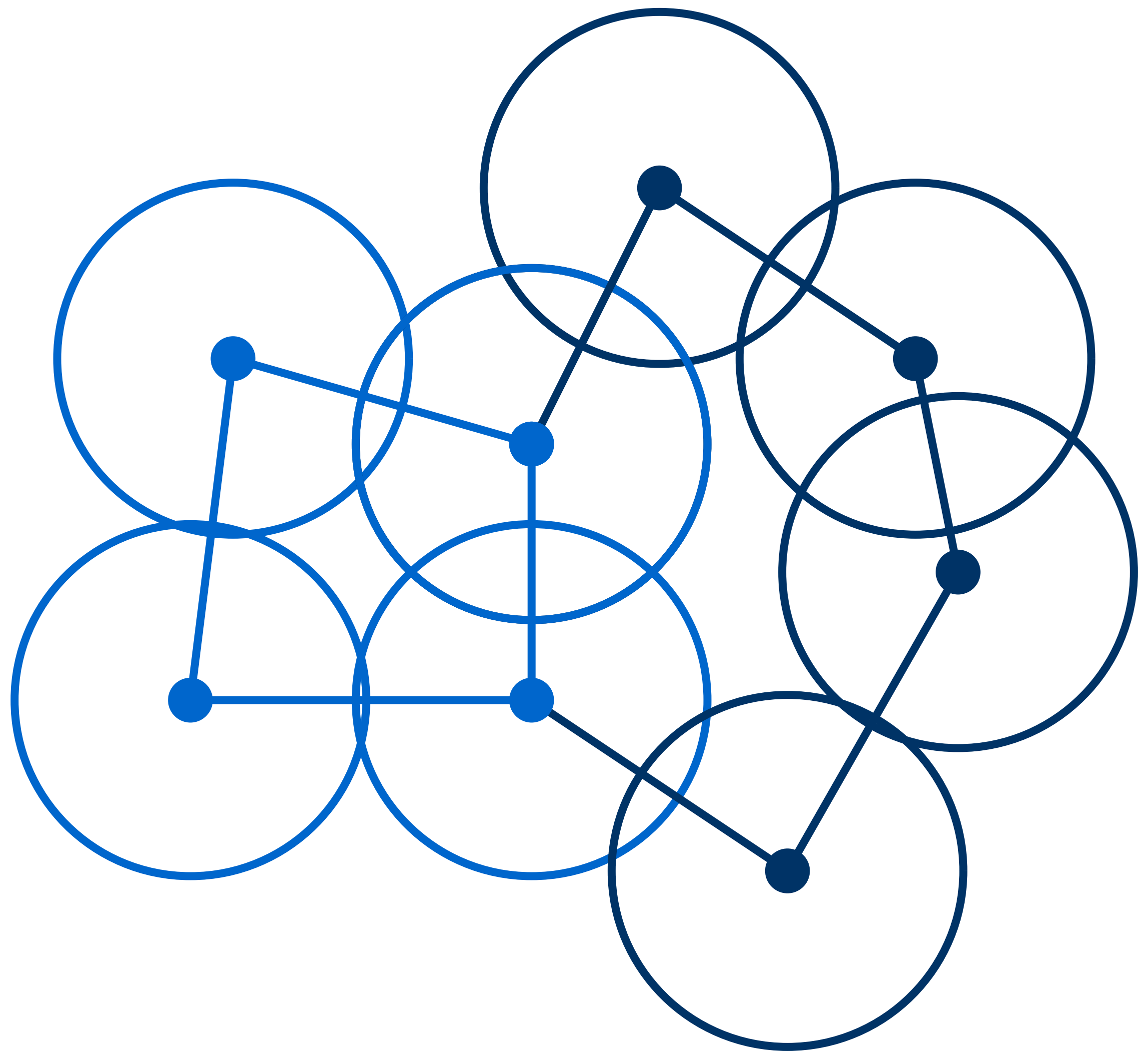
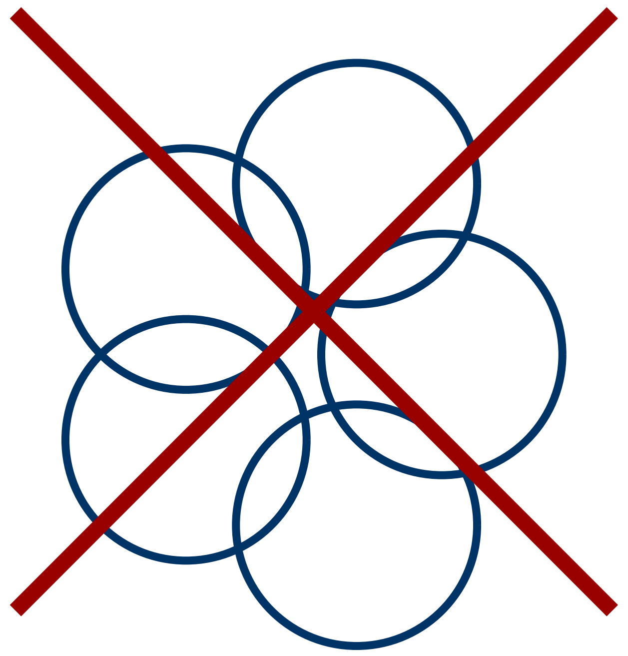
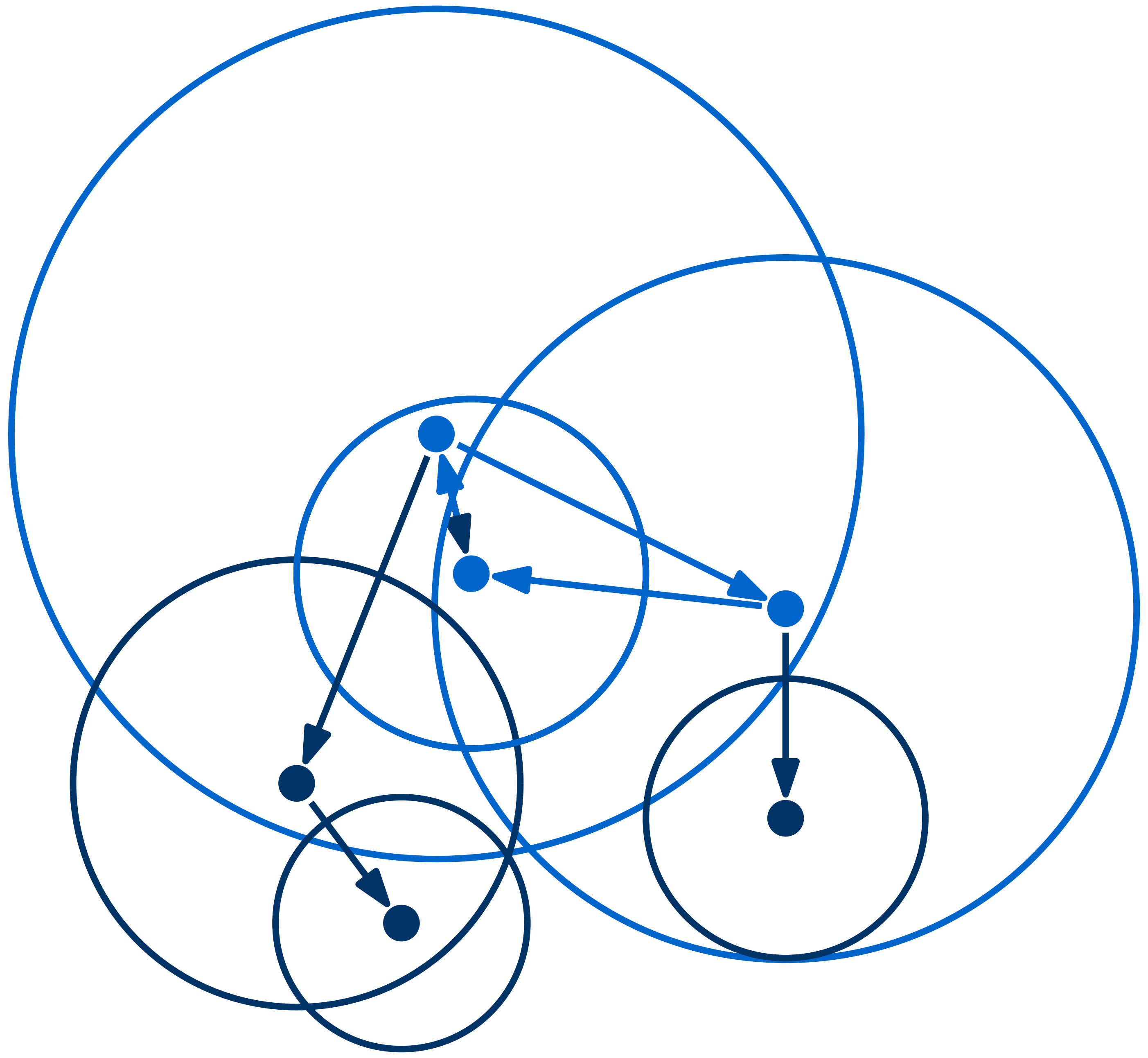
Robust Algorithms for Finding Triangles and Computing the Girth in Unit Disk and Transmission Graphs







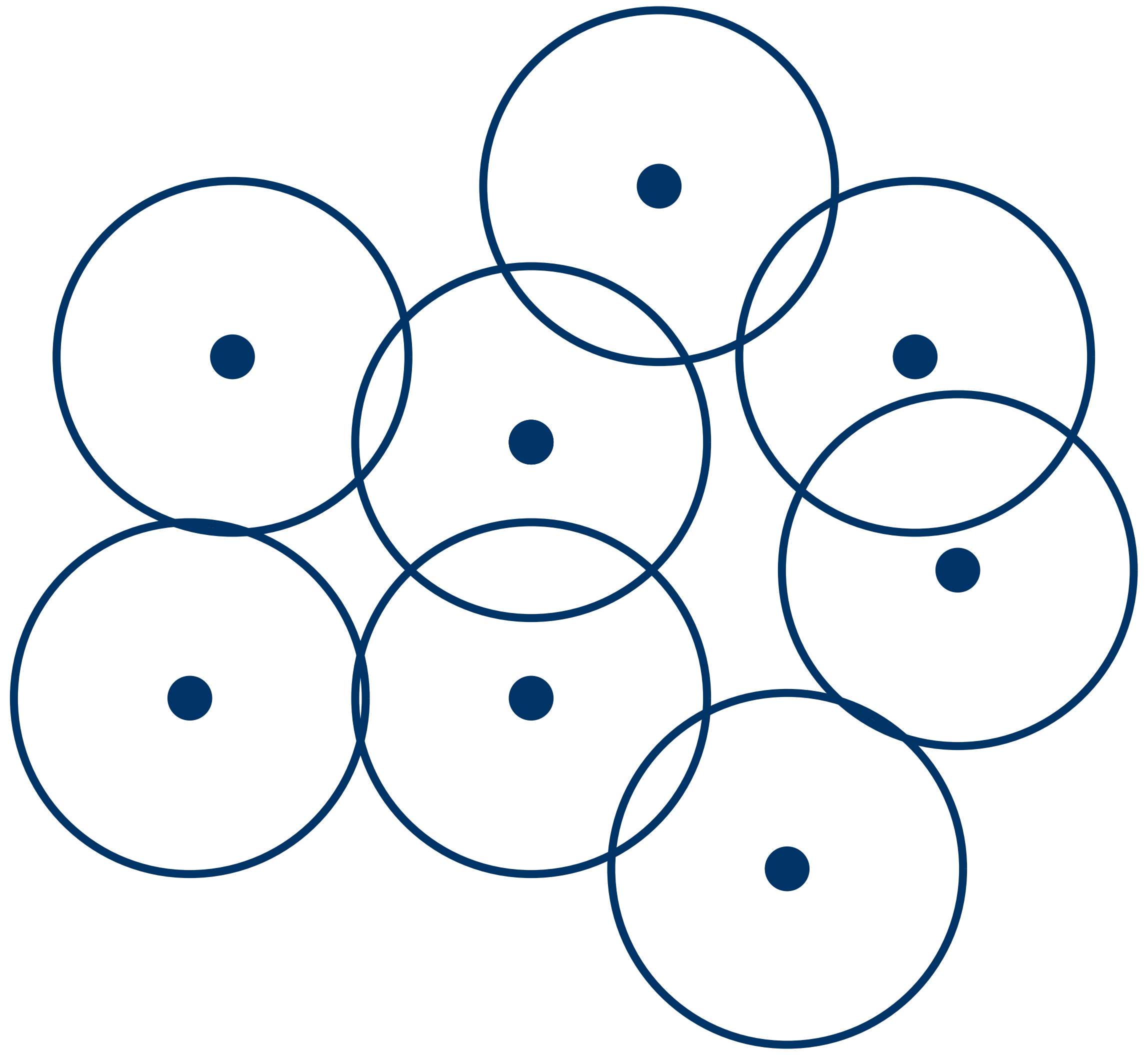
Katharina Klost and Wolfgang Mulzer

EuroCG 2024

Unit Disk Graphs and Transmission Graphs

Unit Disk Graphs

Set of sites $S \subseteq \mathbb{R}^2$



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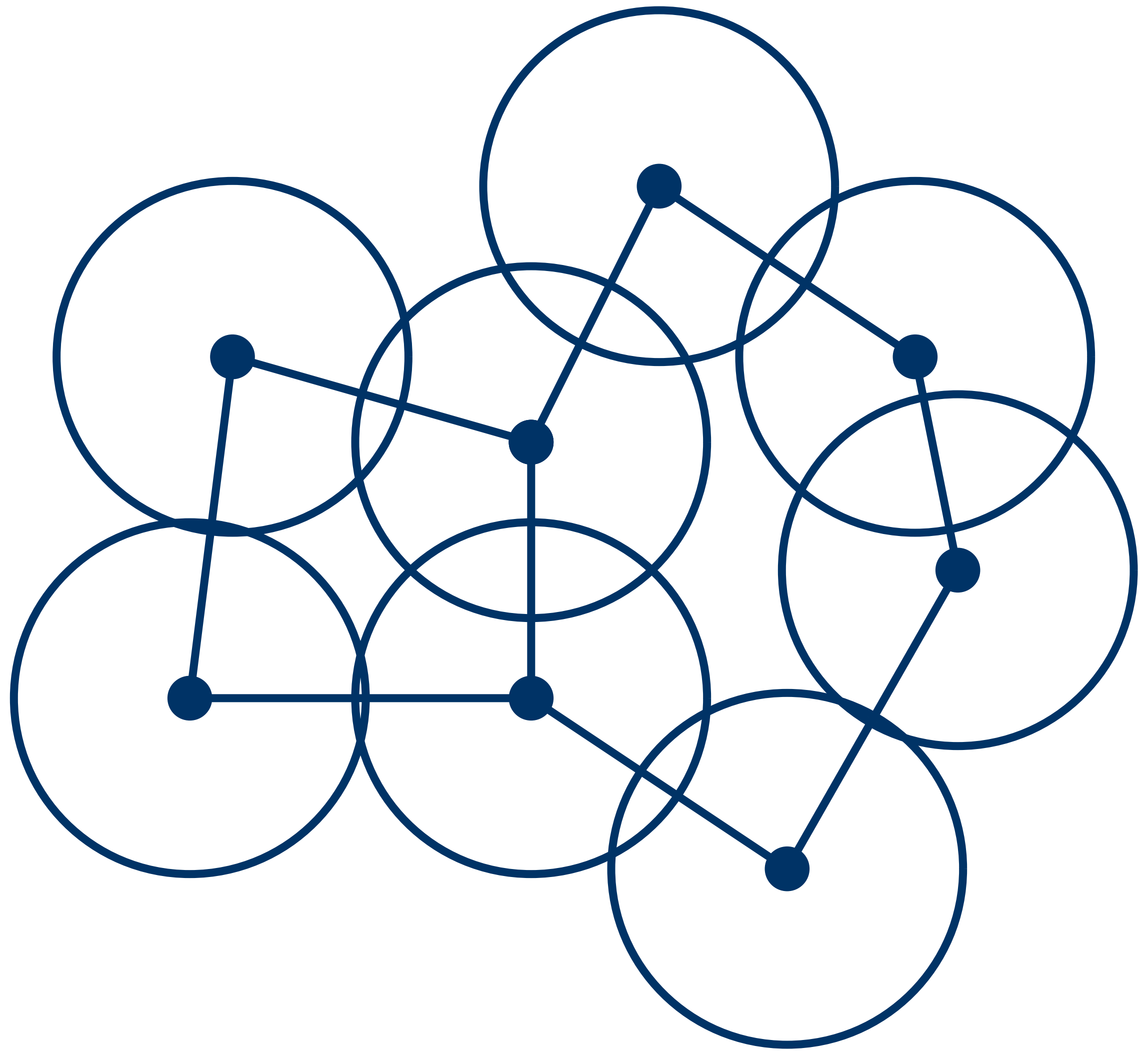

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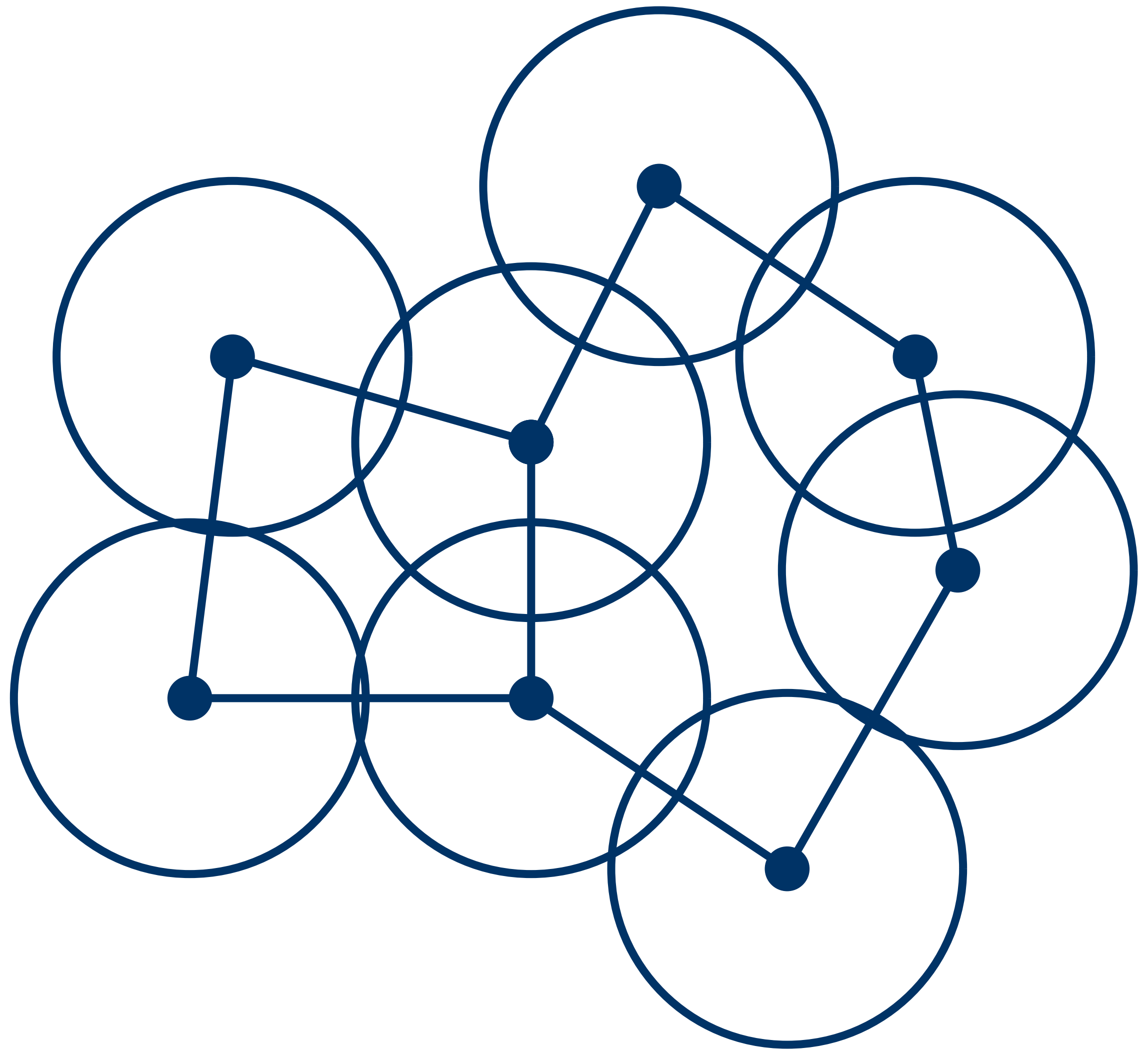

General Disk Graphs

Set of sites $S \subseteq \mathbb{R}^2$ with radii r_s

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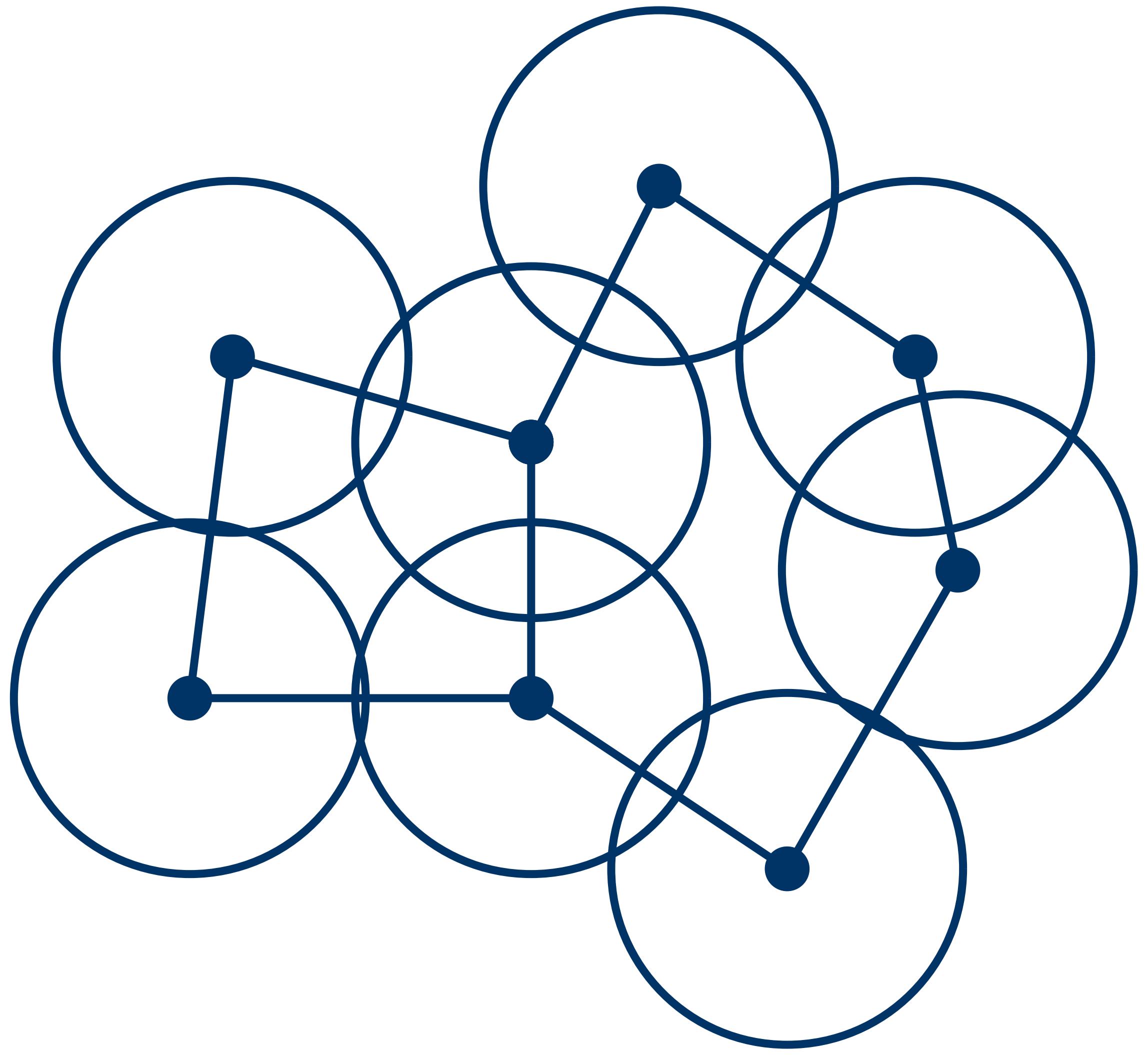

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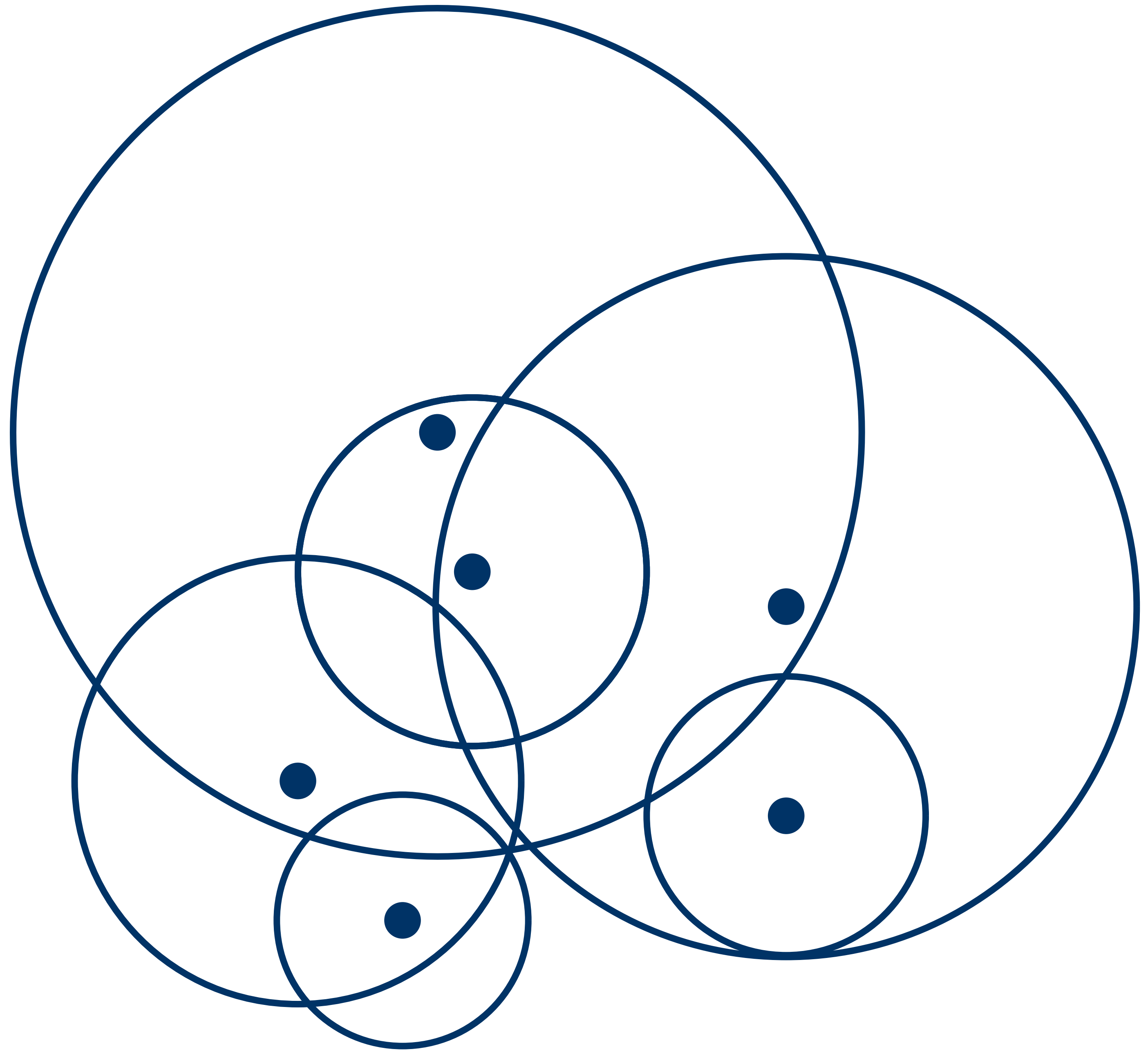

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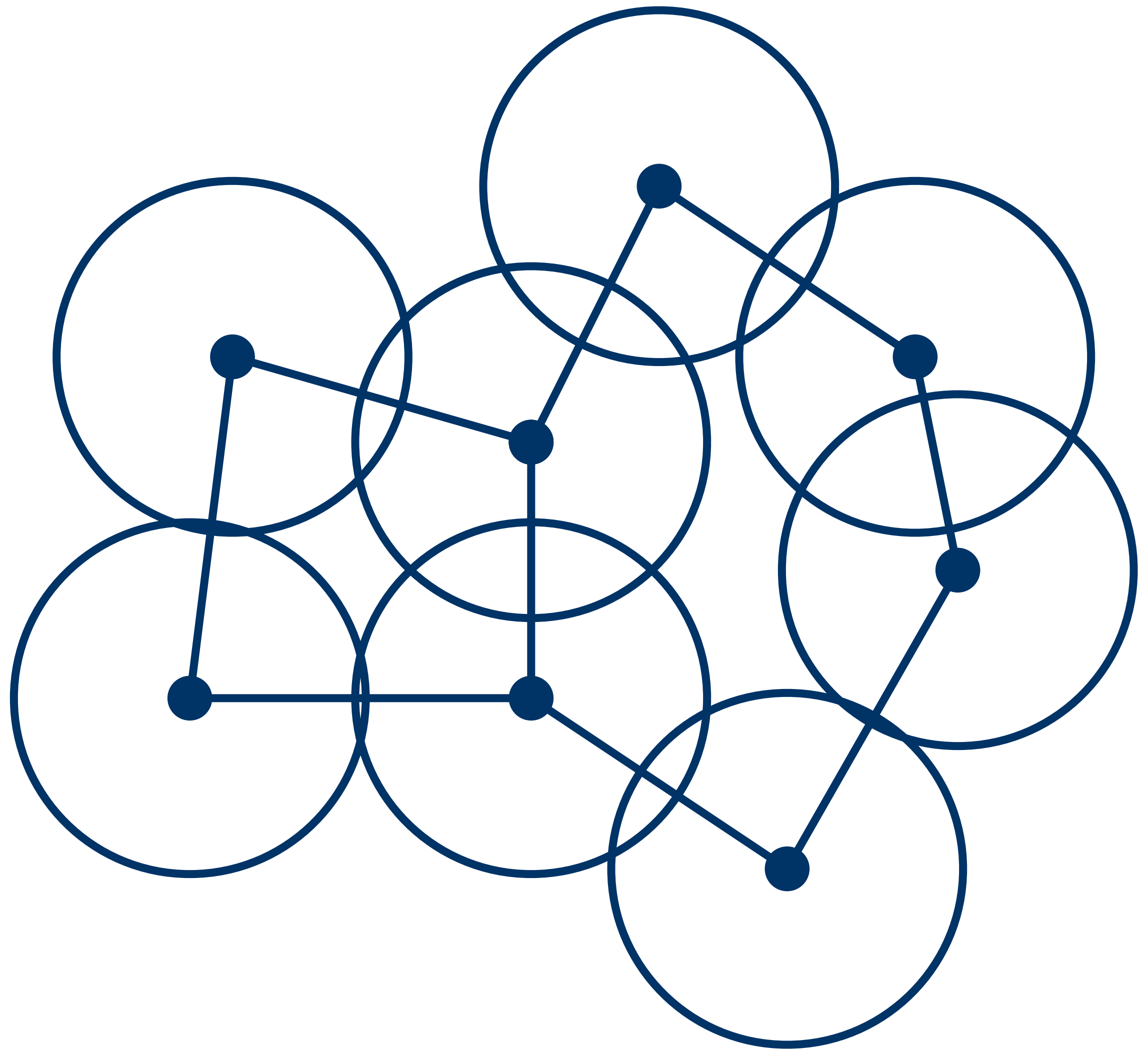

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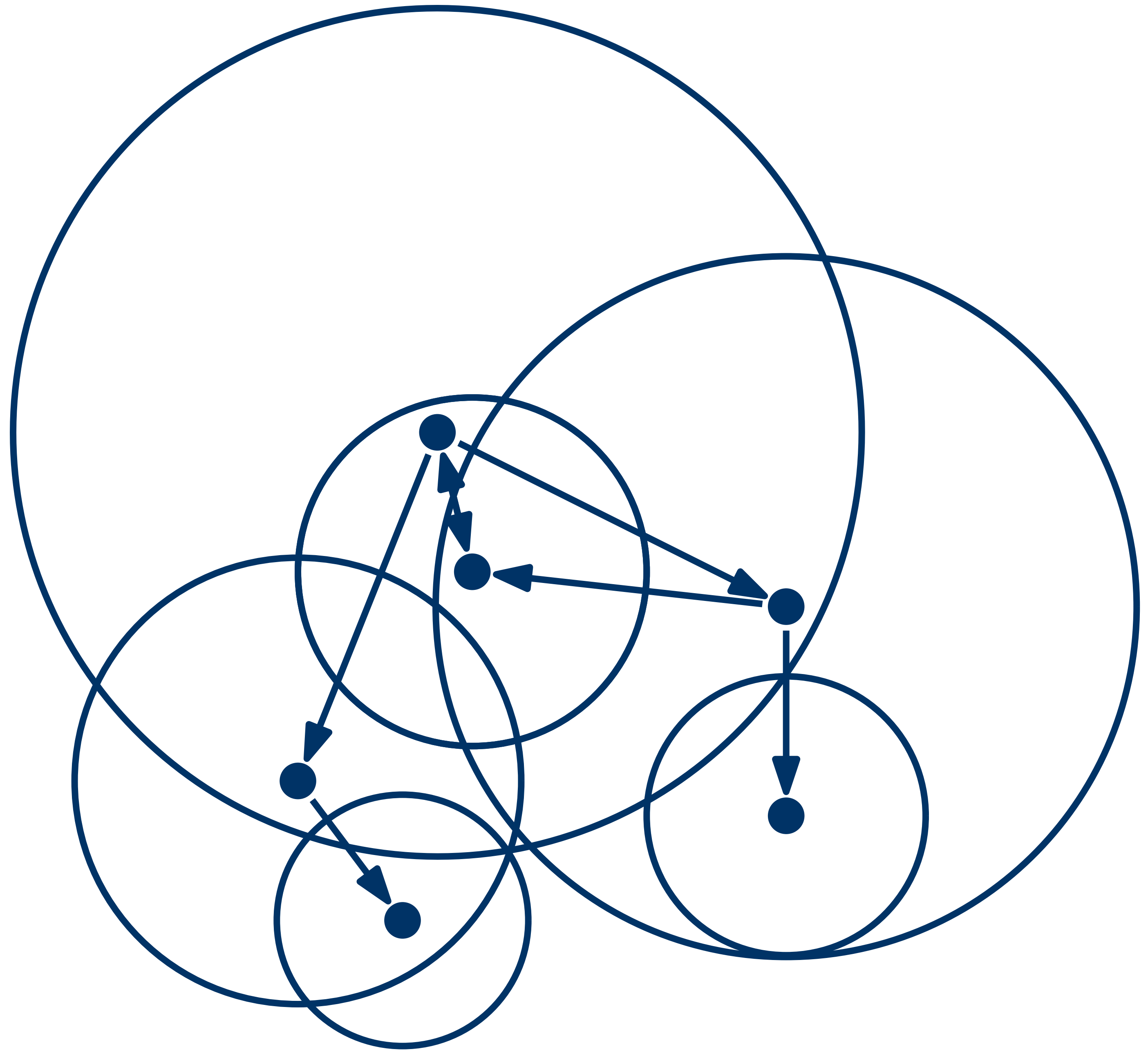

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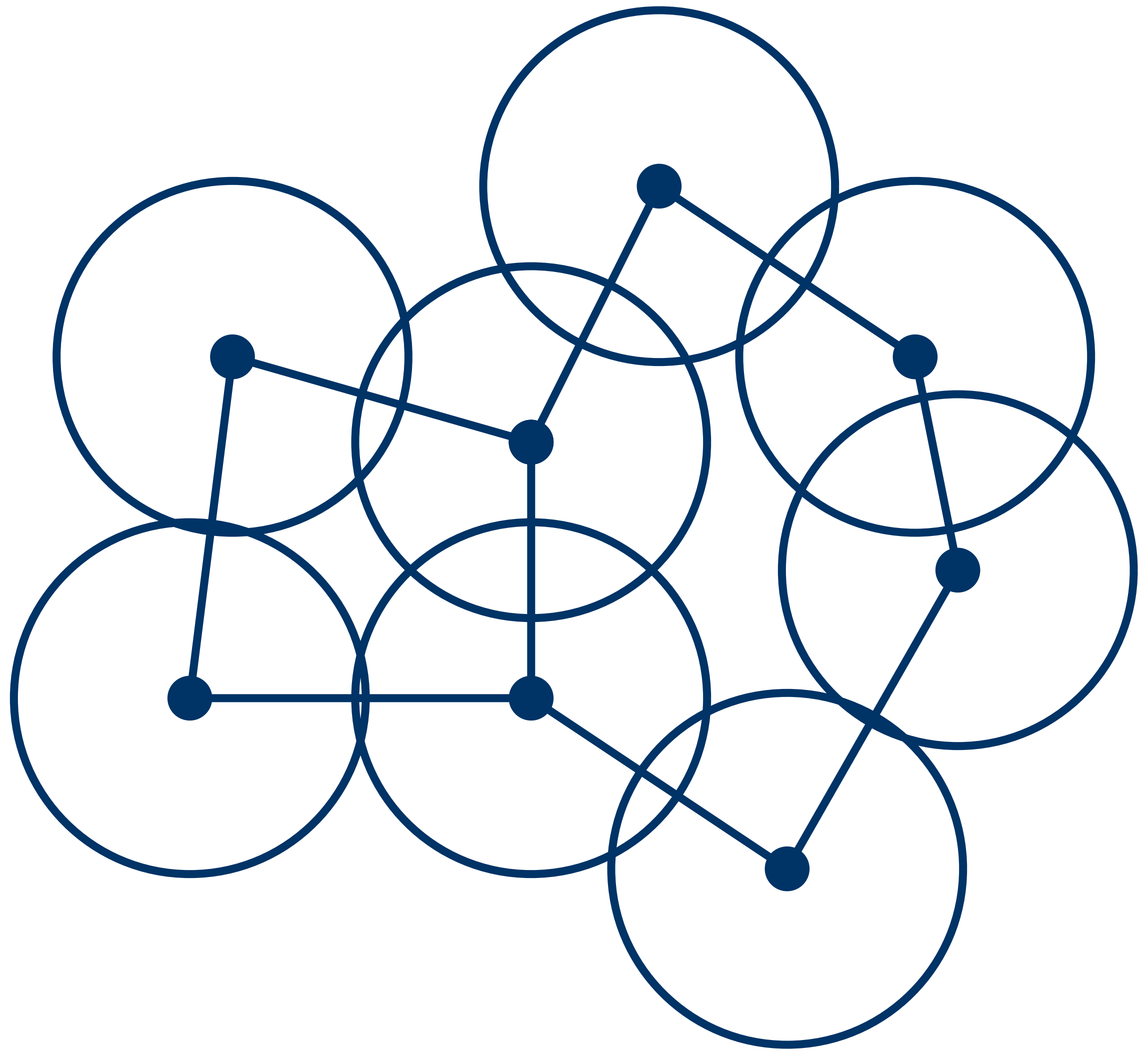

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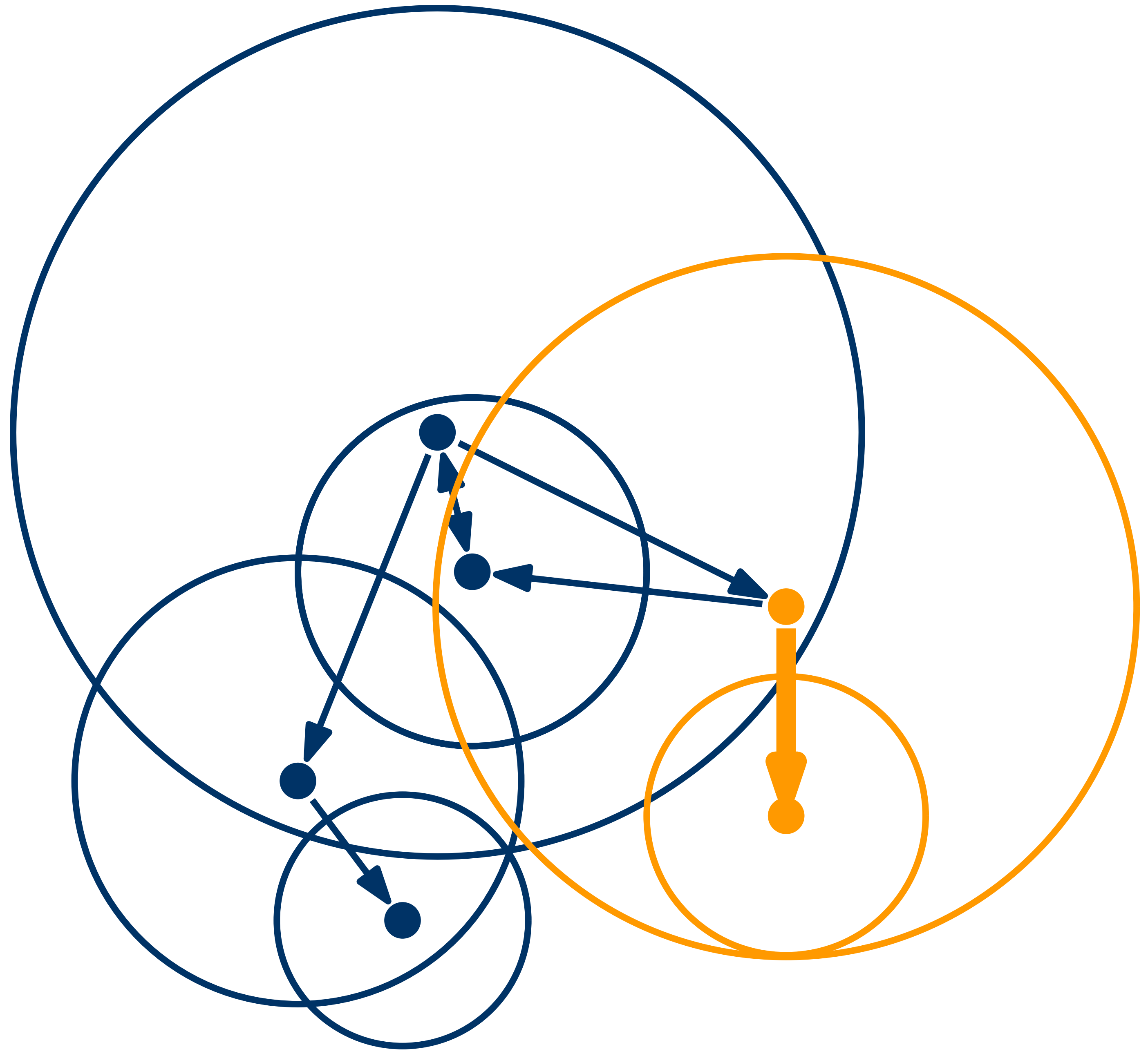

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Challenges

Transfer from geometric setting to robust setting takes $\Omega(n+m)$ time.

