UNIVERSITÄT TRIER

ALGO

On k-Plane Insertion into Plane Drawings

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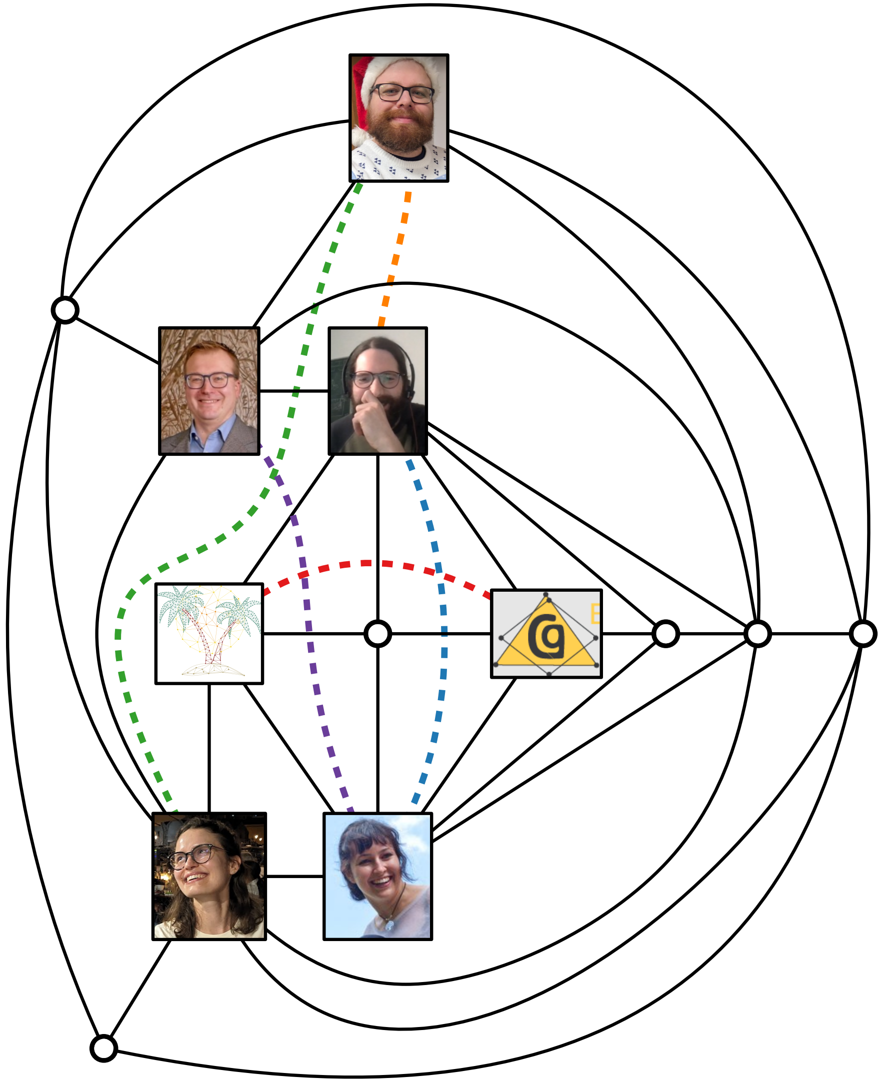
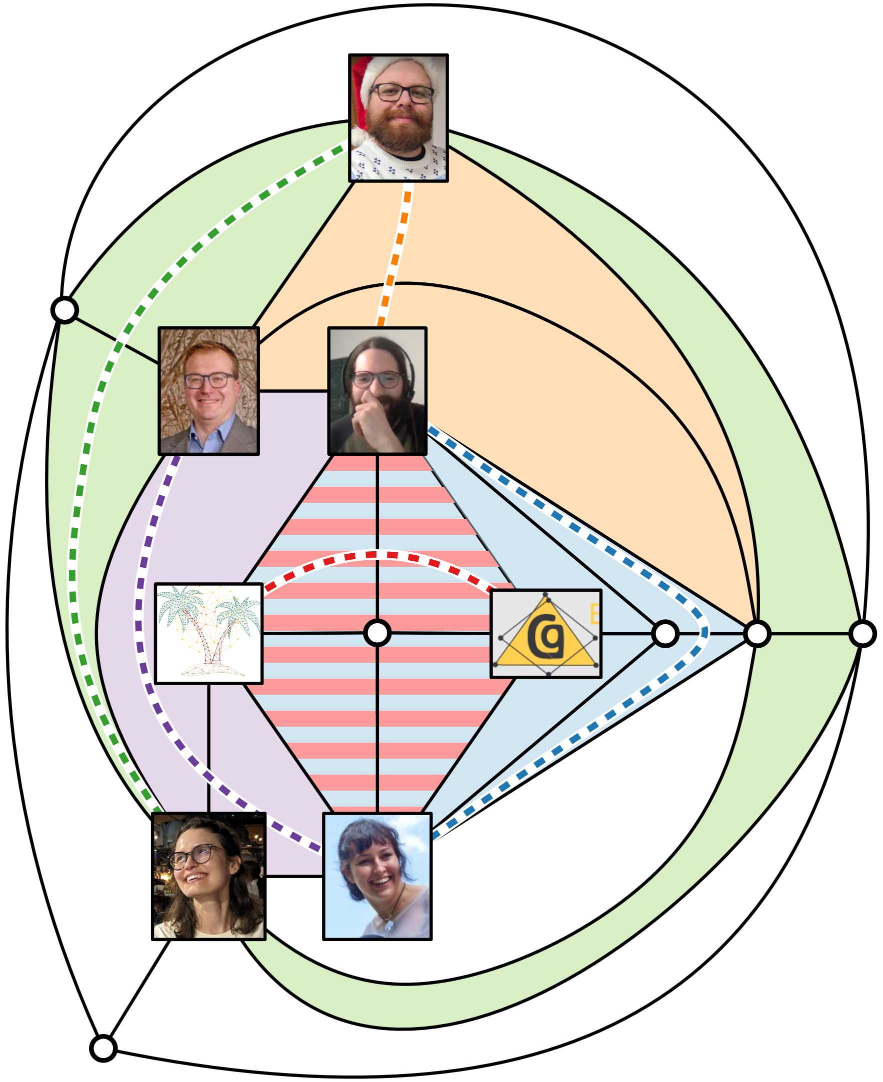
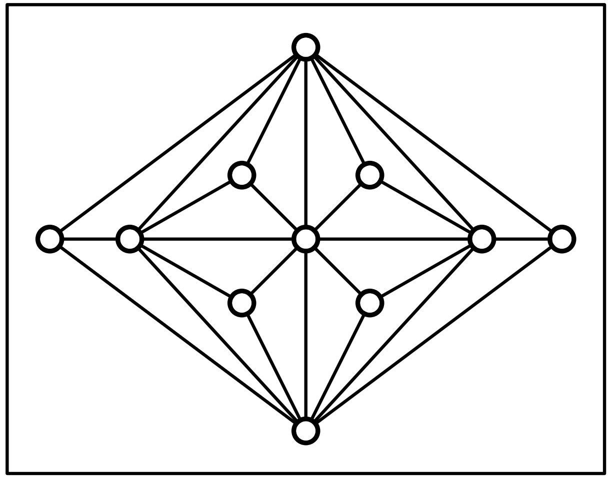
SWGD 2023
Summer Workshop
on Graph Drawing

EuroCG 2024
13-15 March
Ioannina, Greece





Inserting an Edge Into a Planar Graph



planar graph G

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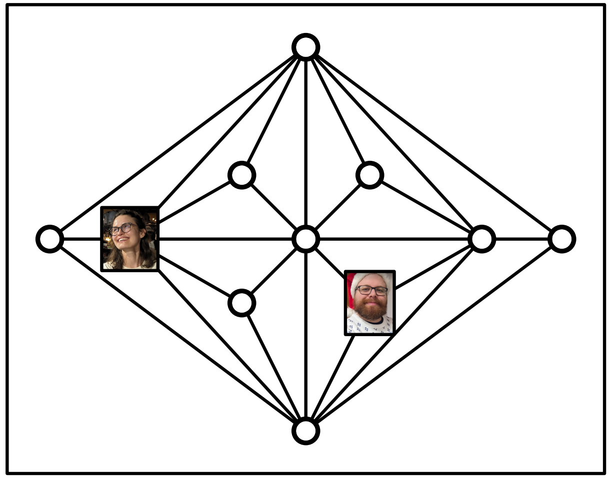
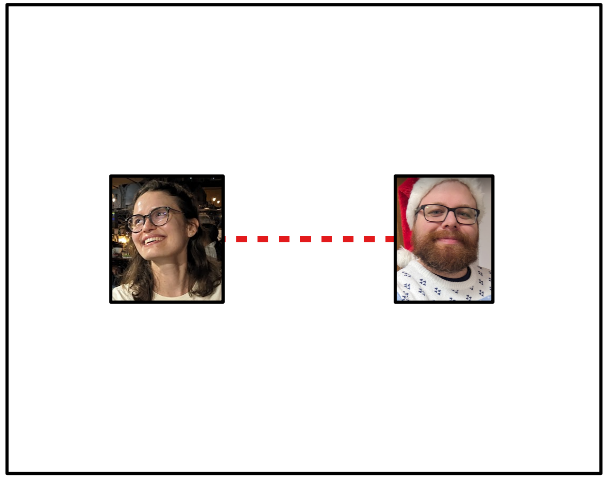


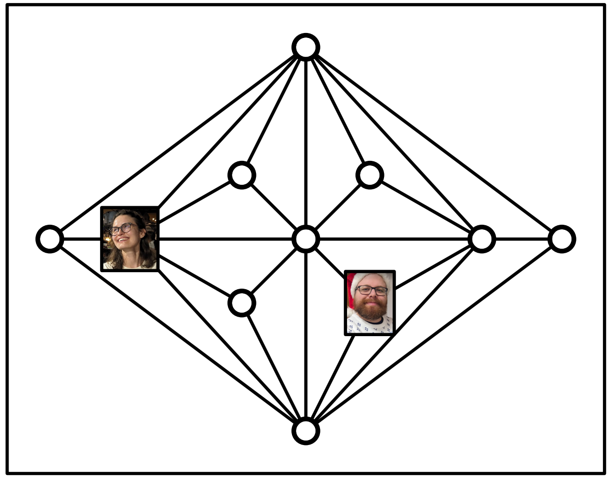
planar graph G

$+$



an edge e btw.
2 vertices of G

Inserting an Edge Into a Planar Graph



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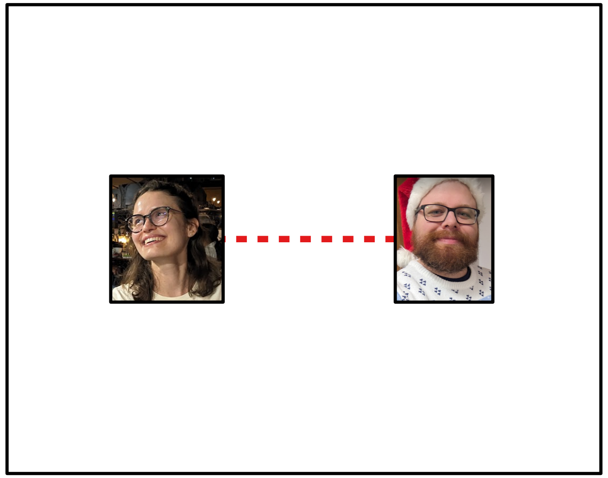

