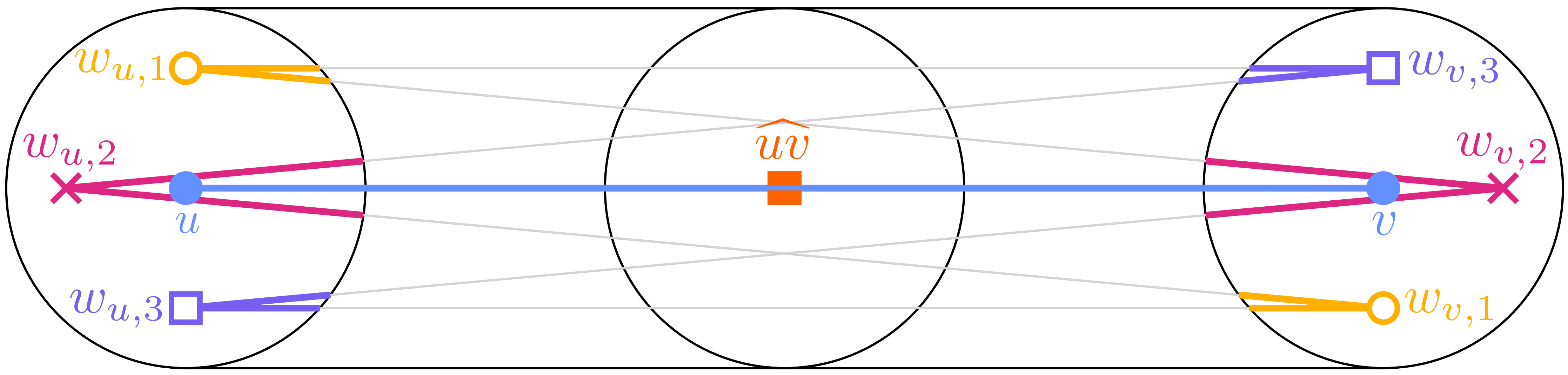

Hardness of approximating $\vec{\delta}_{wG}(G_1, G_2)$ if G_1 is plane

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Corollary: The weak graph distance is NP-hard to approximate within any constant ratio $c \geq 1$ even if G_1 is plane, G_2 is immersed in $\mathbb{R}^2$.

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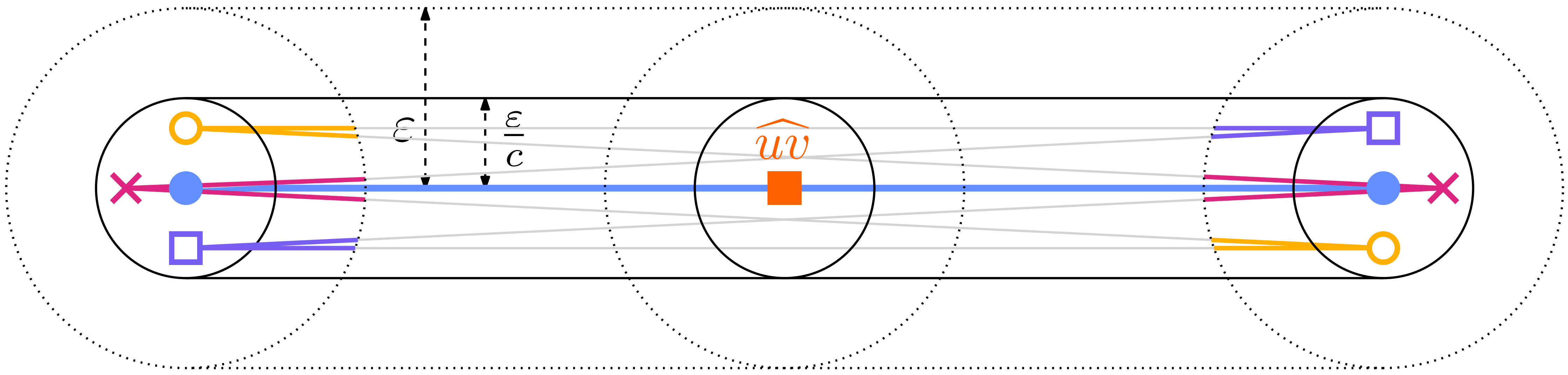