Recognizing a unit disk or transmission graph is $\exists\mathbb{R}$-hard

Triangle Detection and Computing the Girth

Triangle Detection

Computing the Girth

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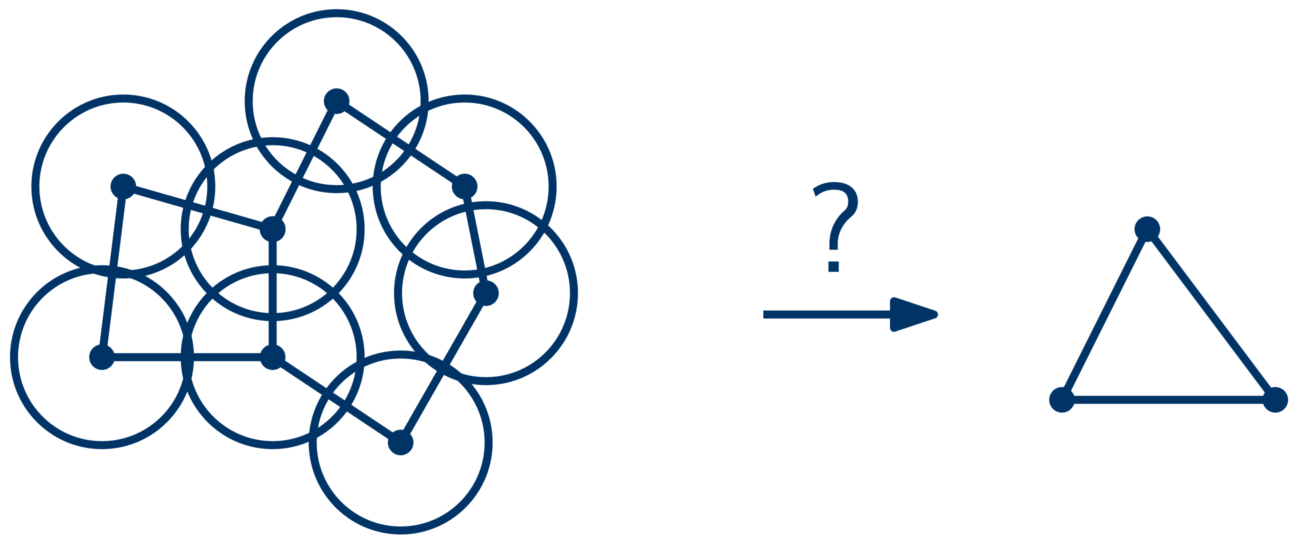

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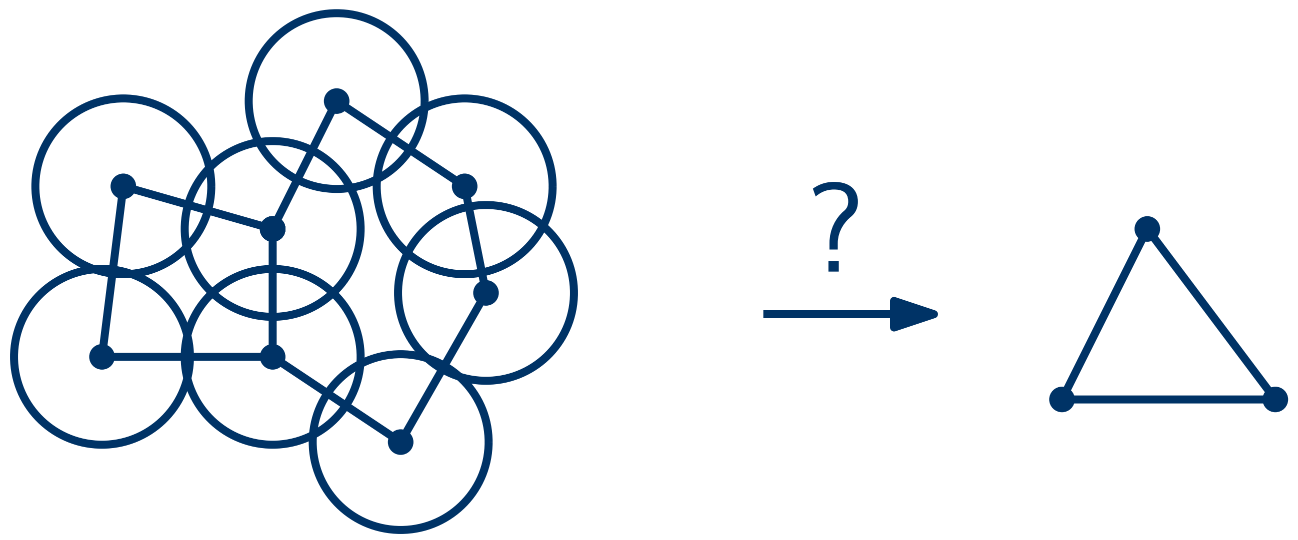

Geometric Setting

(General) Disk Graphs: $O(n \log n)$

[Kaplan et al., 2019]

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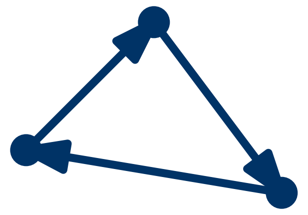

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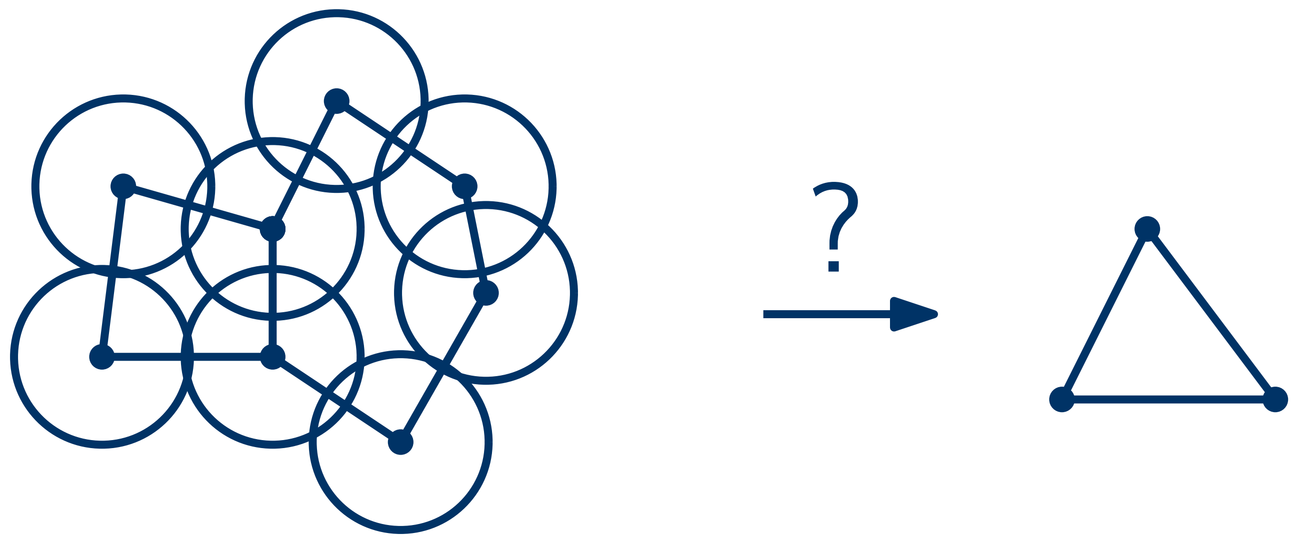

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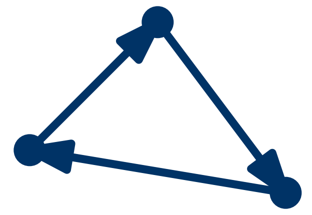

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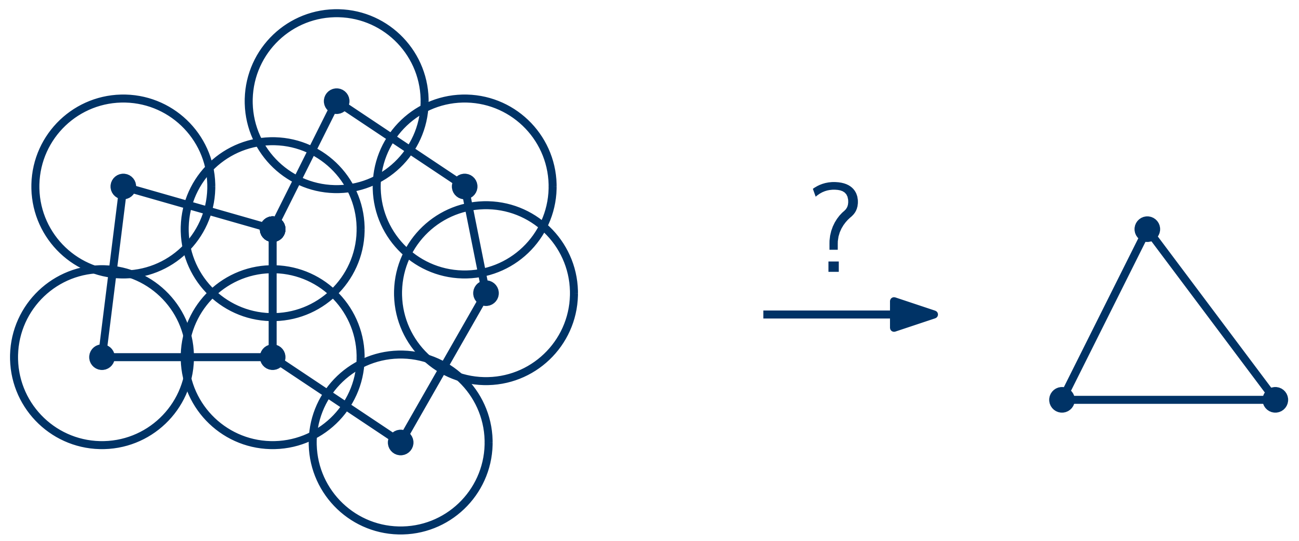

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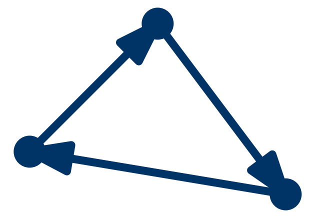

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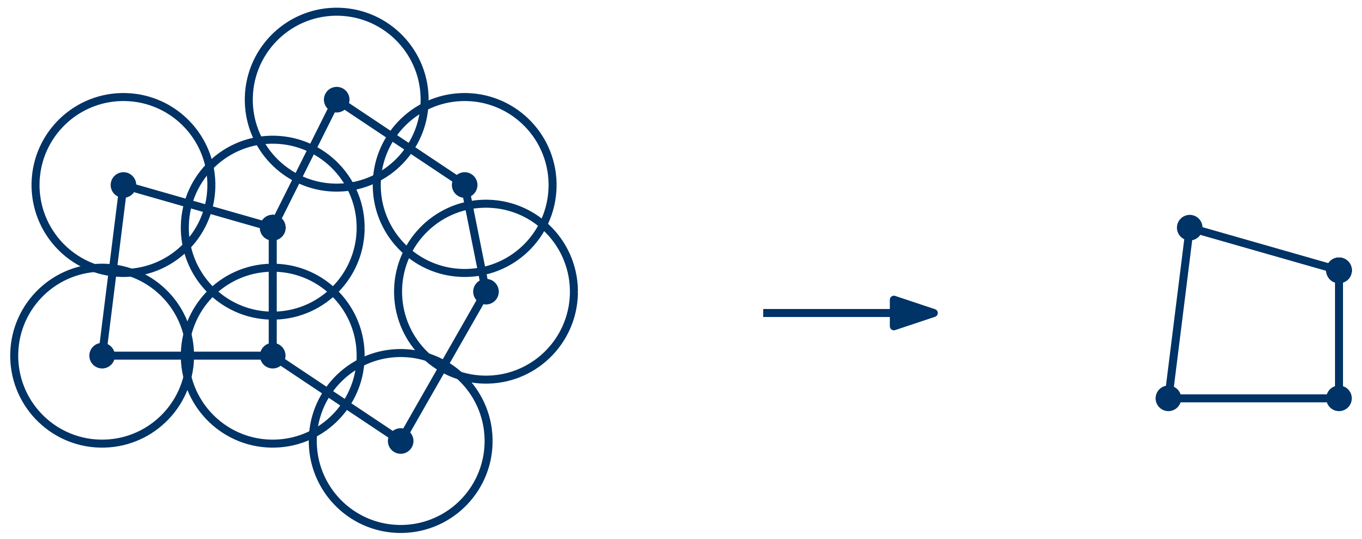

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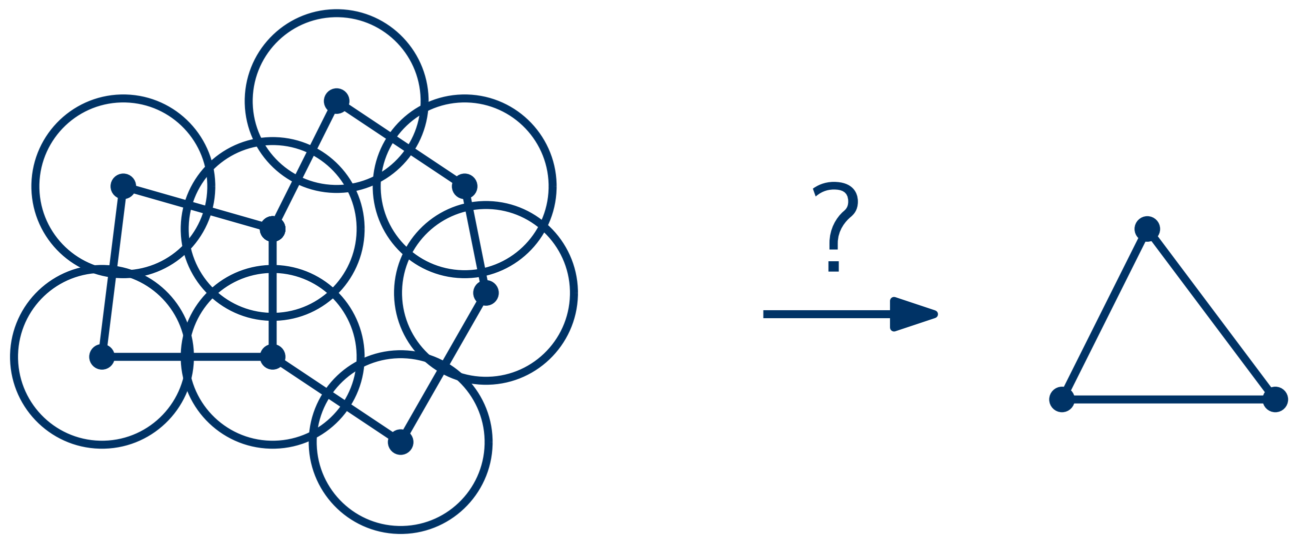

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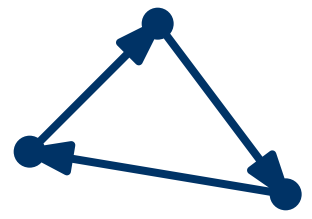

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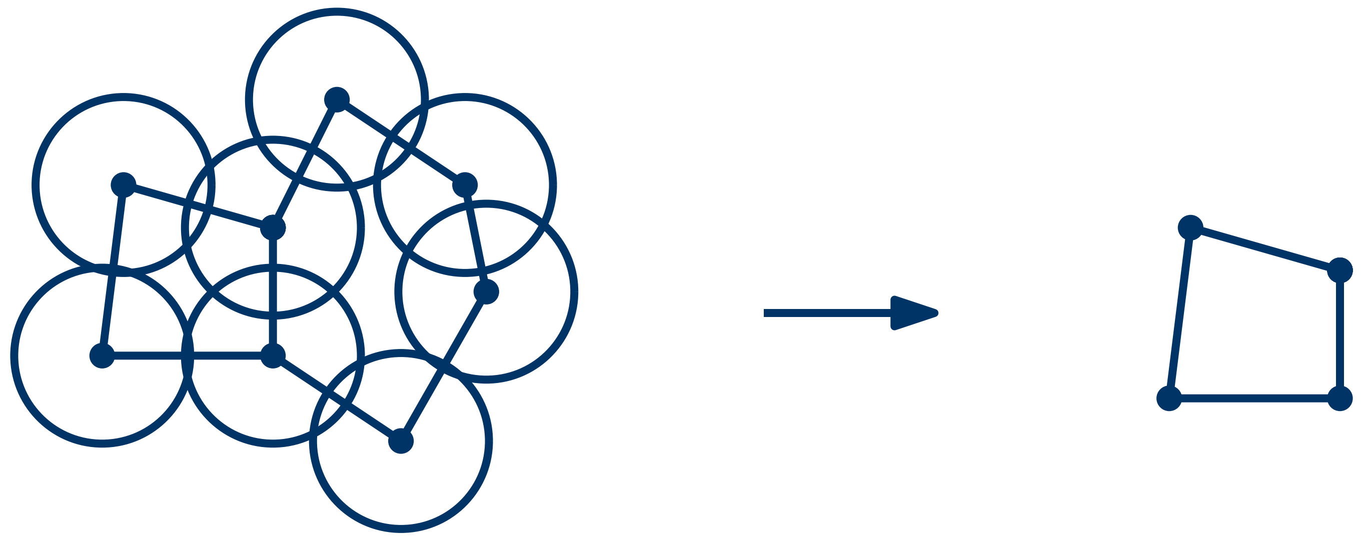

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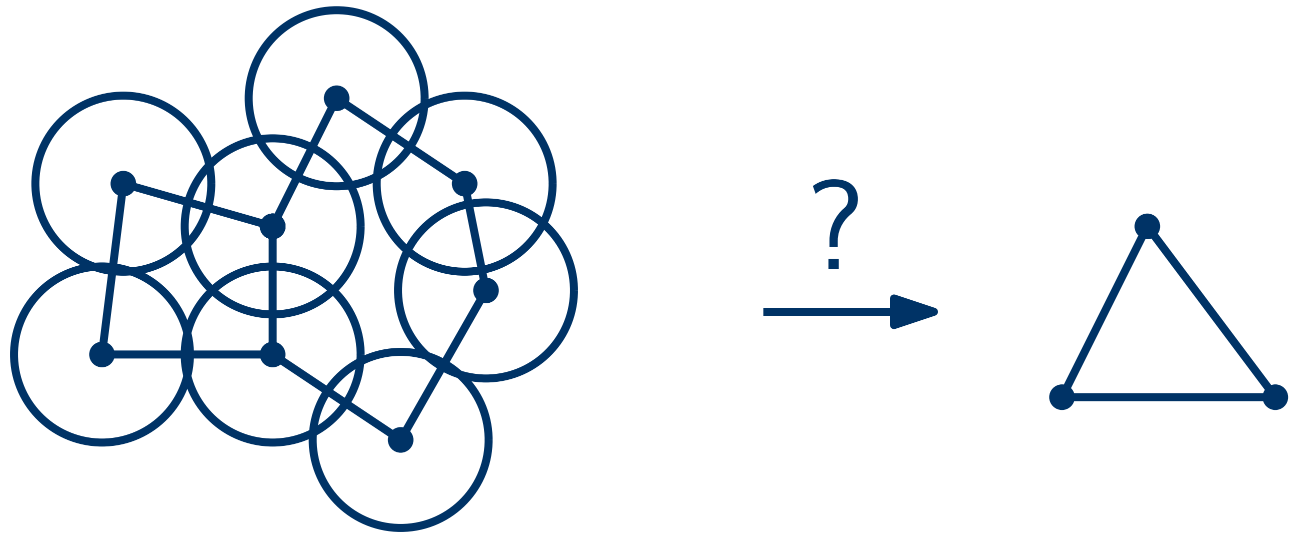

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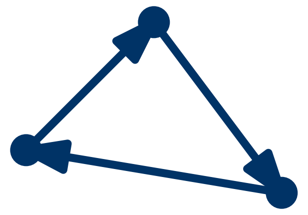

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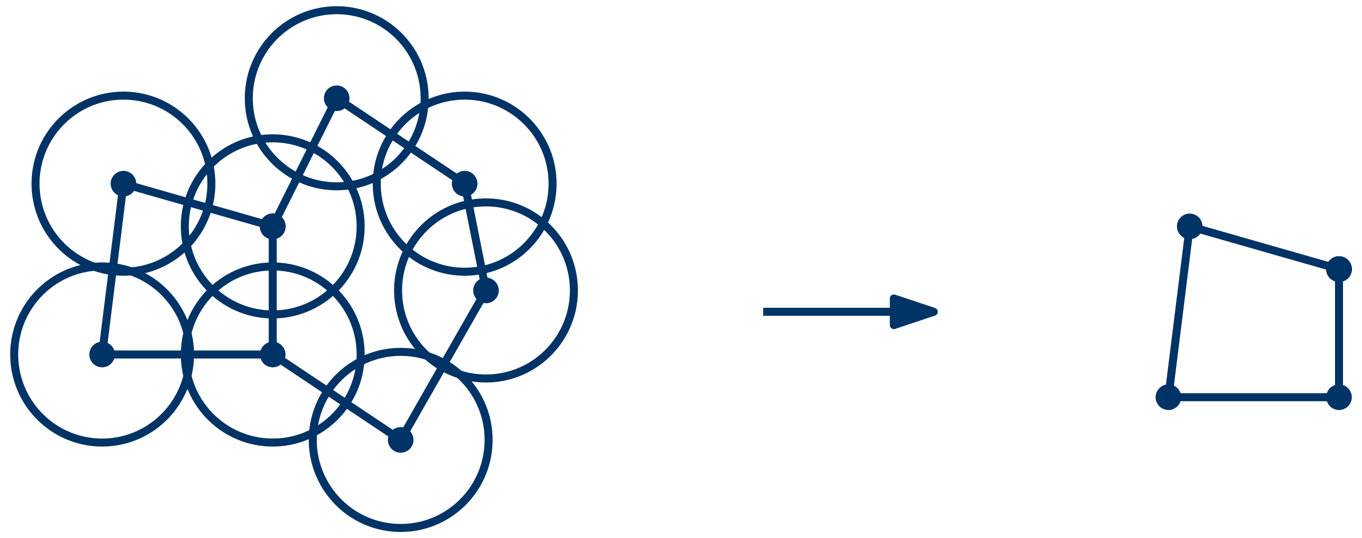

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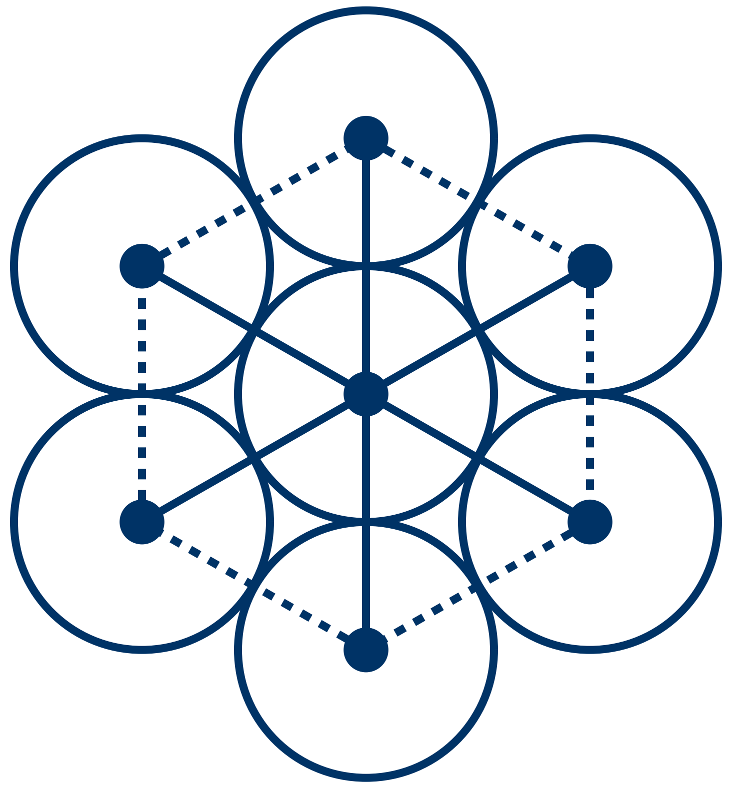

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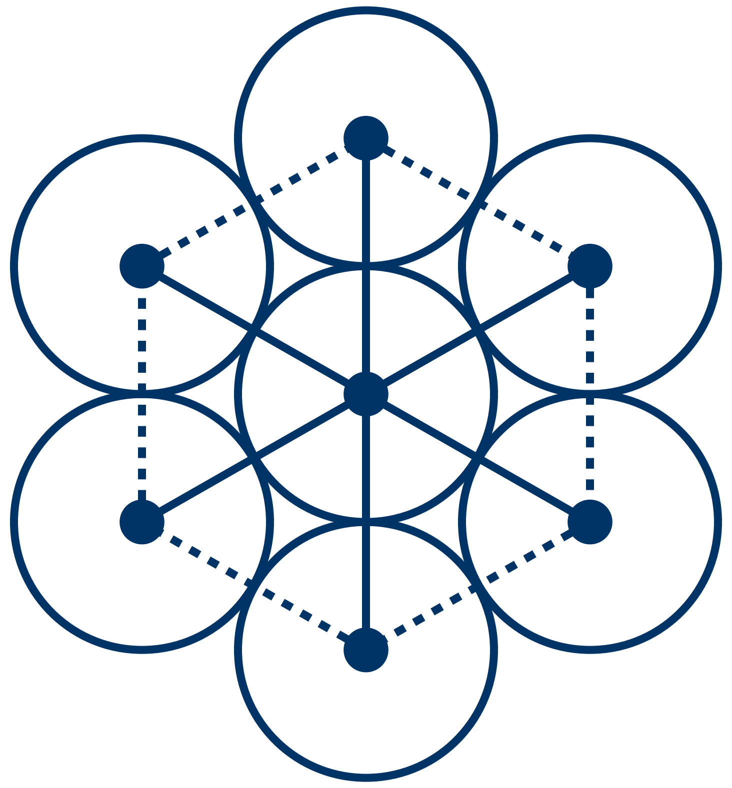

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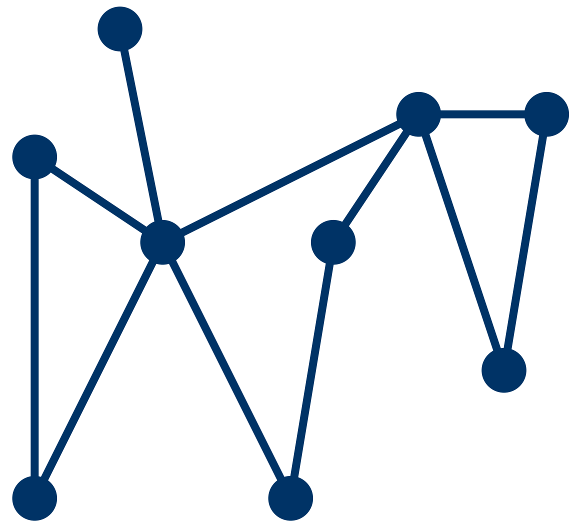

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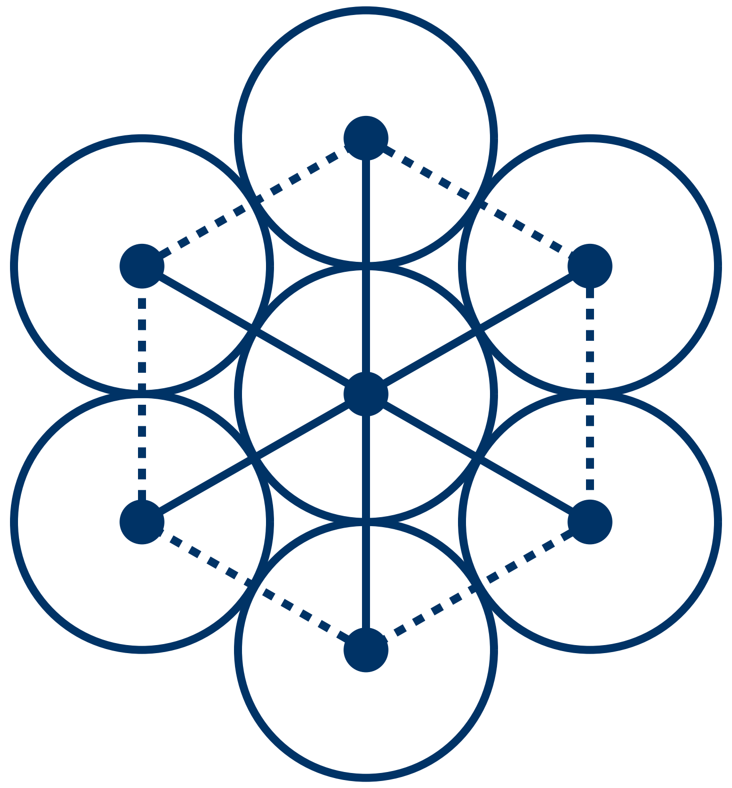

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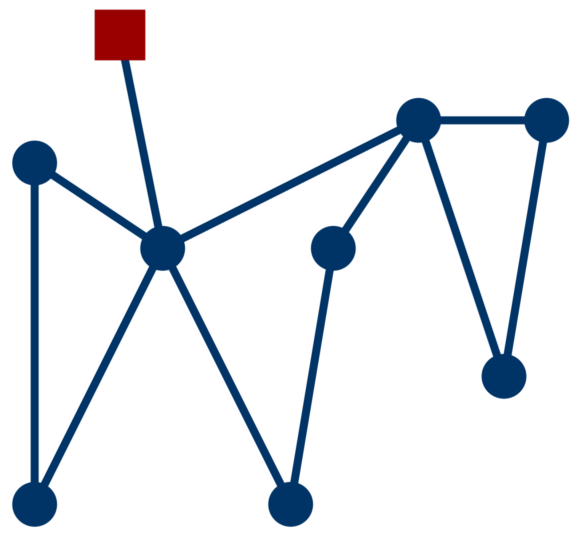

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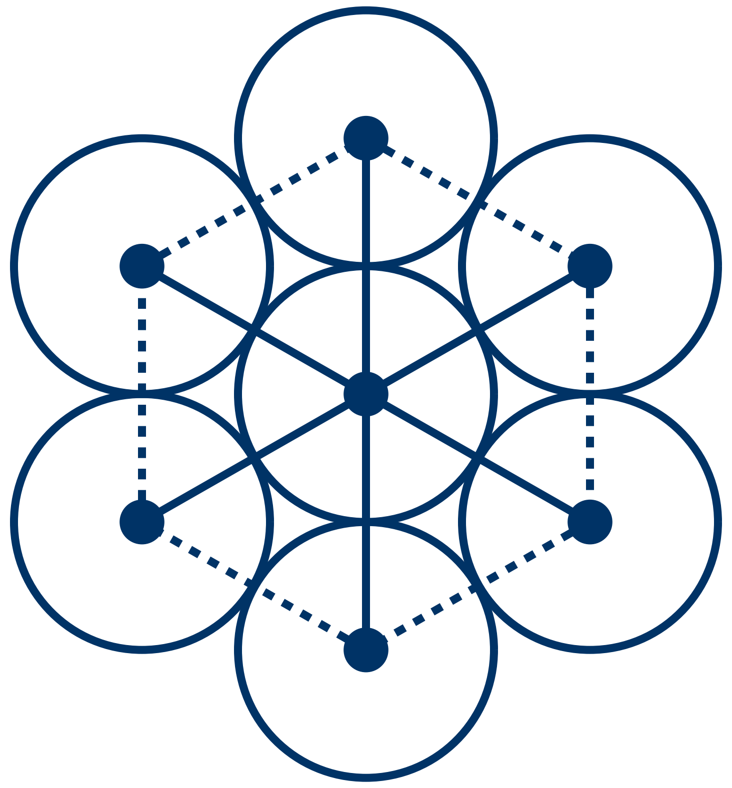

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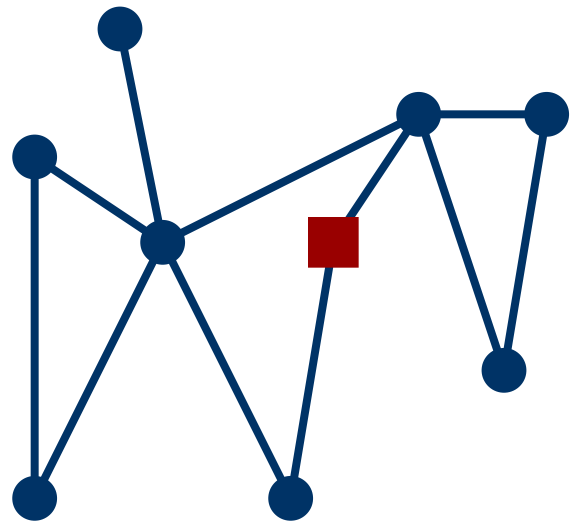

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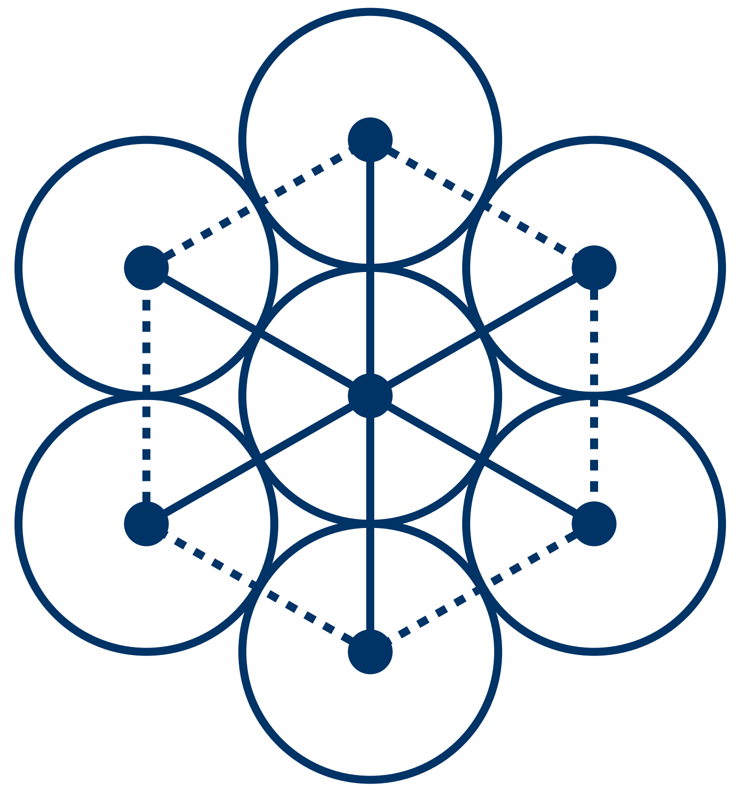

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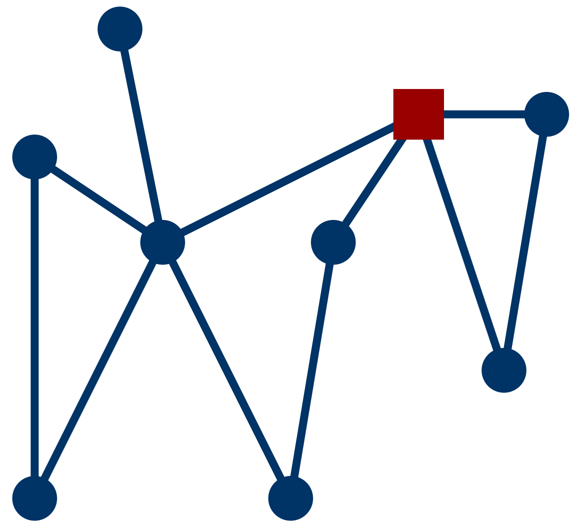

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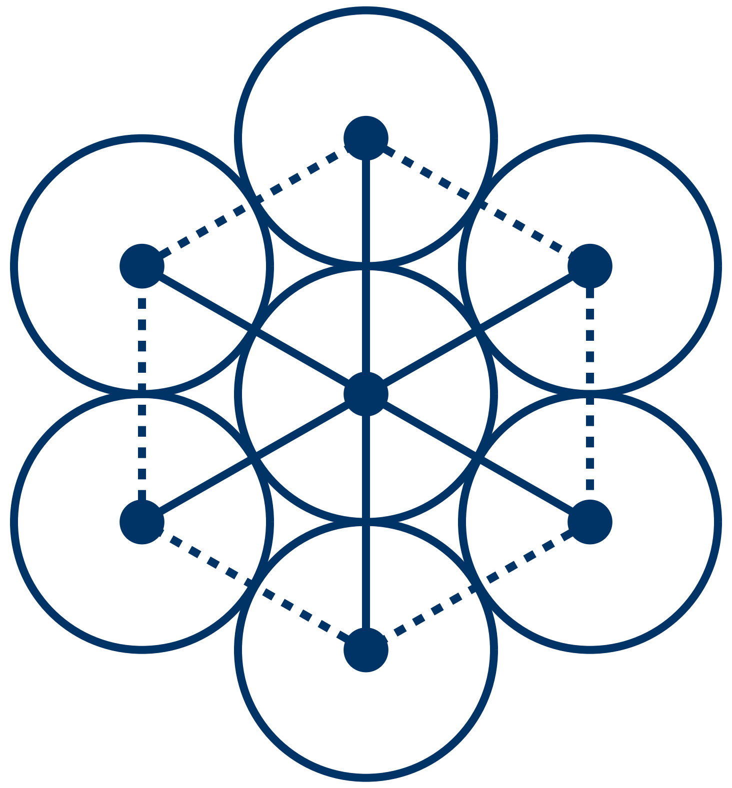

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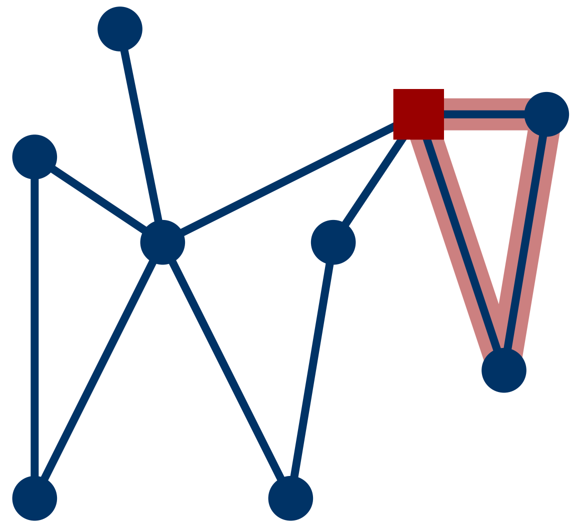