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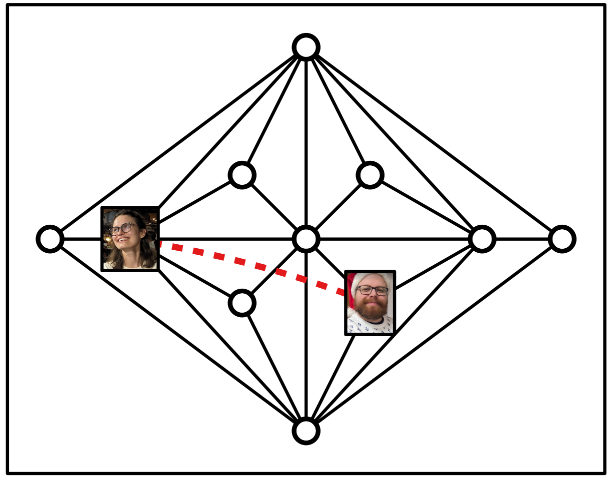
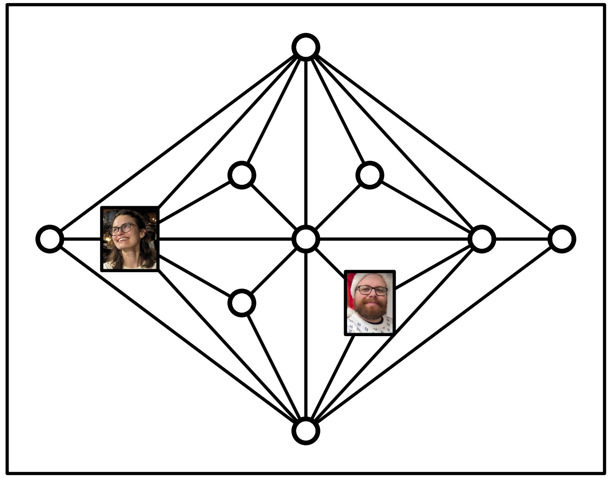


crossing-min. drawing of $G + e$

Inserting an Edge Into a Planar Graph

2 crossings



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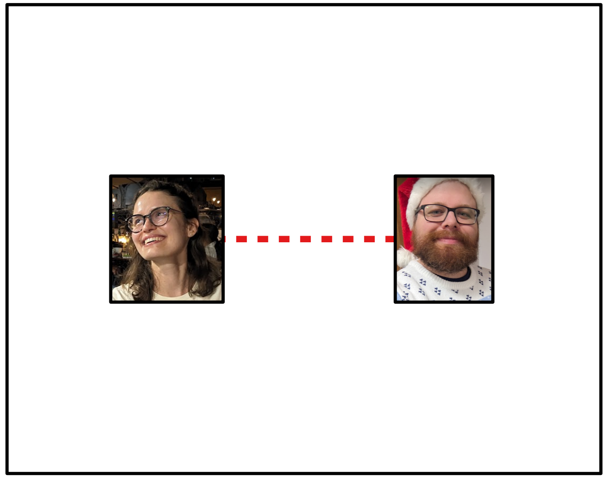

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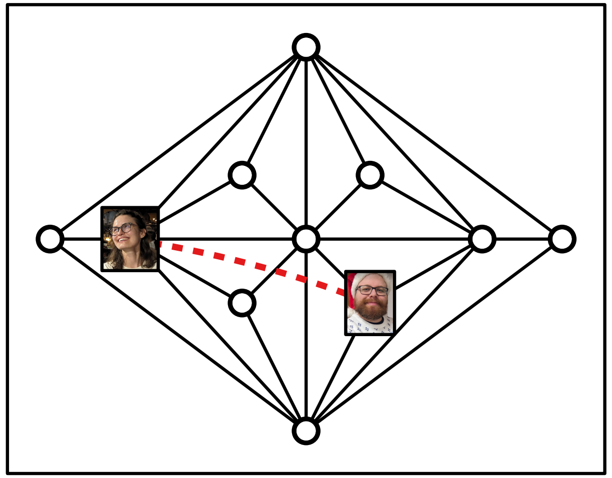

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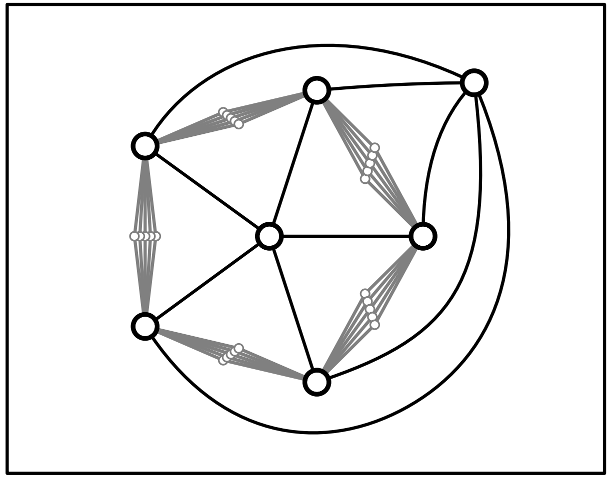

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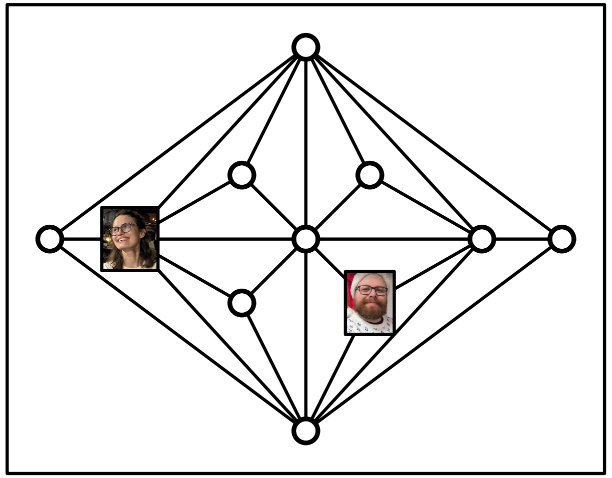

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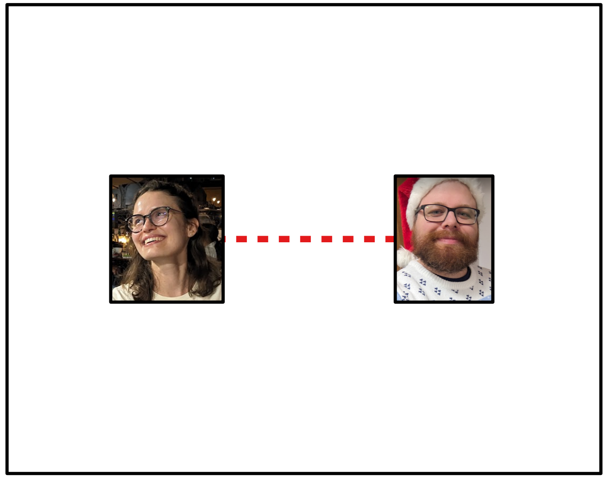

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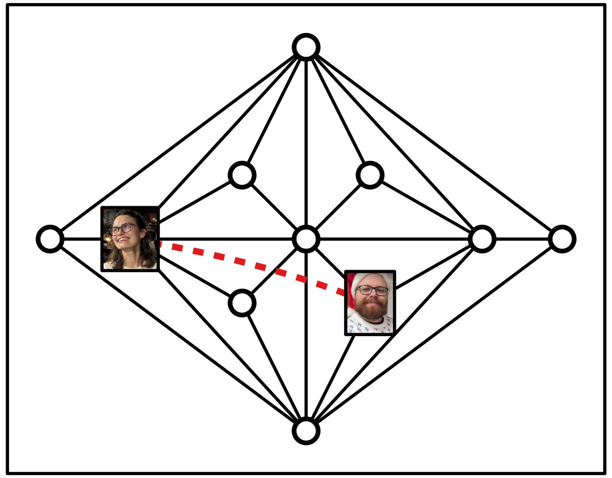

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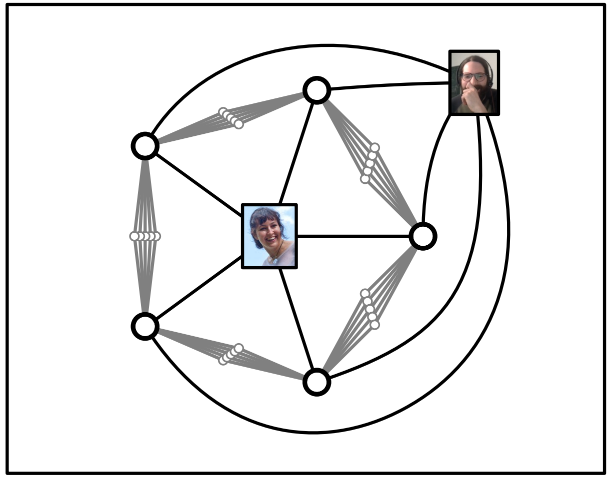

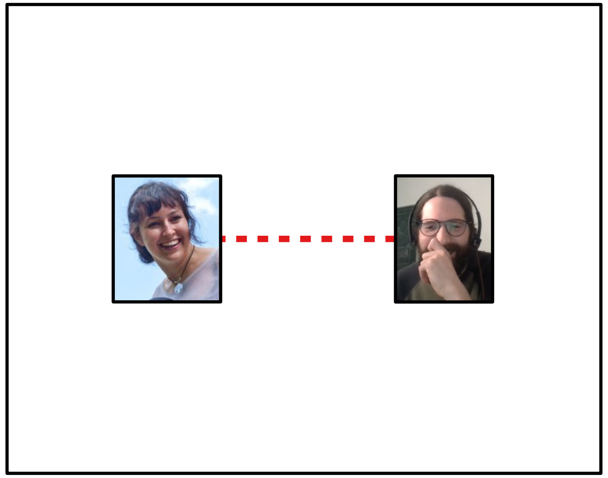

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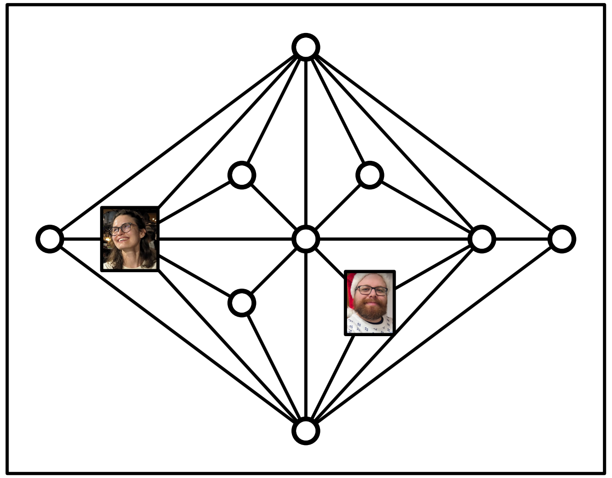

planar graph G

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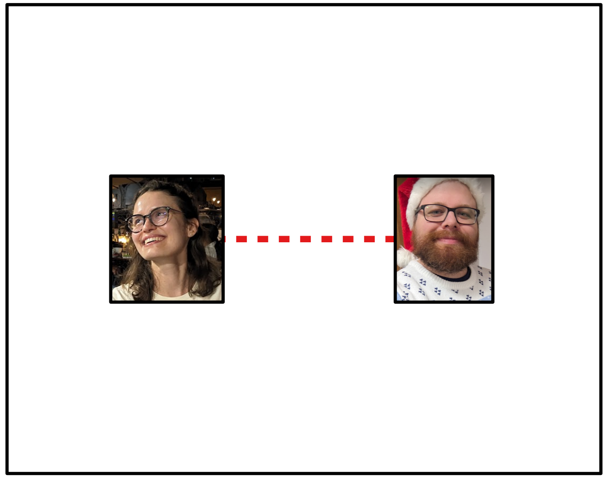

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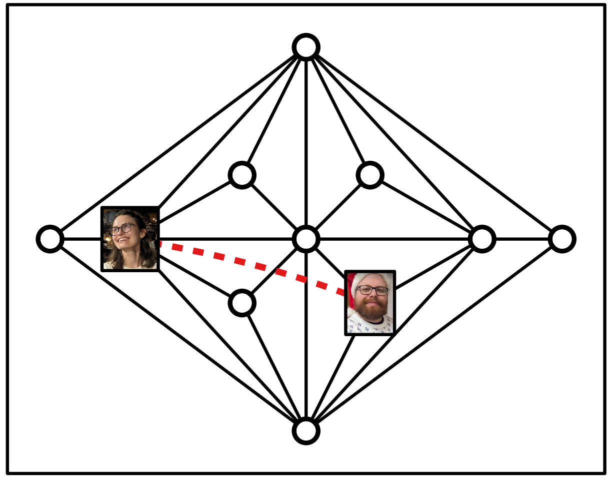