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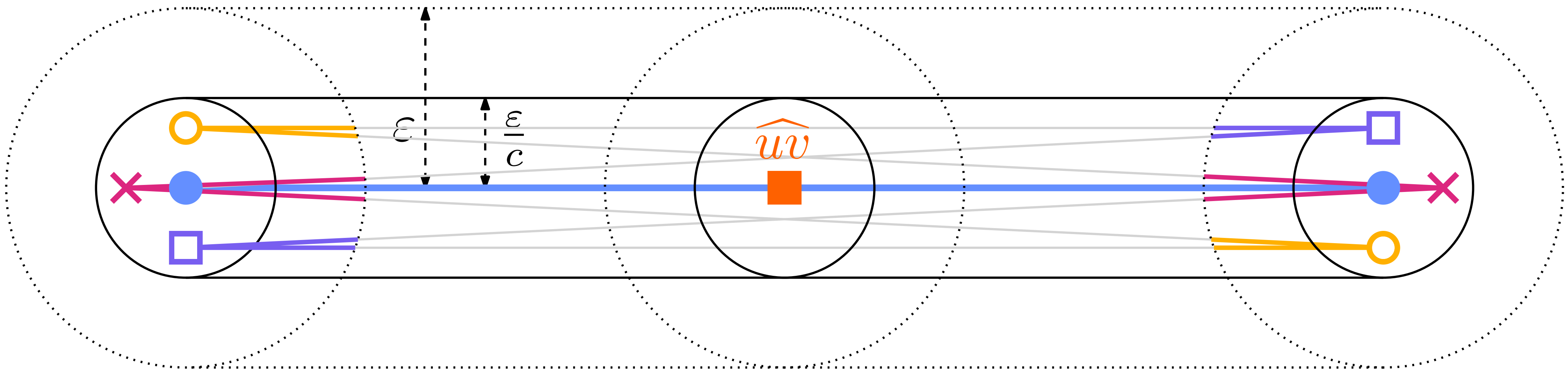

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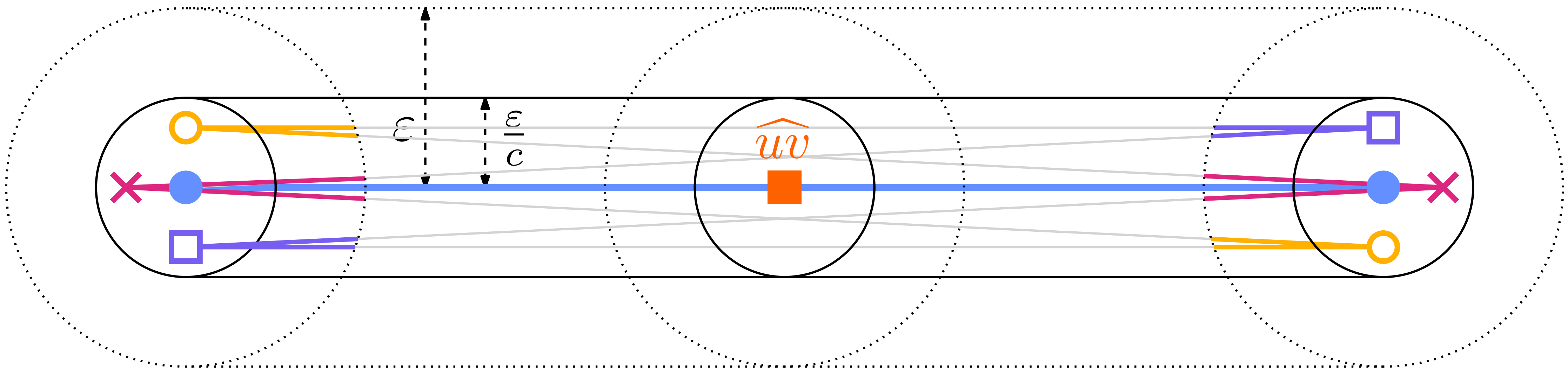