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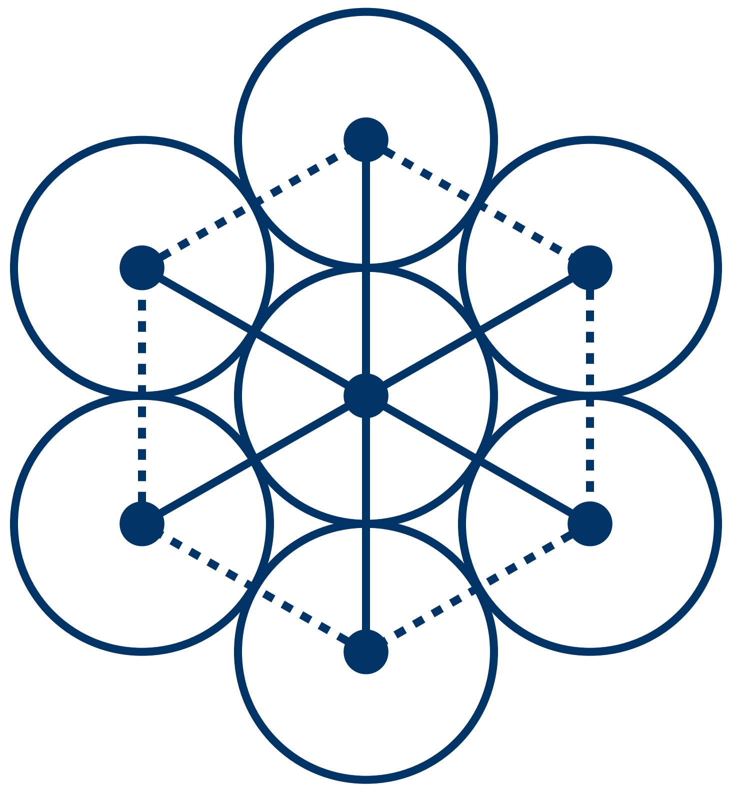

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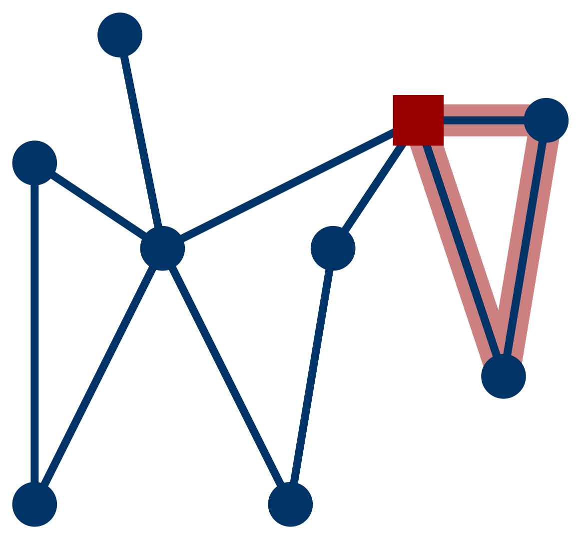

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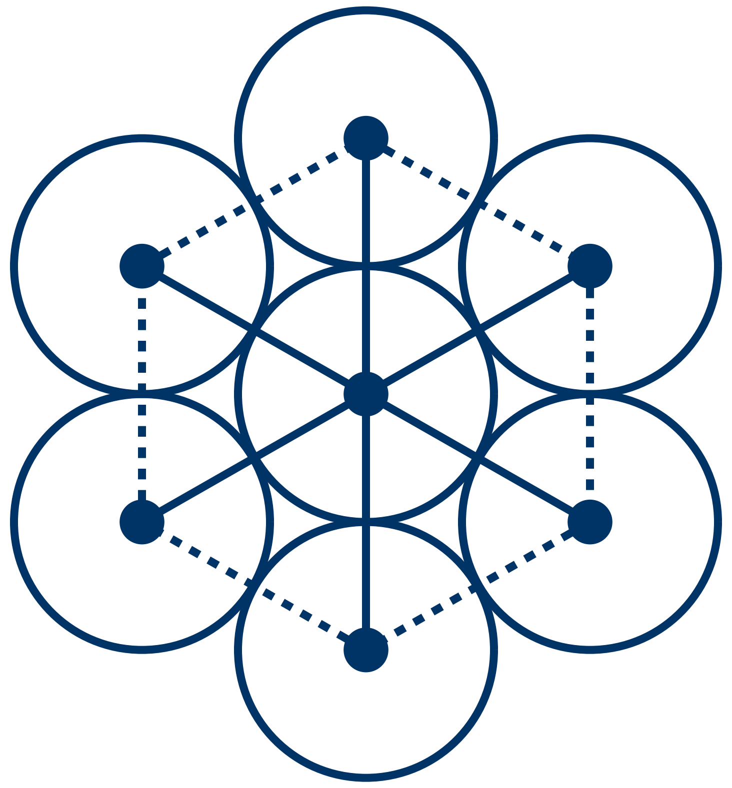

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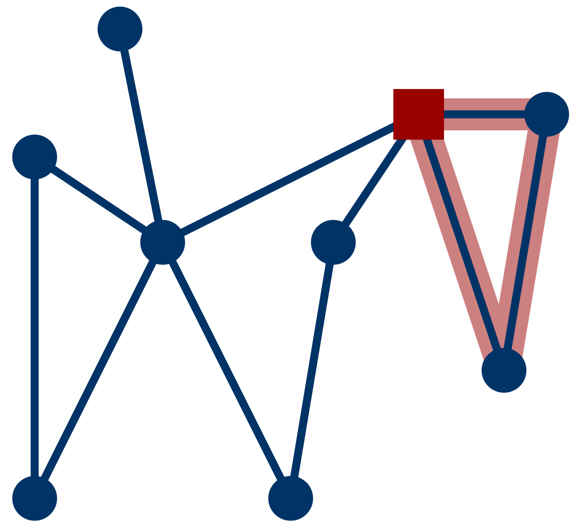

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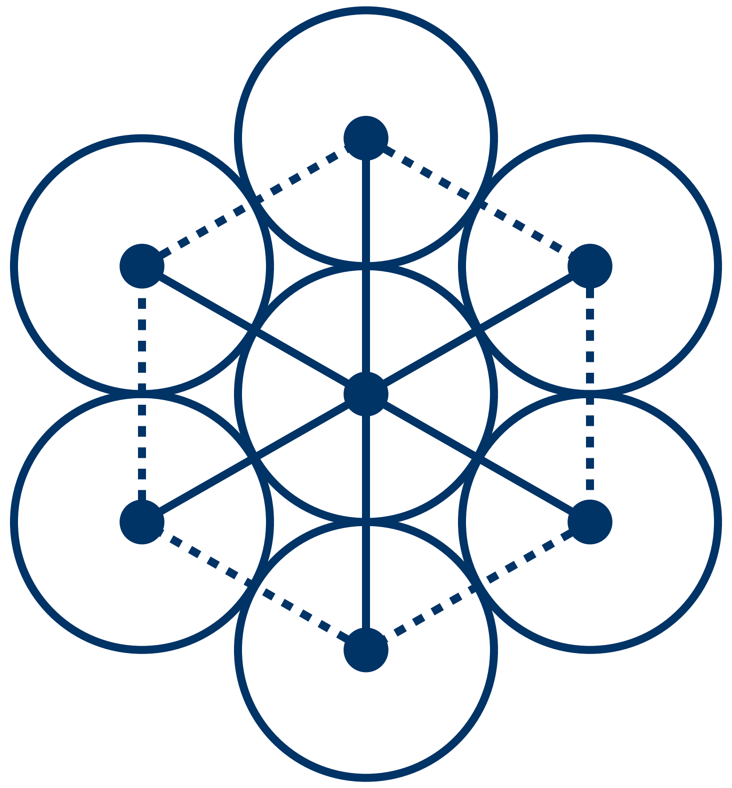

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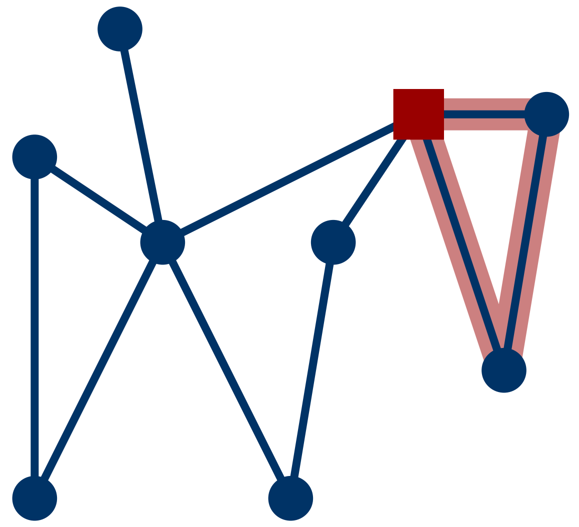

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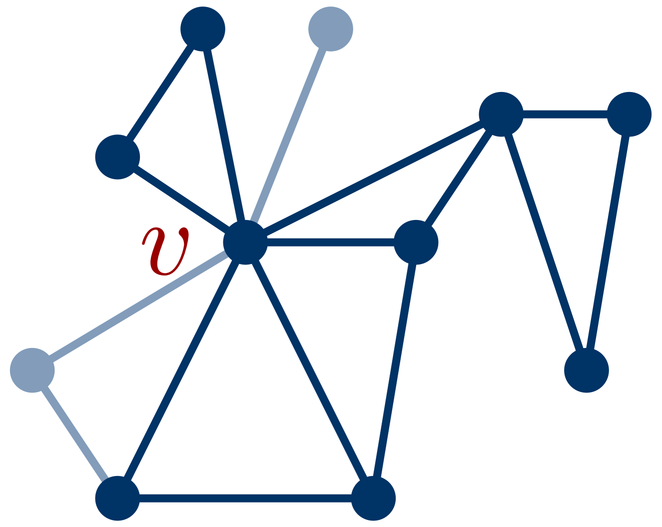

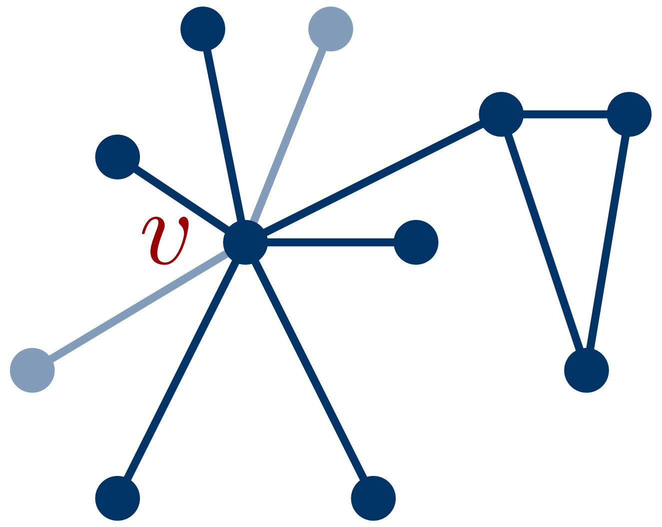

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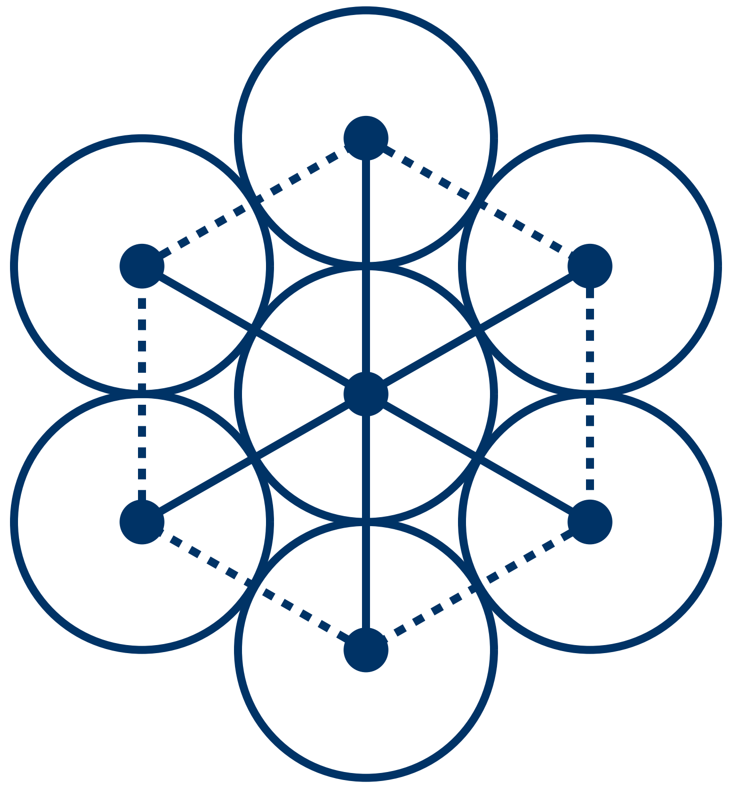

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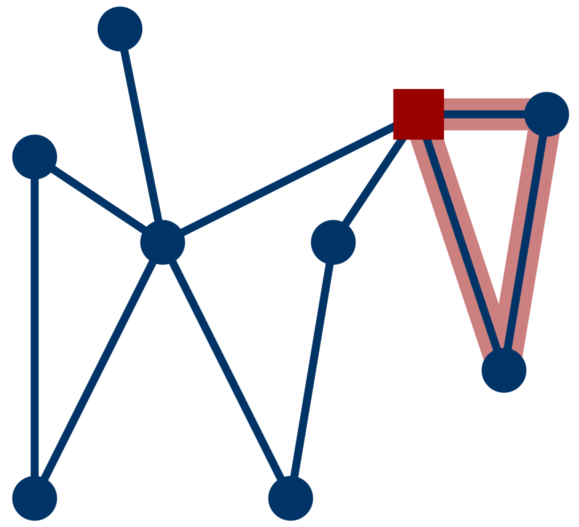

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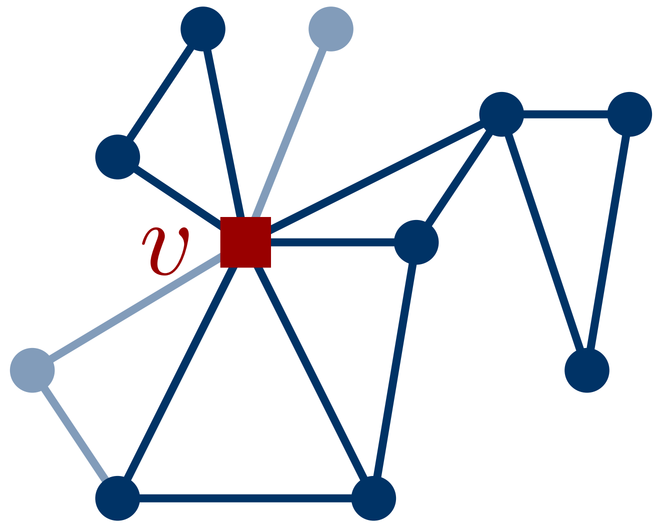

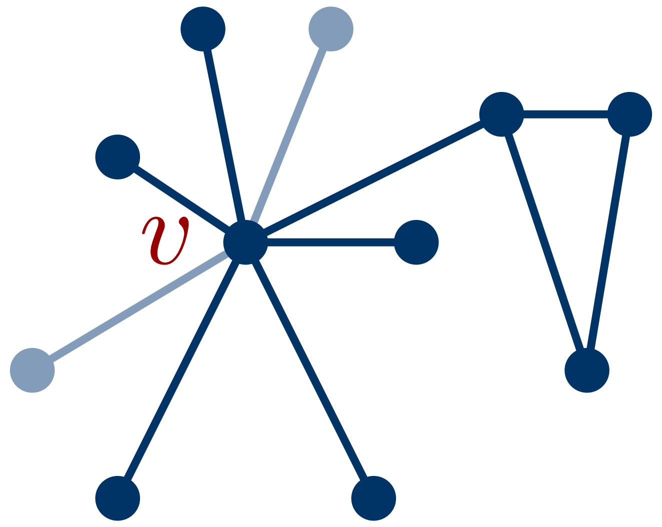

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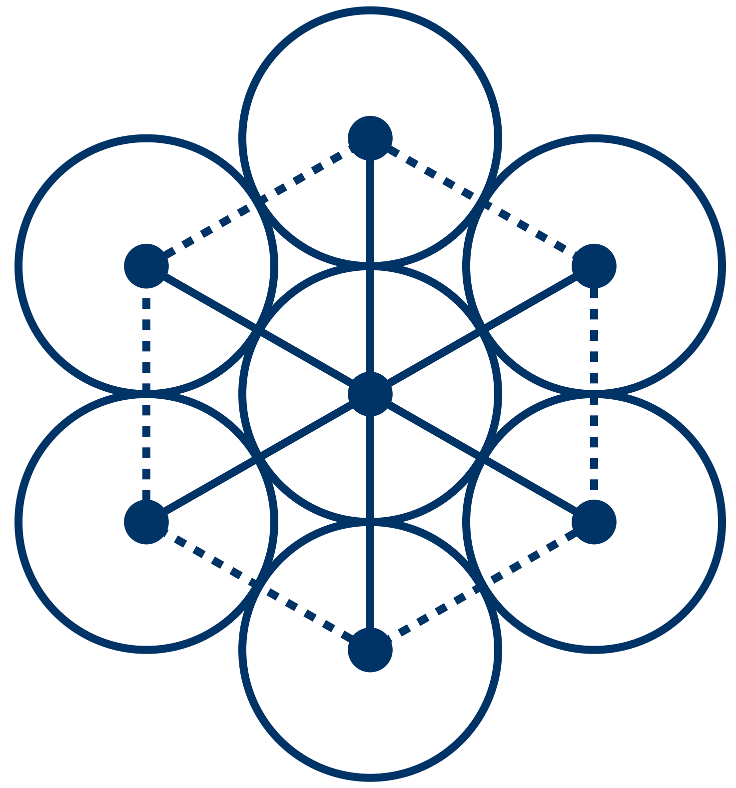

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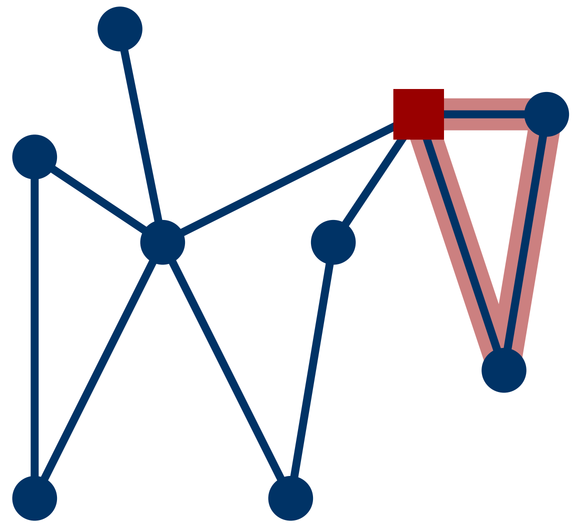

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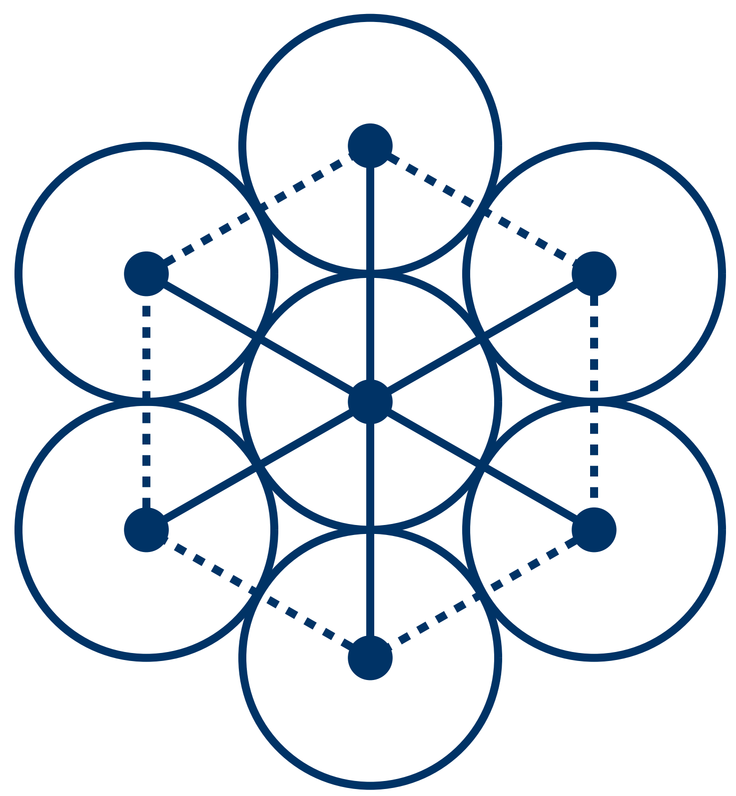

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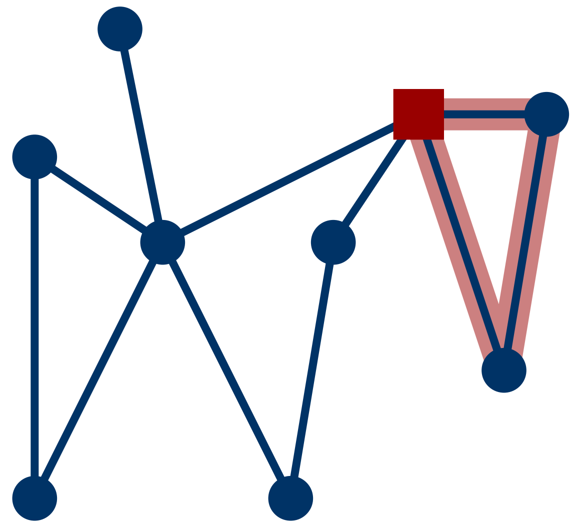

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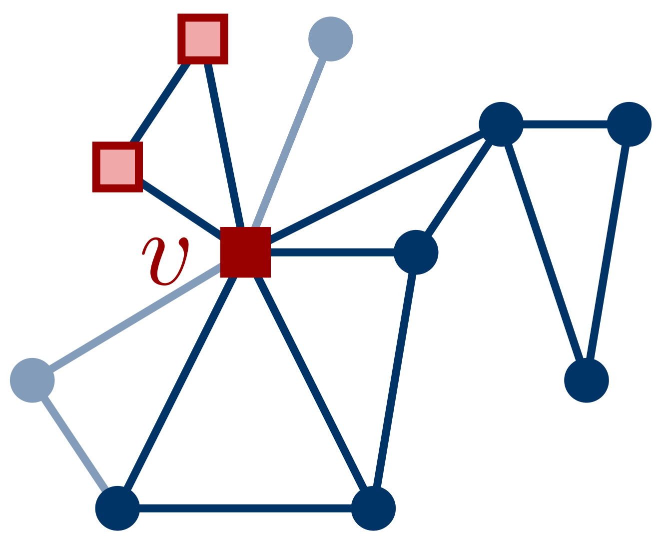
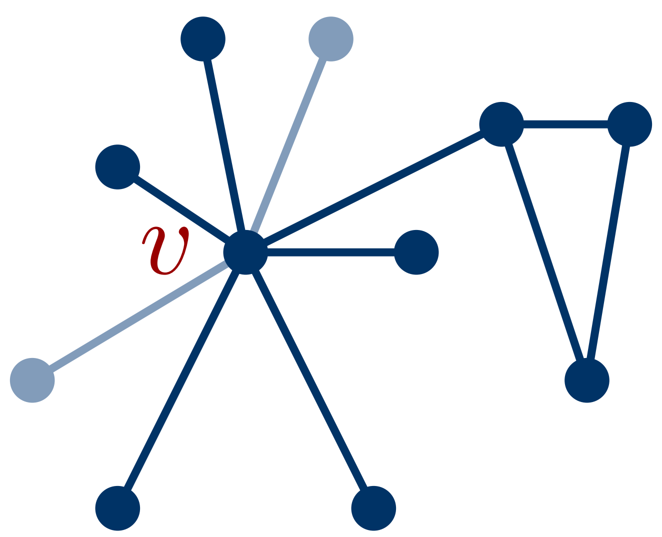
explicitly check

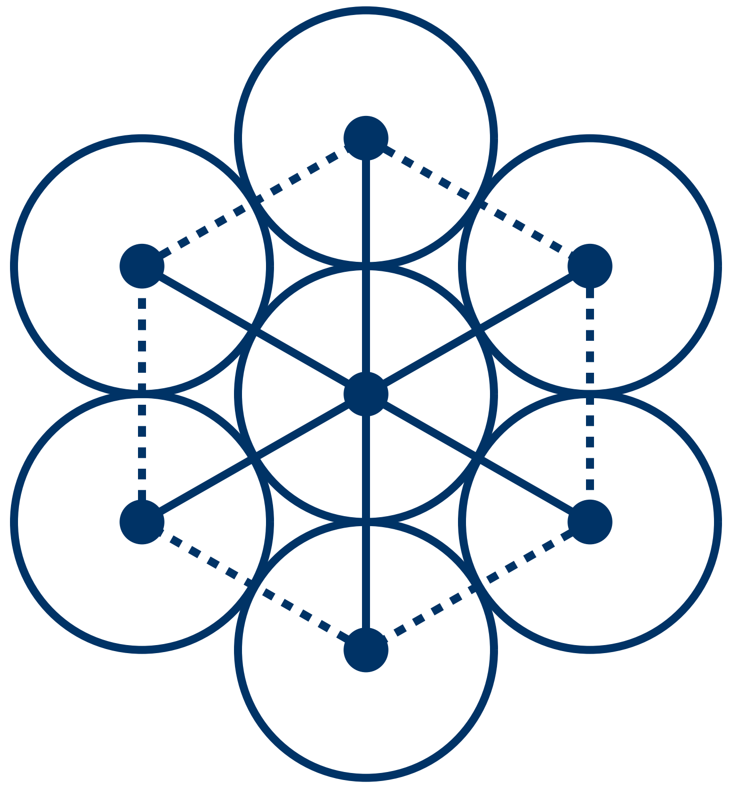
Case 2: At least one vertex v with $\deg(v) \geq 6$





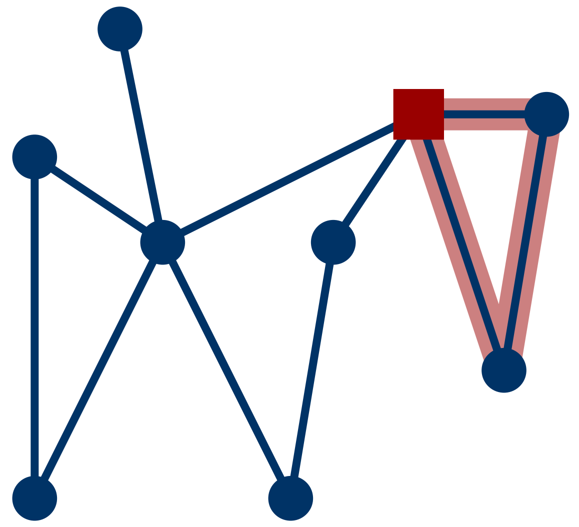
Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

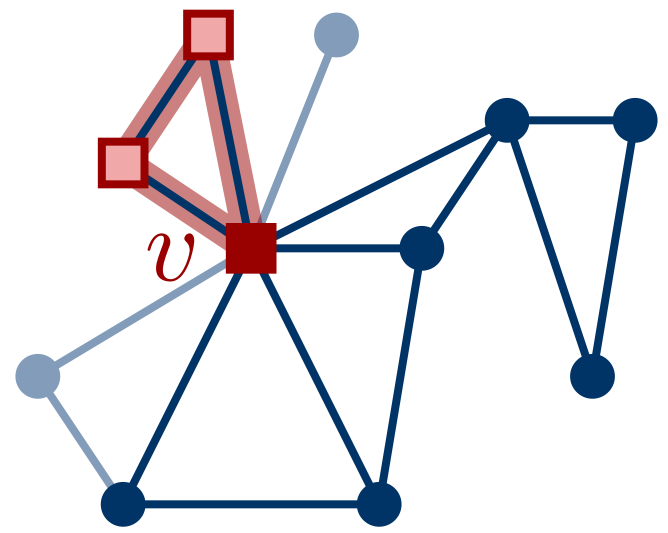
Case 1: All degrees smaller than 6

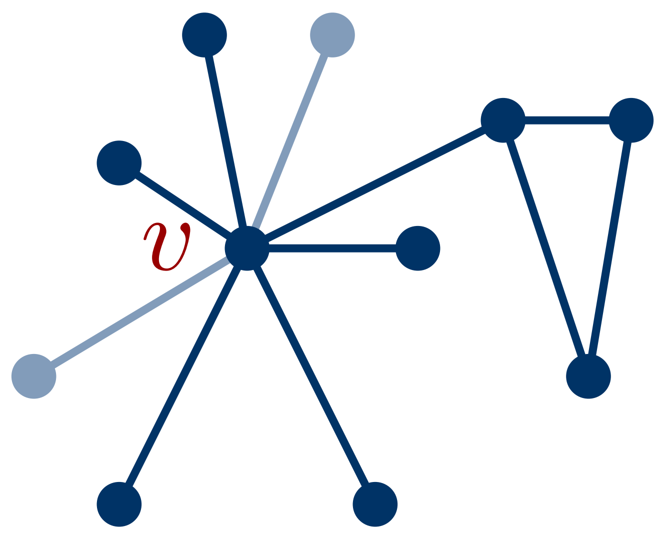


$O(n)$

explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$

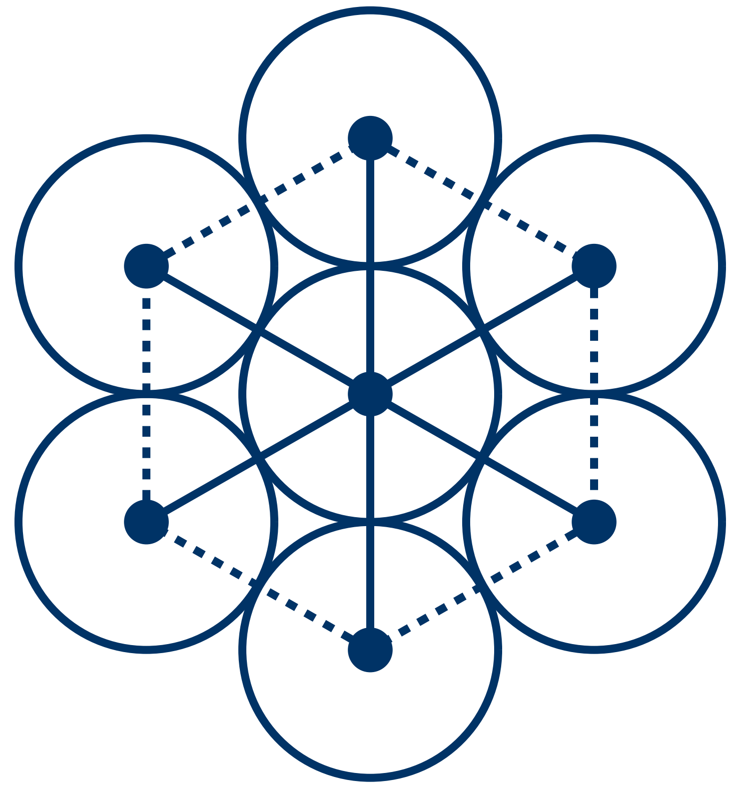




report triangle

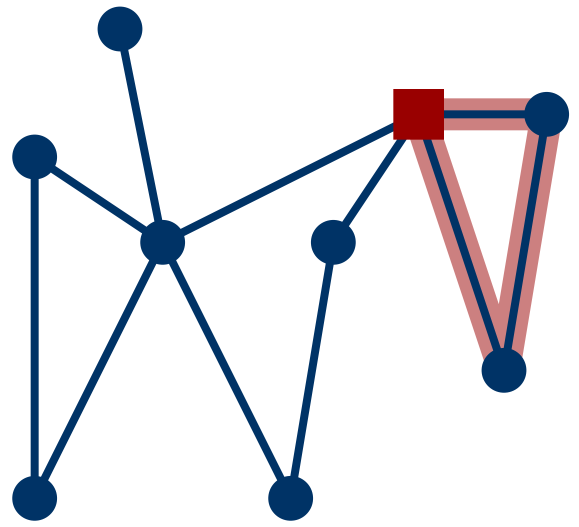
Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

Case 1: All degrees smaller than 6



$O(n)$

explicitly check

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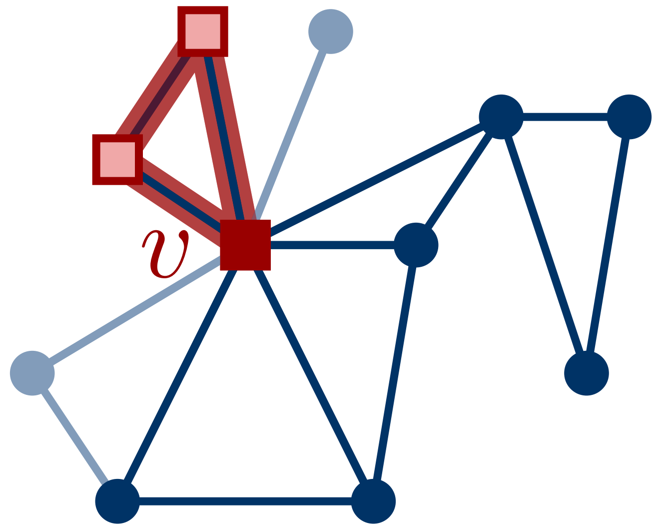
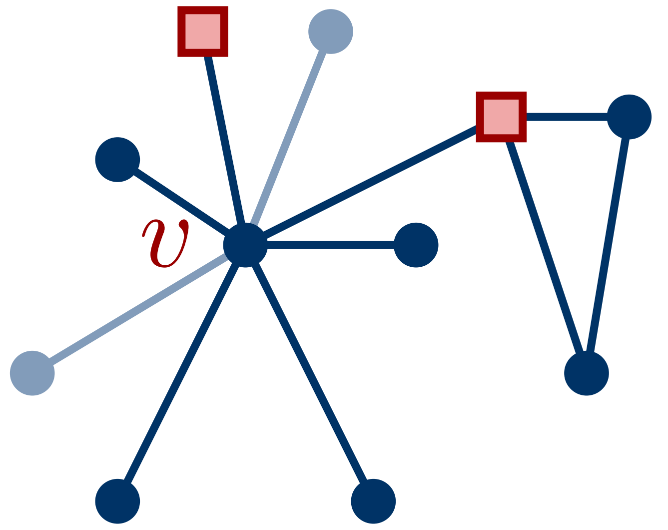
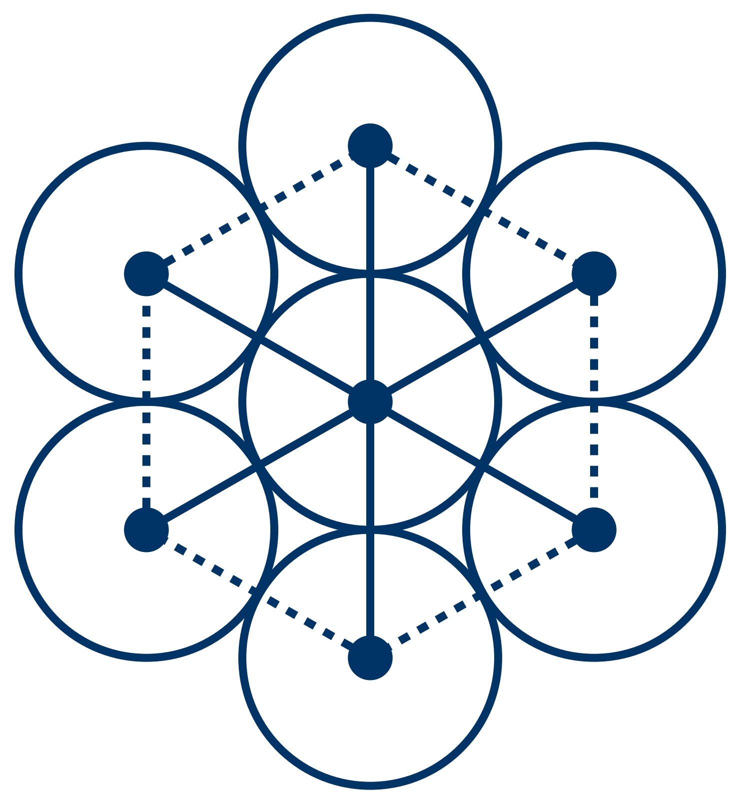




report triangle

Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

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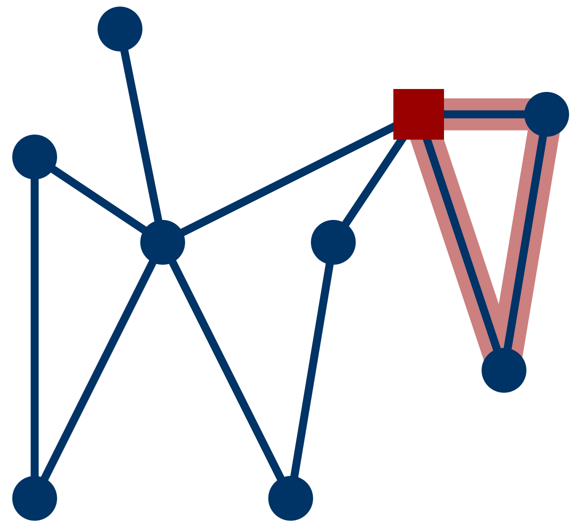

$O(n)$

explicitly check

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report triangle

Triangle Detection in Unit Disk Graphs

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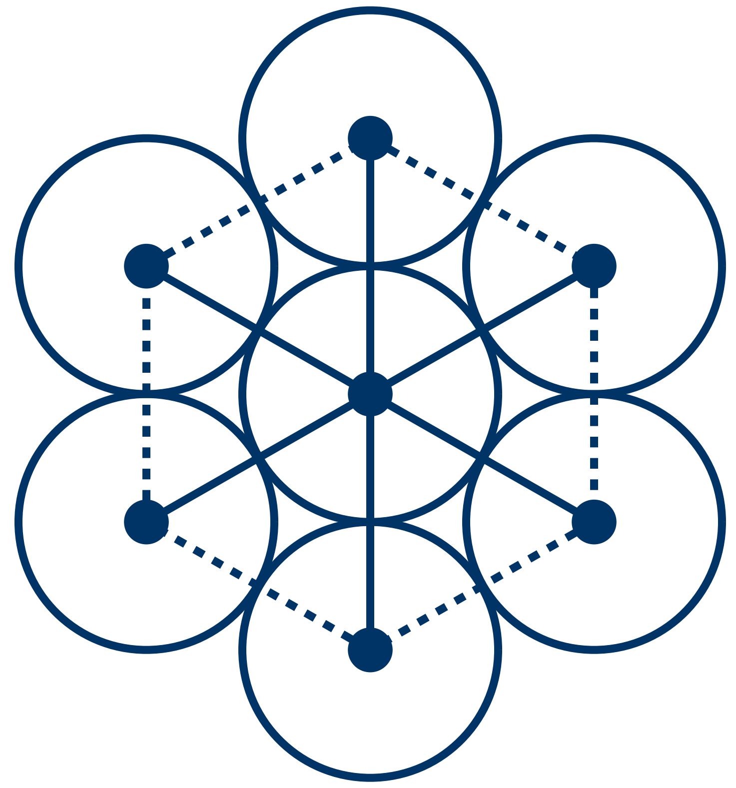

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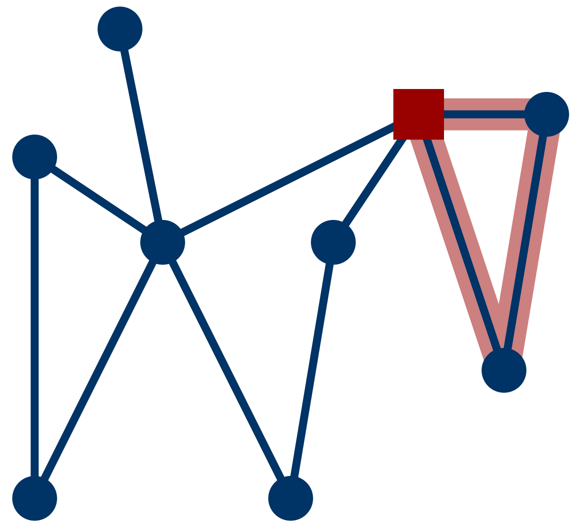