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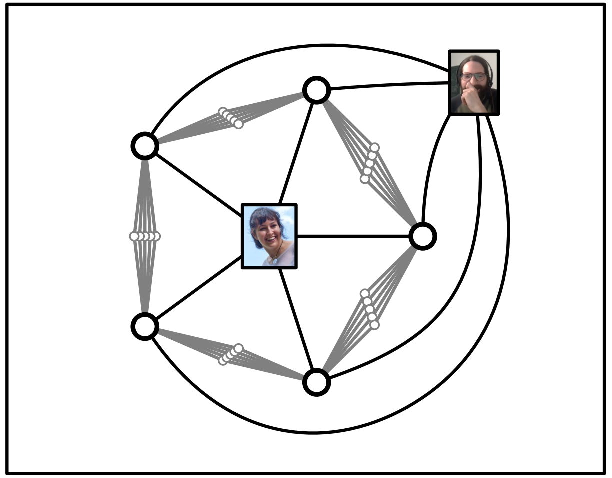

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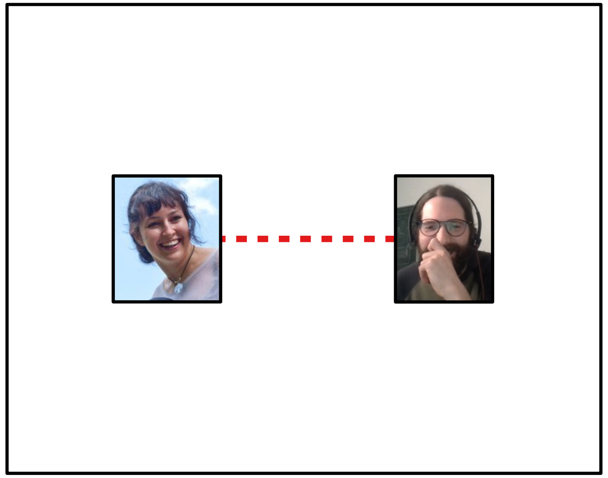

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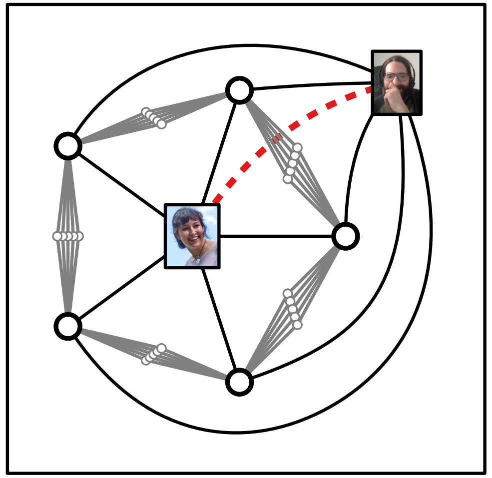
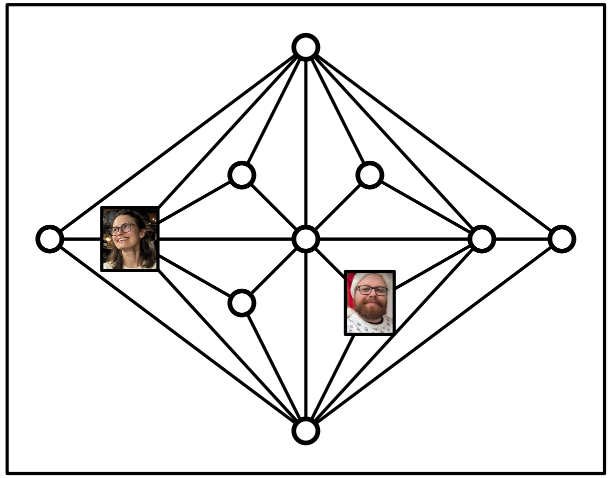
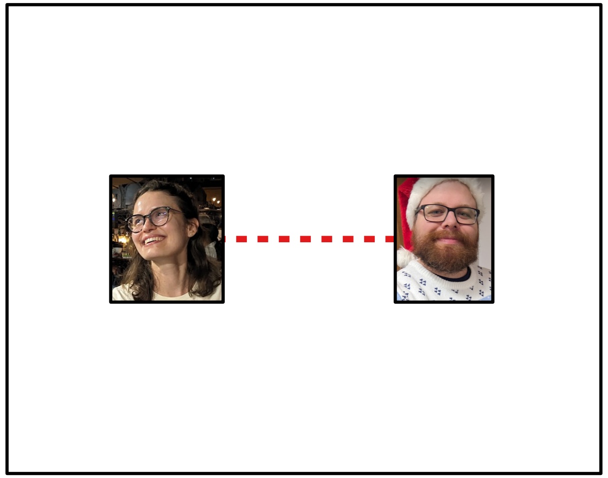