Hardness of the embedded case in $\mathbb{R}^d$, $d \geq 3$

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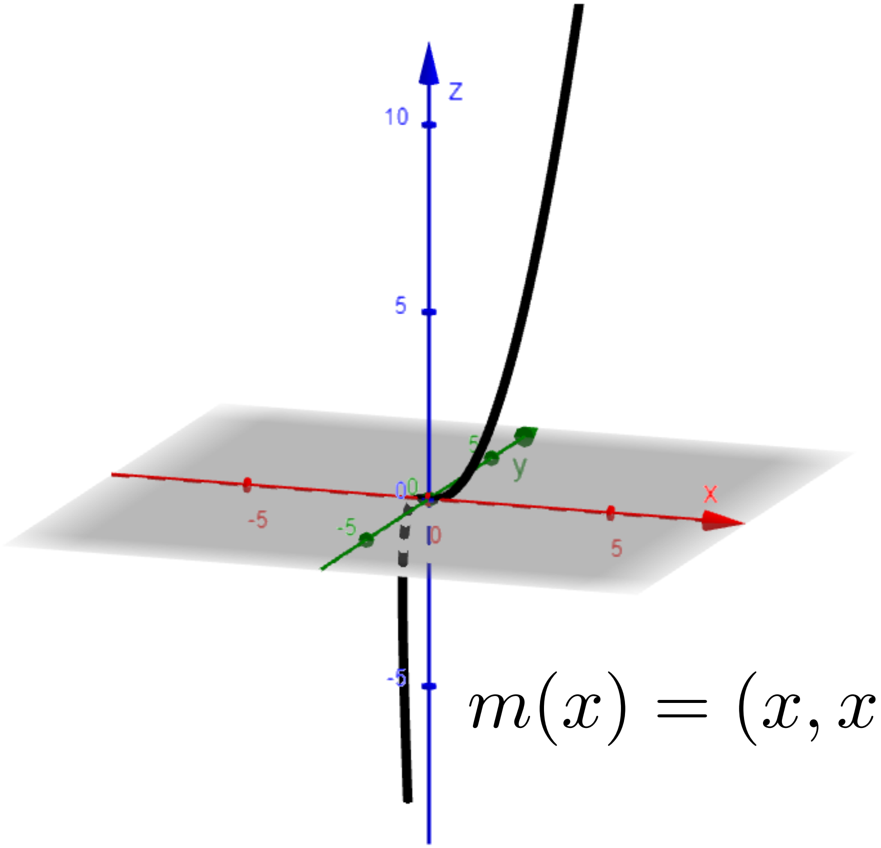
Proof idea:

(Up to details,) embed on the 3-dim moment curve instead



$$m(x) = (x, x^2, x^3)$$

Crossing-rigid weak graph distances

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Now: crossing-rigid weak graph distances

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- up to n crossings is *loosely crossing-rigid*
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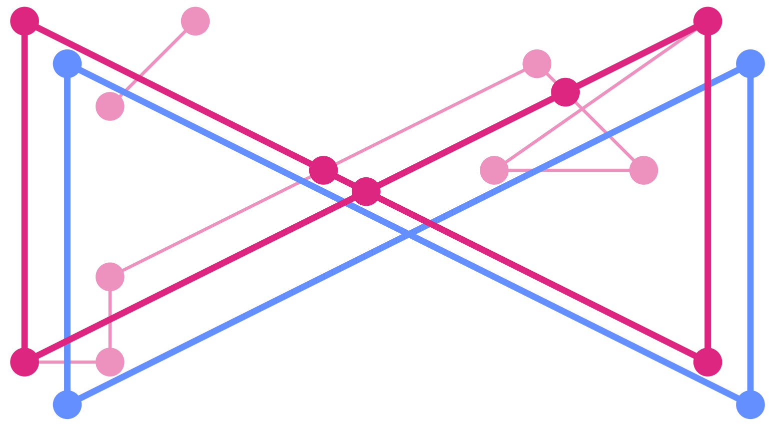