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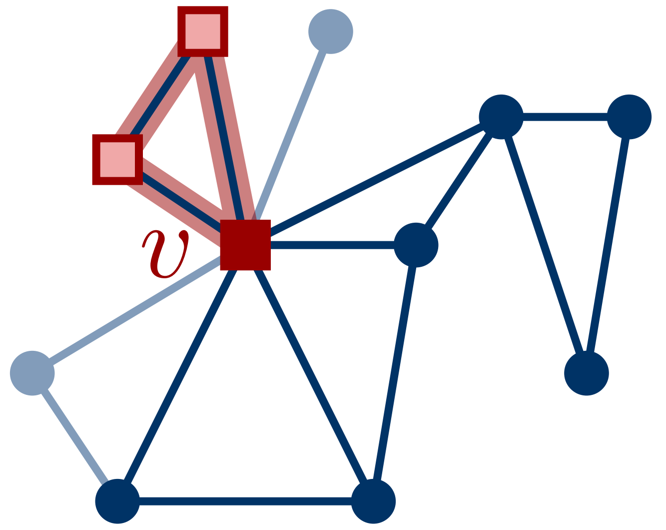

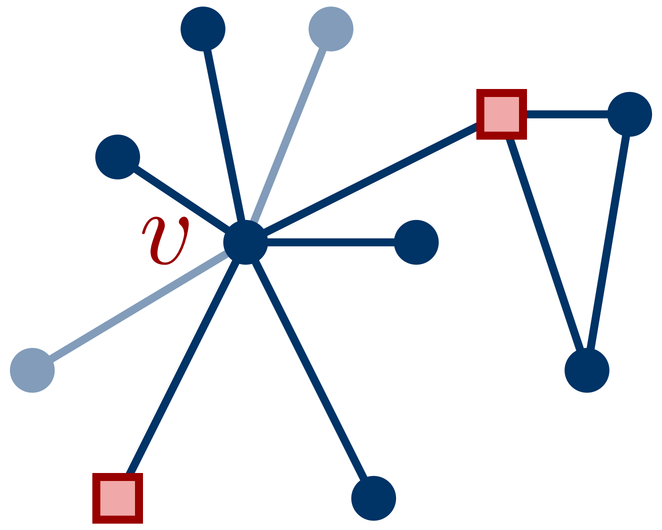

report triangle

Triangle Detection in Unit Disk Graphs

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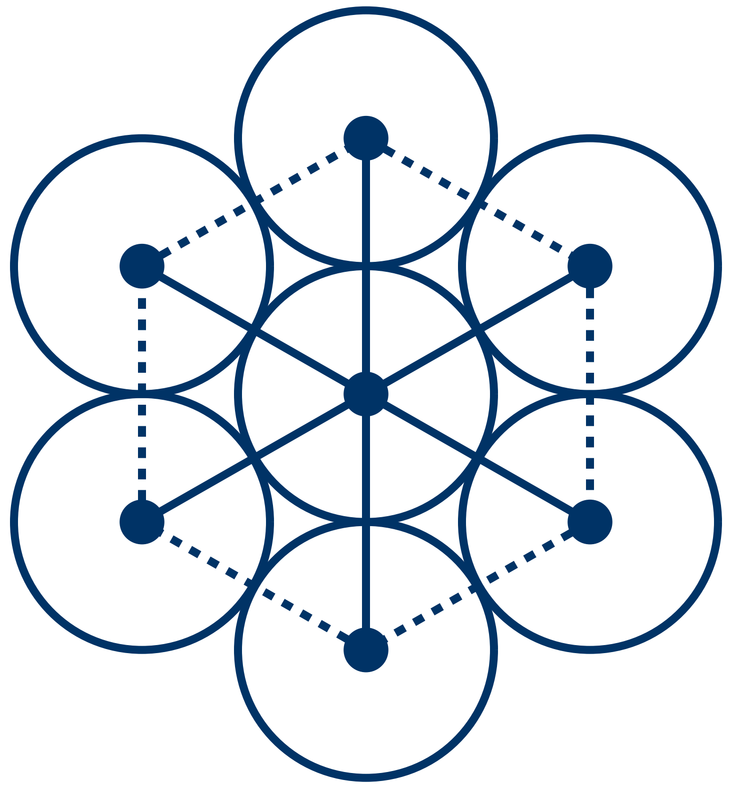

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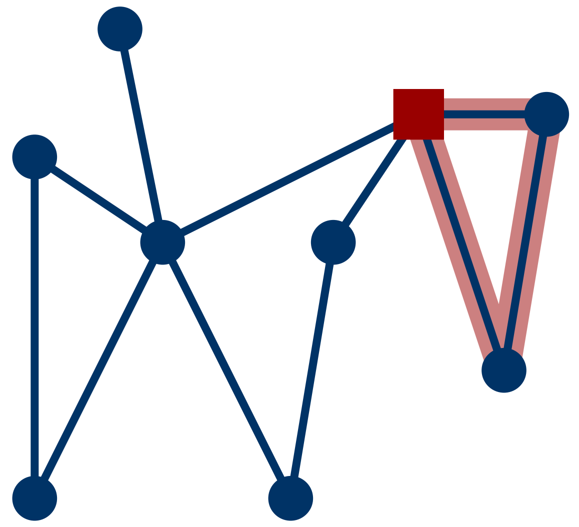

$O(n)$

explicitly check

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report triangle

Triangle Detection in Unit Disk Graphs

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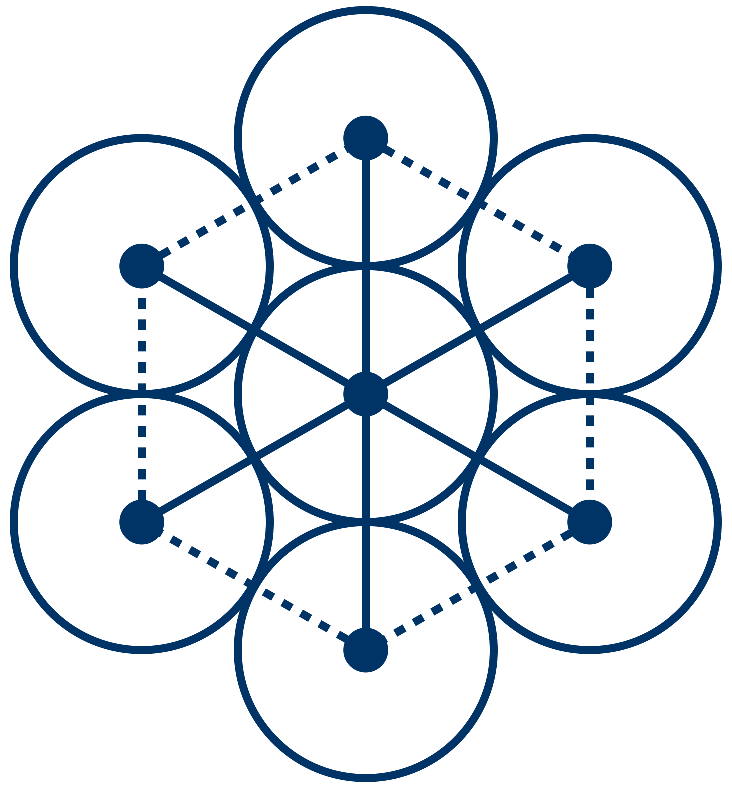

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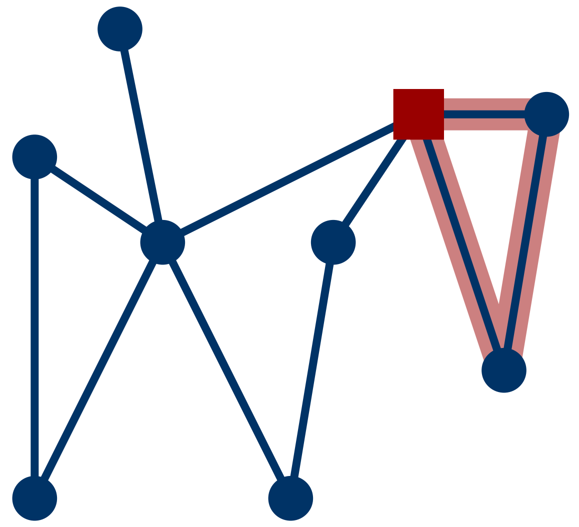
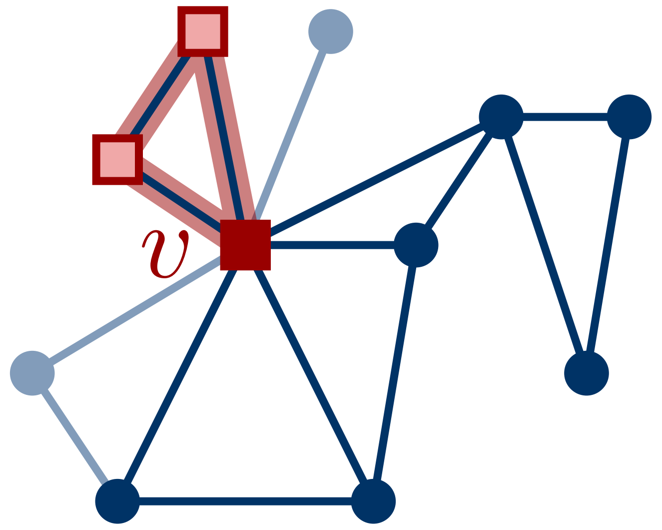
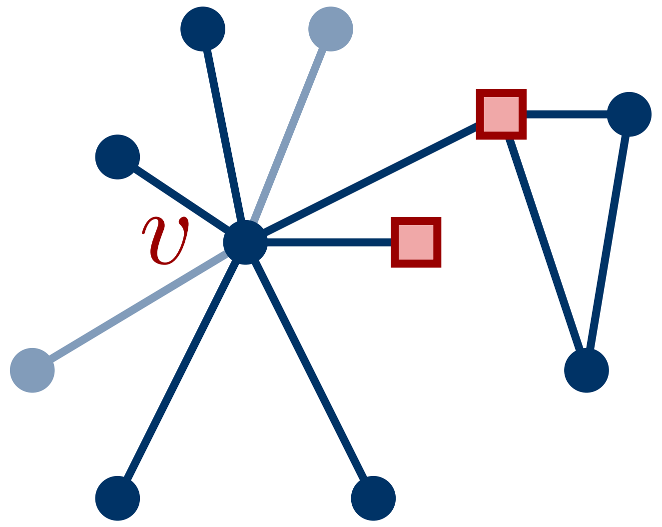


$O(n)$

explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$

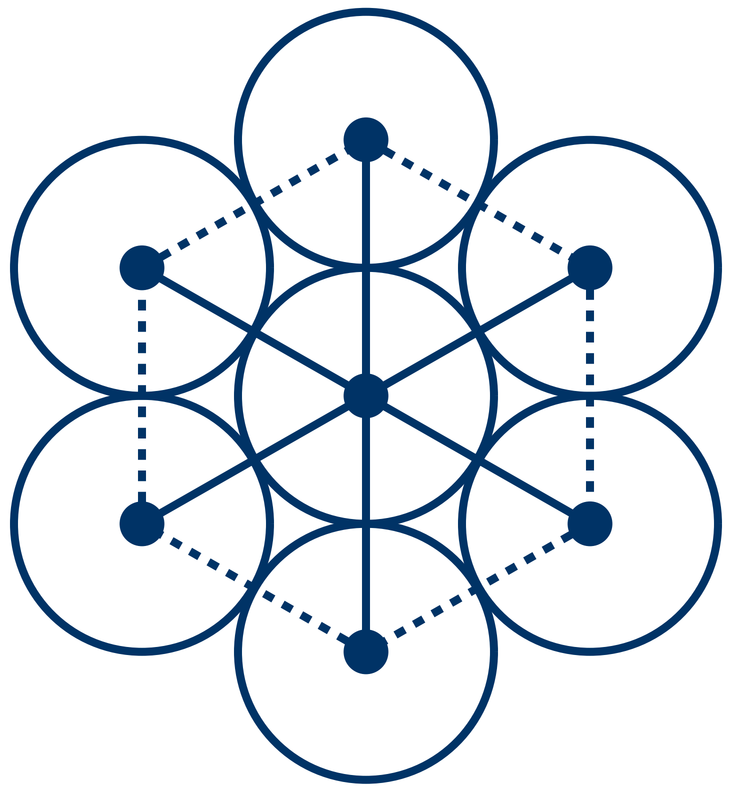




report triangle

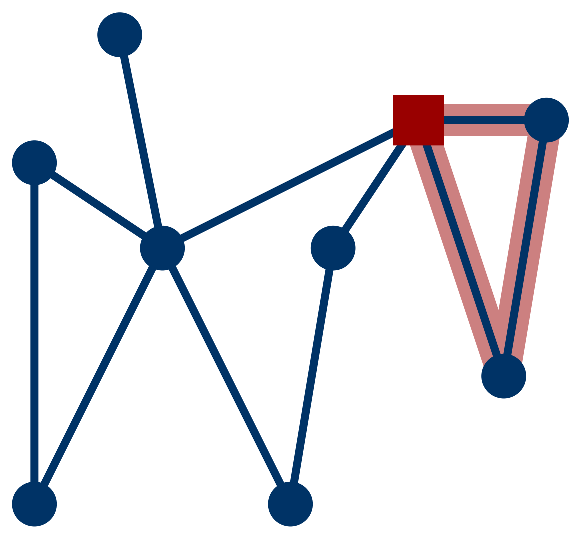
Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

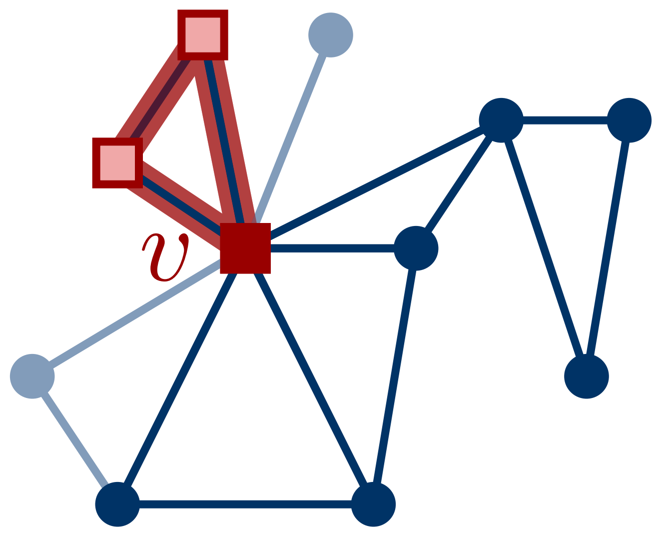
Case 1: All degrees smaller than 6

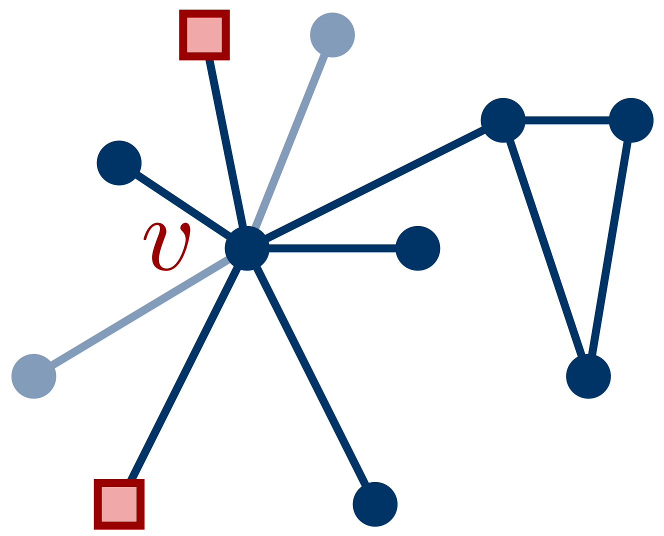


$O(n)$

explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$

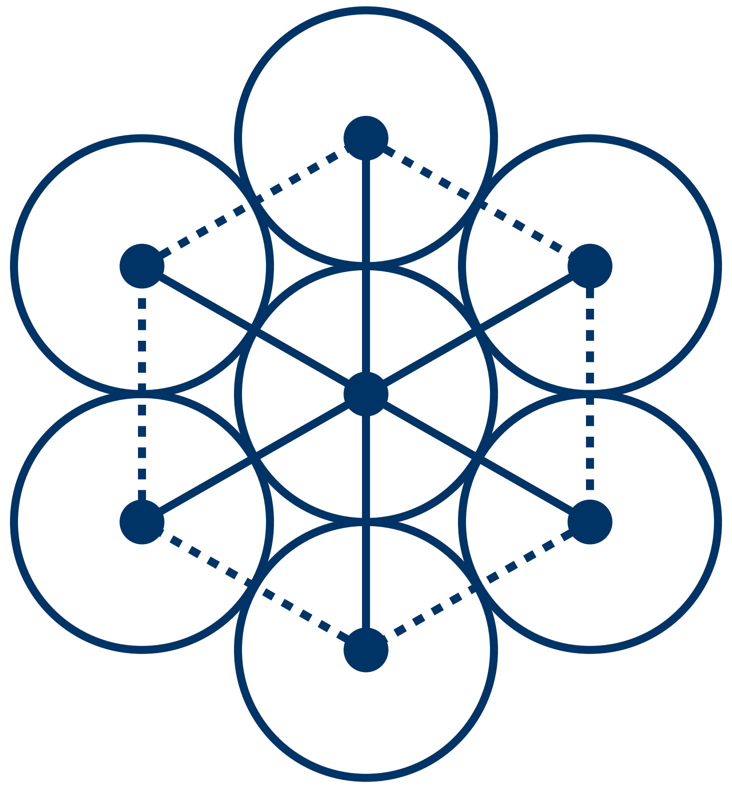




report triangle

Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

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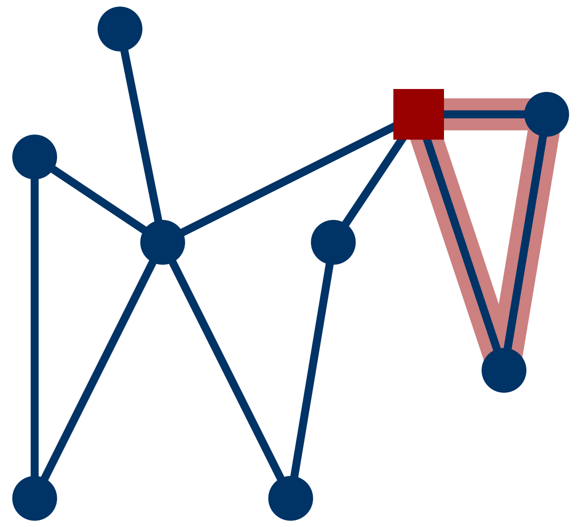
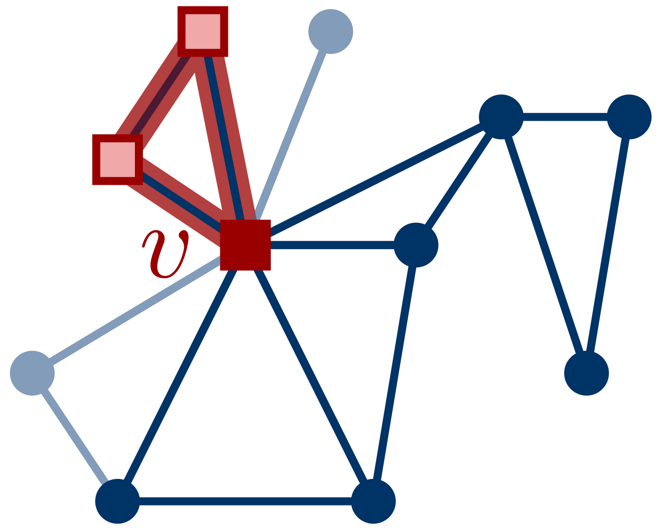


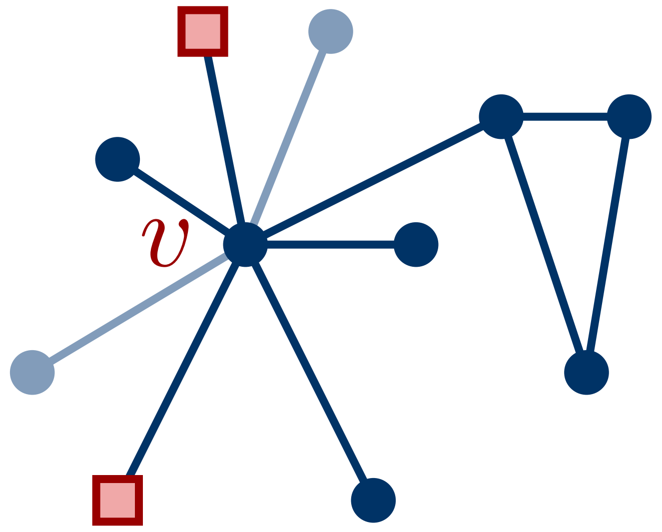
$O(n)$

explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$



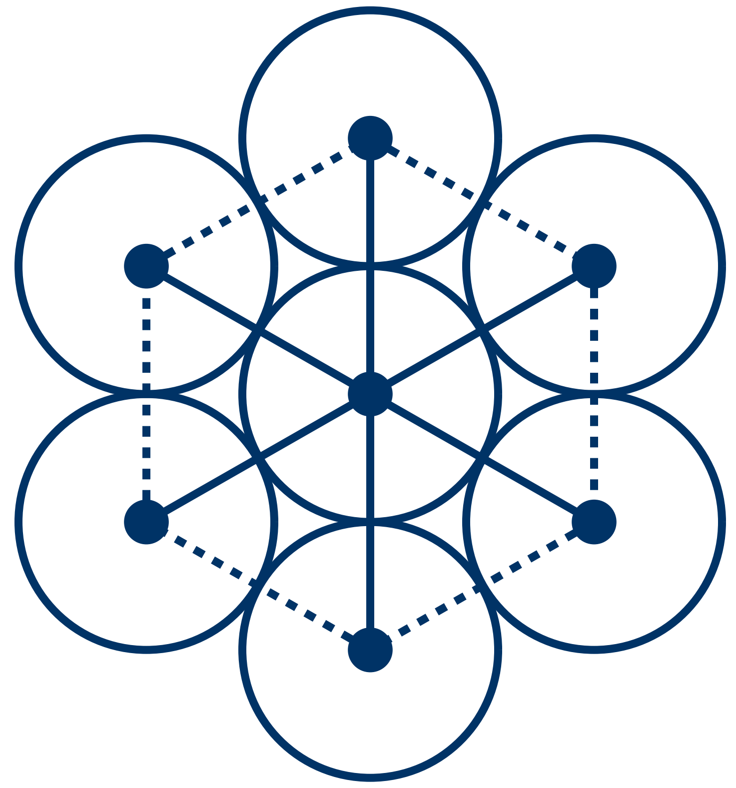
report triangle



not a unit disk graph

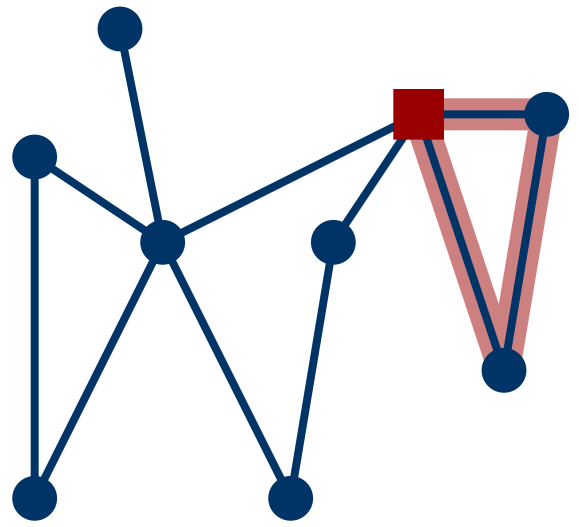
Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.



Algorithm

Case 1: All degrees smaller than 6

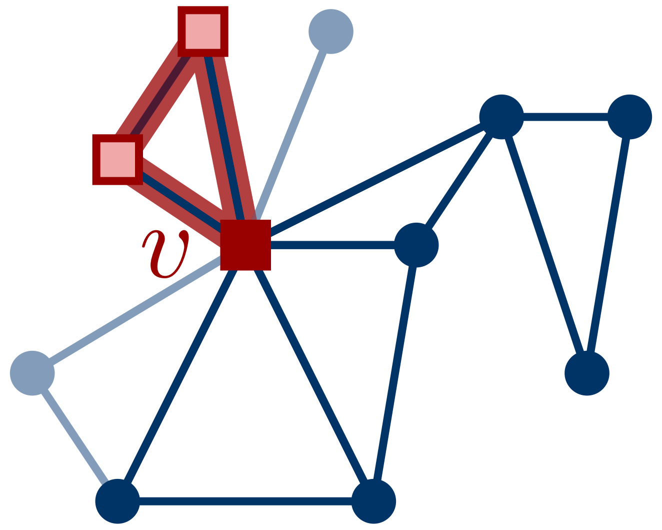


$O(n)$

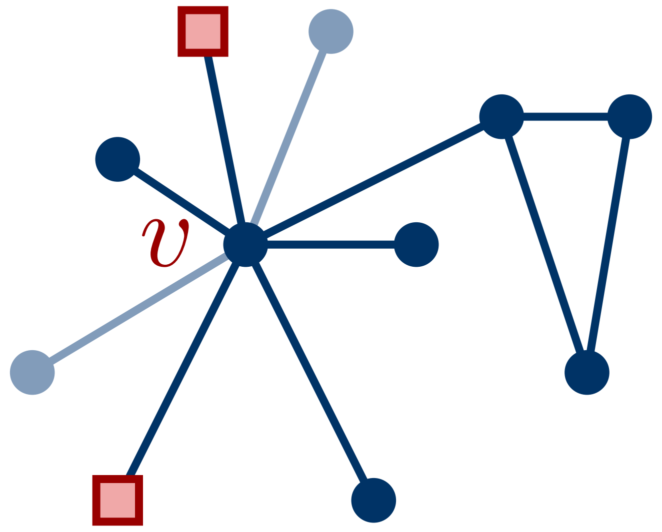
explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$

$O(n)$



report triangle



not a unit disk graph

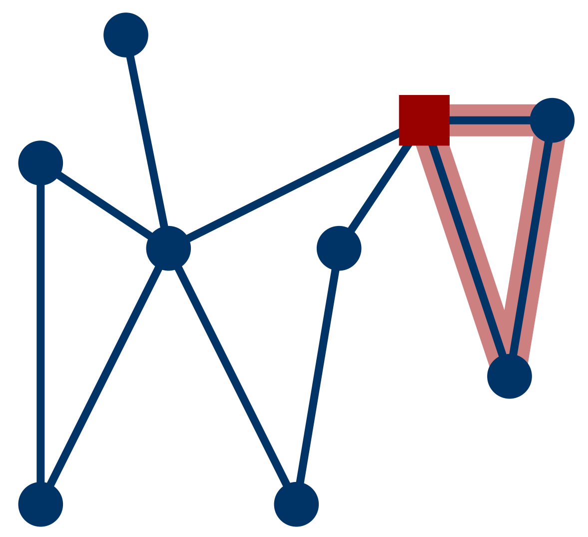
Triangle Detection in Unit Disk Graphs

Lemma Vertex v with $\deg(v) \geq 6$ in unit disk graph has triangle with two of any six neighbors.

Theorem There is a robust algorithm that finds a triangle in a unit disk graph in $O(n)$ time.

Algorithm

Case 1: All degrees smaller than 6

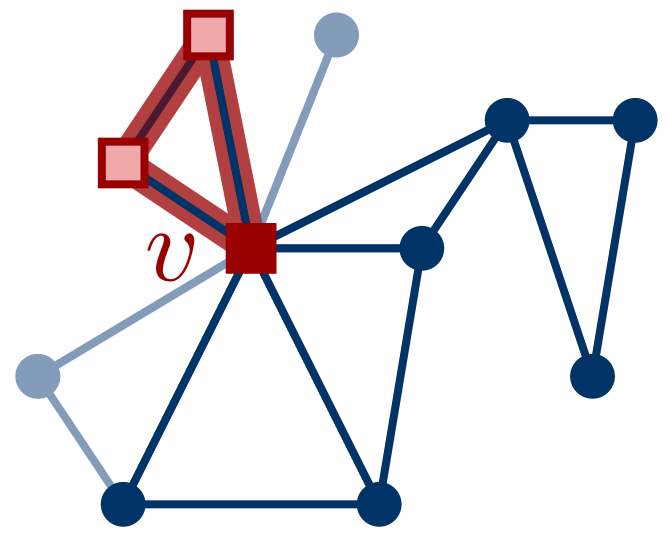


$O(n)$

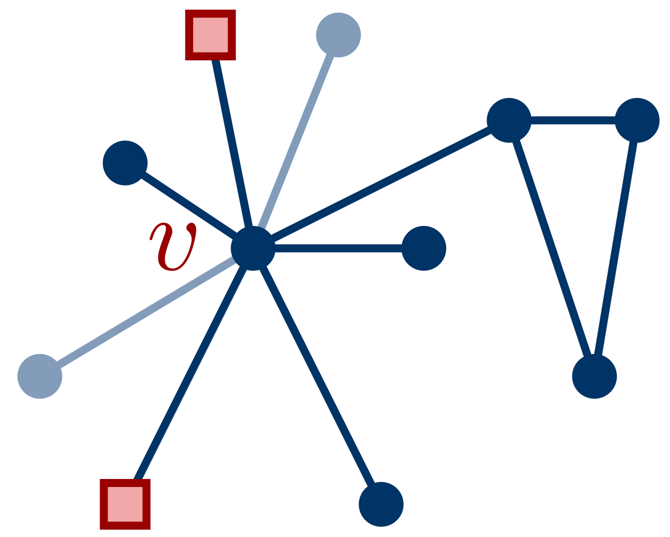
explicitly check

Case 2: At least one vertex v with $\deg(v) \geq 6$

$O(n)$

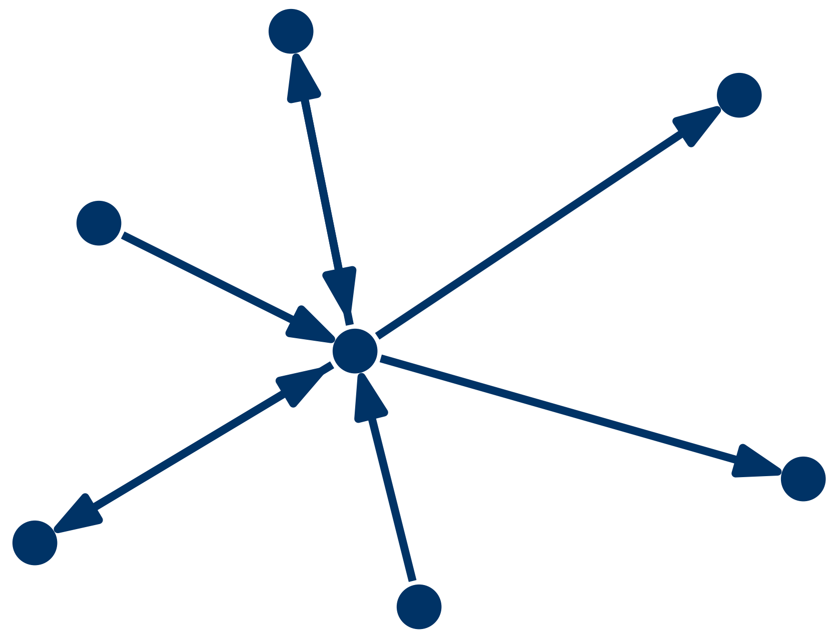


report triangle



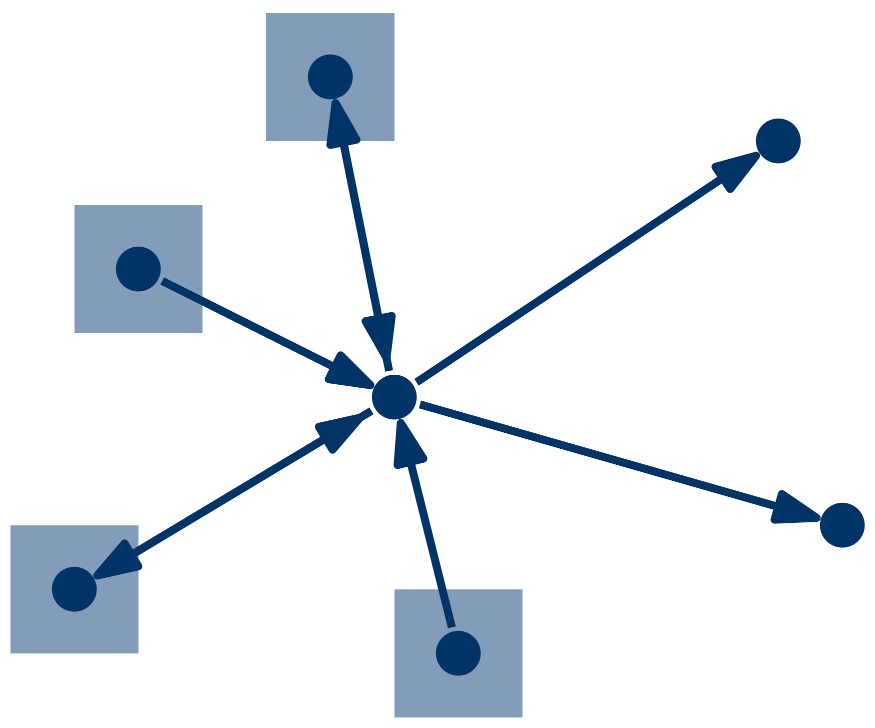
not a unit disk graph

Triangle Detection in Transmission Graphs



Triangle Detection in Transmission Graphs

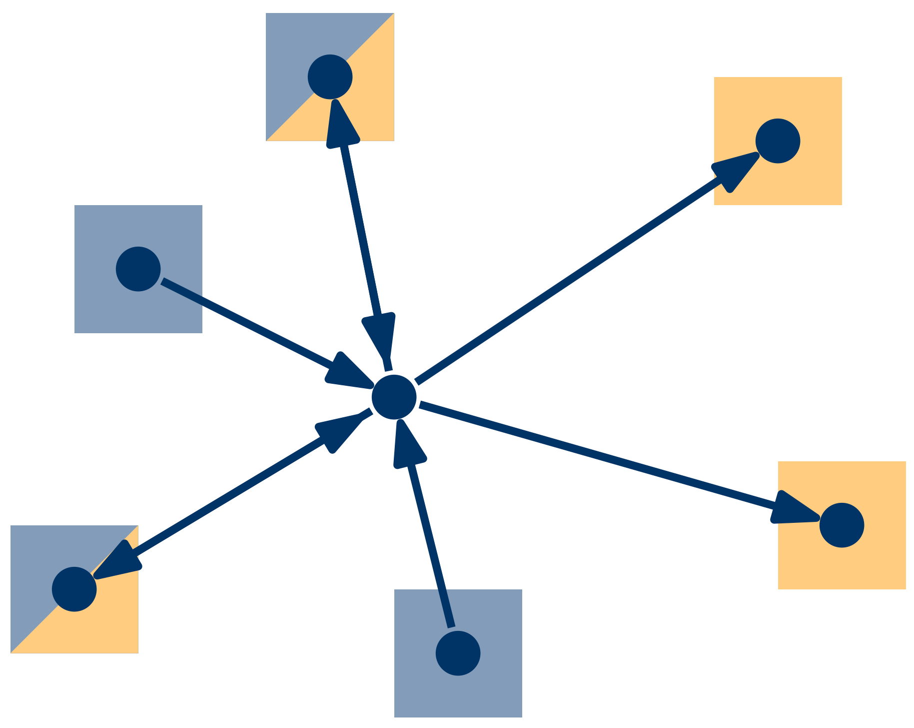
$N_{\text{in}}(v)$ ingoing neighbors



Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

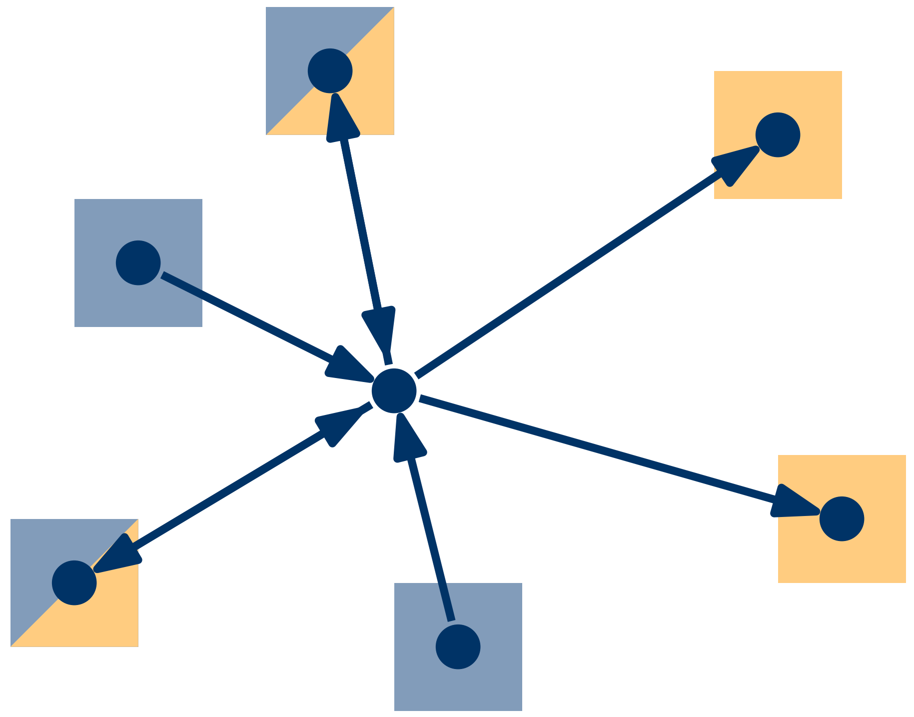


Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

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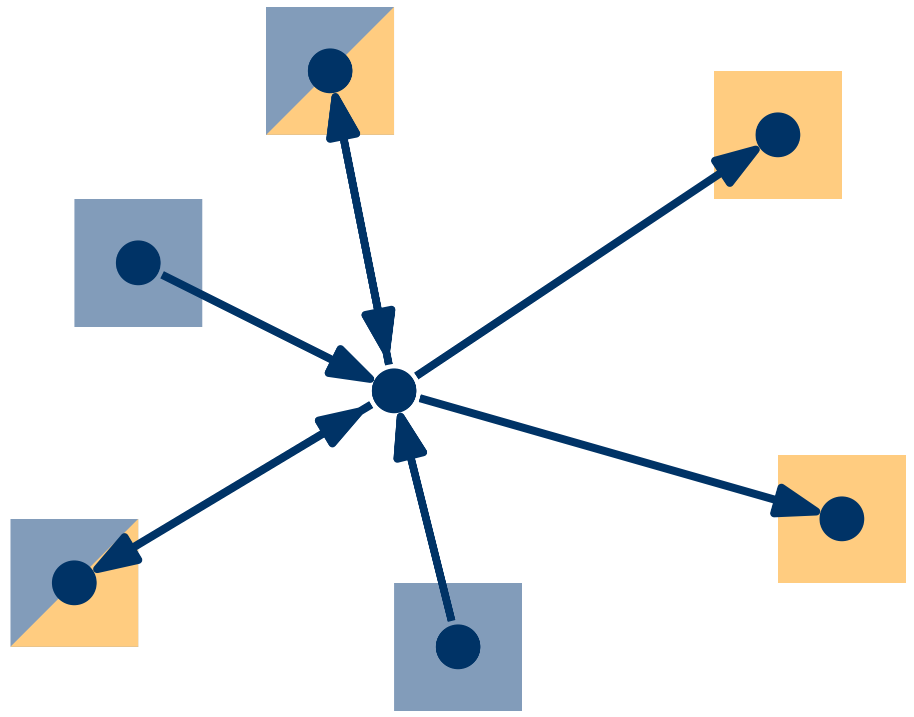

Lemma In a transmission graph, if there is a vertex with $|N_{\text{bi}}(v)| \geq 6$ then two among every six of those vertices form a triangle.

Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

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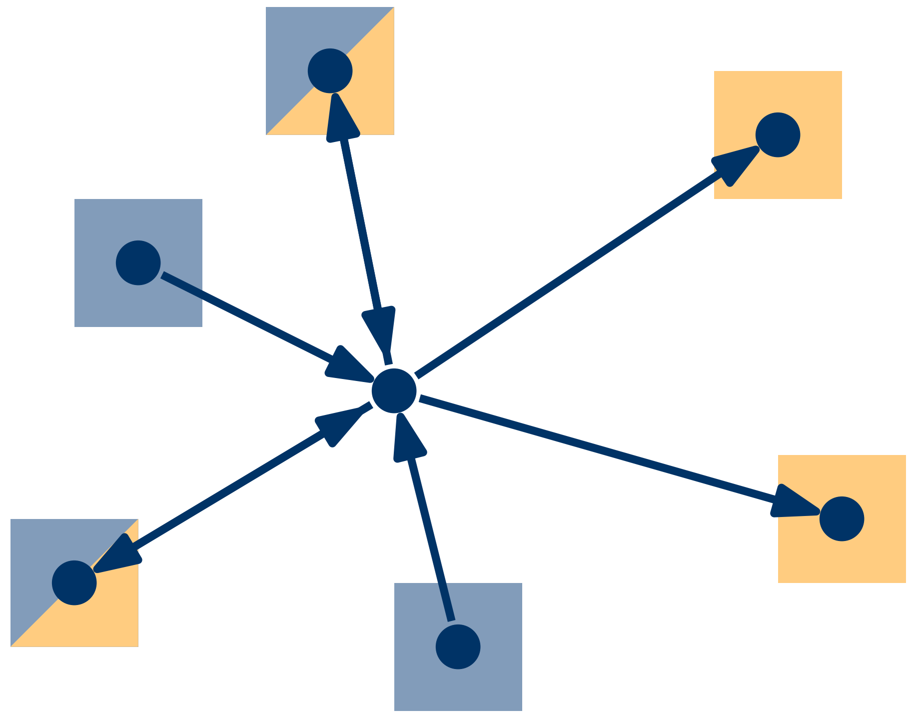

Lemma In a transmission graph, if there is a vertex with $|N_{\text{bi}}(v)| \geq 6$ then two among every six of those vertices form a triangle.

Lemma: Every directed cycle in a transmission graph has at least one $\leftrightarrow$ edge.

Triangle Detection in Transmission Graphs

$N_{\mathrm{in}}(v)$ ingoing neighbors

$N_{\mathrm{out}}(v)$ outgoing neighbors

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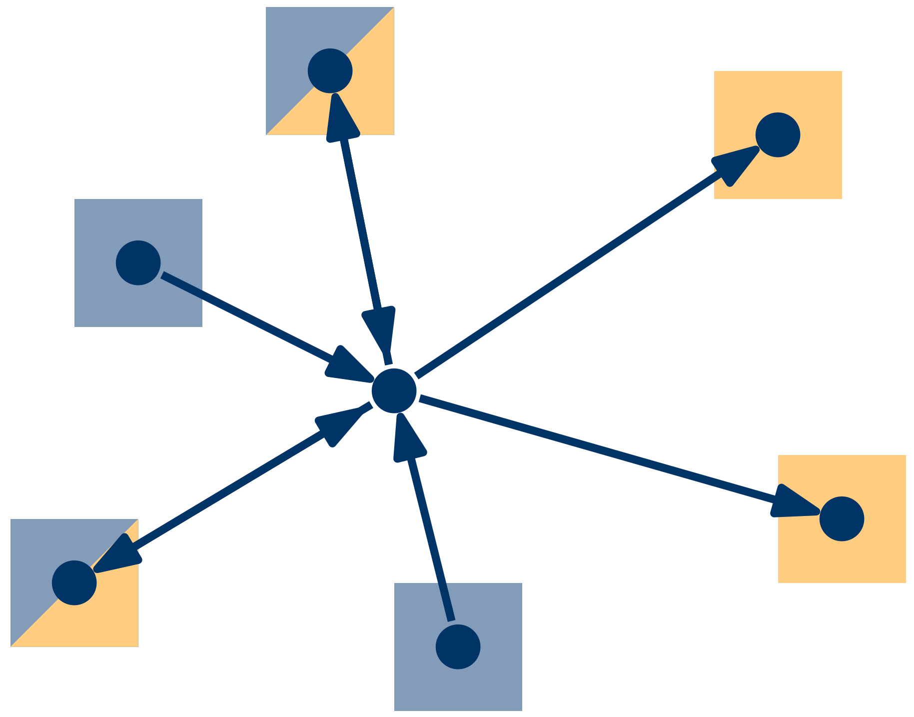

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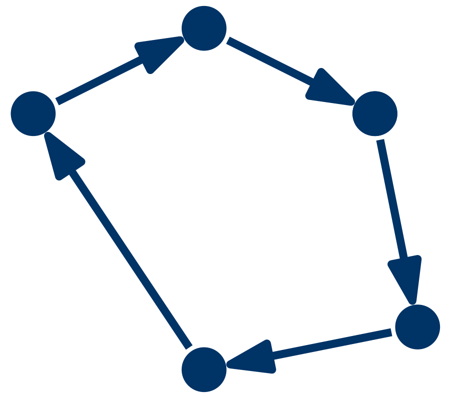

Triangle Detection in Transmission Graphs

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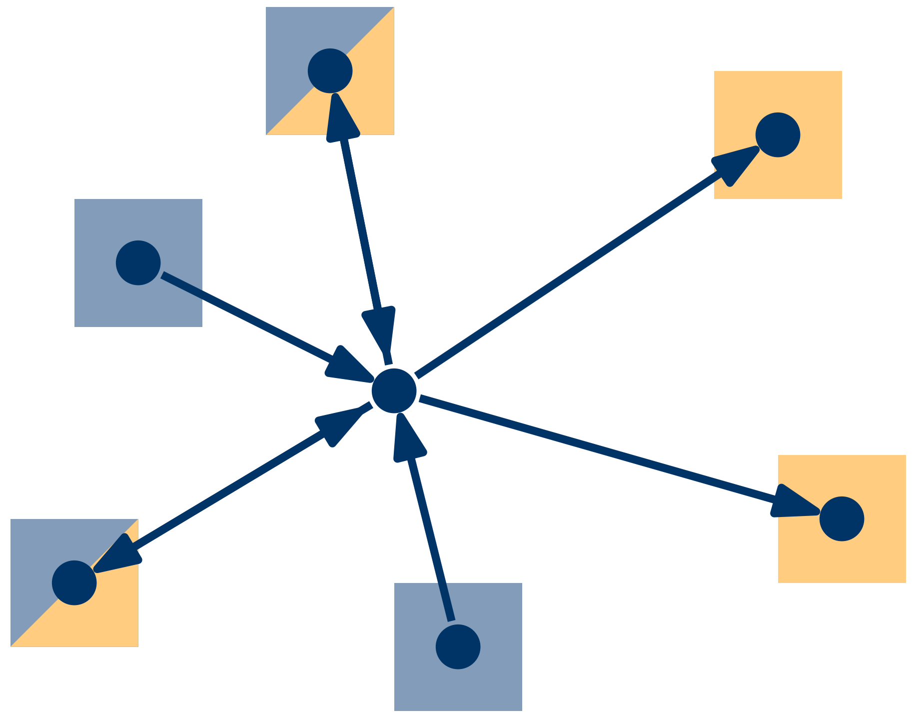

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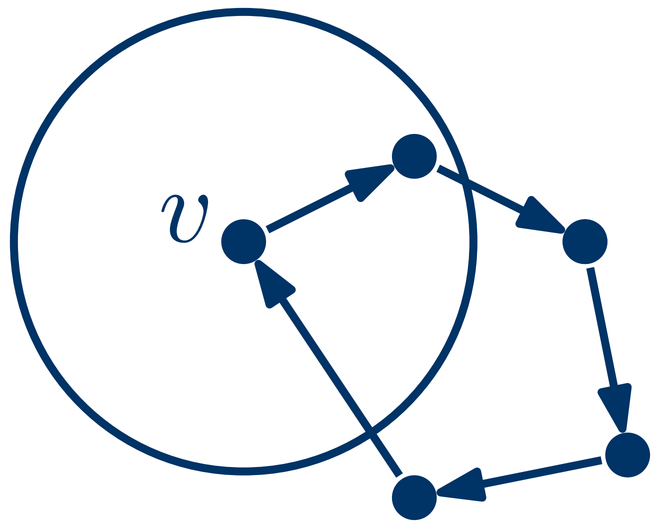


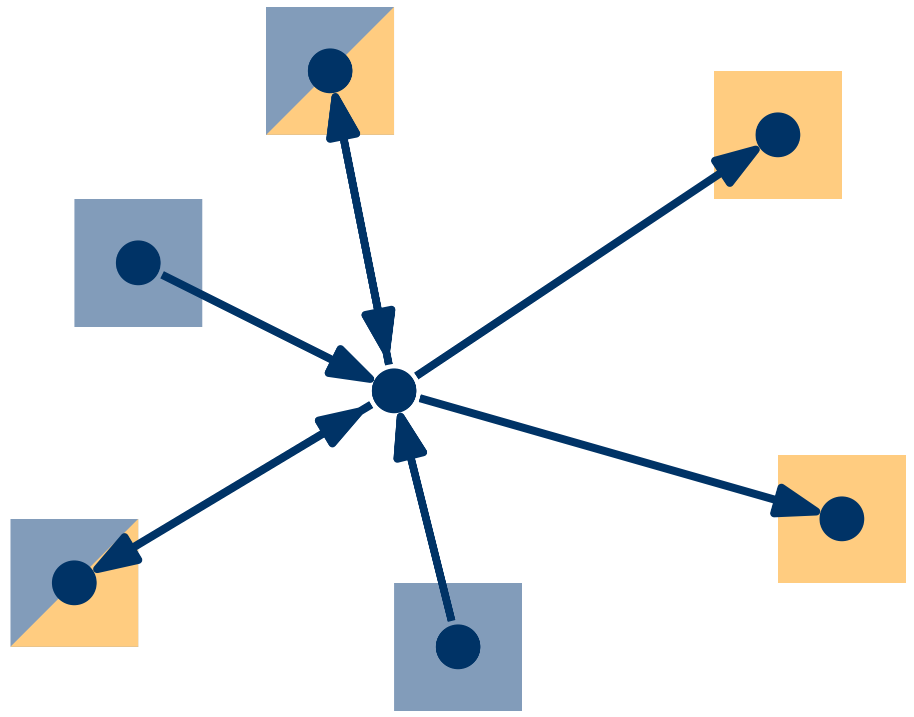
v smallest radius

Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



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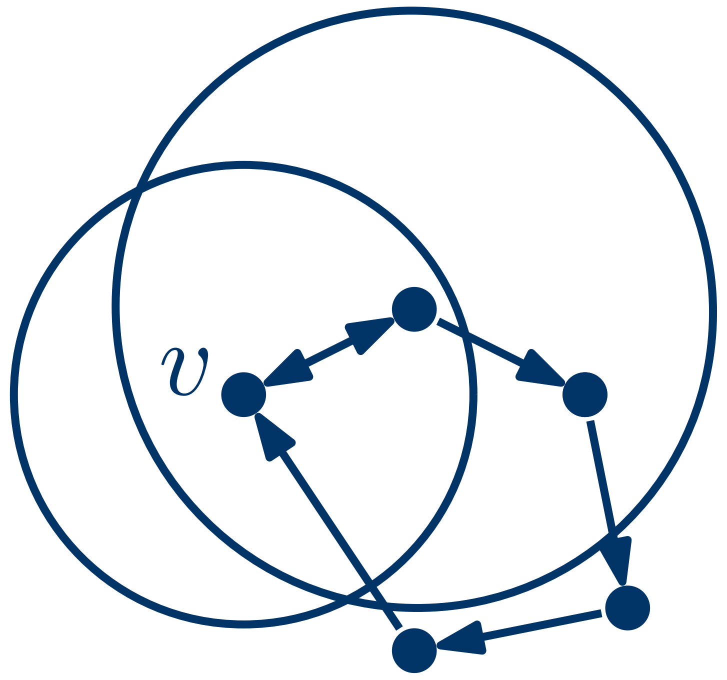

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Triangle Detection in Transmission Graphs

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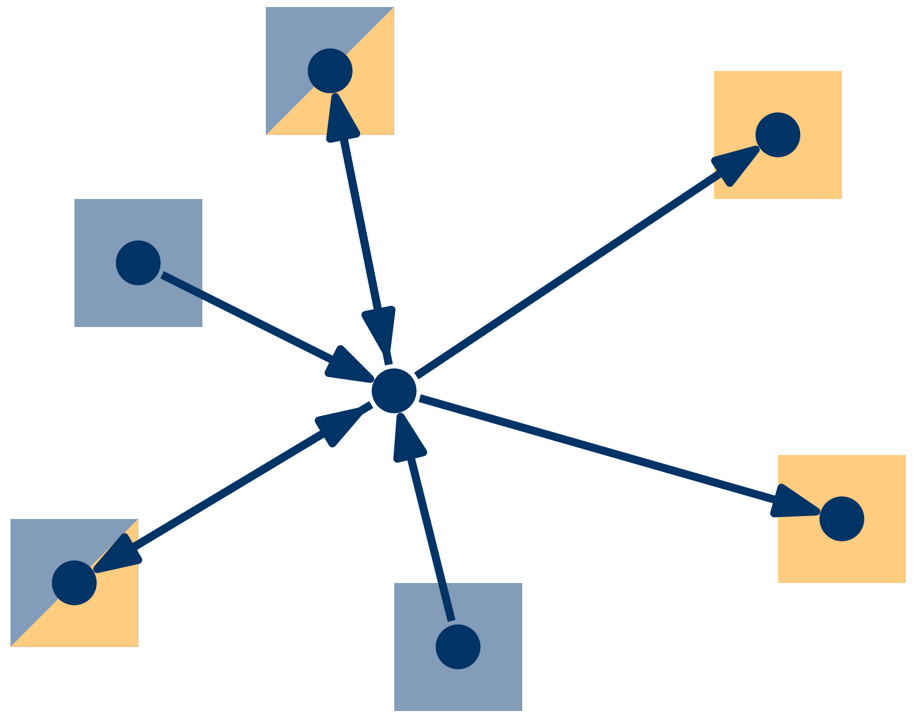

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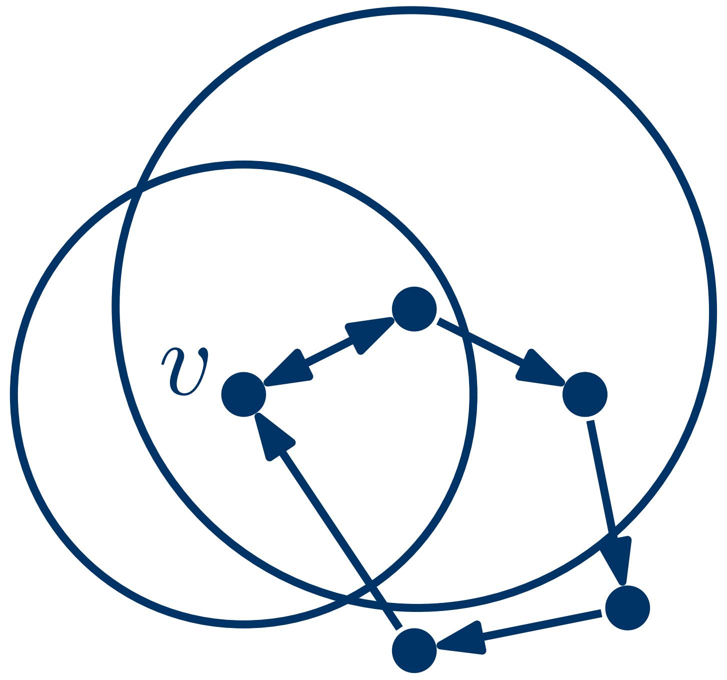


v smallest radius

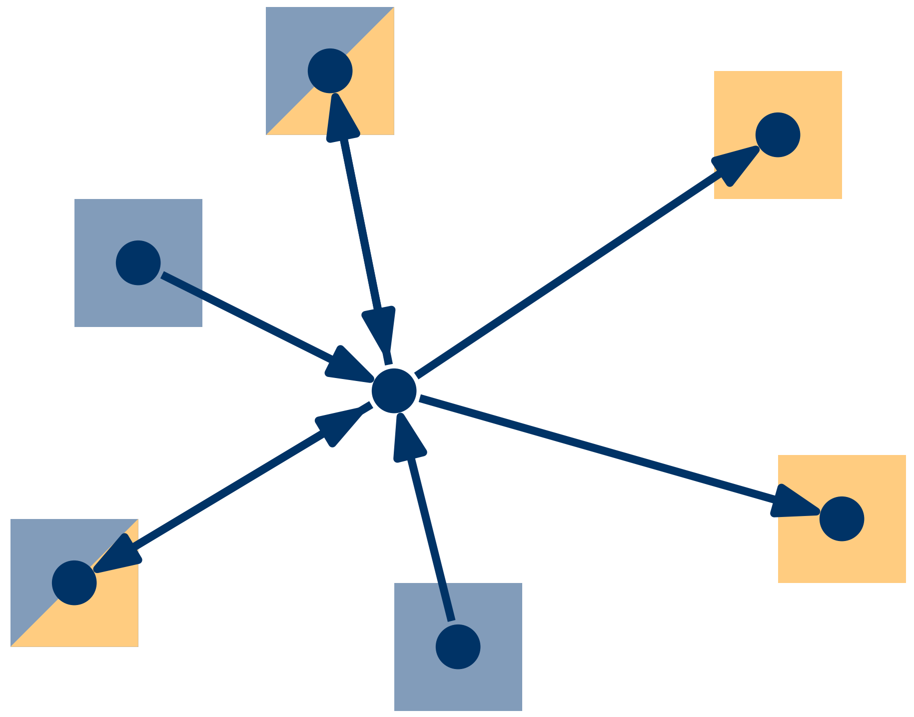
Lemma Given the adjacency list representation of a directed graph, the set $N_{\mathrm{bi}}(v)$ can be computed in $O(n+m)$ time for all $v \in V$.

Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



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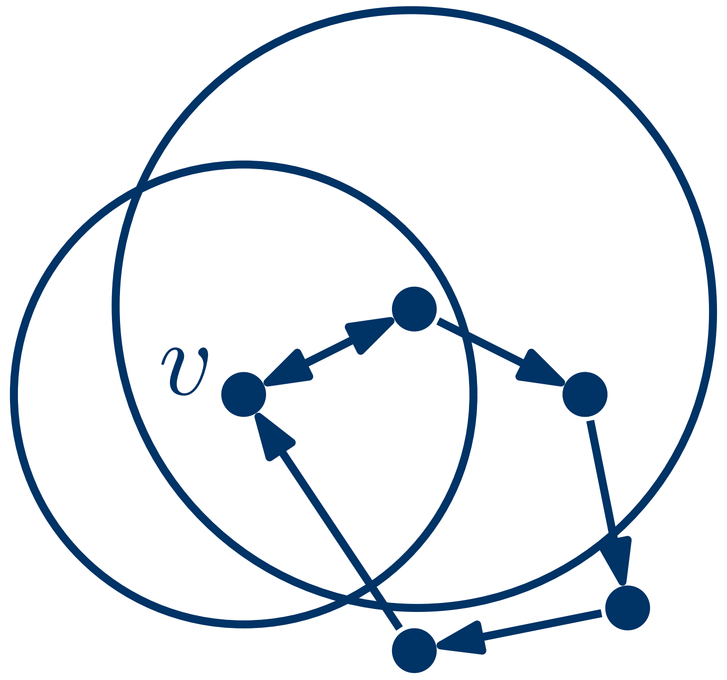

v smallest radius

Sort the adjacency lists by transposing twice

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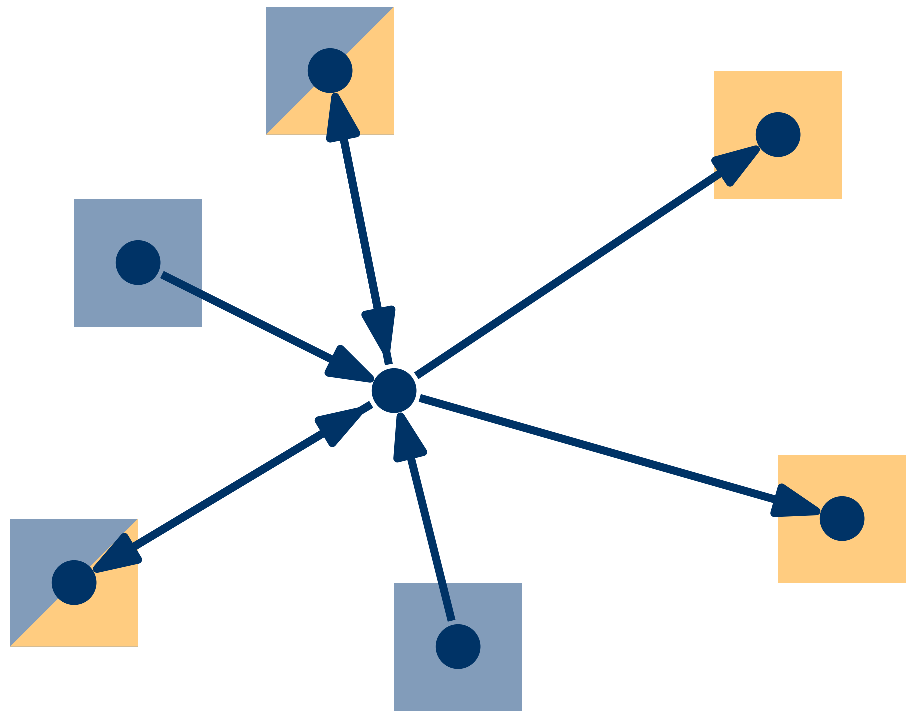
Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



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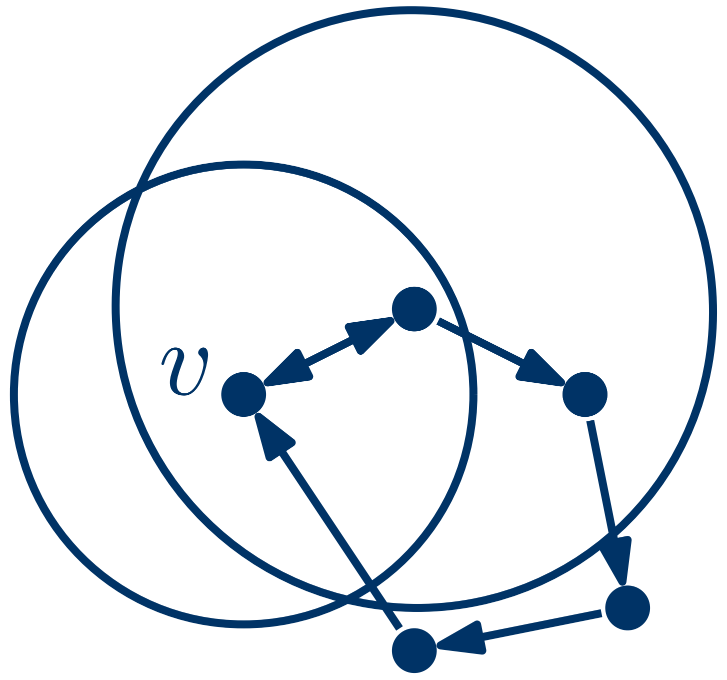

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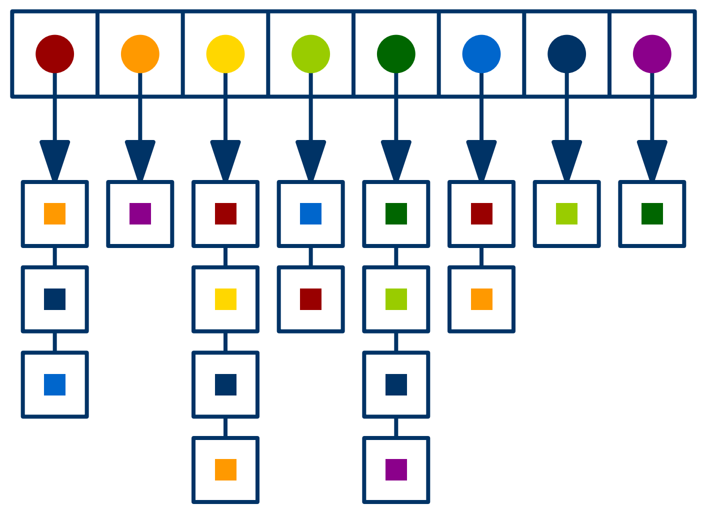
Sort the adjacency lists by transposing twice

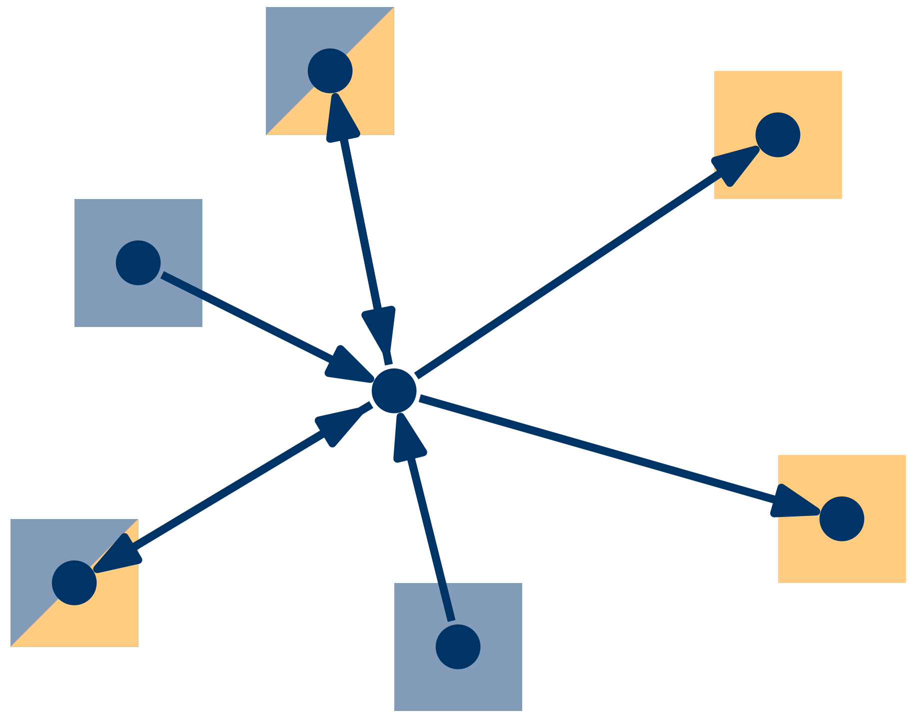


Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

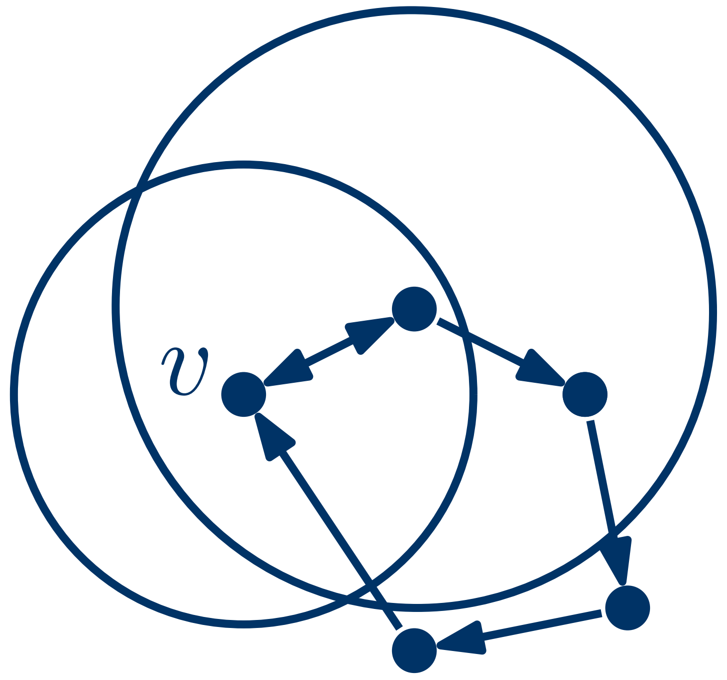
$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



Lemma In a transmission graph, if there is a vertex with $|N_{\text{bi}}(v)| \geq 6$ then two among every six of those vertices form a triangle.

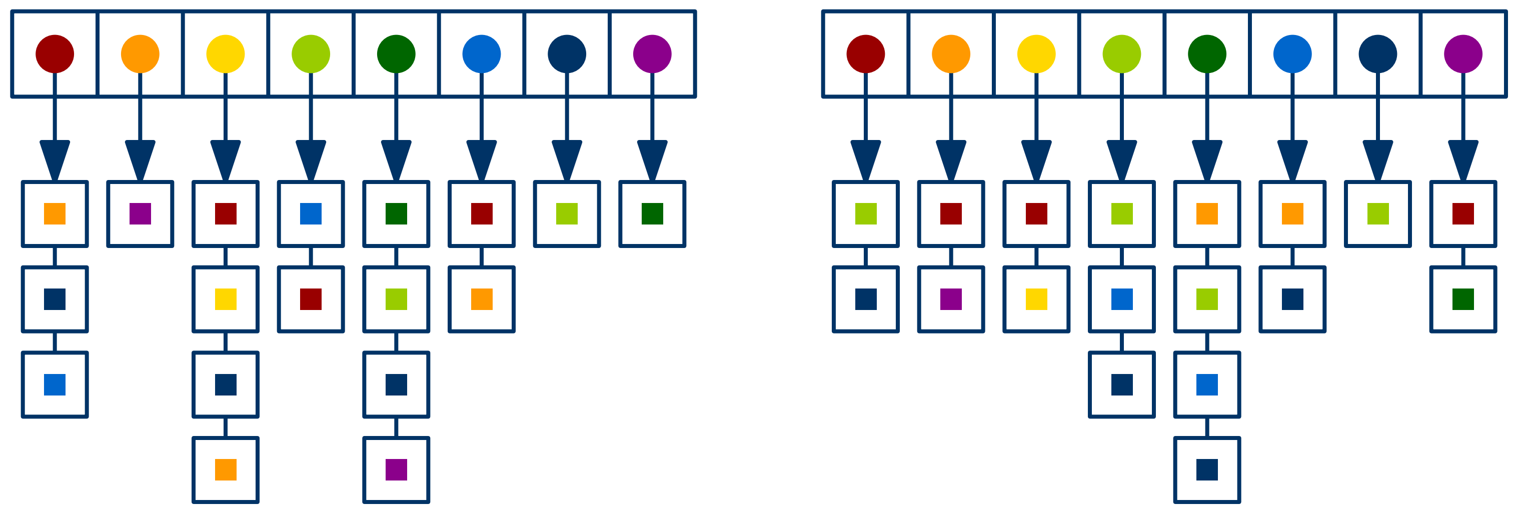
Lemma: Every directed cycle in a transmission graph has at least one ↔ edge.



v smallest radius

Lemma Given the adjacency list representation of a directed graph, the set $N_{\text{bi}}(v)$ can be computed in $O(n+m)$ time for all $v \in V$.

Sort the adjacency lists by transposing twice

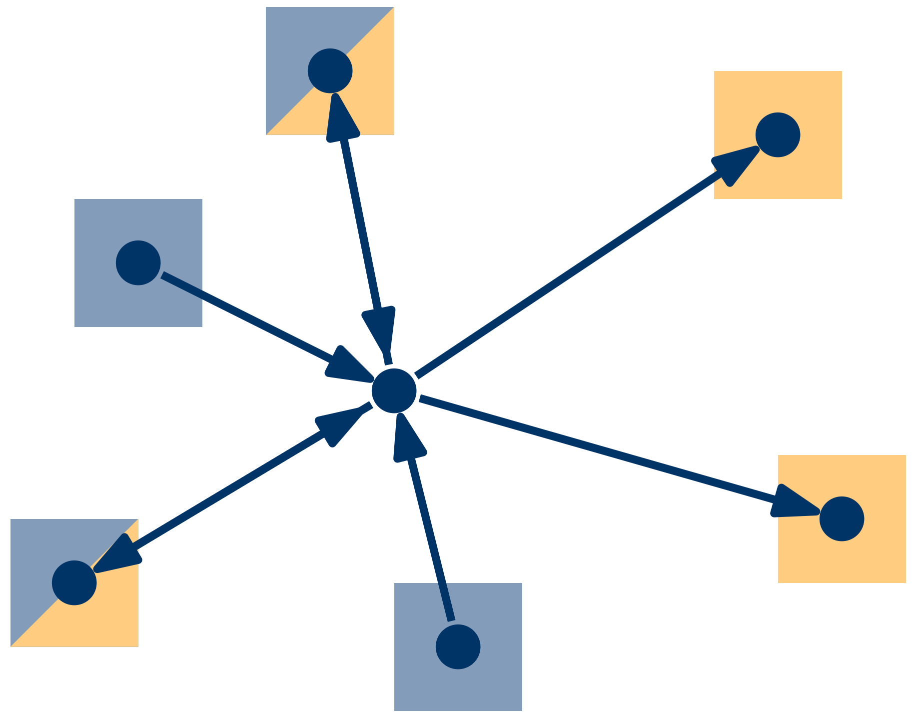


Triangle Detection in Transmission Graphs

$N_{\text{in}}(v)$ ingoing neighbors

$N_{\text{out}}(v)$ outgoing neighbors

$N_{\text{bi}}(v) = N_{\text{in}} \cap N_{\text{out}}$



Lemma In a transmission graph, if there is a vertex with $|N_{\text{bi}}(v)| \geq 6$ then two among every six of those vertices form a triangle.

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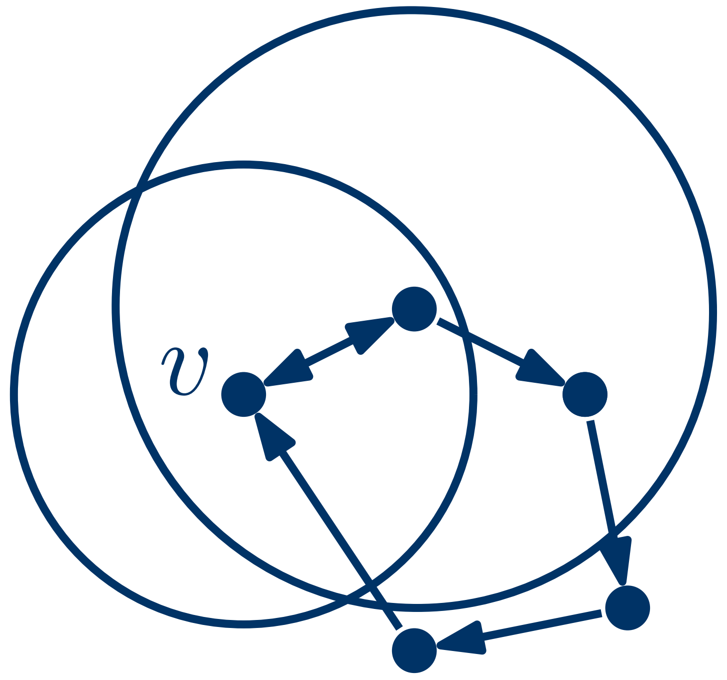

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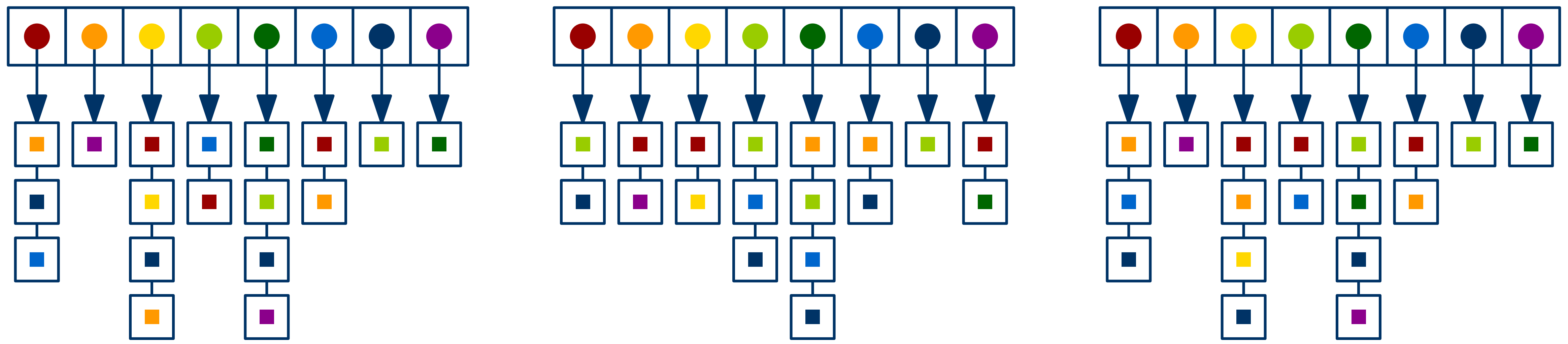


Triangle Detection in Transmission Graphs

Algorithm

Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $\quad O(n + m)$

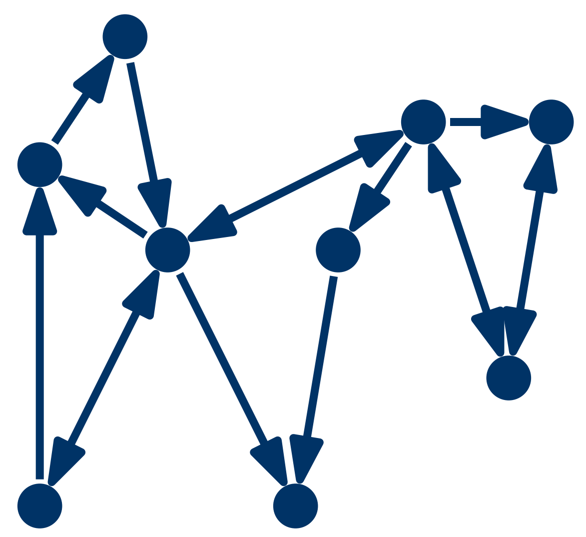
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $\quad O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

a)



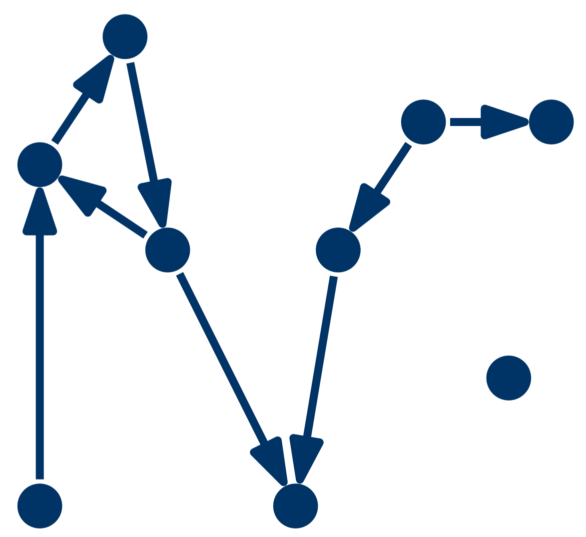
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

a)



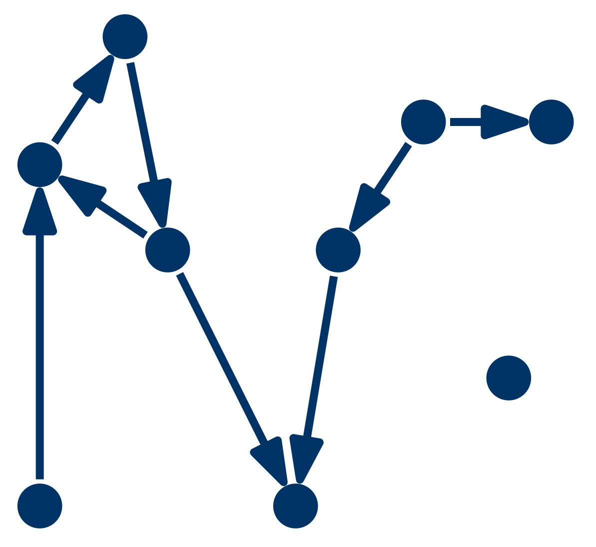
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

a)



if not acyclic $\rightarrow$
no transmission graph

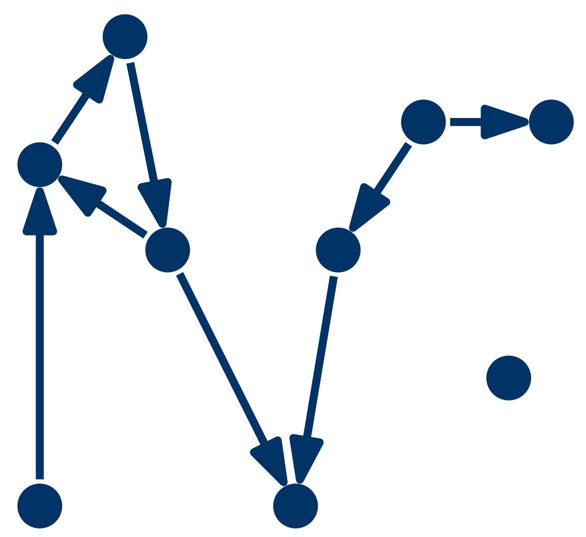
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

a)