k crossings

Inserting an Edge Into a Planar Graph

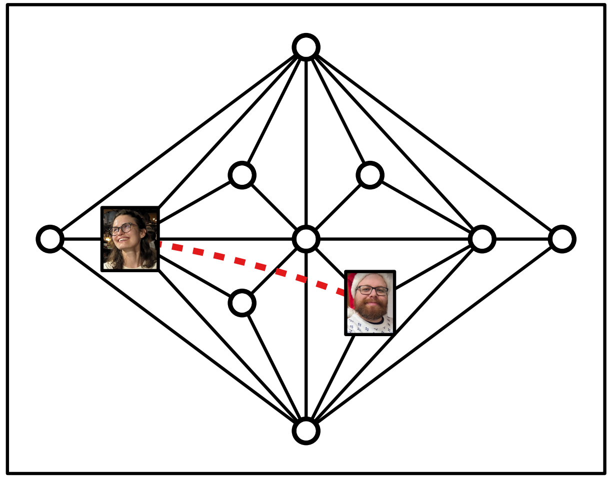


planar graph G

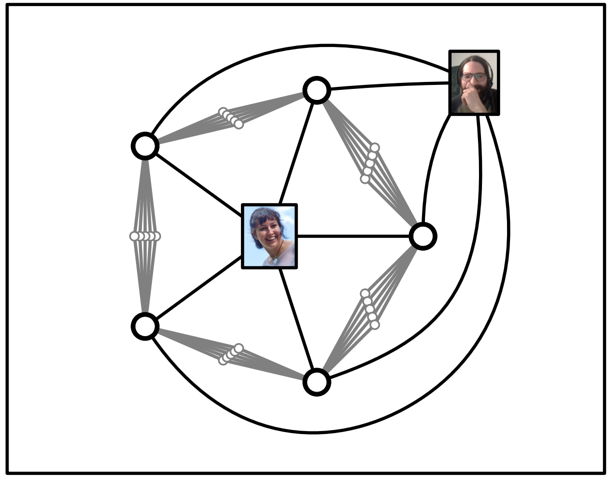


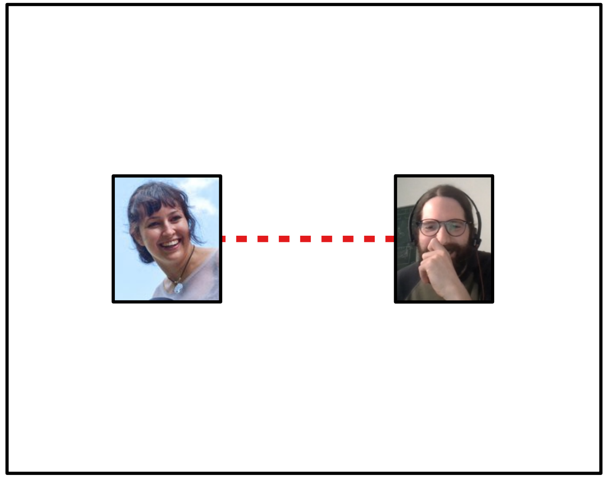
an edge e btw.
2 vertices of G

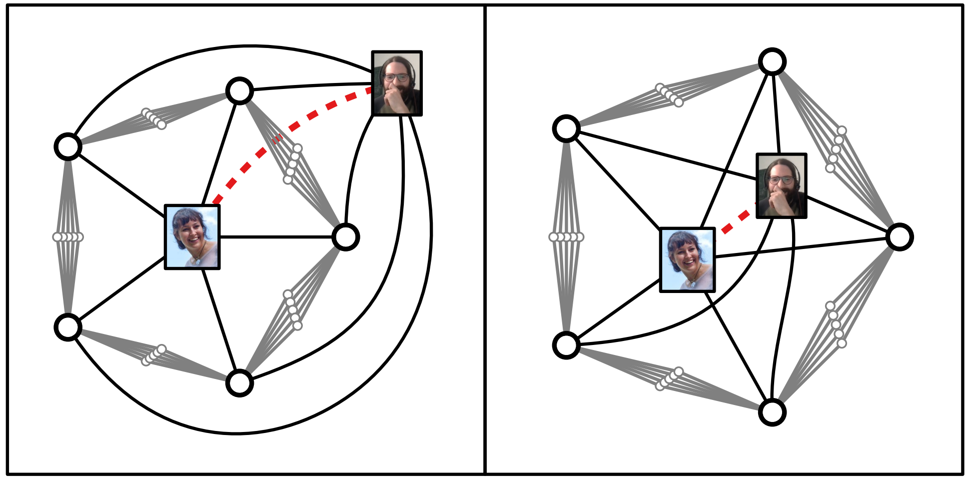
2 crossings



crossing-min. drawing of $G + e$



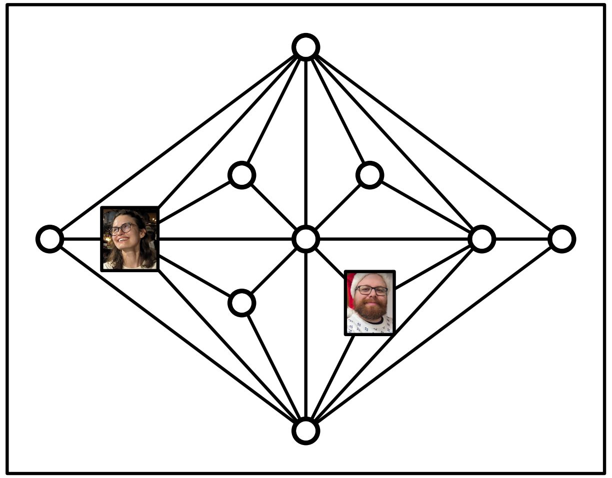




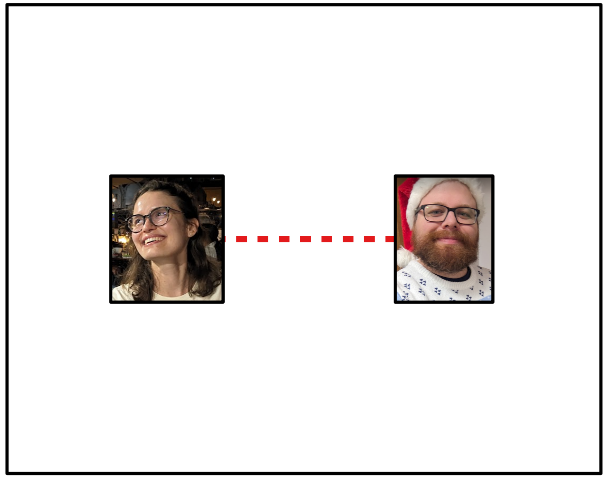
k crossings

4 crossings

Inserting an Edge Into a Planar Graph

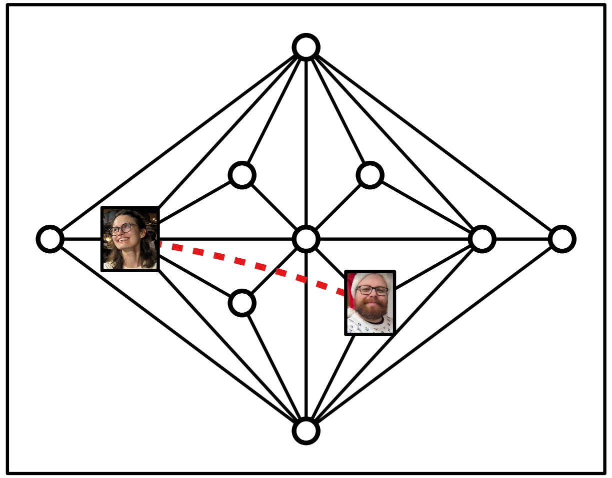


planar graph G

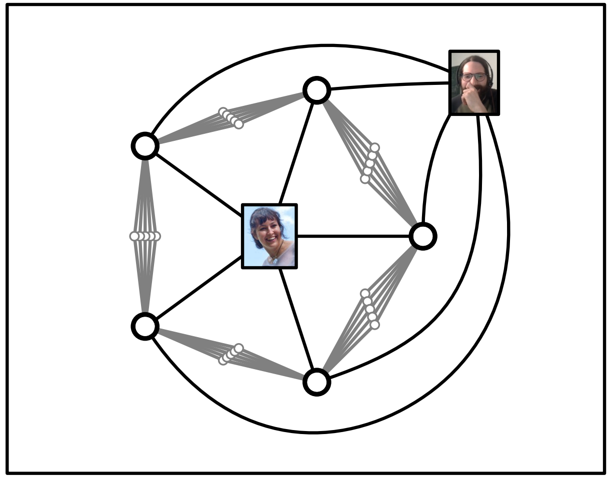


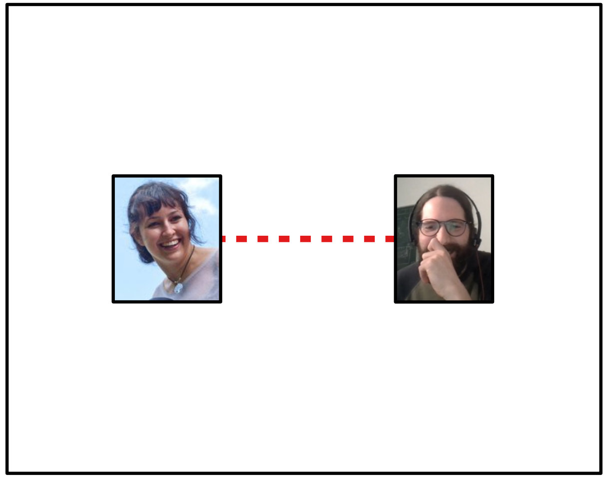
an edge e btw.
2 vertices of G

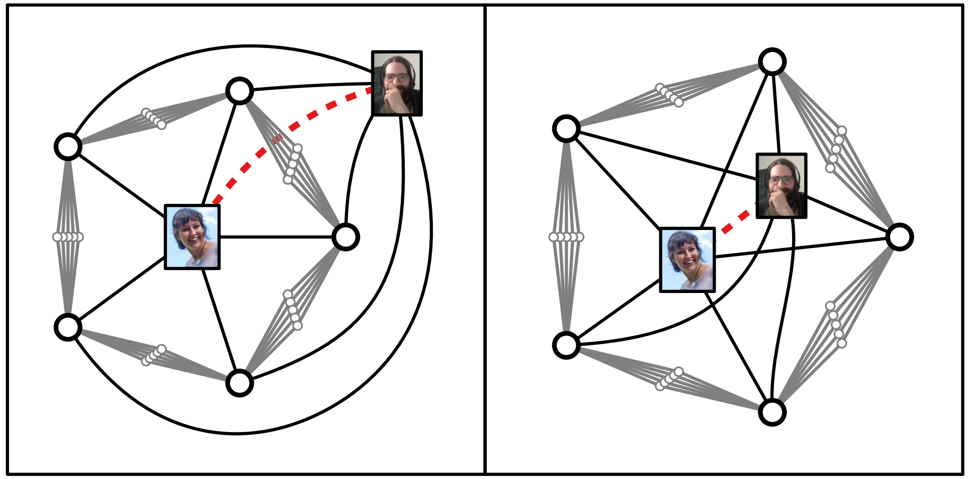
2 crossings



crossing-min. drawing of $G + e$



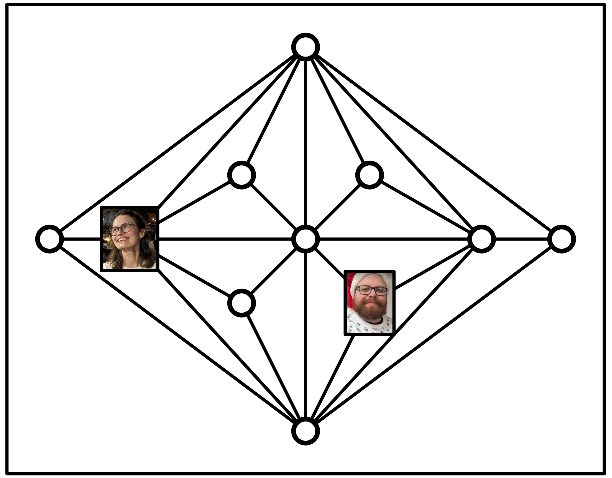




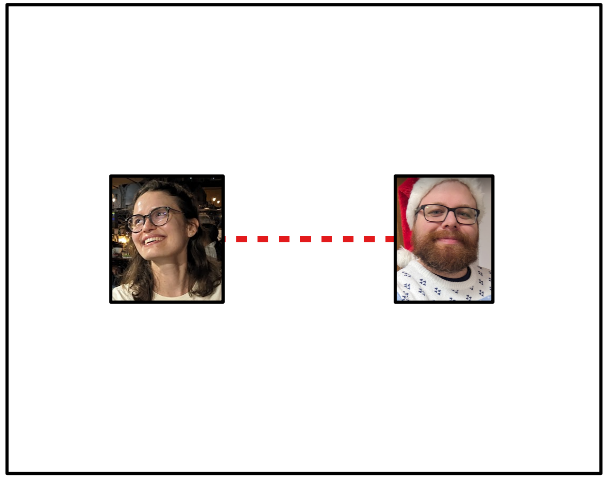
k crossings 4 crossings

This problem is NP-hard. [Cabello & Mohar '08]