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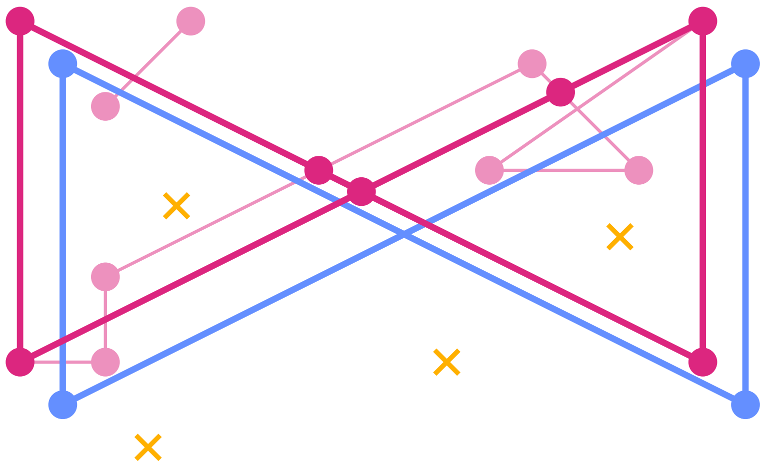
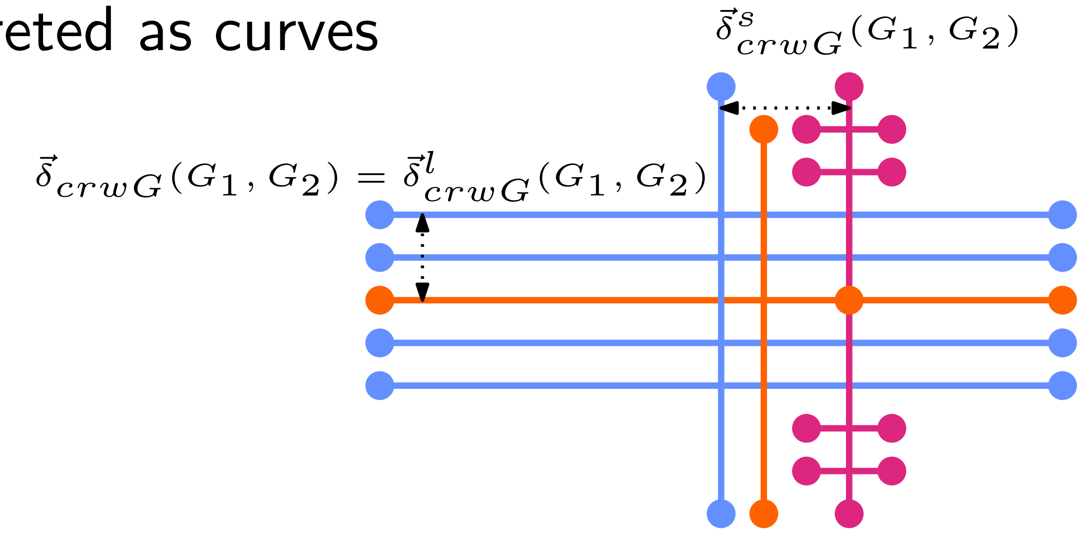


The *directed crossing-rigid weak graph distance* is defined as

$$\vec{\delta}_{crwG}(G_1, G_2) = \inf_{s: G_1 \to G_2} \max_{e \in E} \delta_{wF}(e, s(e))$$

crossing-rigid interpreted as curves

Analogous: $\vec{\delta}^{l}_{crwG}$, $\vec{\delta}^{s}_{crwG}$



Decision algorithm for the existence of crossing-rigid placements

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Caveat: since placements are no longer compact, there might be no ε-placement but still $\delta_{crwG}^{(l)/(r)}(G_1, G_2) = \varepsilon$

Outlook

RUB

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Thank you