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$O(n+m)$

Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

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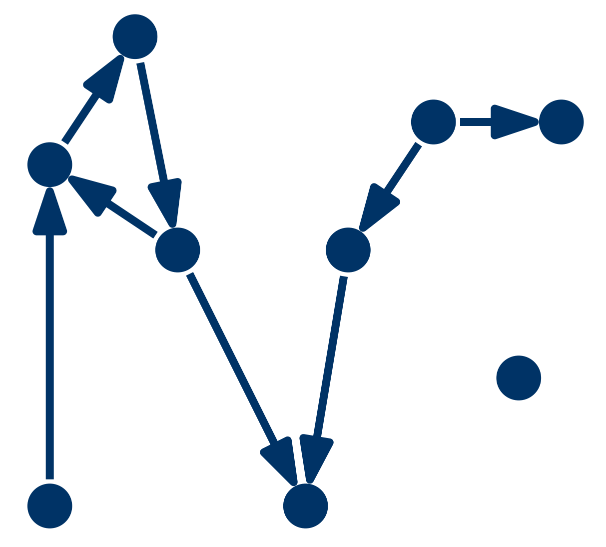
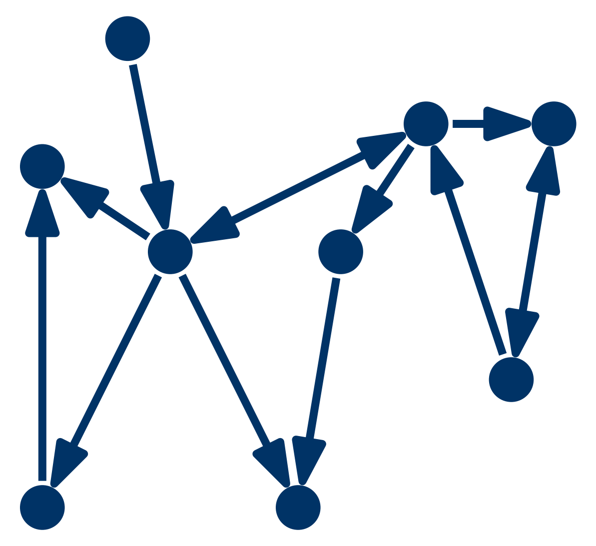


if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



explicitly check for v
with $N_{\mathsf{bi}}(v) \neq \emptyset$

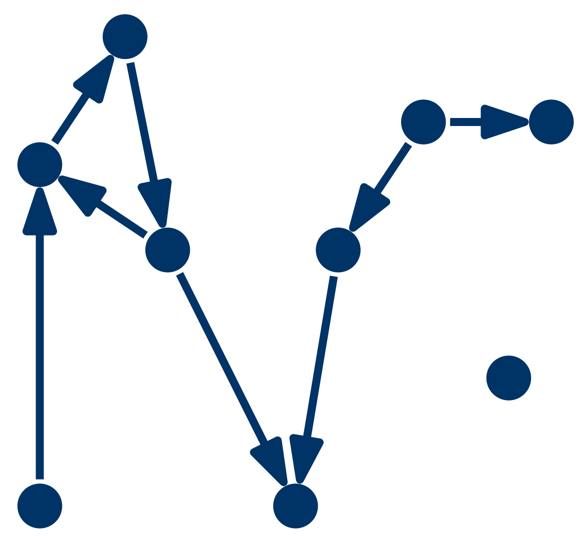
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

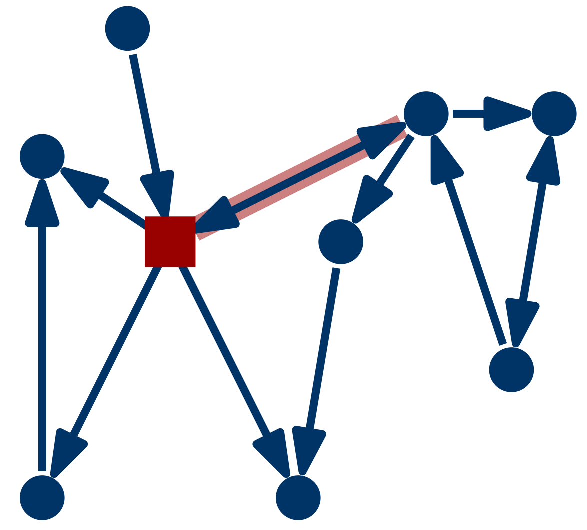
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



explicitly check for v
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Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

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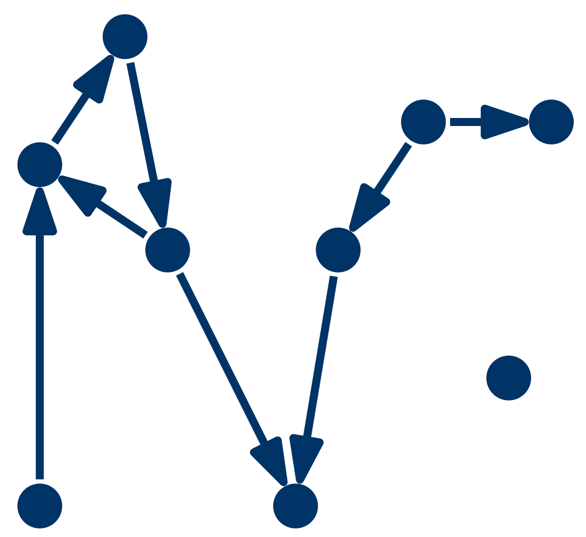
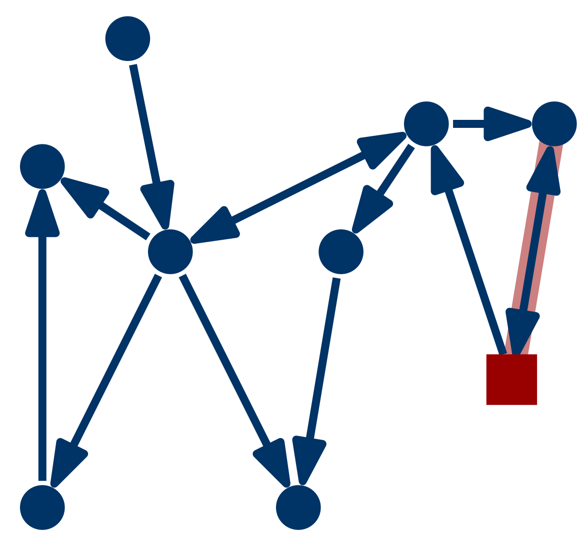


if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

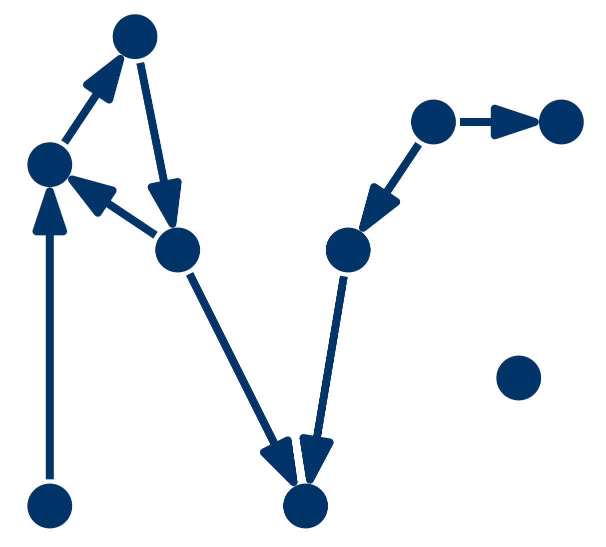
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

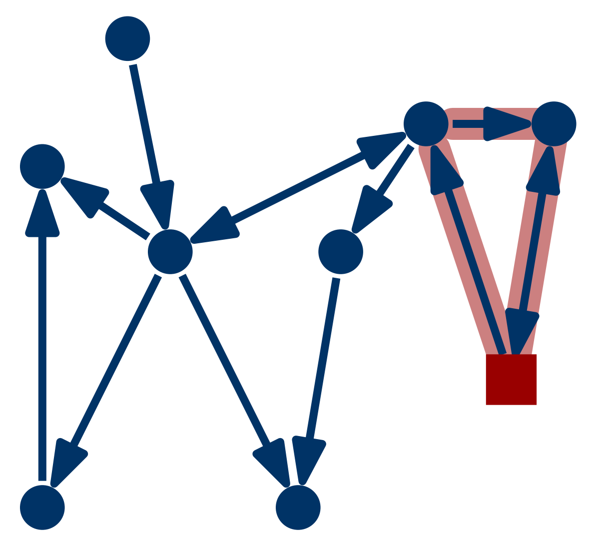
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



explicitly check for v
with $N_{\mathsf{bi}}(v) \neq \emptyset$

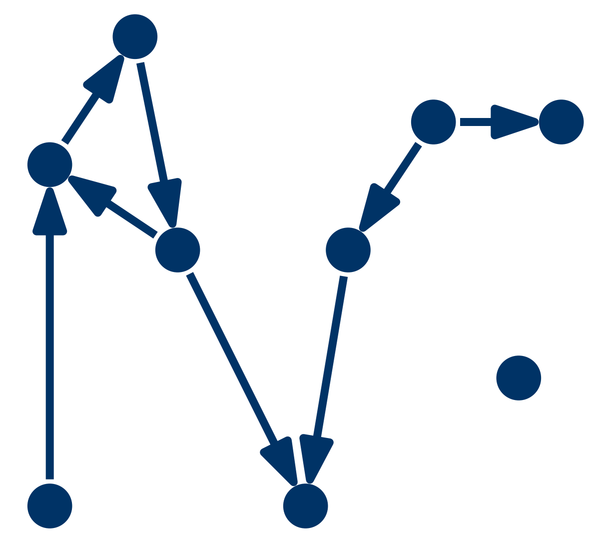
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

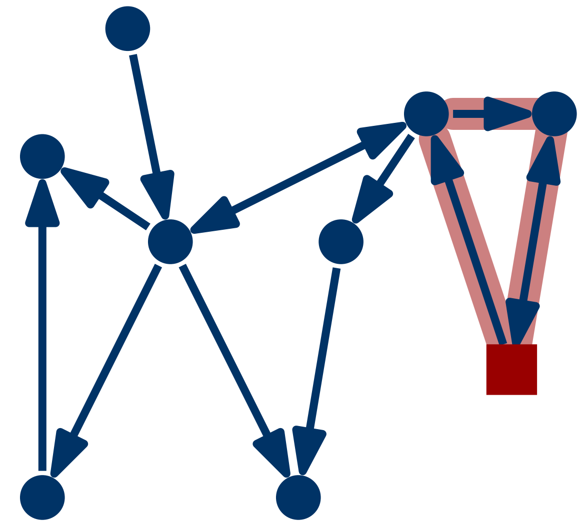
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



explicitly check for v
with $N_{\mathsf{bi}}(v) \neq \emptyset$

$O(n+m)$

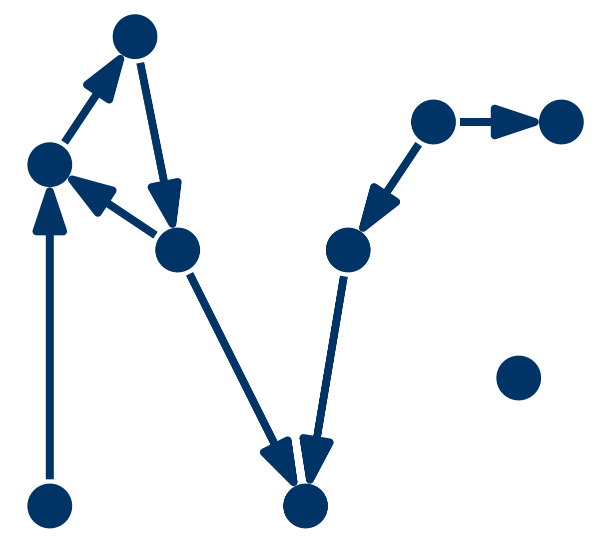
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

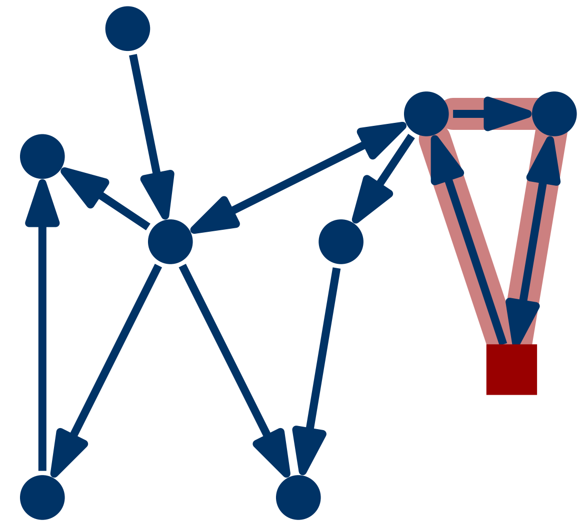
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

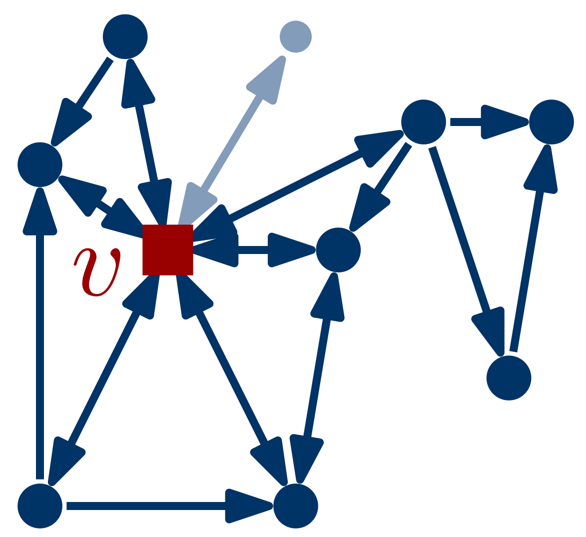


explicitly check for v
with $N_{\mathsf{bi}}(v) \neq \emptyset$

$O(n+m)$

Case 2: At least one vertex v with $|N_{\mathrm{bi}}(v)| \geq 6$

a)



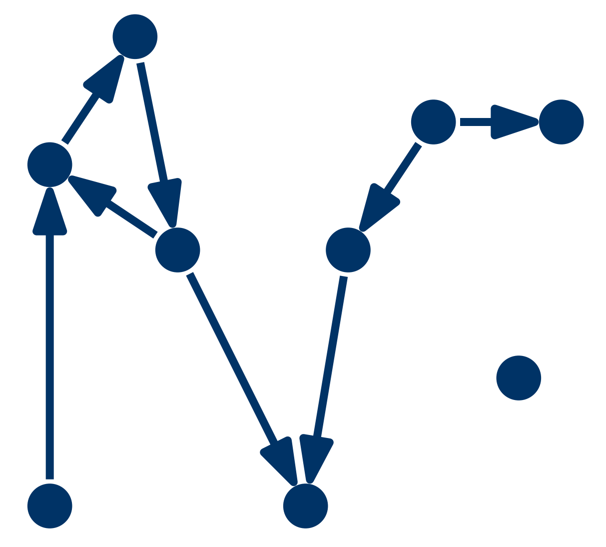
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

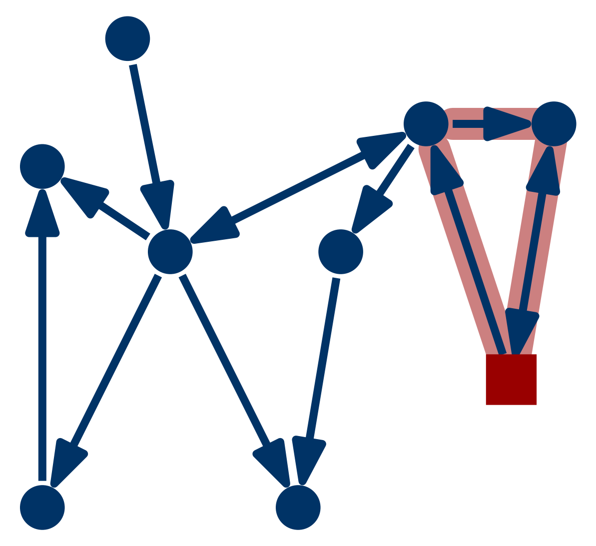
Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

a)



if not acyclic $\rightarrow$
no transmission graph
$O(n+m)$

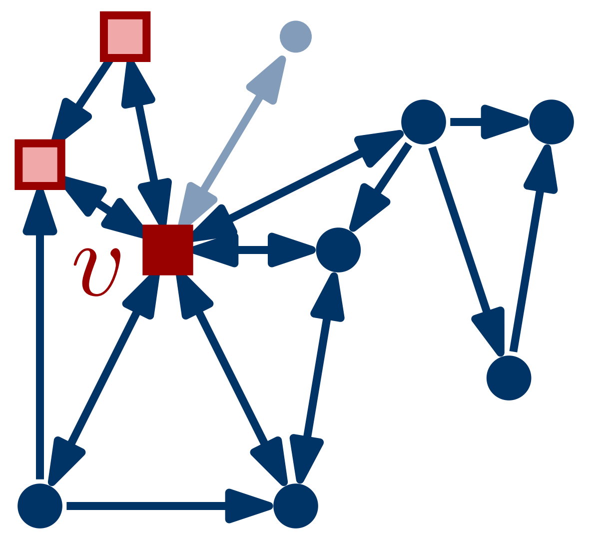
b)



explicitly check for v
with $N_{\mathrm{bi}}(v) \neq \emptyset$
$O(n+m)$

Case 2: At least one vertex v with $|N_{\mathrm{bi}}(v)| \geq 6$

a)



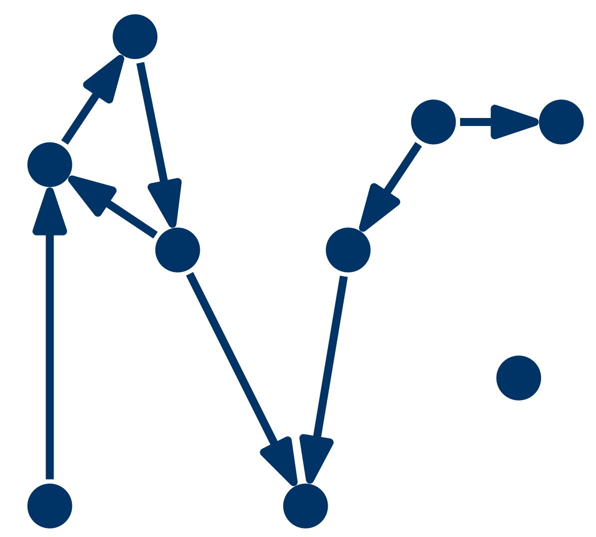
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\mathrm{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\mathrm{bi}}(v)| < 6$ for all vertices

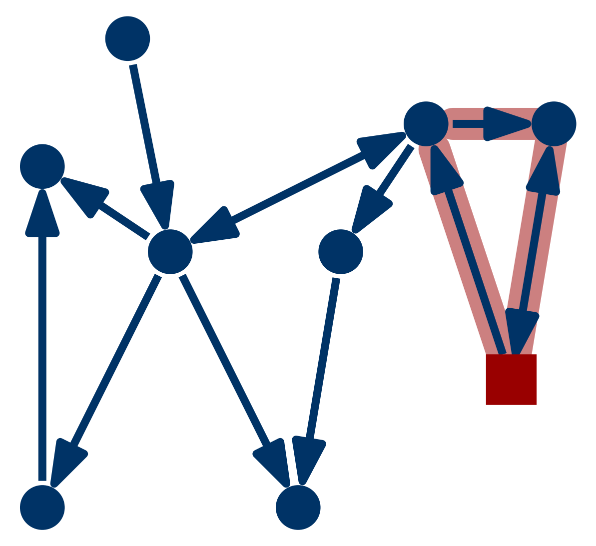
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

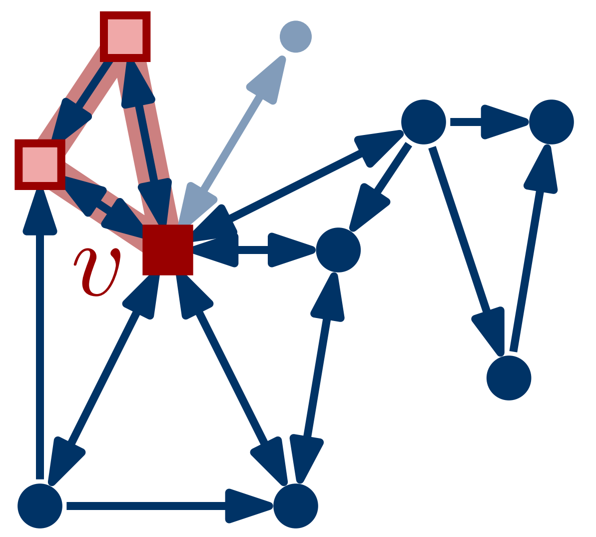


explicitly check for v
with $N_{\mathrm{bi}}(v) \neq \emptyset$

$O(n+m)$

Case 2: At least one vertex v with $|N_{\mathrm{bi}}(v)| \geq 6$

a)



report triangle

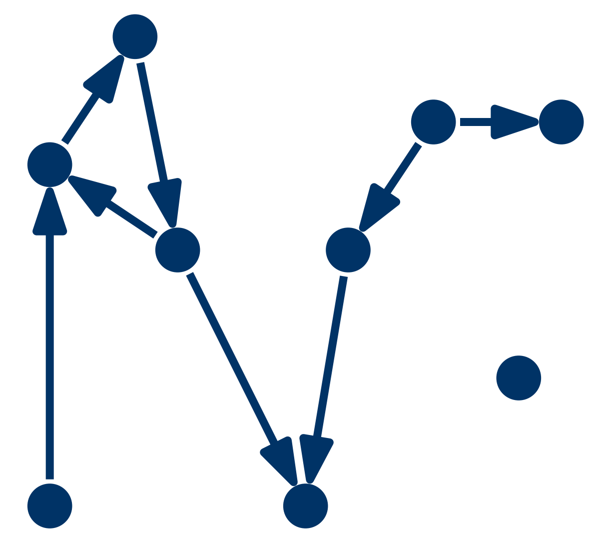
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

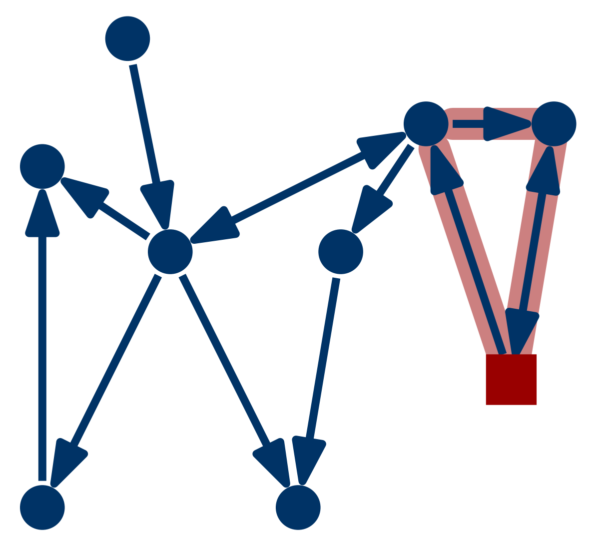
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

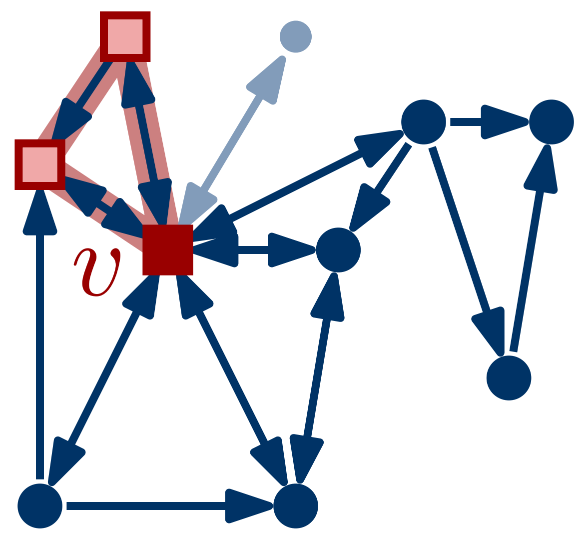


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

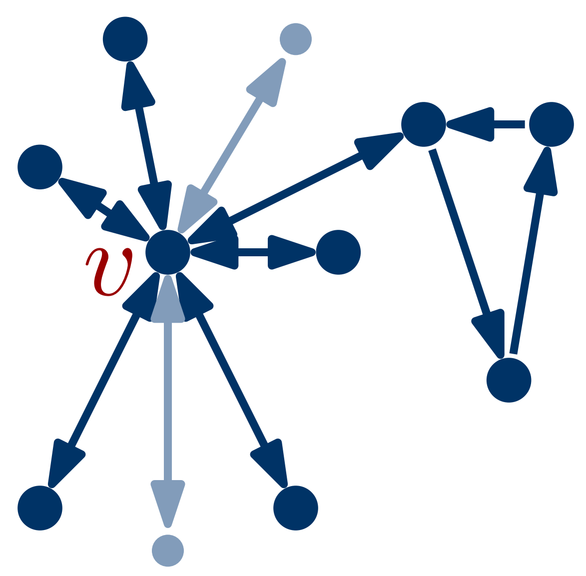
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



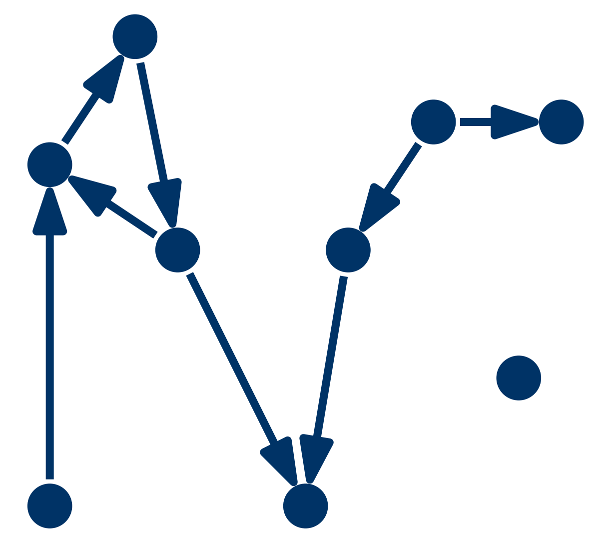
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

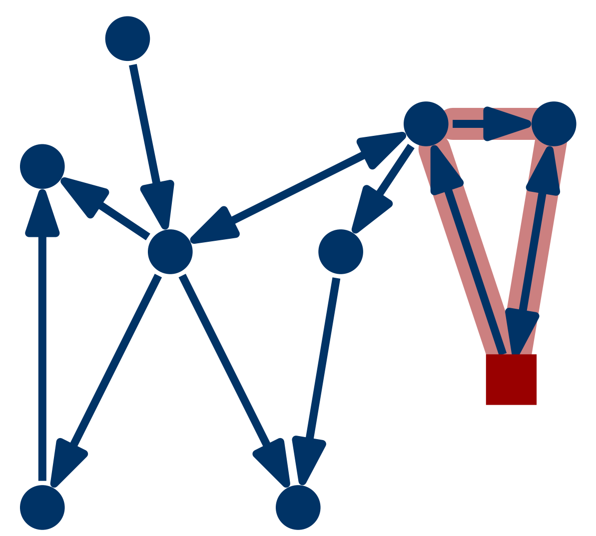
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

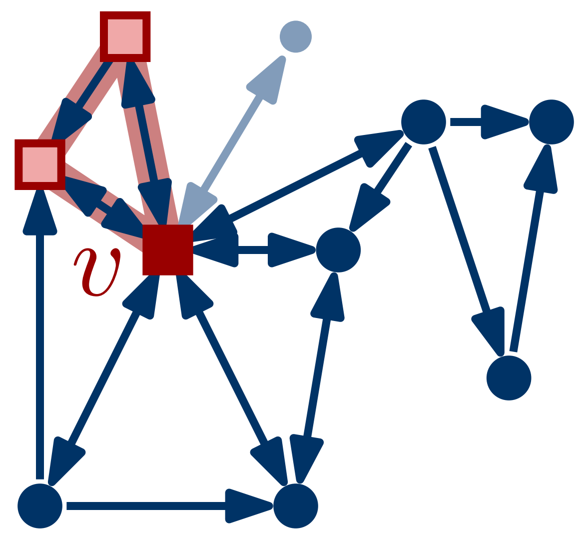


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

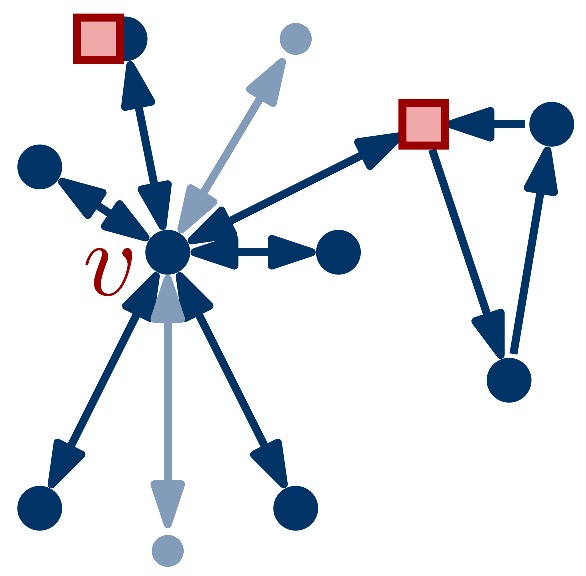
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



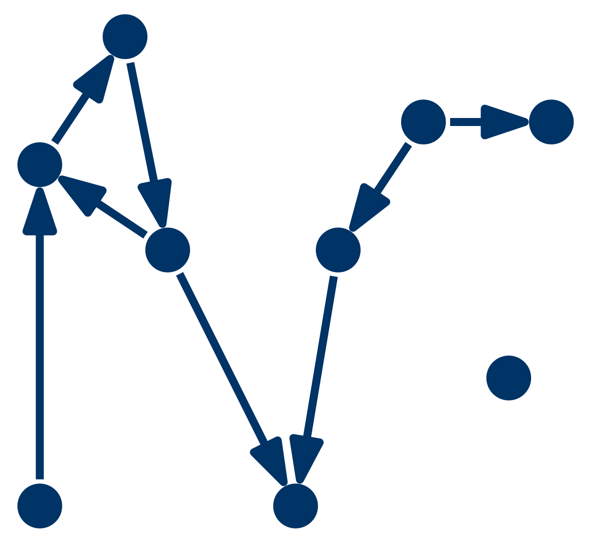
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

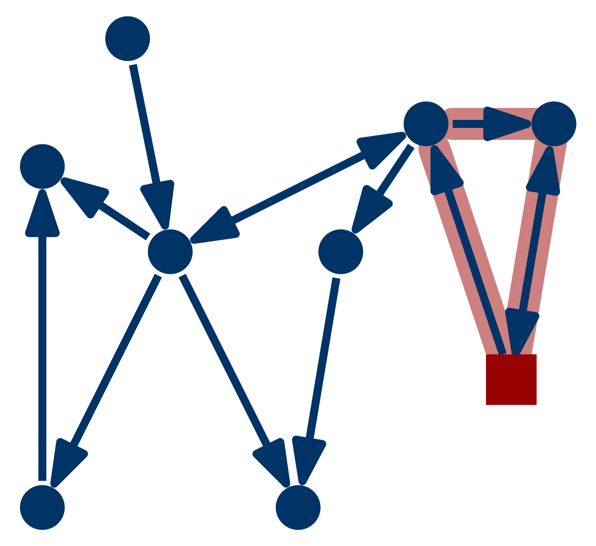
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

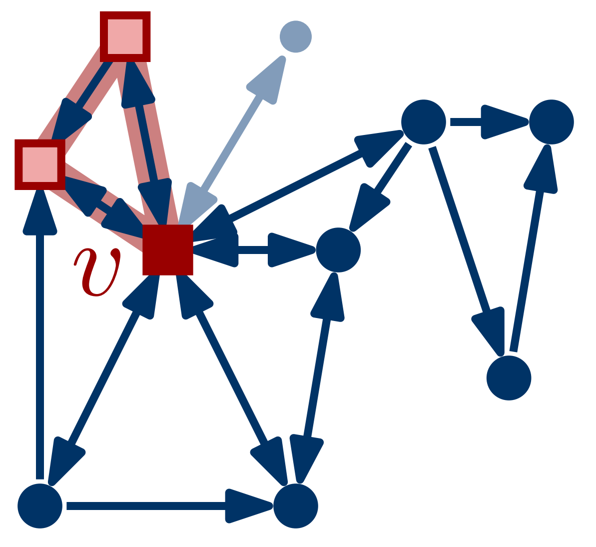


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

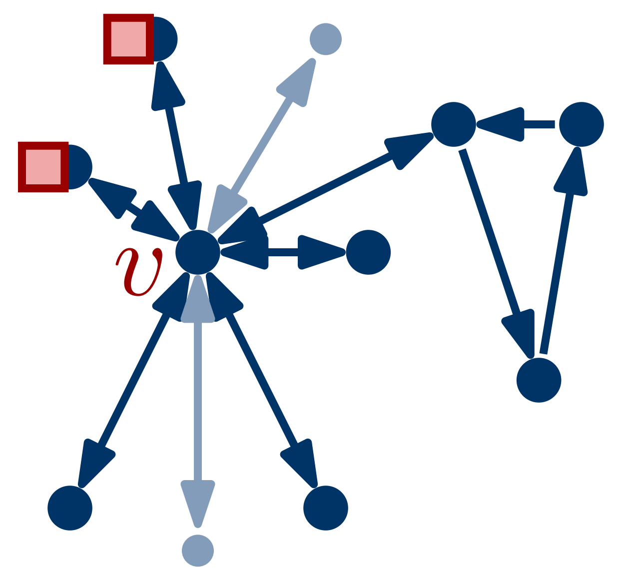
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



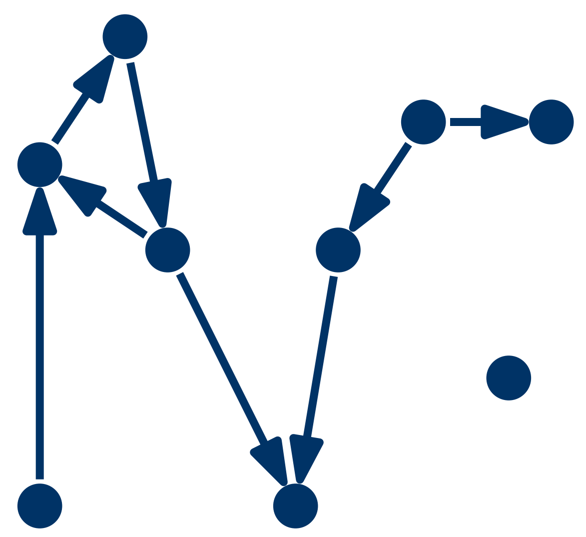
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

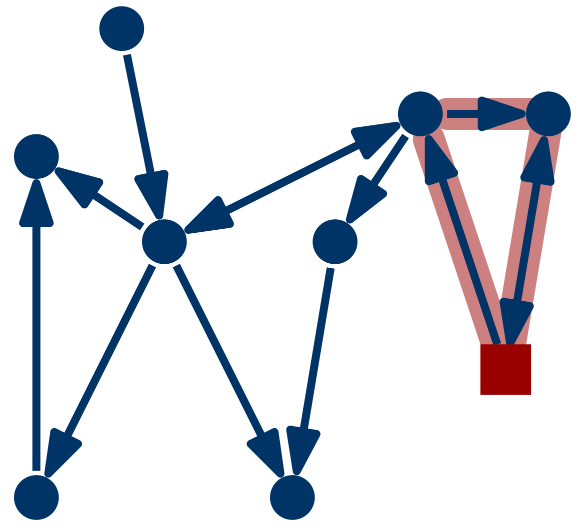
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

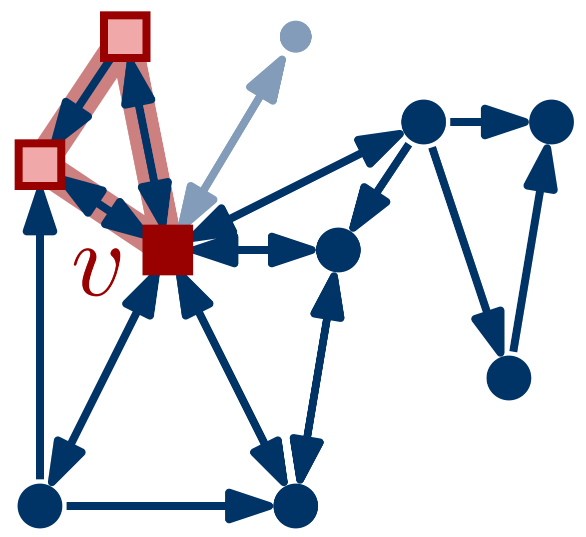


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

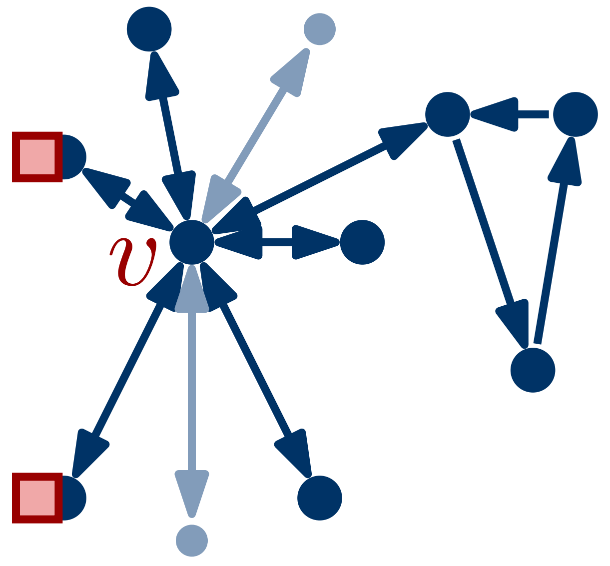
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



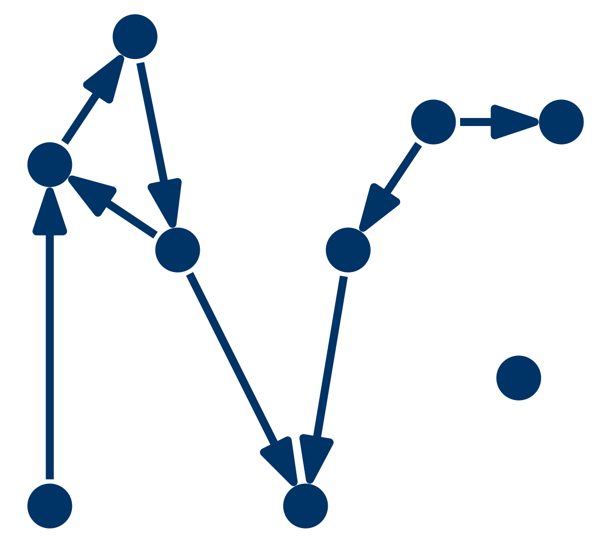
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

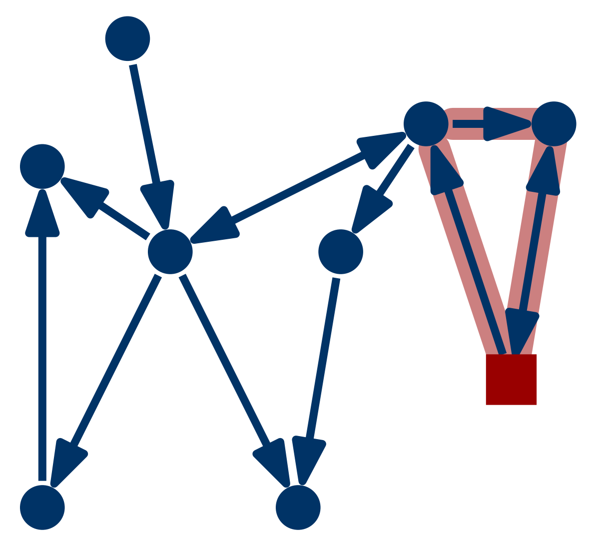
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