Inserting an Edge Into a Planar Graph II

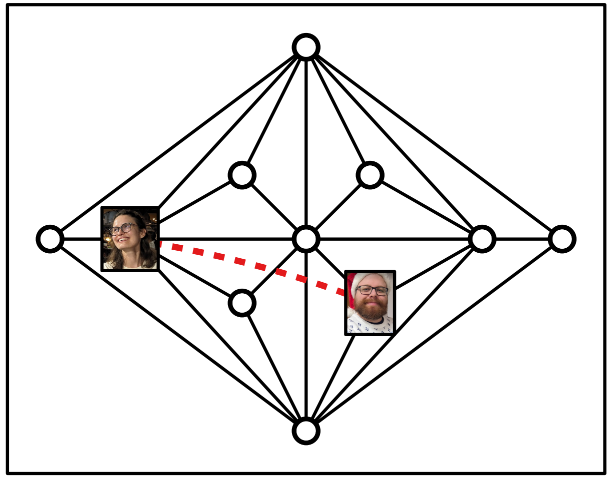


planar graph G

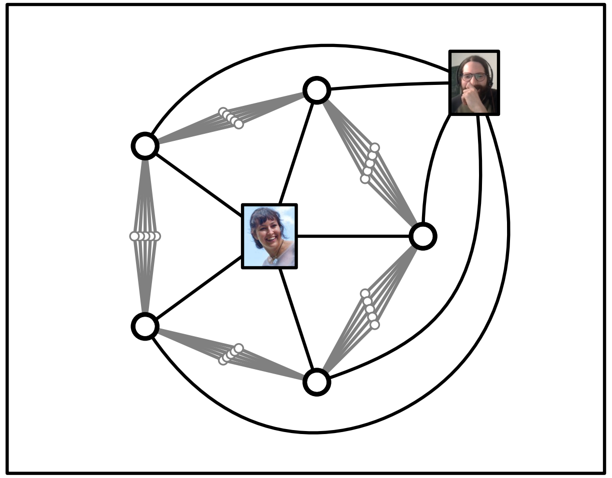


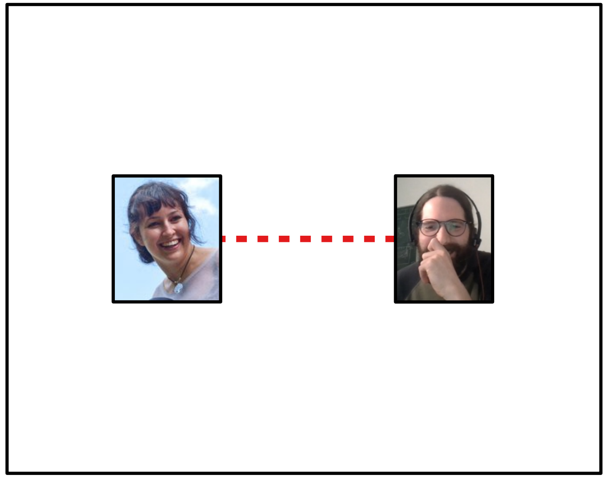
an edge e btw.
2 vertices of G

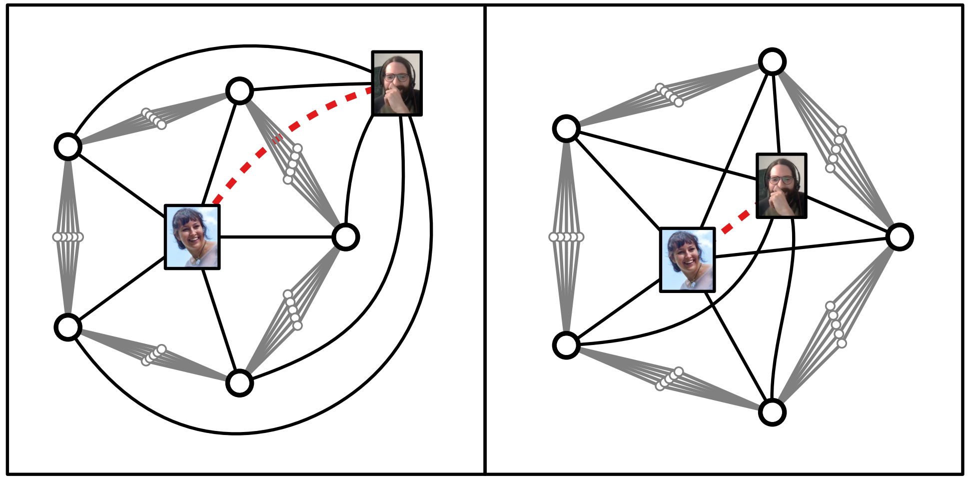
2 crossings



crossing-min. drawing of $G + e$
s.t. G is drawn planar





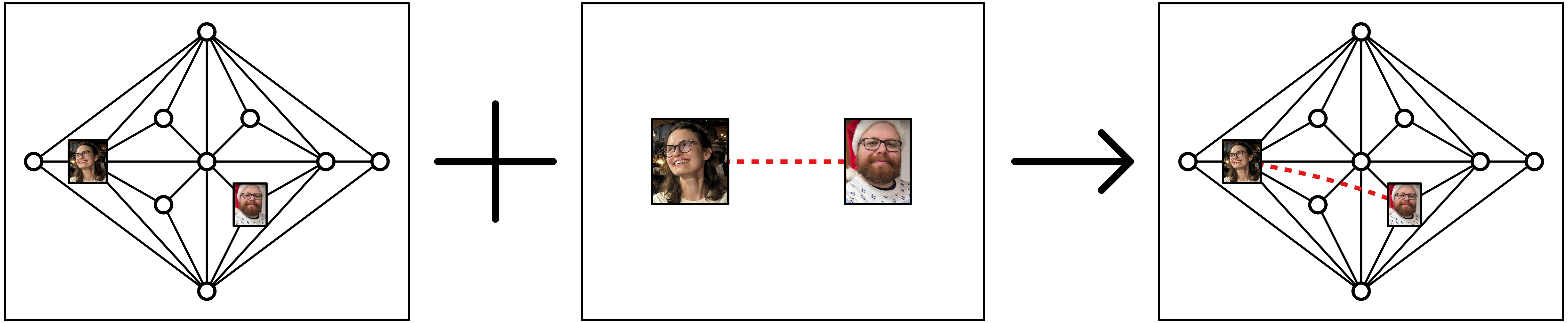


k crossings

4 crossings

Inserting an Edge Into a Planar Graph II

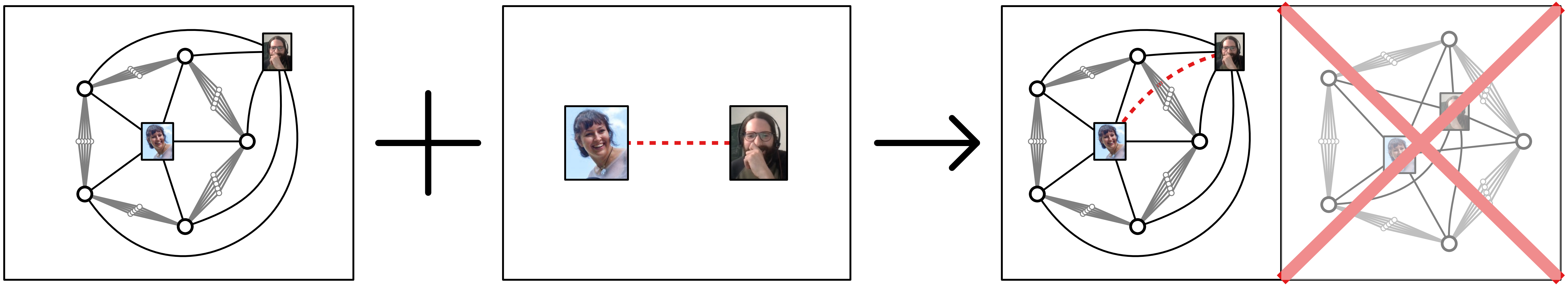
2 crossings



planar graph G

an edge e btw.
2 vertices of G

crossing-min. drawing of $G + e$
s.t. G is drawn planar



k crossings

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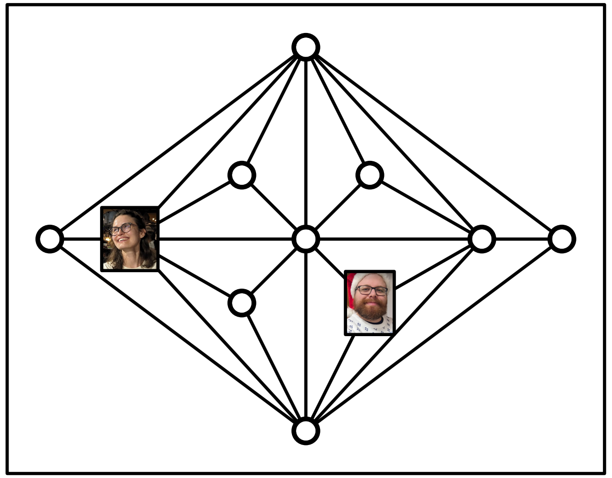
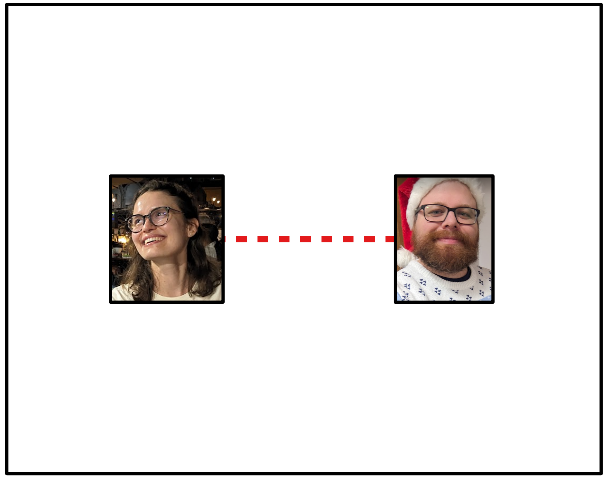
Inserting an Edge Into a Planar Graph II



planar graph G



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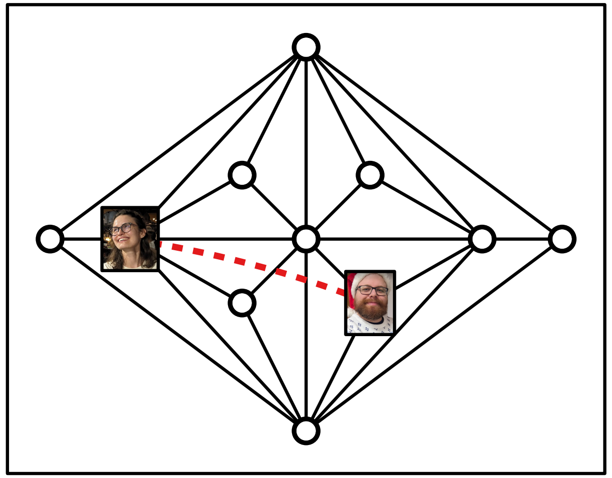

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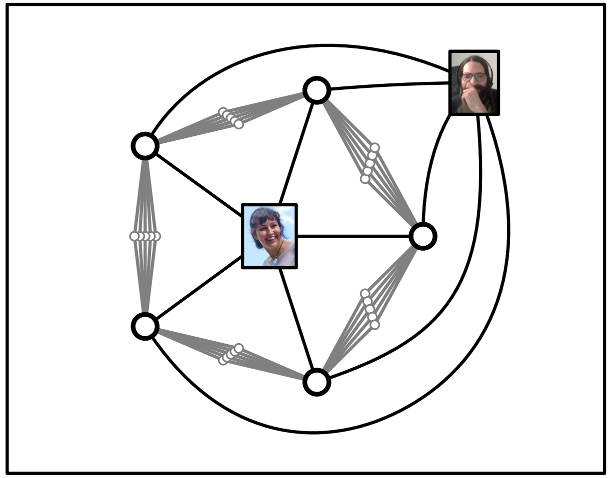
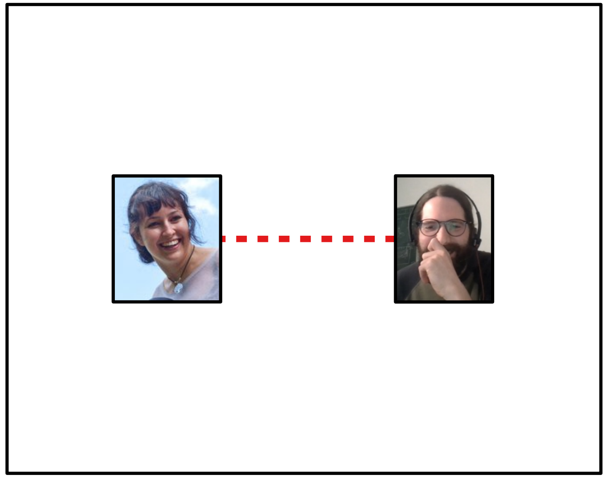
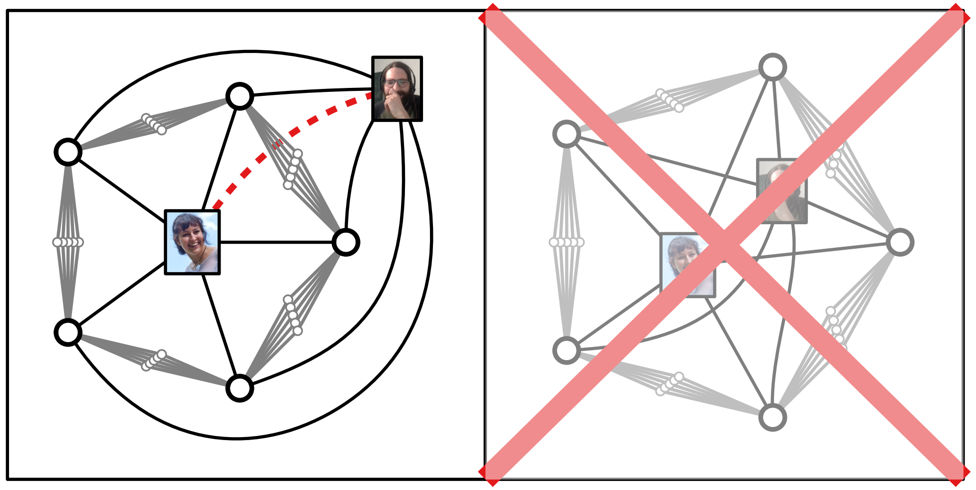




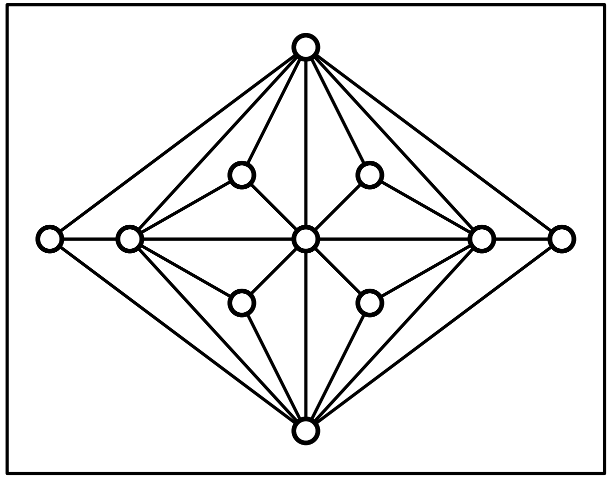


k crossings

4 crossings

This problem can be solved in $\mathcal{O}(n)$ time. [Gutwenger, Mutzel & Weiskircher '05]

Inserting a **Vertex** Into a Planar Graph



planar graph G

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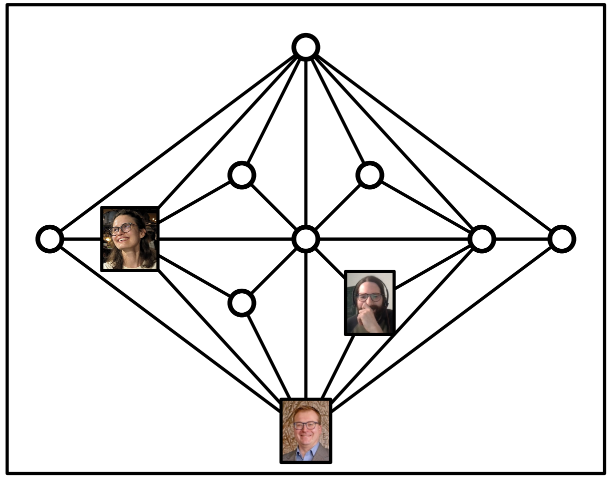

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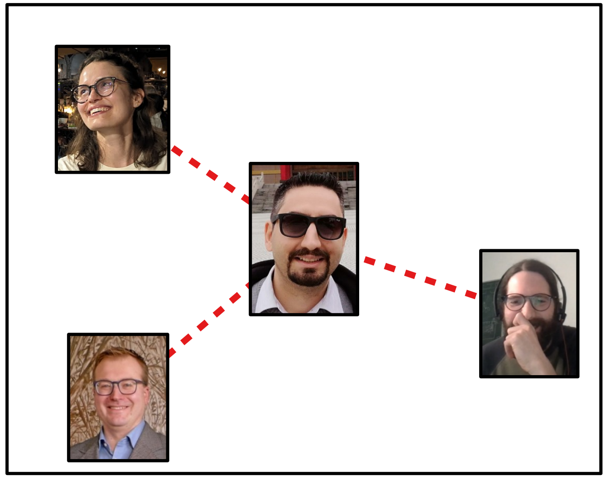

a star S with
its leaves in G

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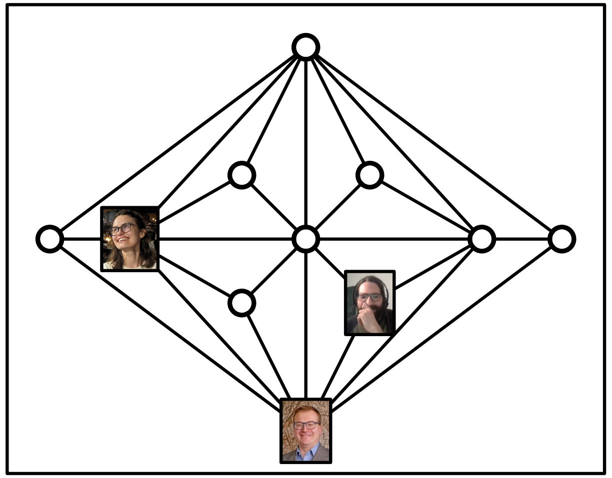

