Connected matchings

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Joint work with
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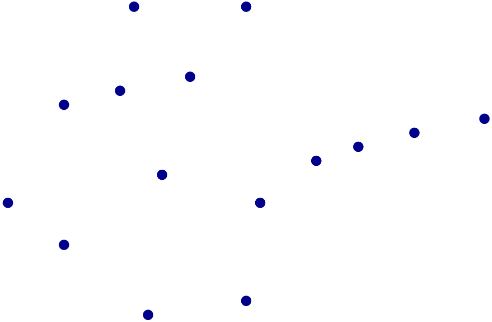
The problem

- P a set of points in the plane, general position
- Consider connected matchings for P



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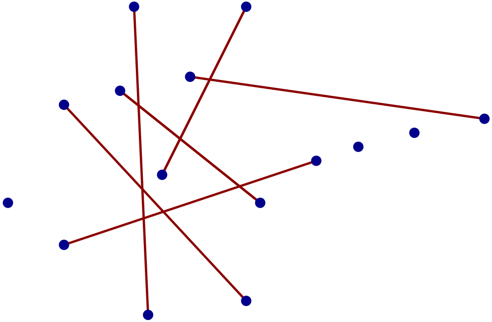

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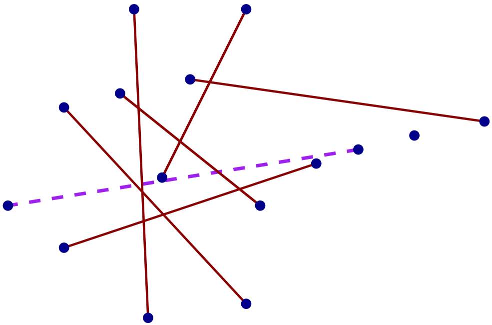

- Question: find the largest $f(n)$ such that any point set P with n points has a connected matching with at least $f(n)$ segments

What do we show?

- There exists a set of n points where each connected matching has at most $\sim \frac{n}{3} = .3333\ldots n$ segments
- Each set of n points has a connected matching with $\frac{5}{27}n = .185185\ldots n$ segments
  - better than $\frac{1}{6}n = .1666\ldots n$
  - computable in $O(n \log n)$ time
- An interesting discrete geometry result: balanced separator with two edges spanned by P
  - readily implies the $\frac{1}{6}n = .1666\ldots n$ bound
  - computable in $O(n)$ time
  - remake of a result of Ábrego and Fernández-Merchant
    The rectilinear local crossing number of K_n [JCTA 2017]

Related work?

- Crossing families: find the largest $g(n)$ such that any point set P with n points has at least $g(n)$ segments that pairwise cross
  - connected vs all pairs intersect

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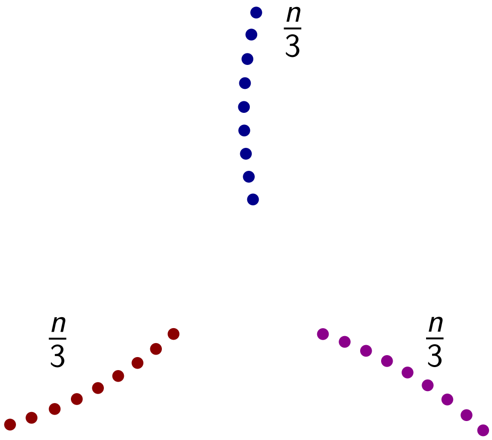
- $\Omega(\sqrt{n})$ by Aronov, Erdőss, Goddard, Kleitman, Klugerman, Pach, and Schulman
  Crossing families [Combinatorica 1994]
- at least $n^{1-o(1)}$ by Pach, Rubin, and Tardos
  Planar point sets determine many pairwise crossing segments [Adv. Math. 2021]
- at most $\sim 8n/41$ by Aichholzer, Kyncl, Scheucher, Vogtenhuber, and Valtr
  On crossing-families in planar point sets [Comput. Geom. 2022]

Upper bound

best connected matching has $\sim n/3$ segments



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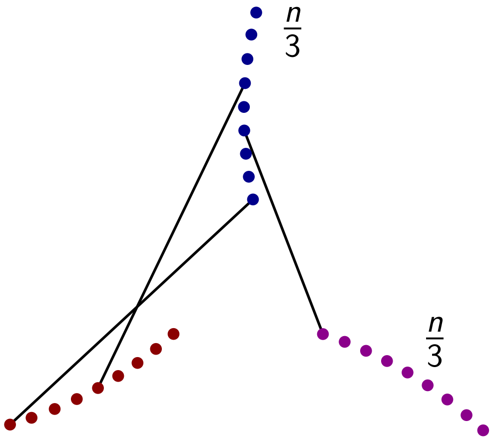