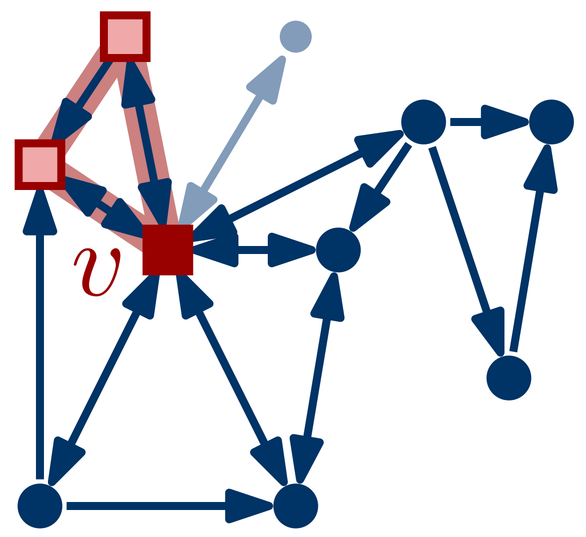


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

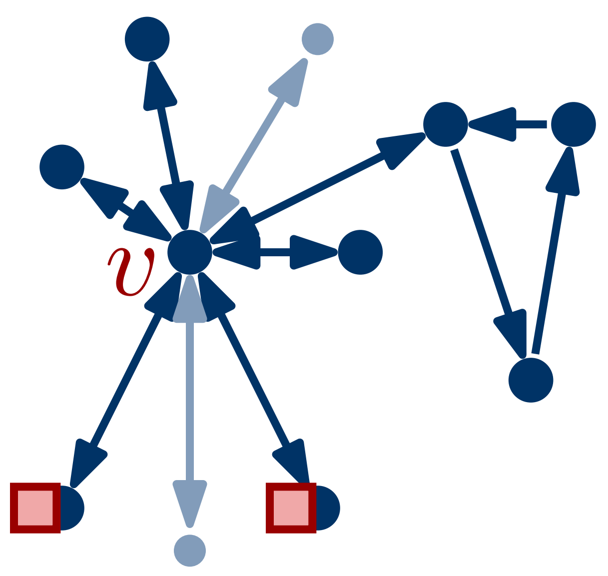
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



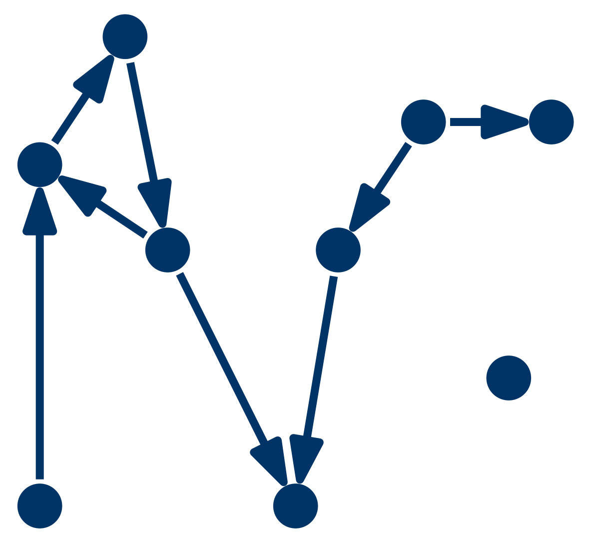
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

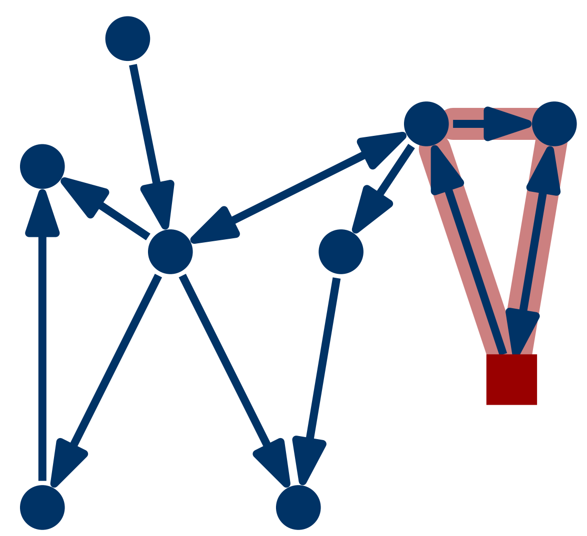
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

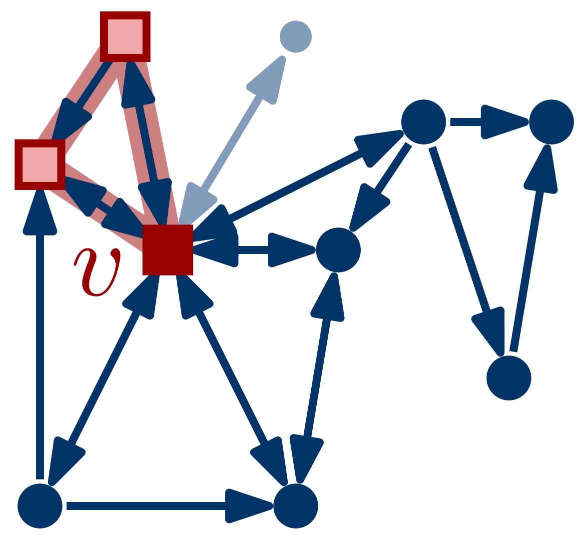


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

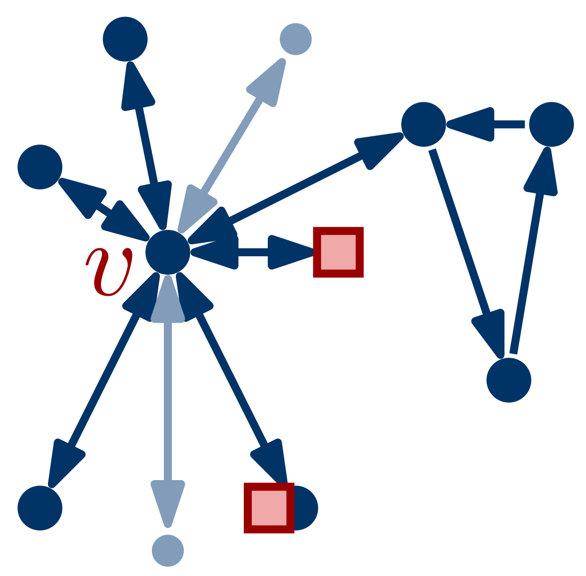
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



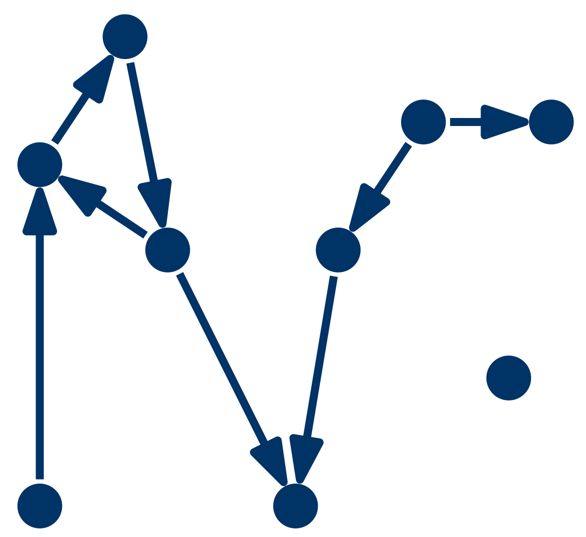
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

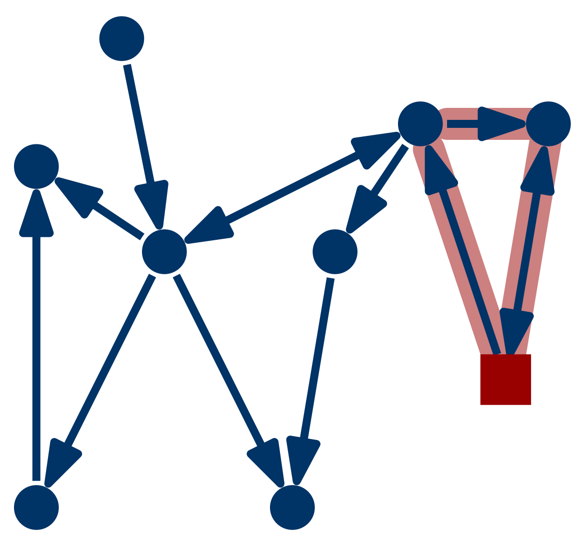
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)

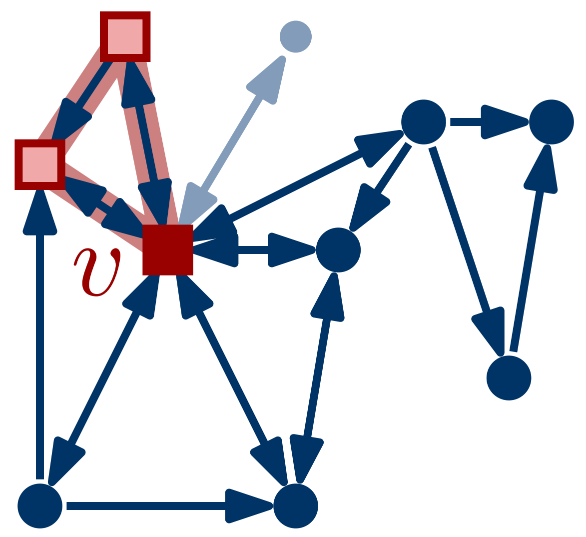


explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

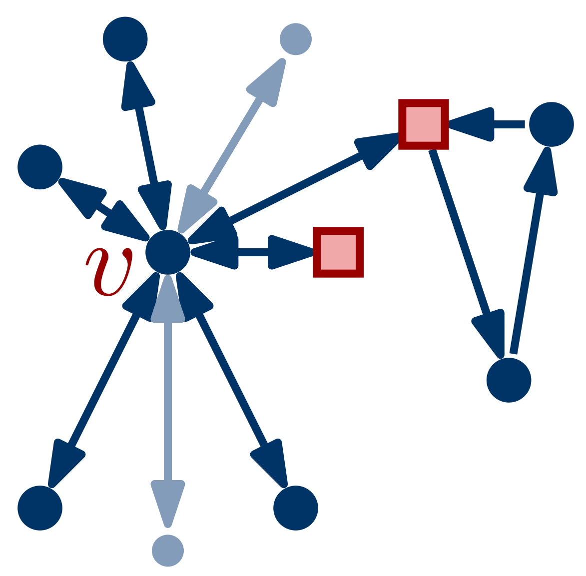
Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

a)



report triangle

b)



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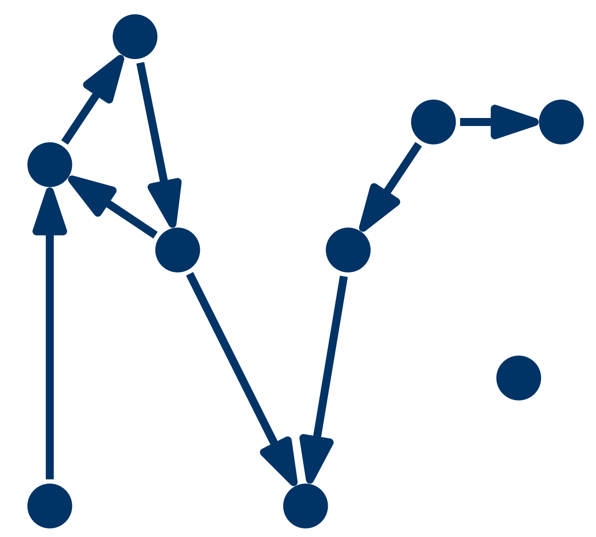
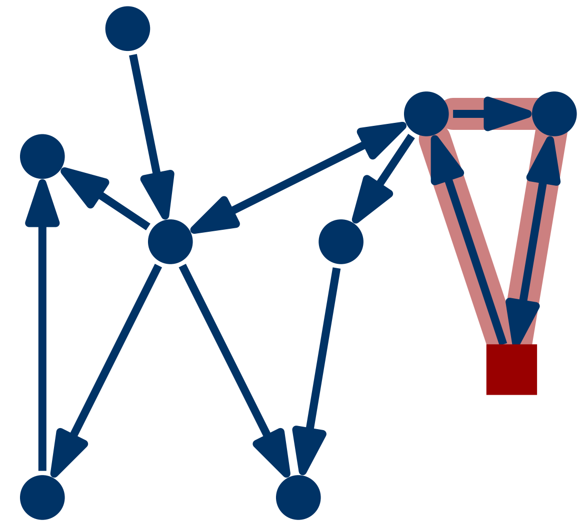


if not acyclic $\rightarrow$
no transmission graph
$O(n+m)$

b)



explicitly check for v
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$O(n+m)$

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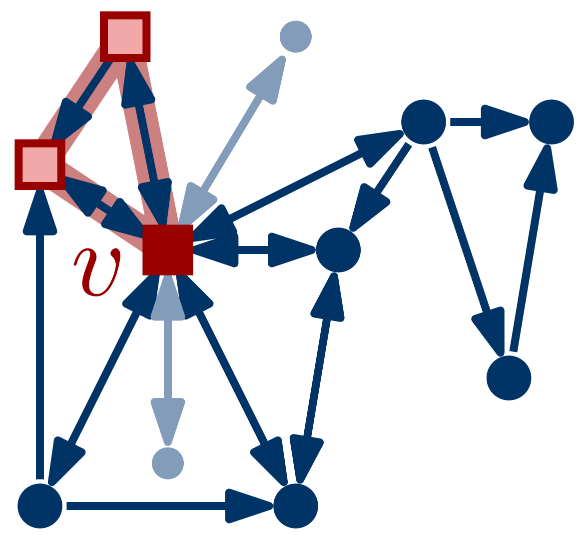
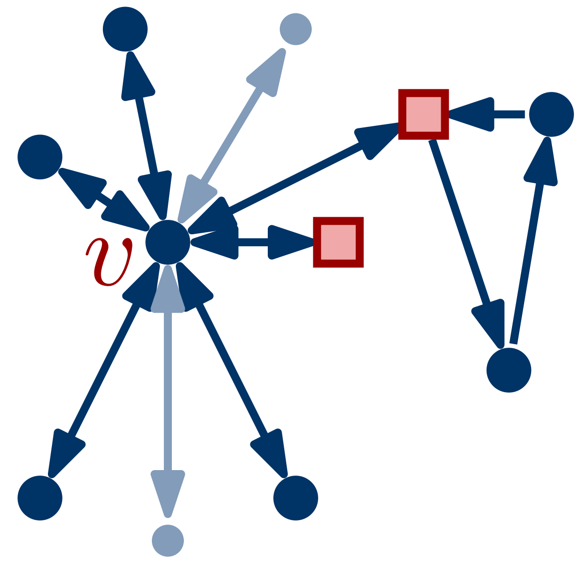
a)



report triangle

b)



no transmission graph

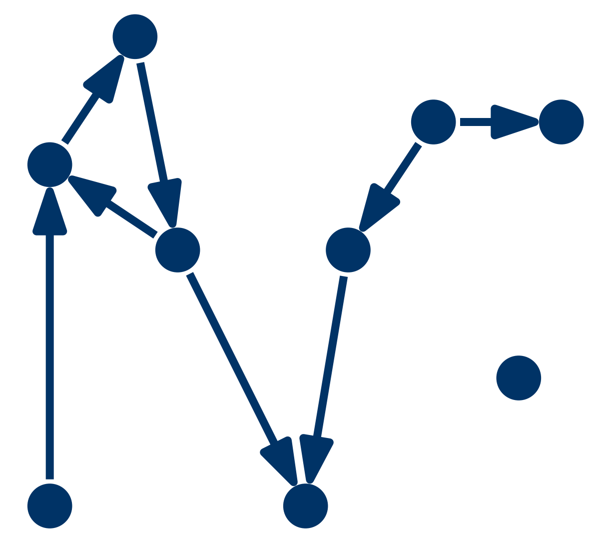
Triangle Detection in Transmission Graphs

Algorithm

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

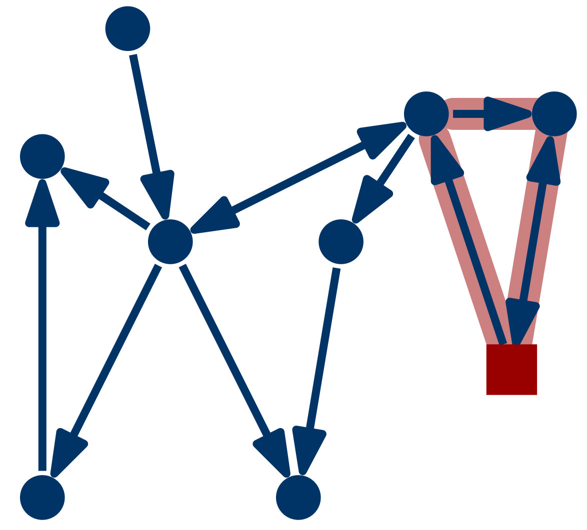
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



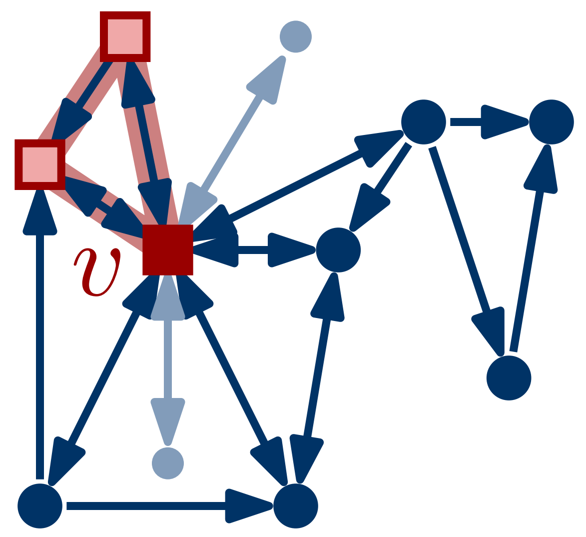
explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

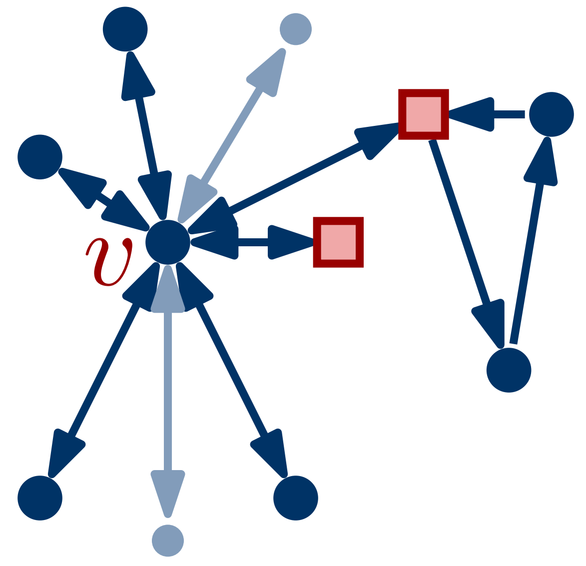
$O(n+m)$

a)



report triangle

b)



no transmission graph

Triangle Detection in Transmission Graphs

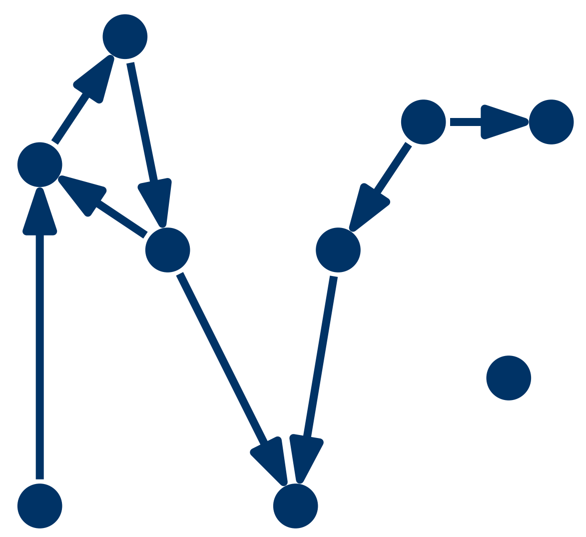
Algorithm

Theorem There is a robust algorithm that finds a triangle in a transmission graph in $O(n+m)$ time.

Compute $N_{\text{bi}}(v)$ for all $v \in V$ $\quad O(n+m)$

Case 1: $|N_{\text{bi}}(v)| < 6$ for all vertices

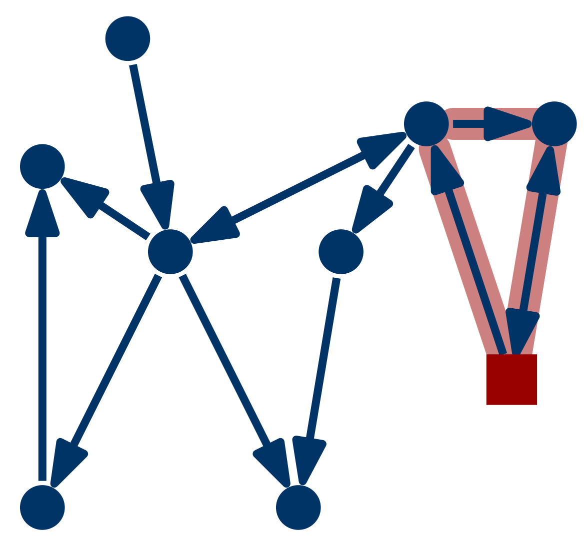
a)



if not acyclic $\rightarrow$
no transmission graph

$O(n+m)$

b)



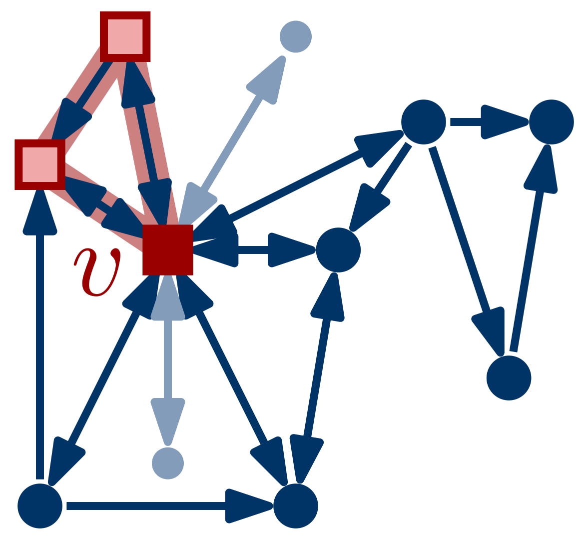
explicitly check for v
with $N_{\text{bi}}(v) \neq \emptyset$

$O(n+m)$

Case 2: At least one vertex v with $|N_{\text{bi}}(v)| \geq 6$

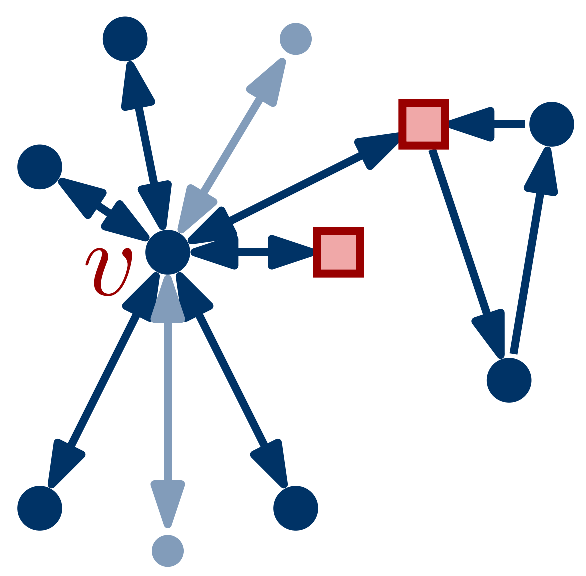
$O(n+m)$

a)



report triangle

b)



no transmission graph

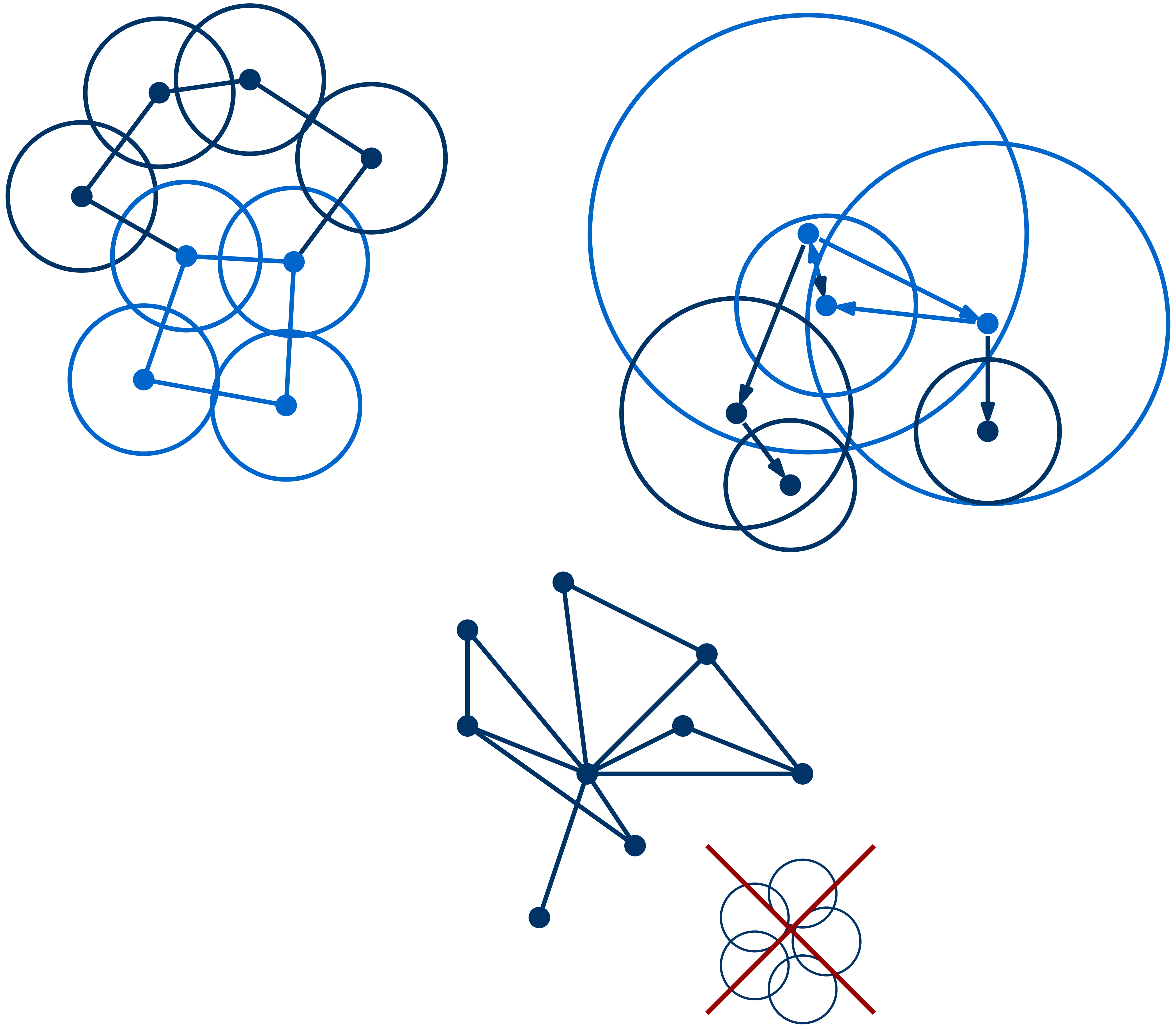
Conclusion

Robust Algorithms for:

Triangles in unit disk graphs $O(n)$

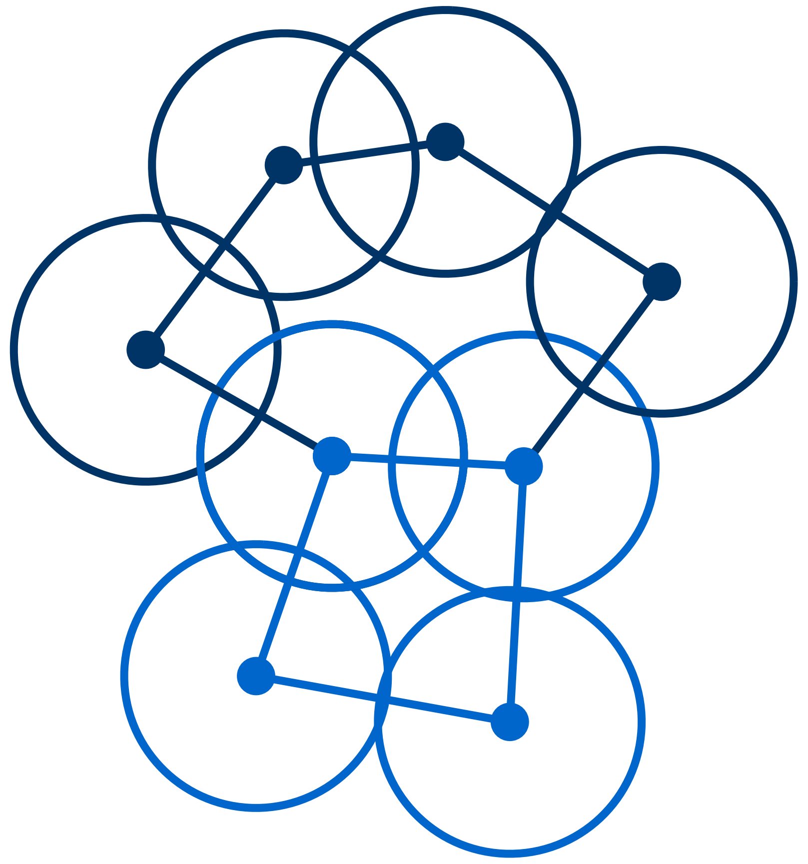
Triangles in transmission graphs $O(n+m)$

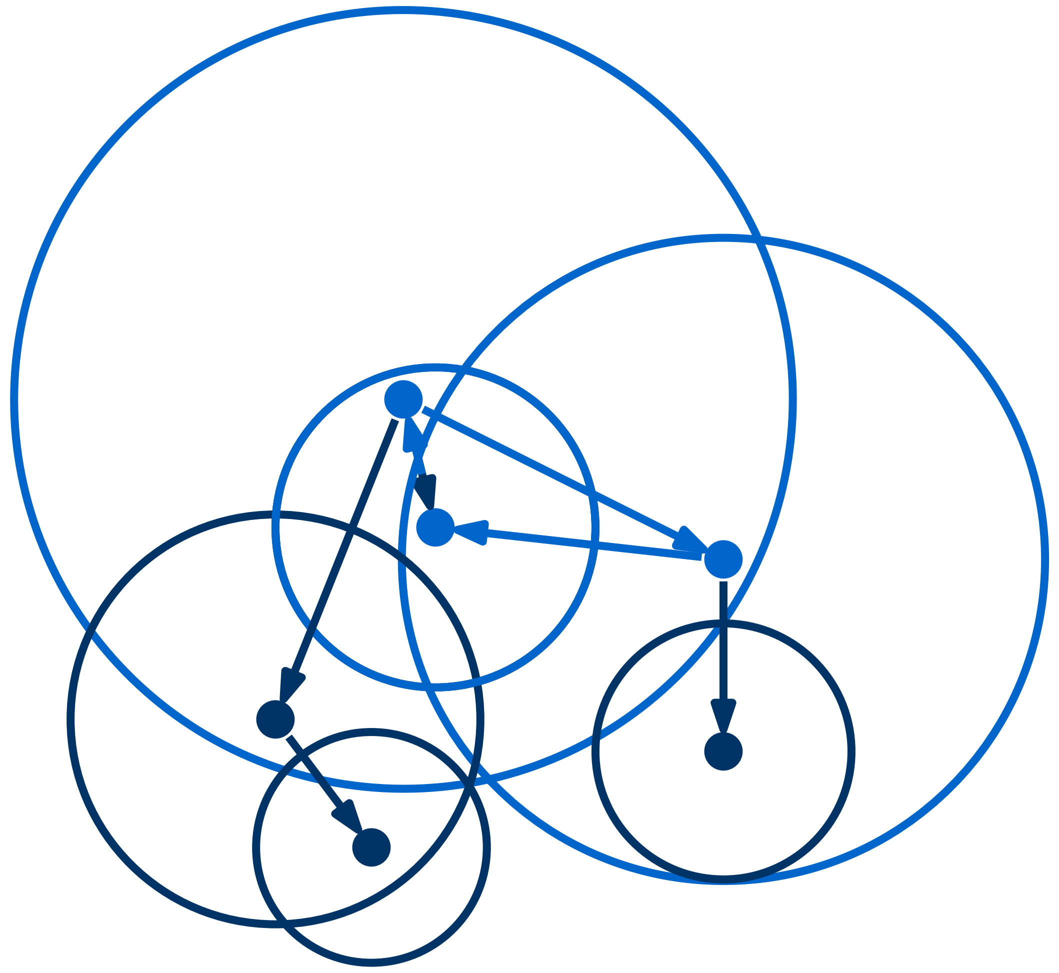
Girth in unit disk graphs $O(n)$



Conclusion

Robust Algorithms for:

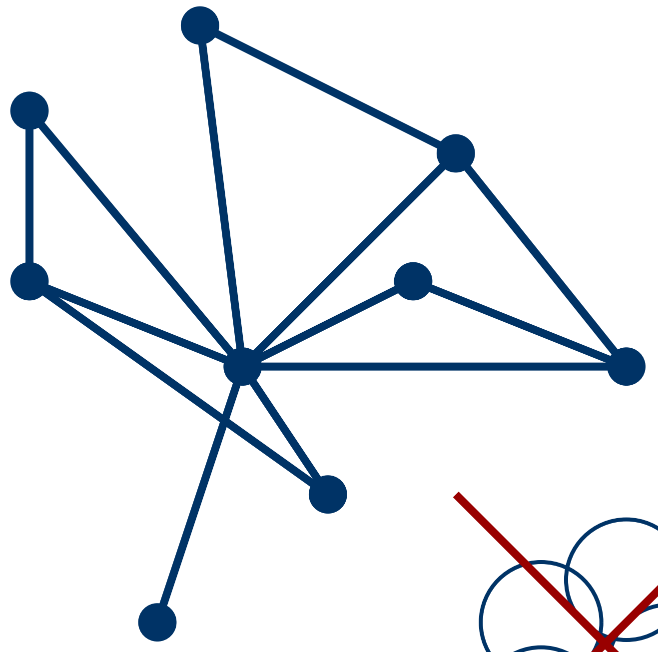




Triangles in unit disk graphs $O(n)$

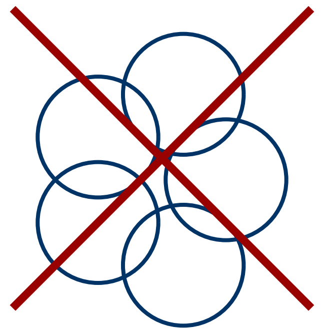
Triangles in transmission graphs $O(n+m)$

Girth in unit disk graphs $O(n)$



Open Questions:

Efficient robust algorithms for general disk graphs

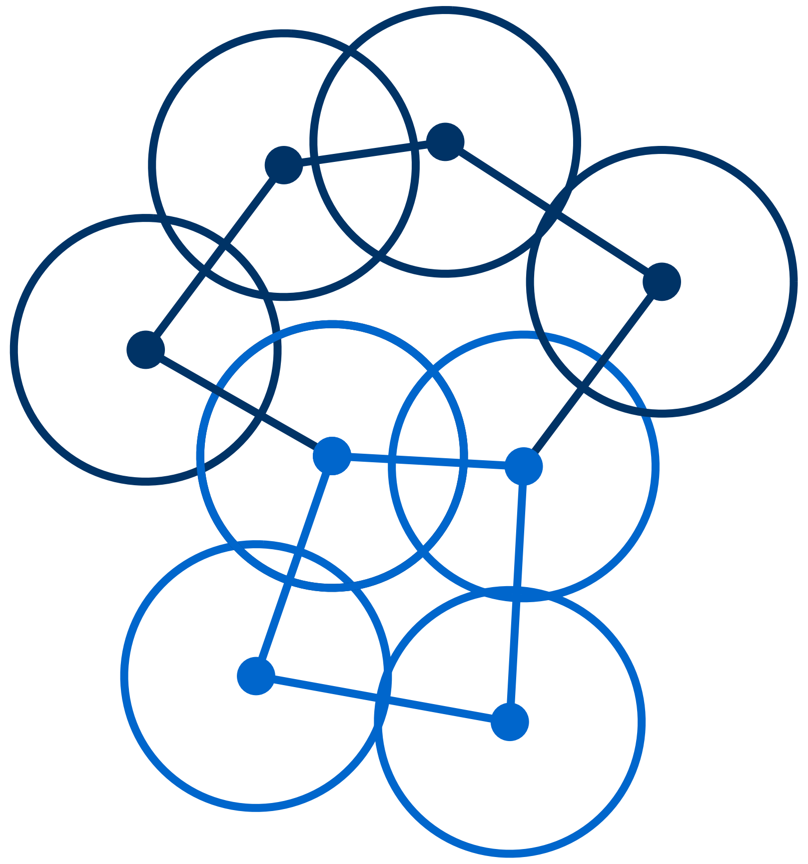
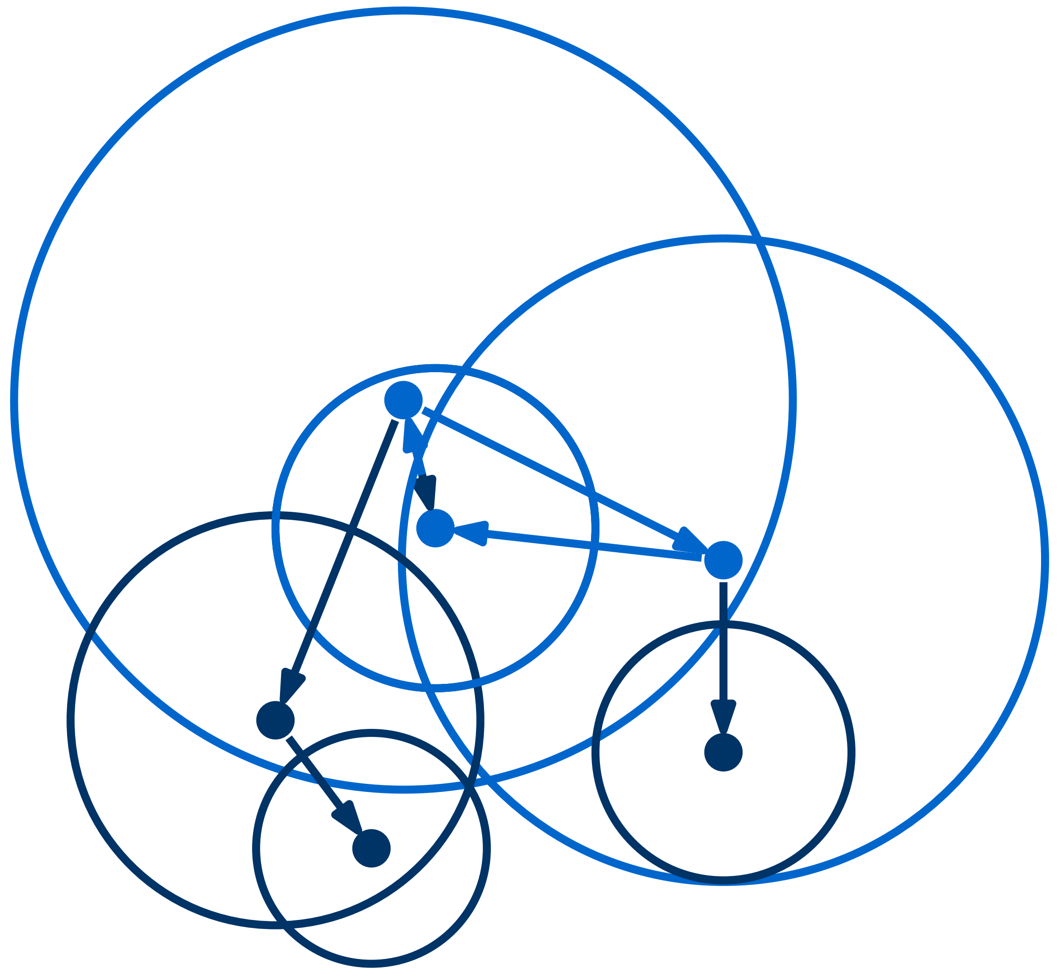


Efficient robust algorithms for the girth in transmission graphs

Conclusion

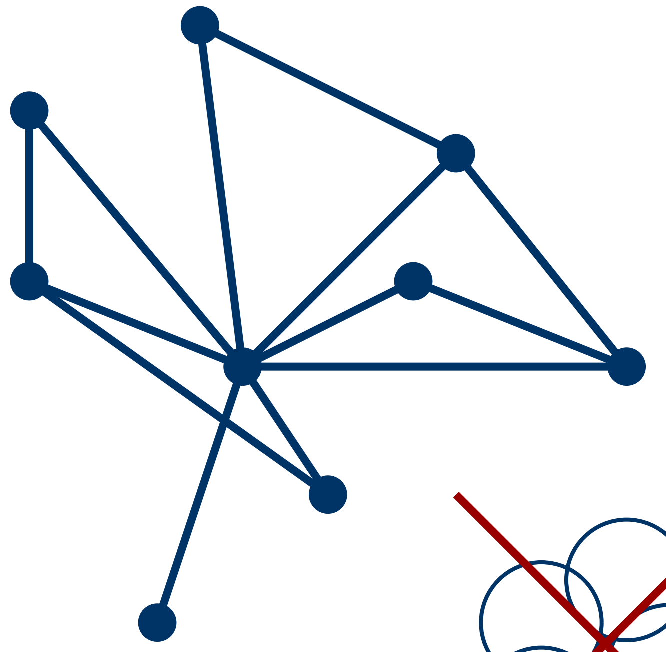
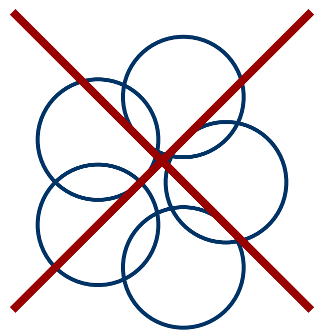
Robust Algorithms for:

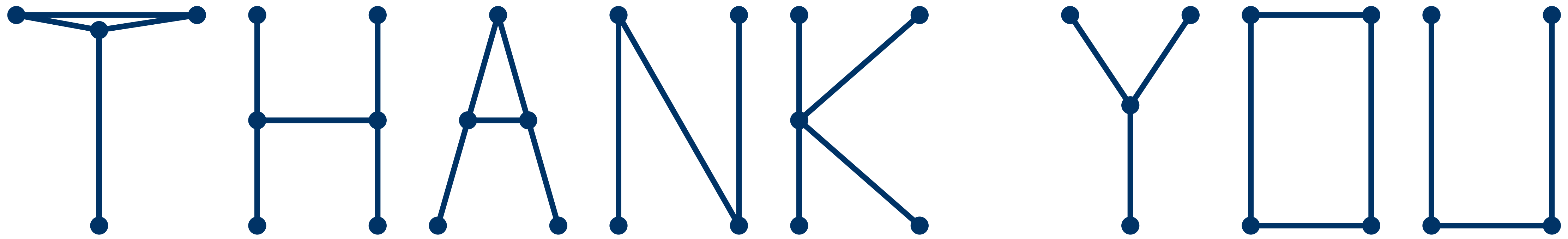
- Triangles in unit disk graphs $O(n)$
- Triangles in transmission graphs $O(n+m)$
- Girth in unit disk graphs $O(n)$

Open Questions:

- Efficient robust algorithms for general disk graphs
- Efficient robust algorithms for the girth in transmission graphs



Computing the Girth in Unit Disk Graphs

Lemma[Evans et al., 2016] If a (unit) disk graph has no triangle, it is planar.