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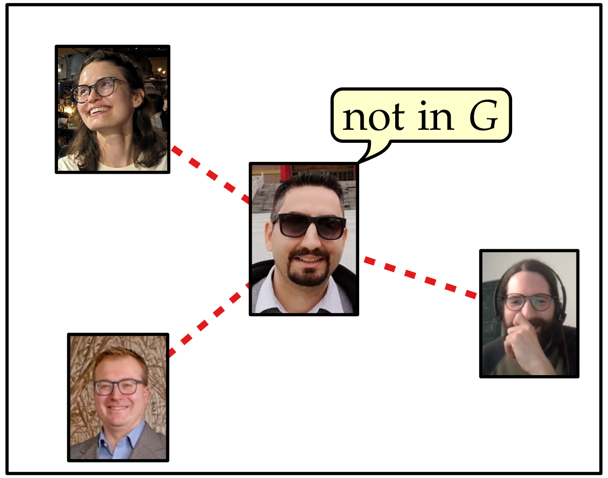

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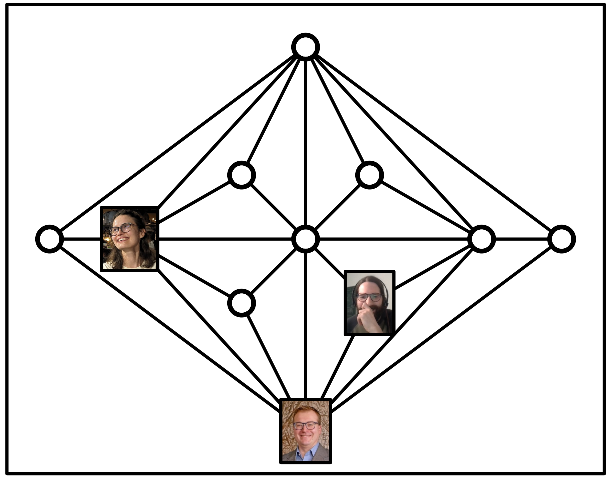

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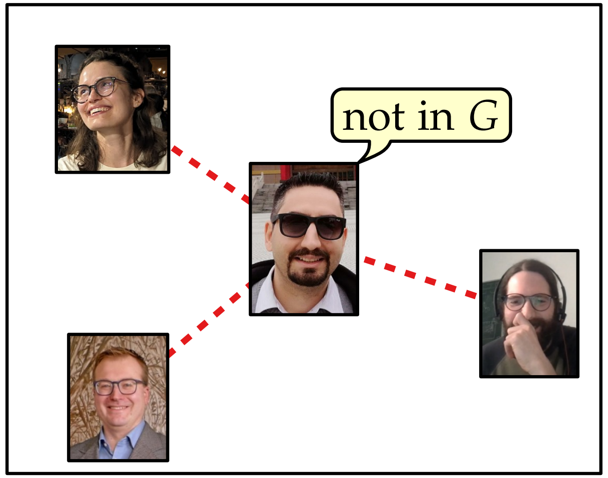

a star S with
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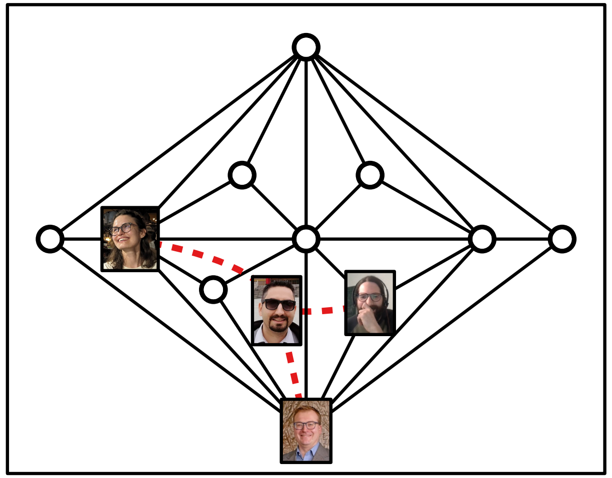
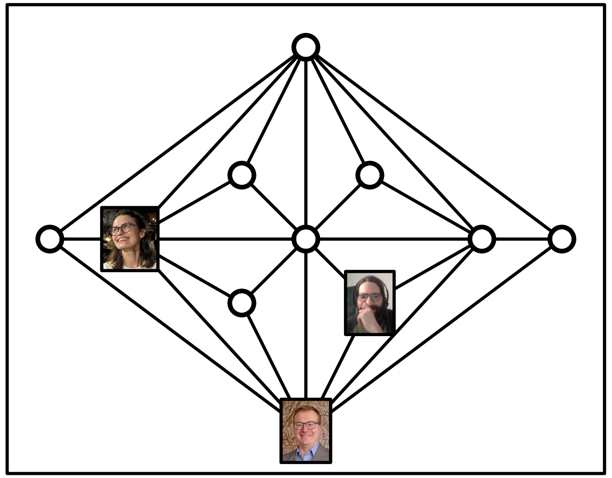
2 crossings



crossing-min. drawing of $G + S$
s.t. G is drawn planar

Inserting a **Vertex** Into a Planar Graph



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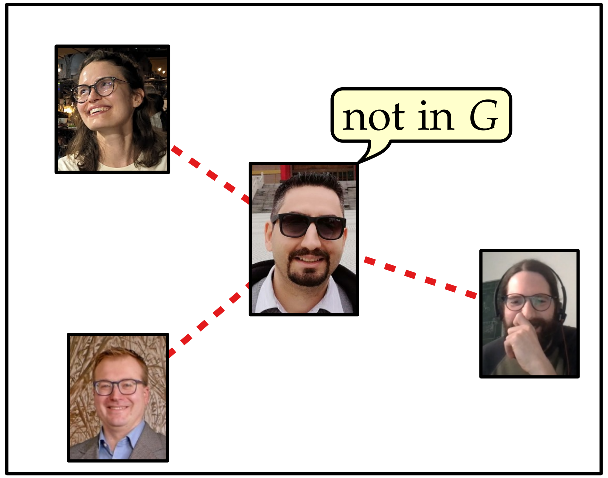

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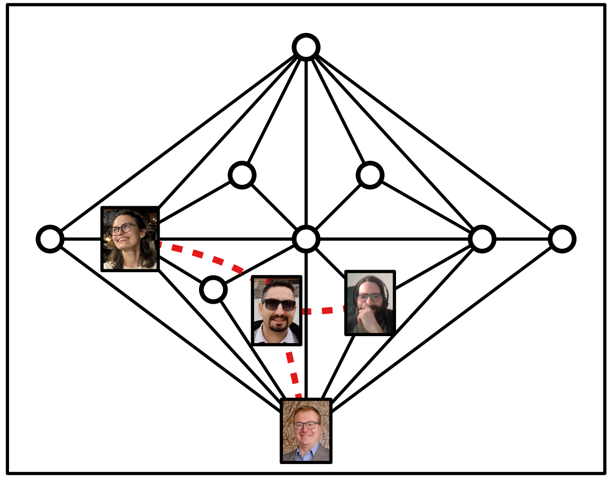
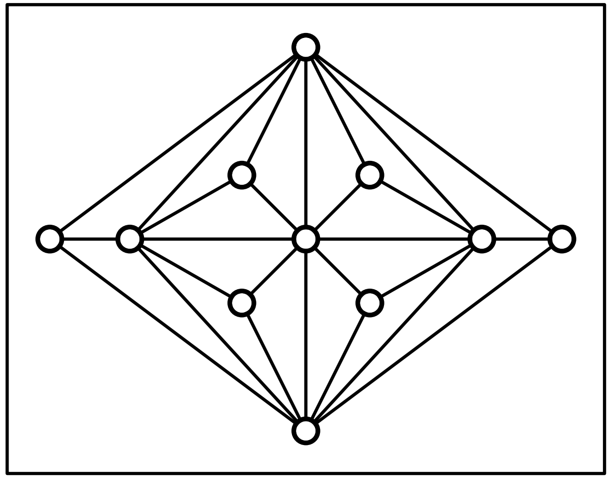


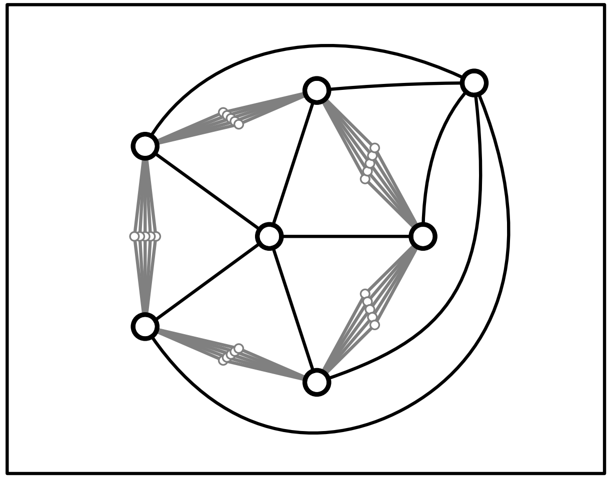
crossing-min. drawing of $G + S$
s.t. G is drawn planar

This problem can be solved in $\mathcal{O}(n^7)$ time. [Chimani, Gutwenger, Mutzel & Wolf '05]

Inserting an Edge Into a **Plane** Graph



plane graph G
(planar graph
+ planar embedding)



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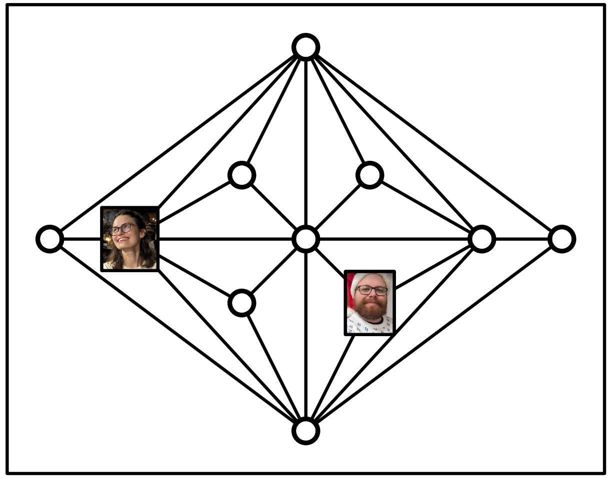