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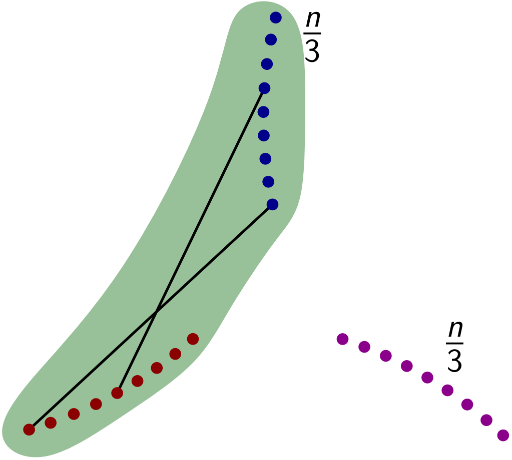
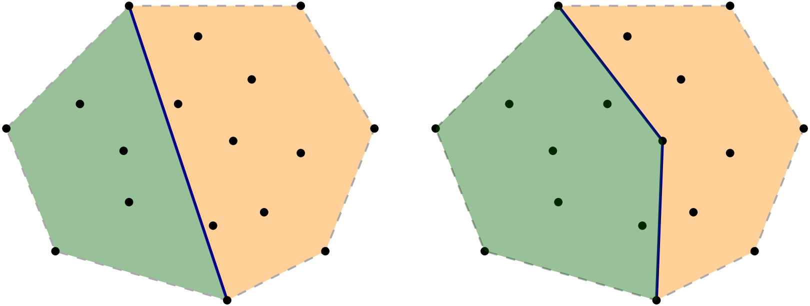


Towards lower bound: 2-edge separator

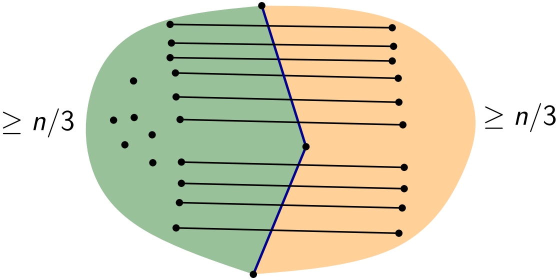
P any set of n points in general position in the plane
There exists a path with the following properties

- at most two edges
- vertices at P
- boundary of $CH(P)$ to boundary of $CH(P)$
- balanced separation: at least $\sim n/3$ points on each side



Lower bound

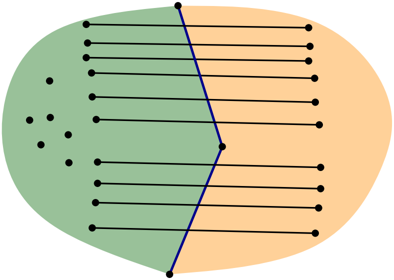
- now it is easy to get a connected matching with $\sim n/6$ edges



Lower bound

- now it is easy to get a connected matching with $\sim n/6$ edges
- how to get $\sim n/6 + n/100$ edges

$\geq 2n/3 - n/100$



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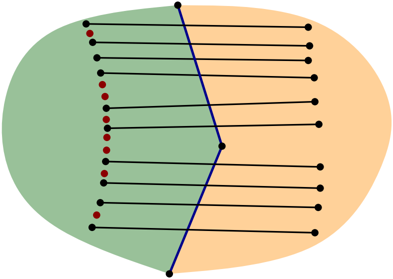

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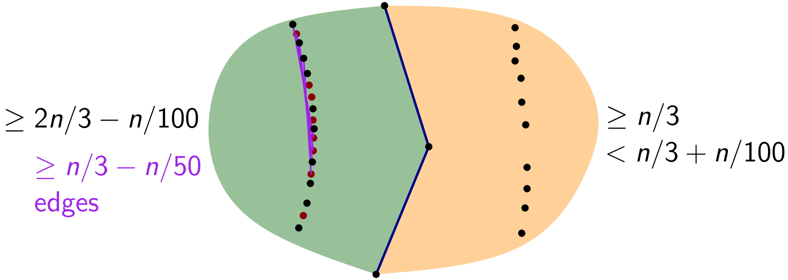

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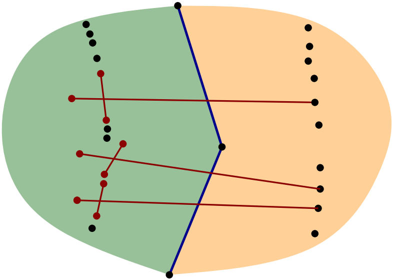

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- Optimize the idea from $n/100$ to $n/54$

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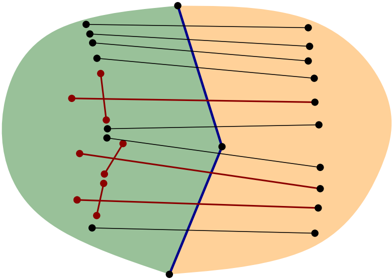

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Conclusions

- New problem: connected matching
- Upper bound $\sim n/3$
- Lower bound $\sim 5n/27$
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- Colored version

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THANKS for your time!!