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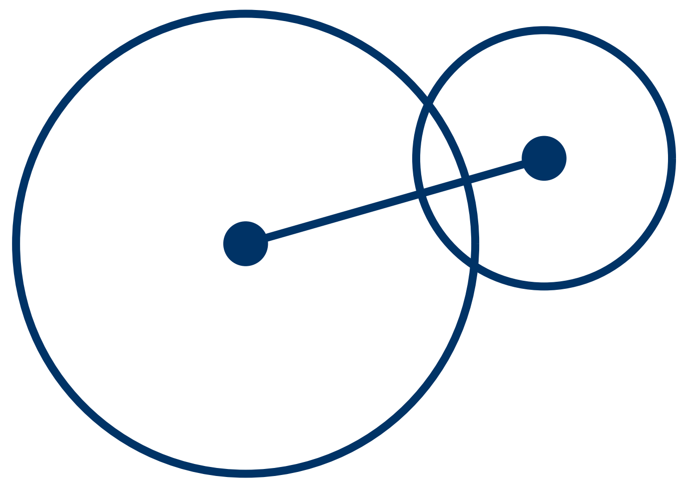

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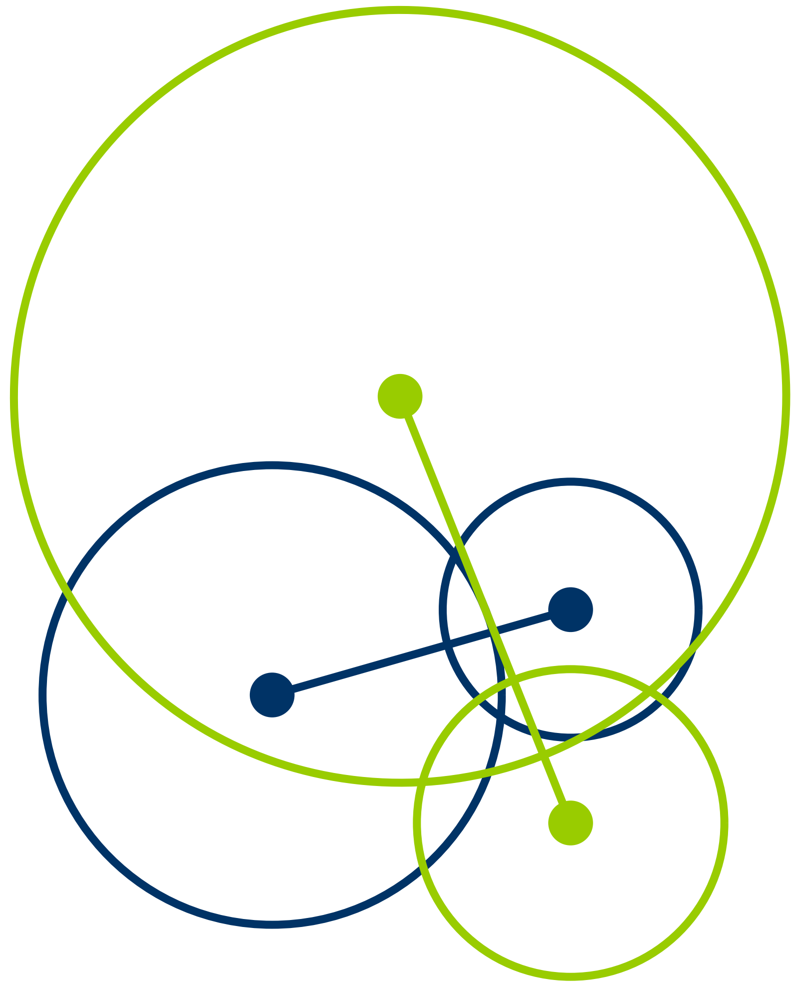

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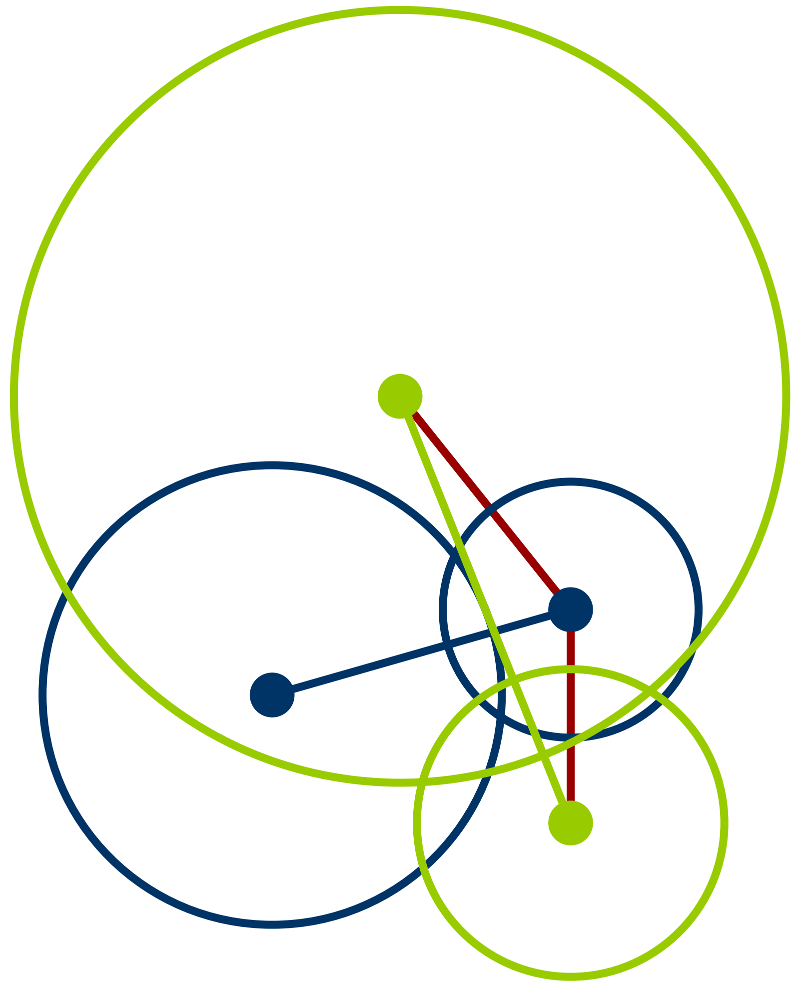

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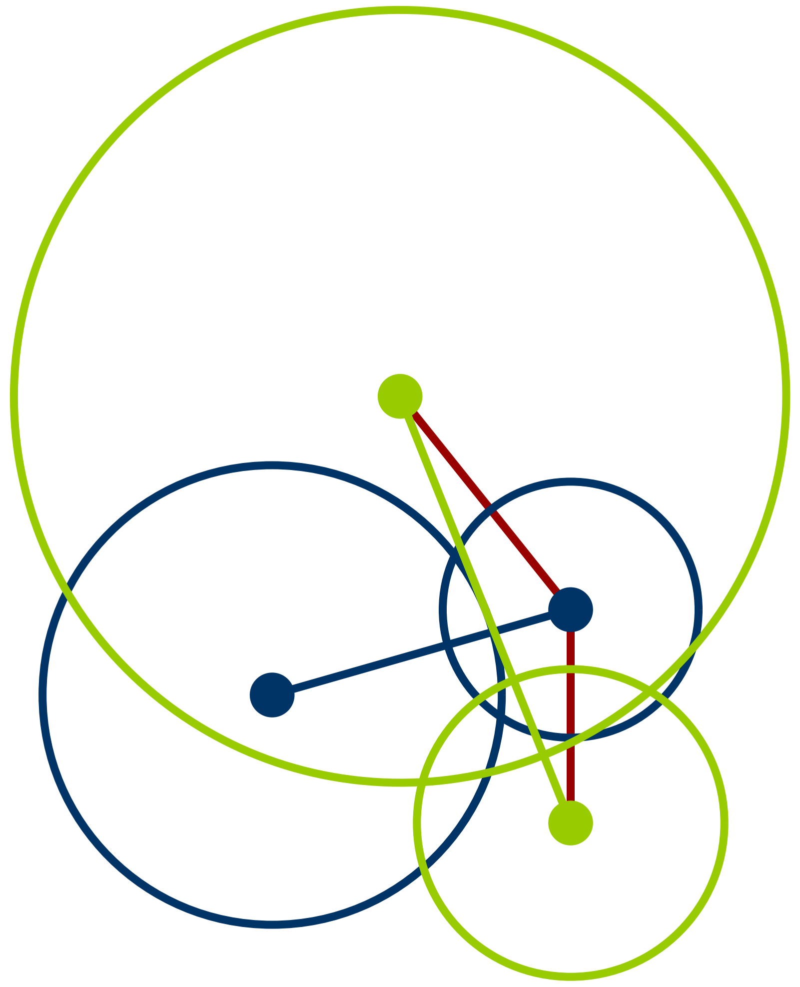
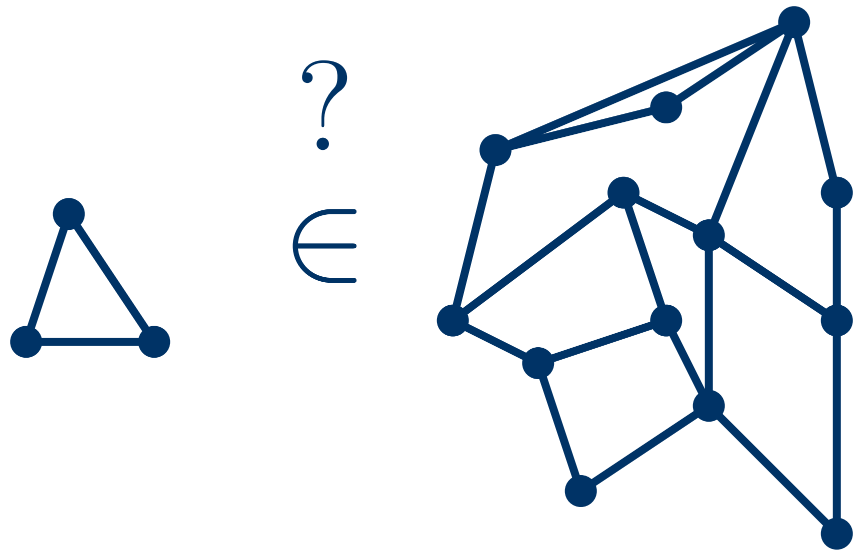


Algorithm



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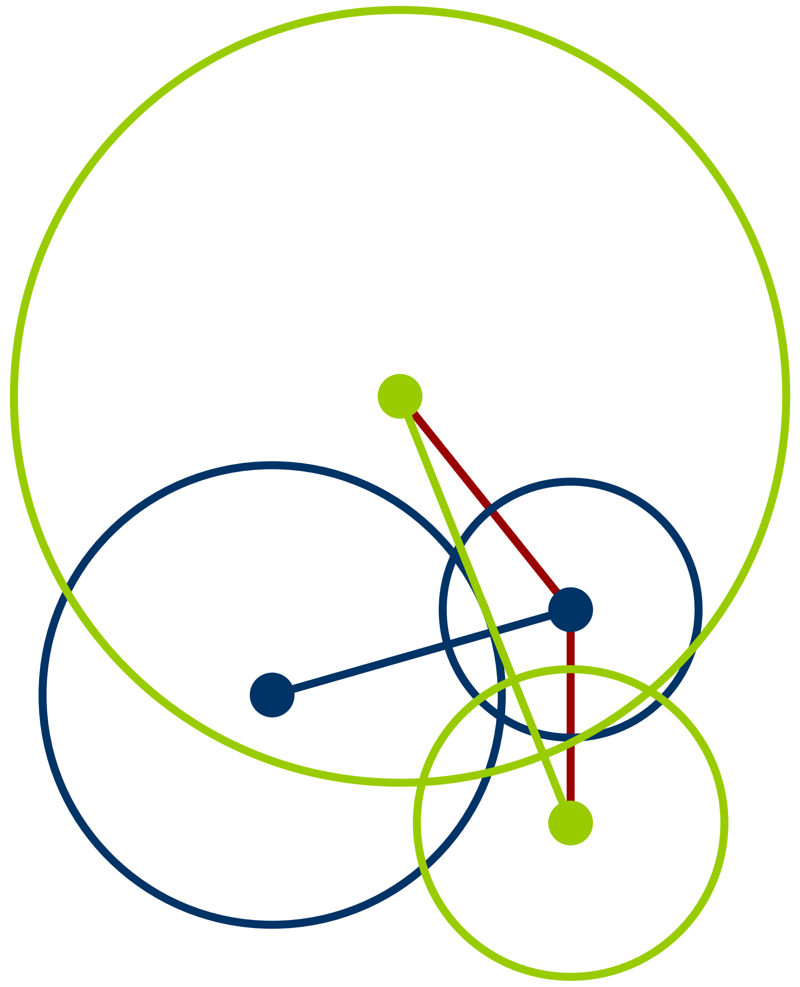

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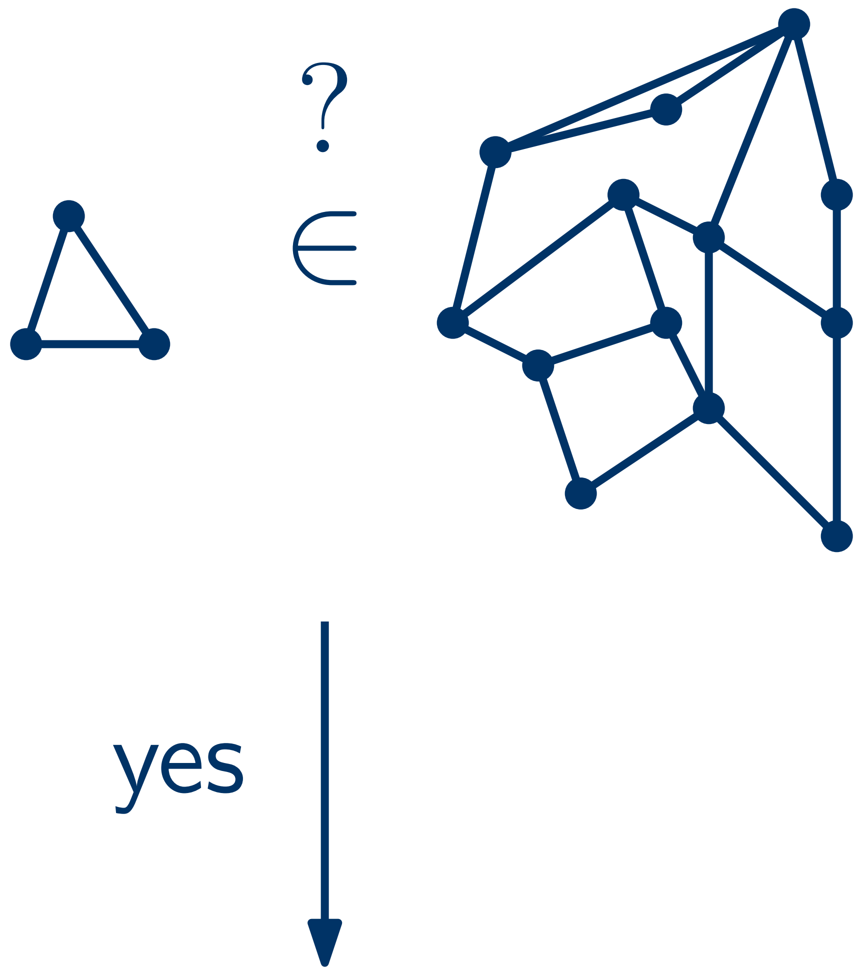
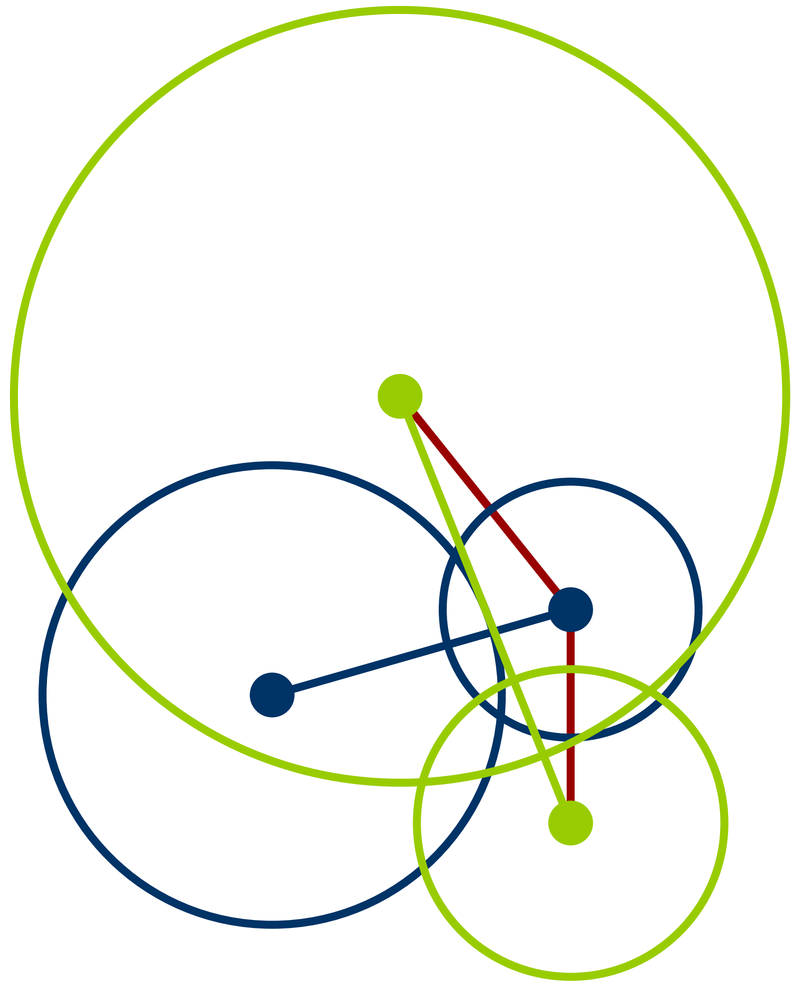


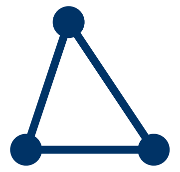
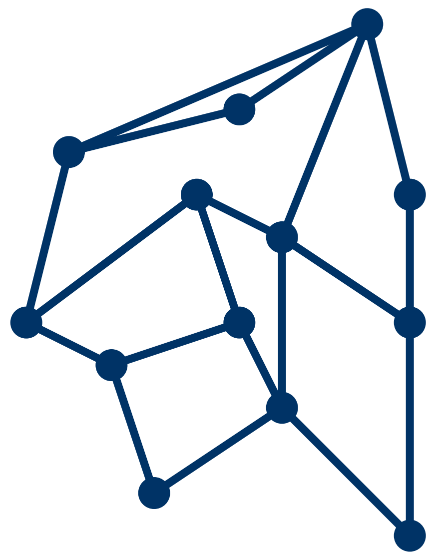
return „girth is 3“

Computing the Girth in Unit Disk Graphs

Lemma[Evans et al., 2016] If a (unit) disk graph has no triangle, it is planar.



Algorithm

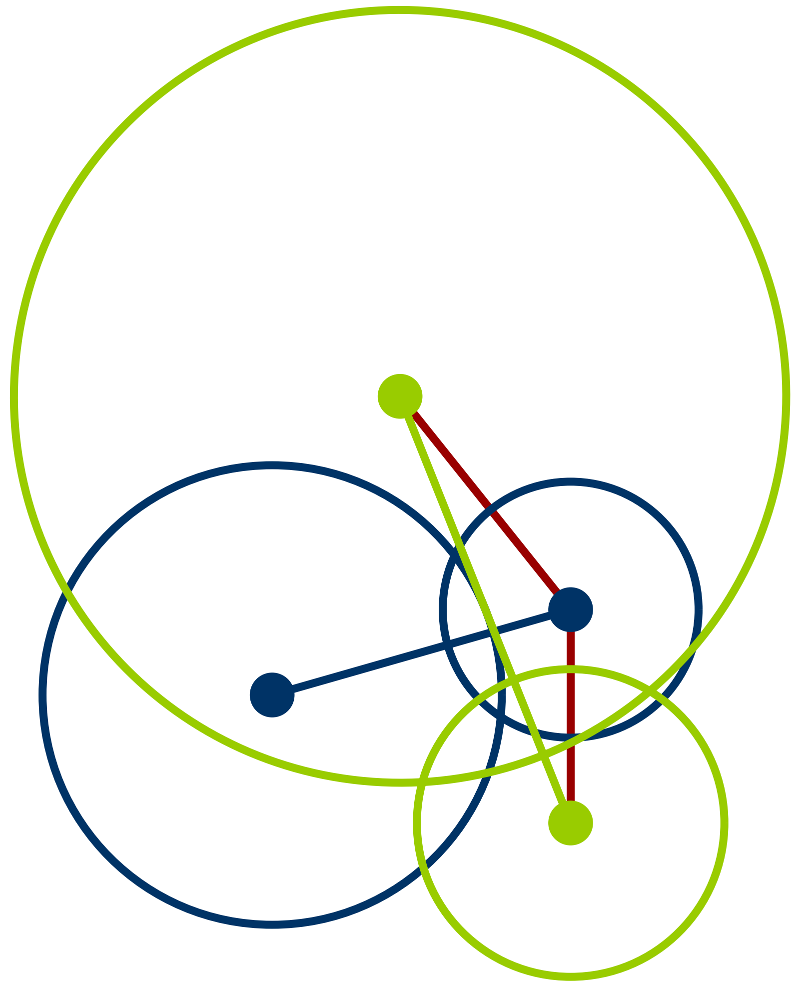
 $\overset{?}{\in}$  —no→ planar?

↓ yes

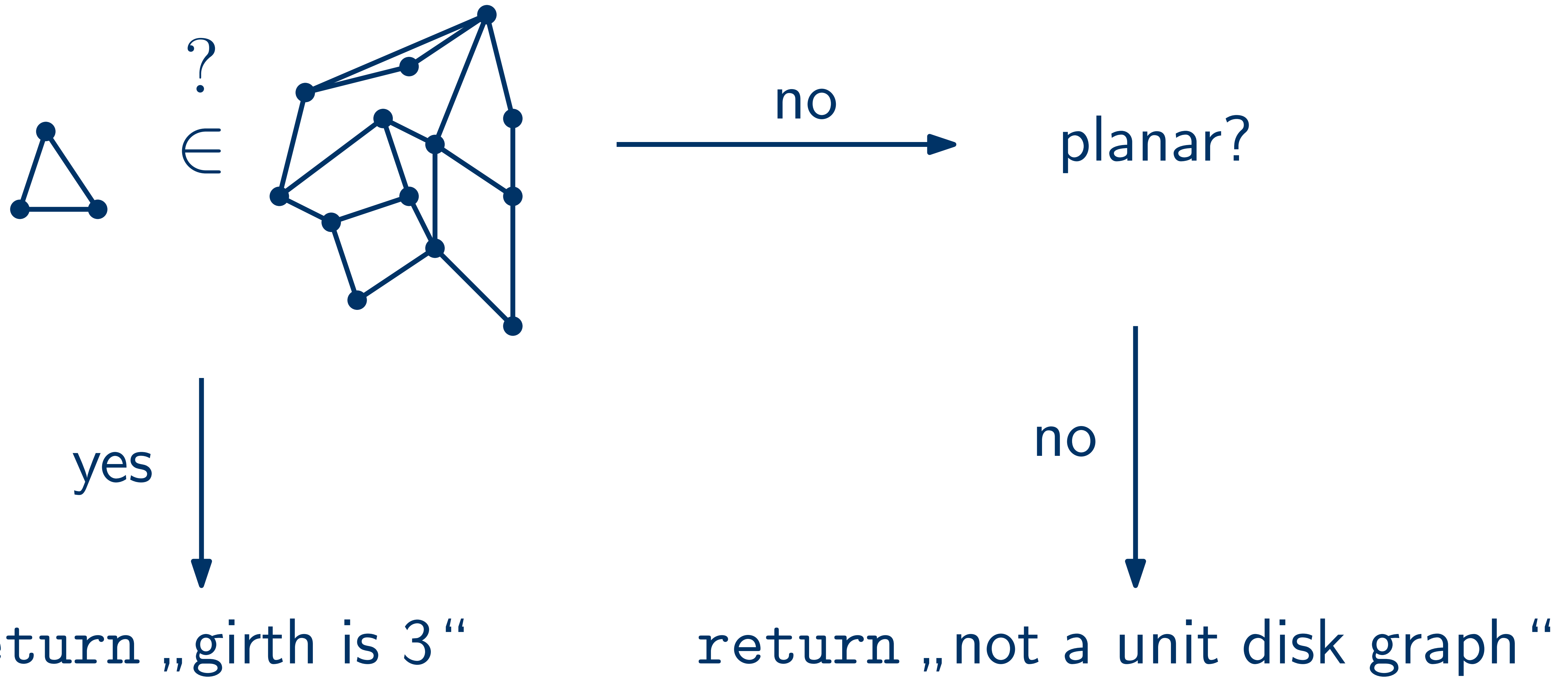
`return` „girth is 3“

Computing the Girth in Unit Disk Graphs

Lemma[Evans et al., 2016] If a (unit) disk graph has no triangle, it is planar.



Algorithm



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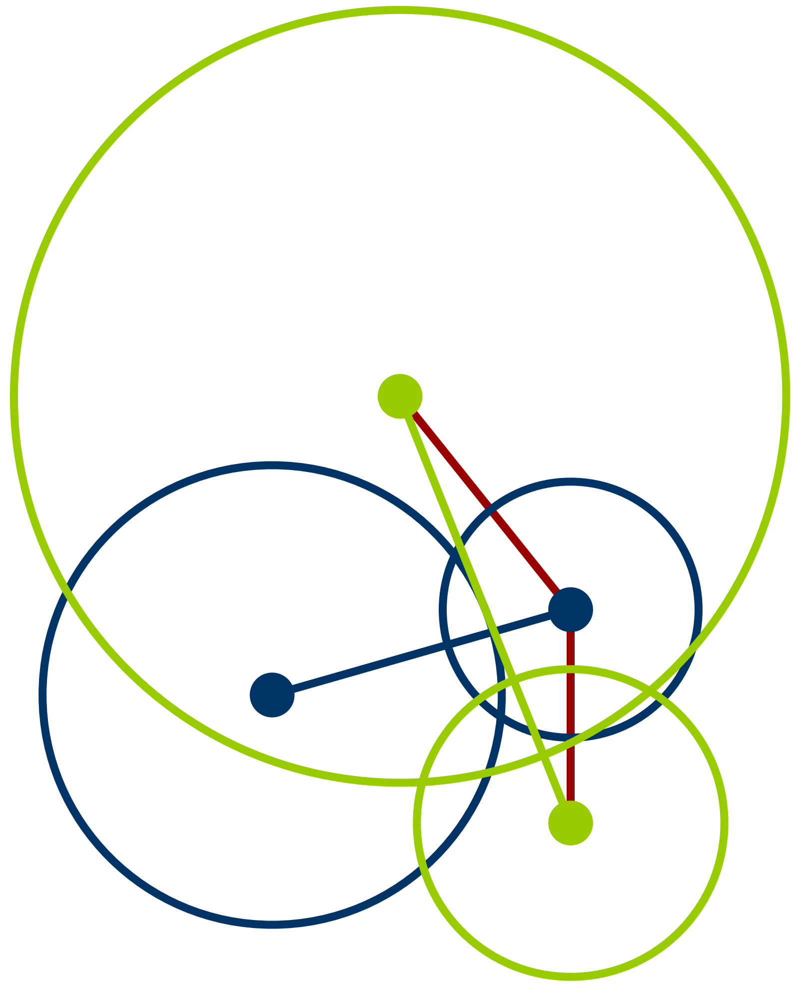

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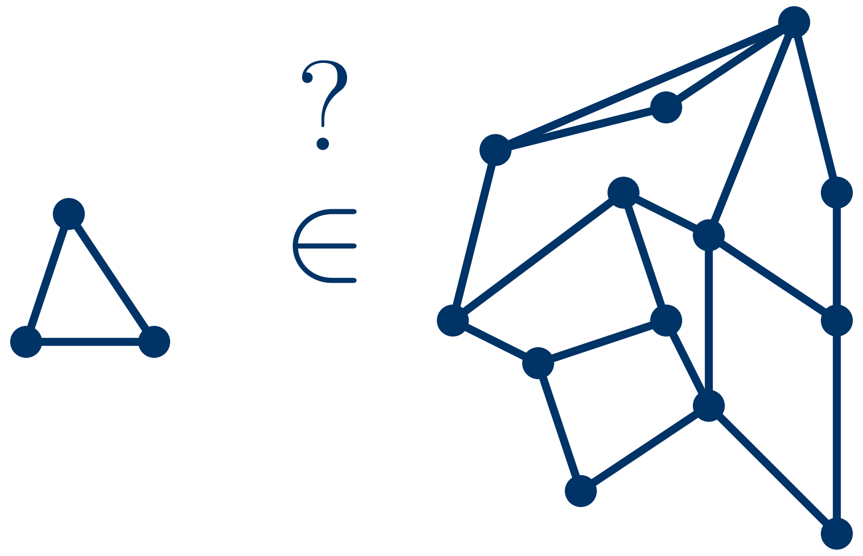


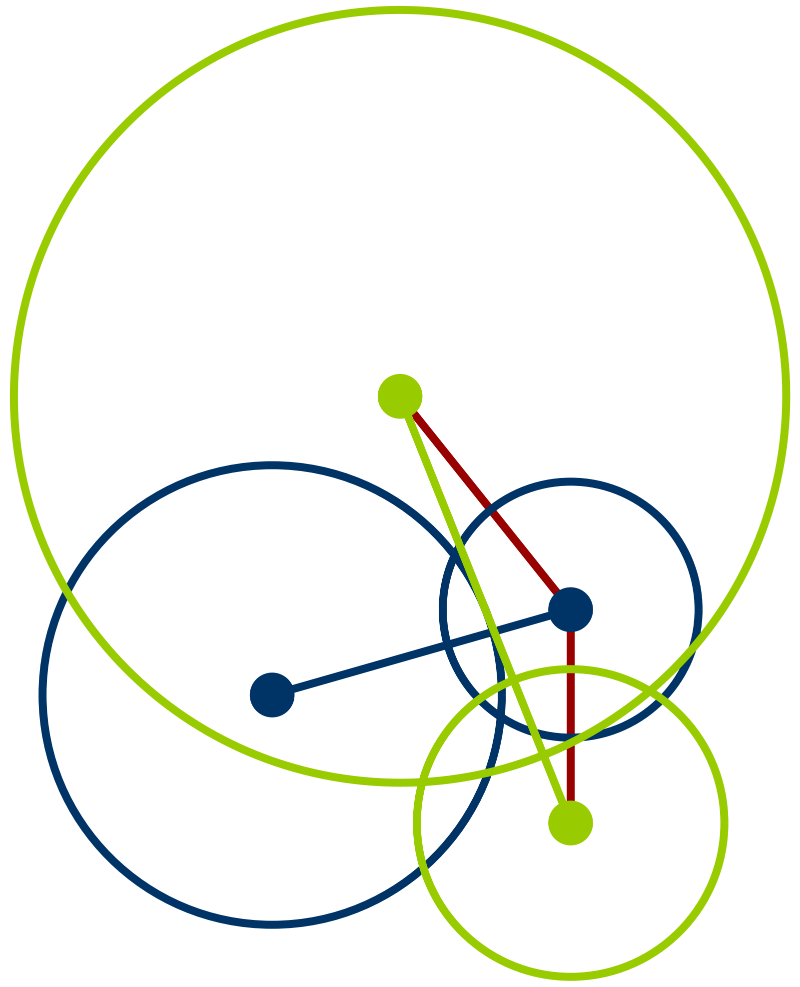
no → planar? yes → girth algorithm for planar graphs [Chang, Lu; 2013]

(triangle in graph?) yes ↓ `return` „girth is 3“

(planar?) no ↓ `return` „not a unit disk graph“

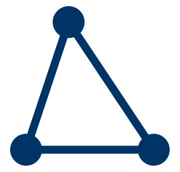
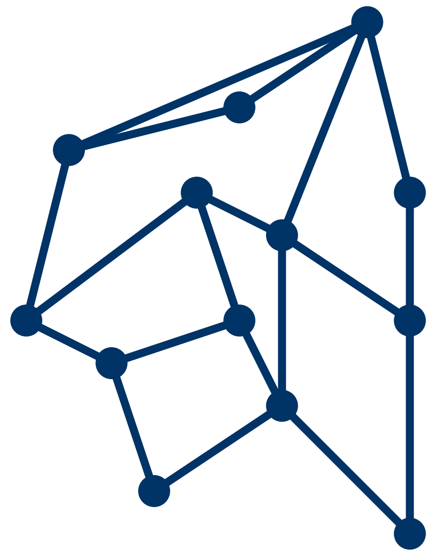
Computing the Girth in Unit Disk Graphs

Lemma[Evans et al., 2016] If a (unit) disk graph has no triangle, it is planar.



Algorithm

$O(n)$

 $\overset{?}{\in}$ 

no →

$O(n)$

planar?

yes →

$O(n)$

girth algorithm for planar graphs

[Chang, Lu; 2013]

yes ↓

`return` „girth is 3“

no ↓

`return` „not a unit disk graph“

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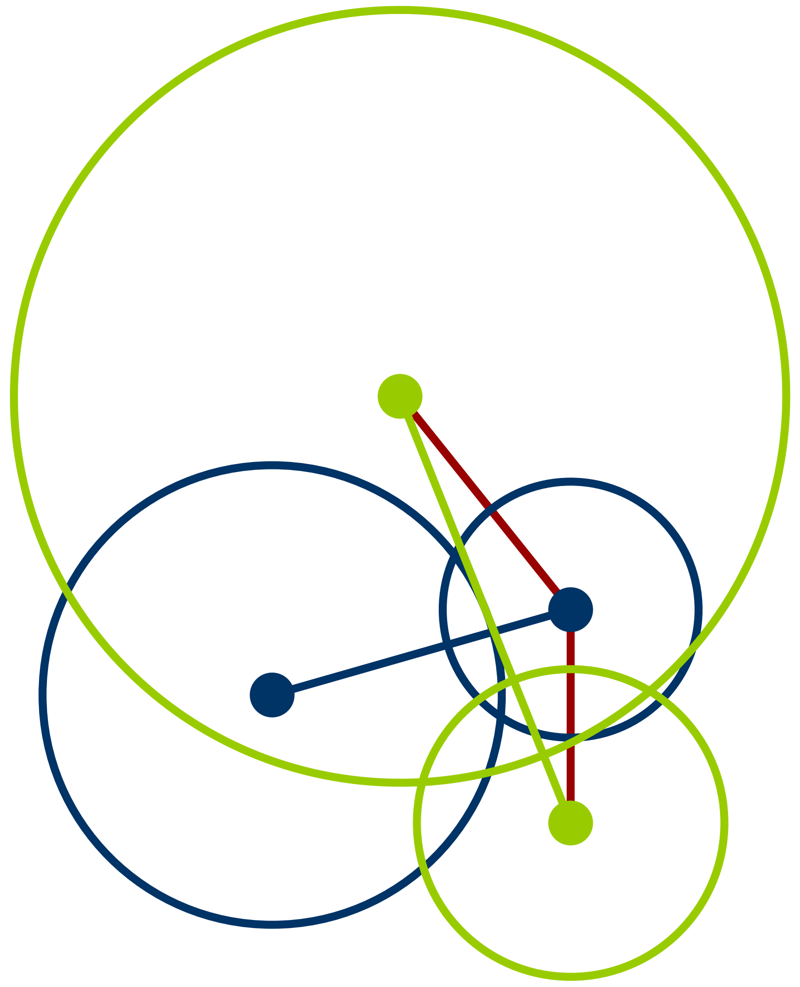
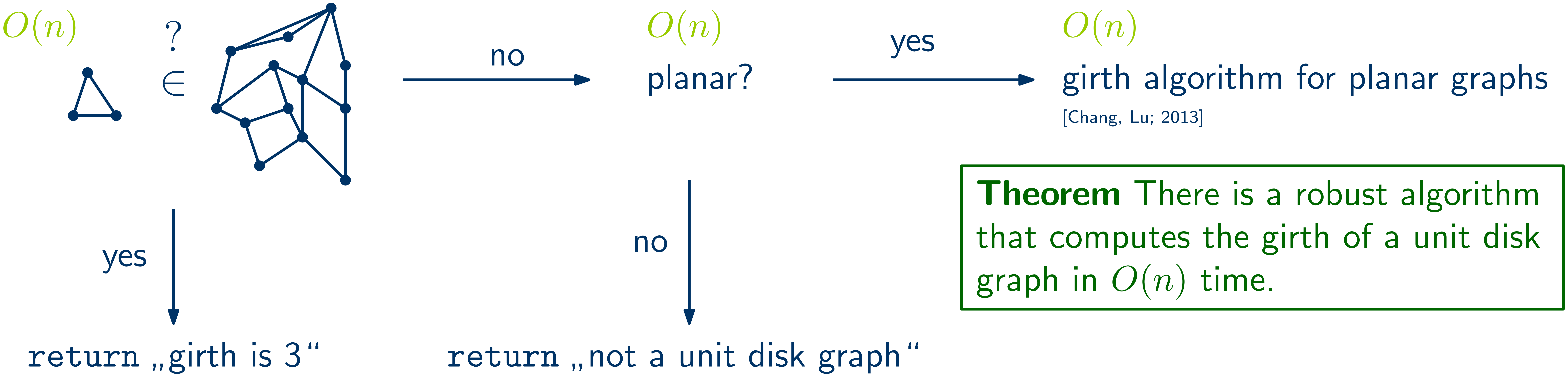
Lemma[Evans et al., 2016] If a (unit) disk graph has no triangle, it is planar.



Algorithm



Theorem There is a robust algorithm that computes the girth of a unit disk graph in $O(n)$ time.