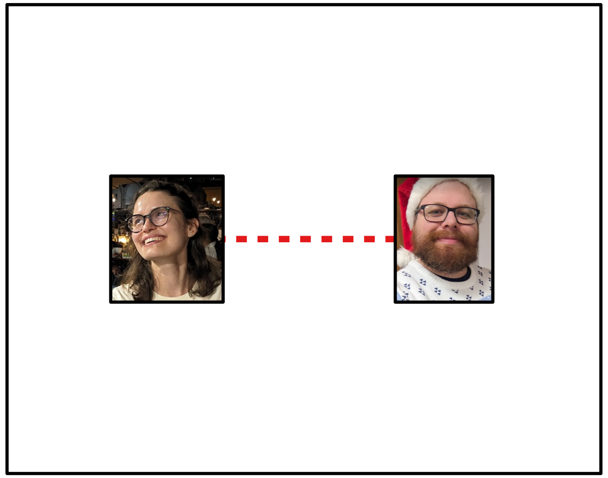

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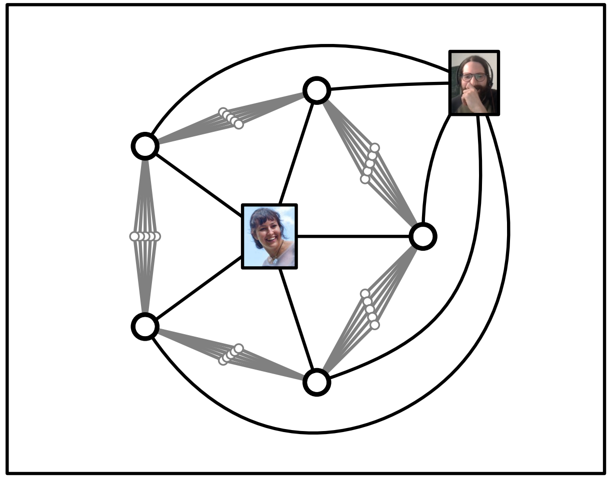
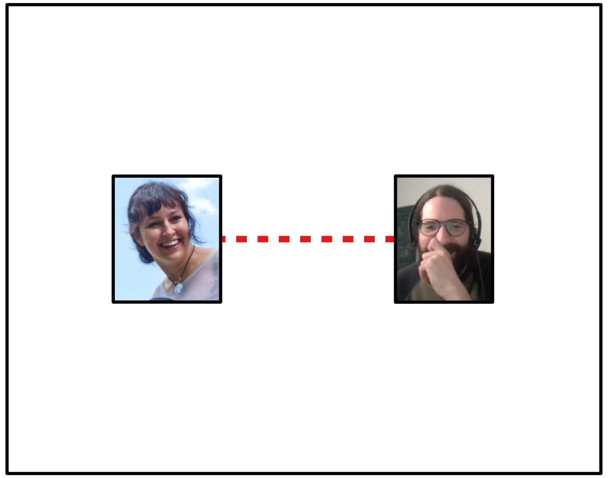
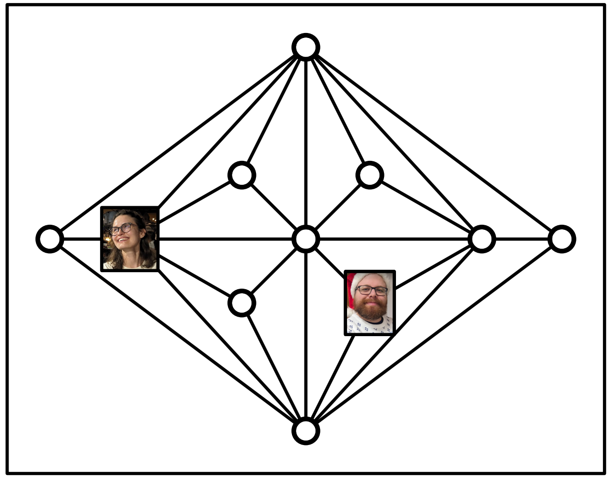
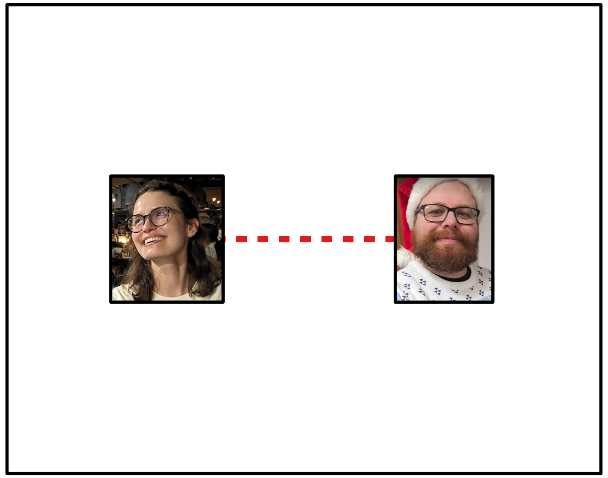


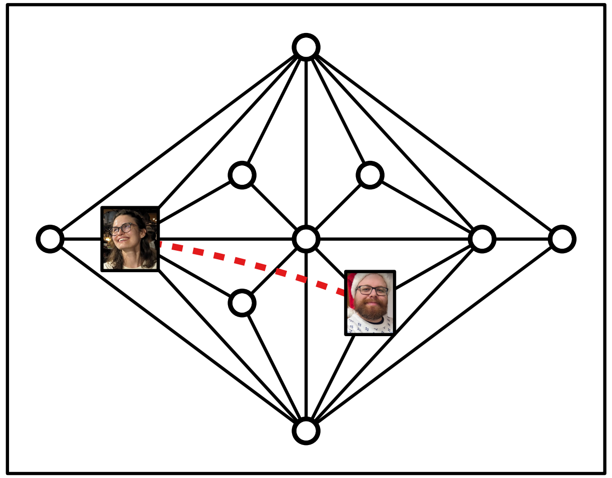


Inserting an Edge Into a **Plane** Graph

2 crossings







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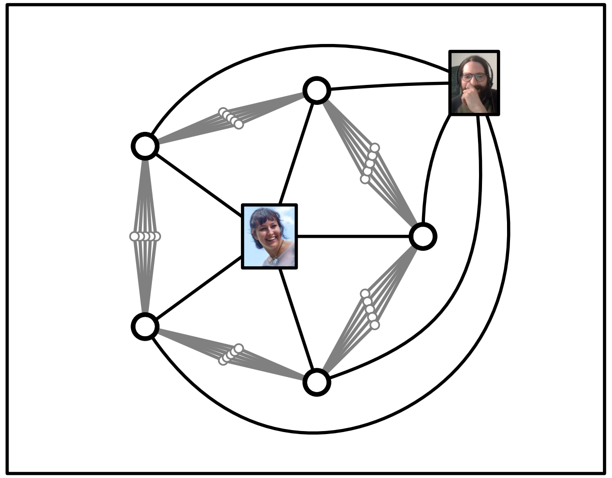
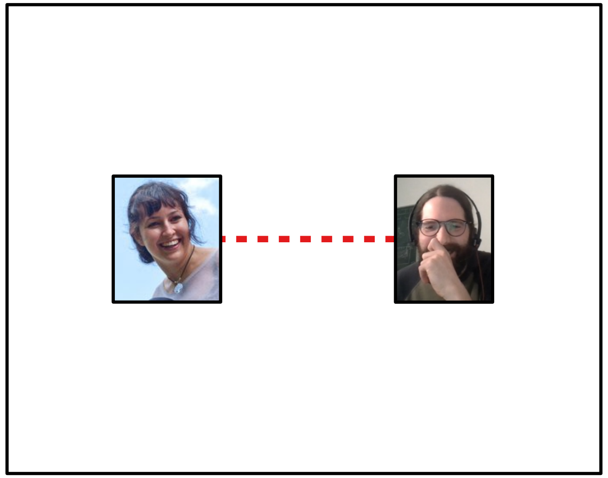
+ planar embedding)

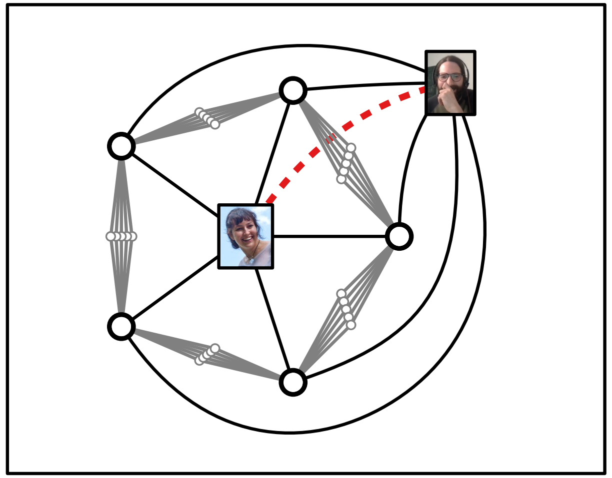
an edge e btw.
2 vertices of G

crossing-min. drawing of $G + e$
that keeps the embedding of G

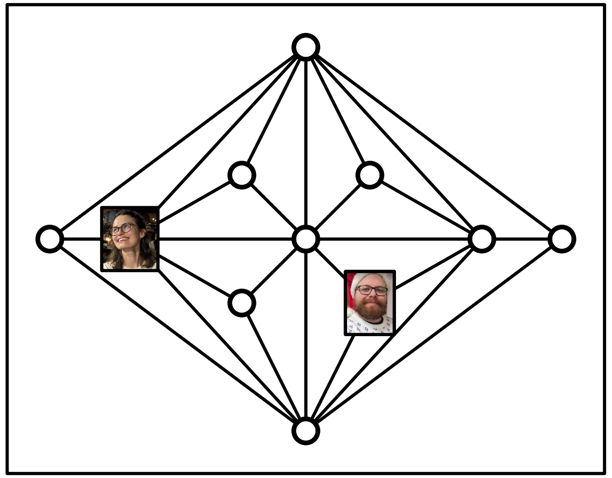
k crossings



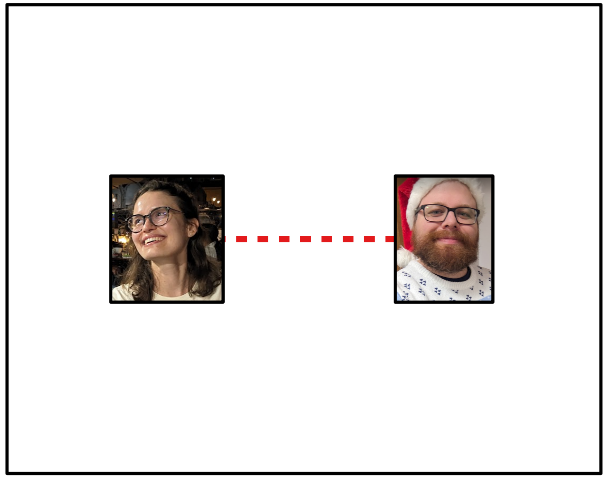




Inserting an Edge Into a **Plane** Graph

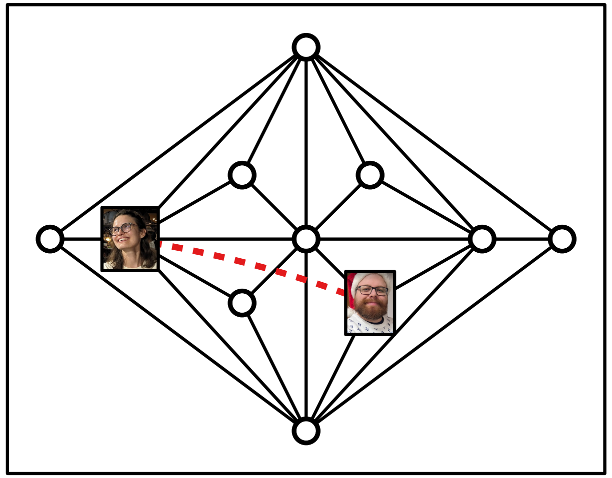


plane graph G
(planar graph
+ planar embedding)

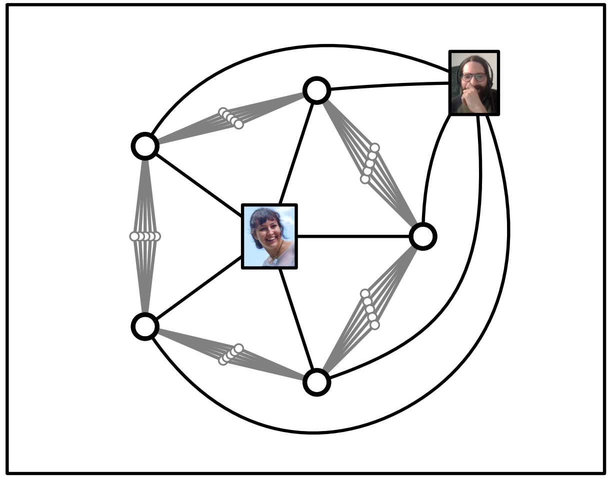


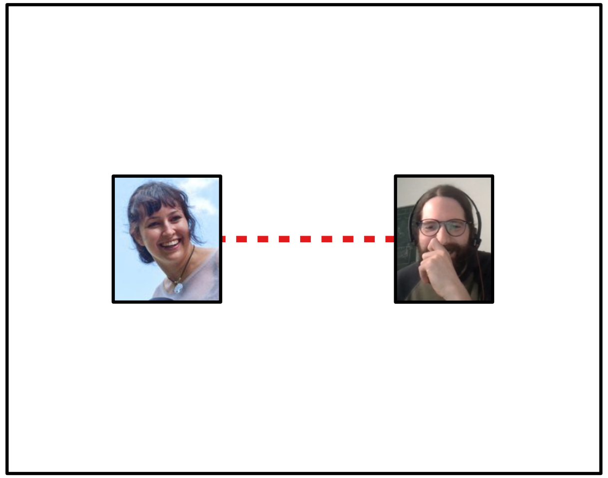
an edge e btw.
2 vertices of G

2 crossings

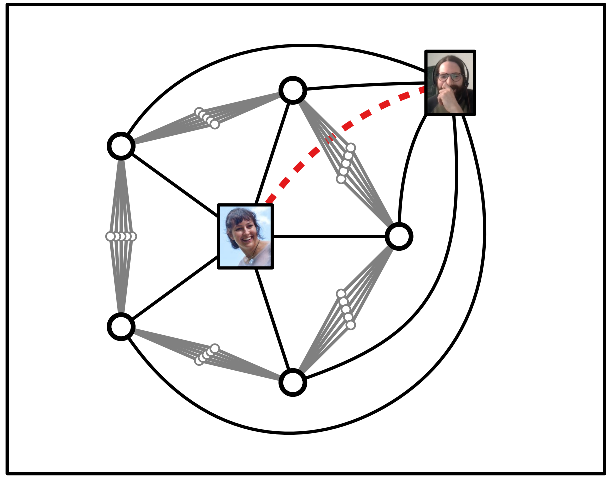


crossing-min. drawing of $G + e$
that keeps the embedding of G



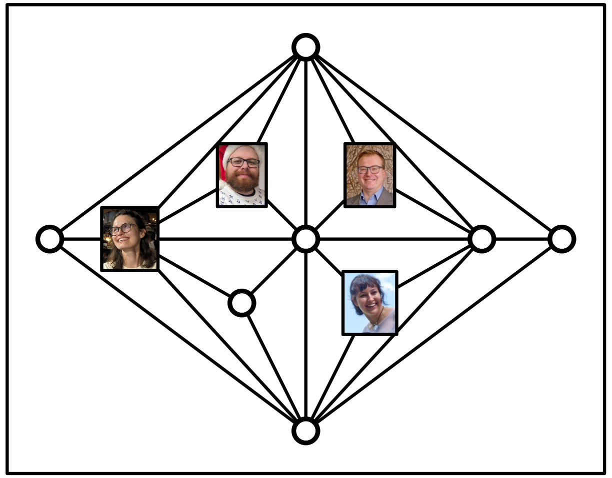


k crossings



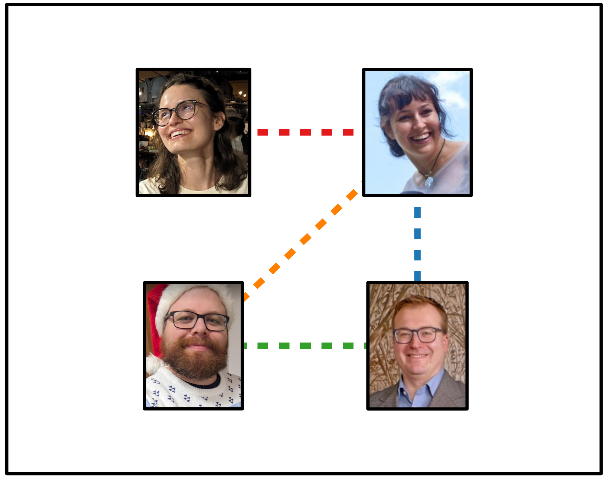
This problem can be solved in $\mathcal{O}(n)$ time. [BFS]

Inserting **Edges** Into a Plane Graph



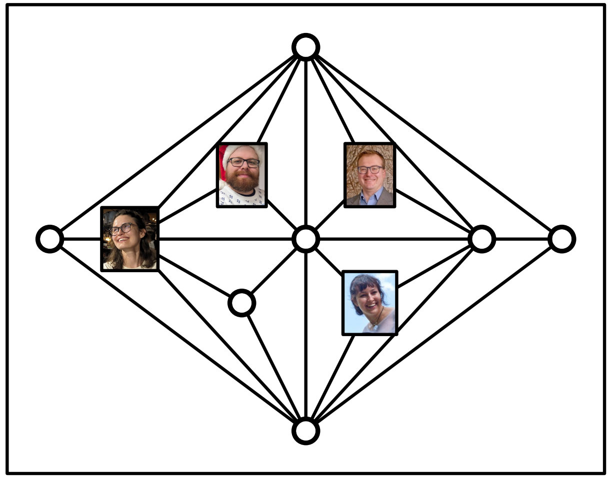
plane graph G

+

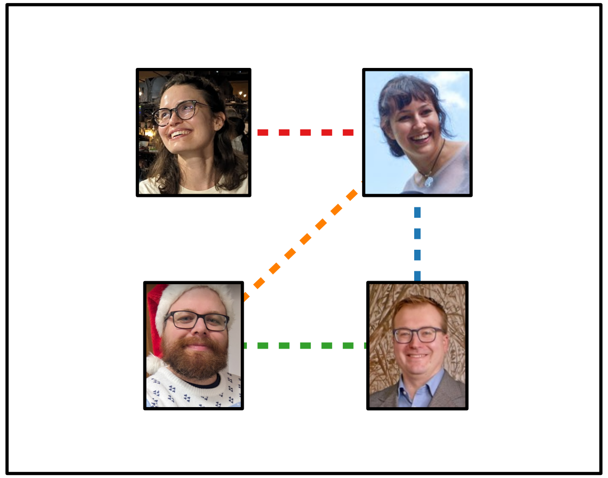


edges E' btw. vtcs in G

Inserting Edges Into a Plane Graph

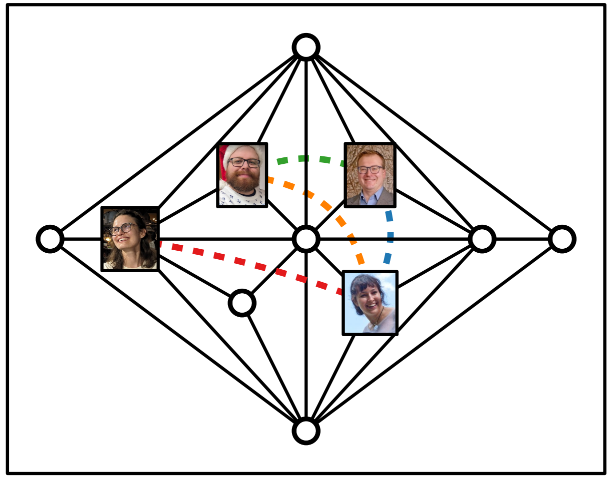


plane graph G



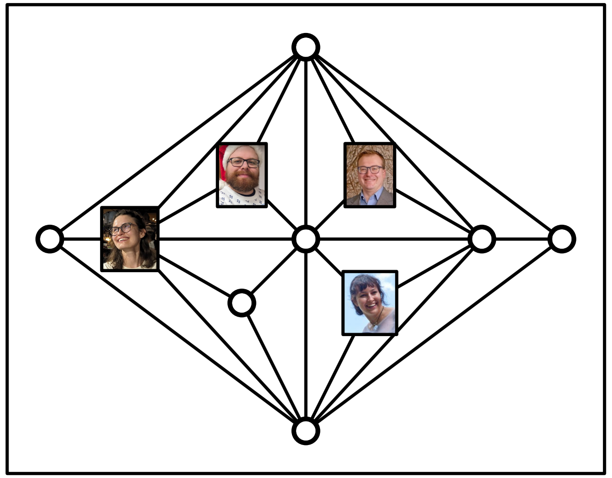
edges E' btw. vtcs in G

7 crossings

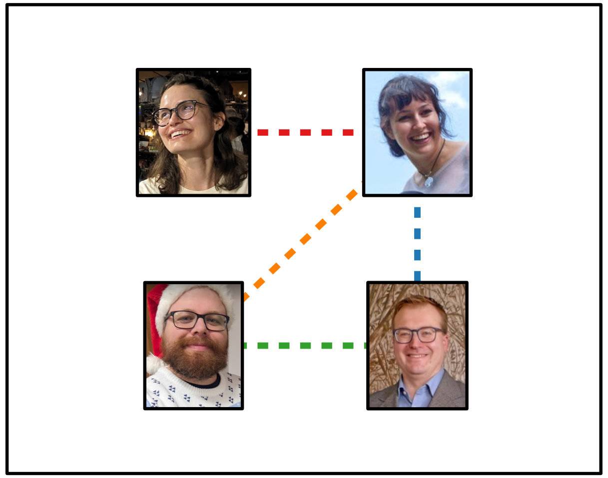


crossing-min. drawing of $G + E'$ that keeps the embedding of G

Inserting Edges Into a Plane Graph



plane graph G



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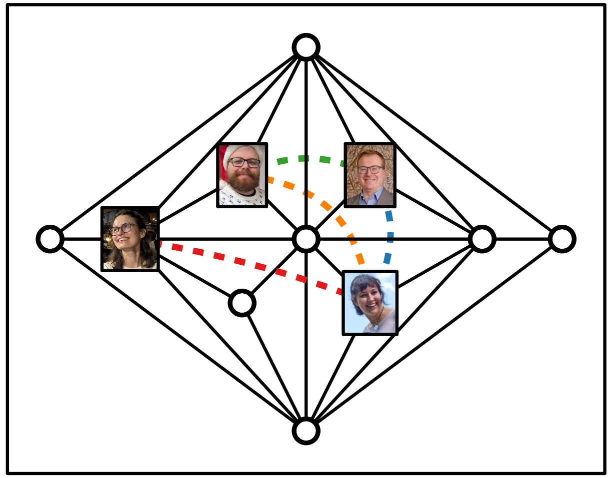
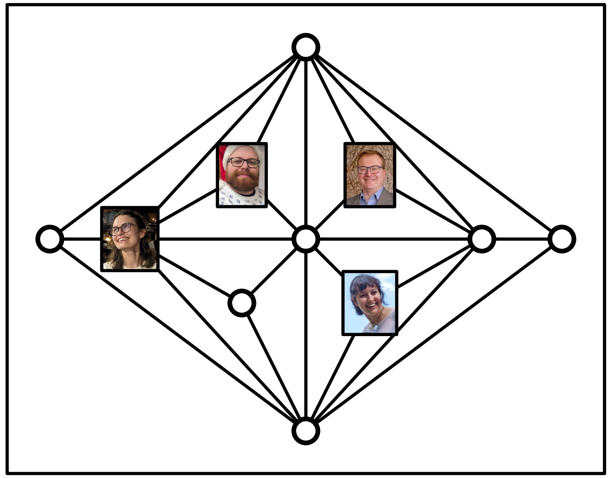


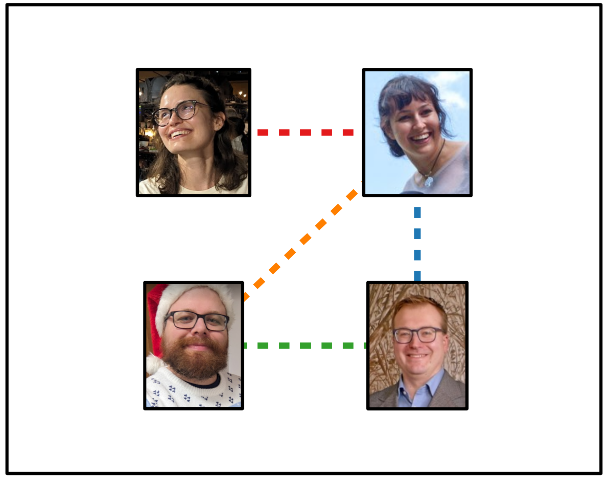
crossing-min. drawing of $G + E'$ that keeps the embedding of G

This problem is NP-hard. [Ziegler '01]

Inserting **Edges** Into a Plane Graph



plane graph G



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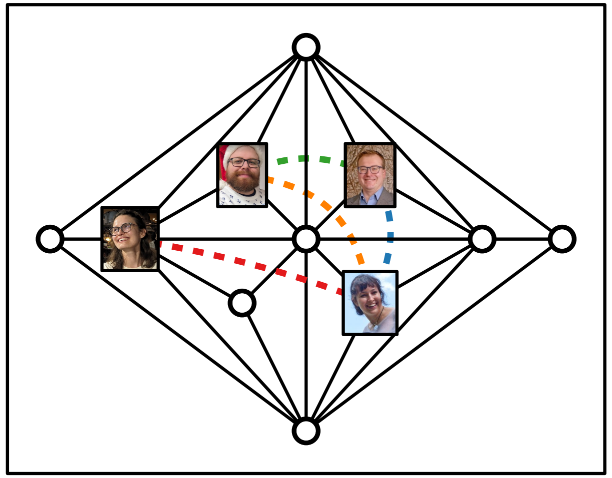

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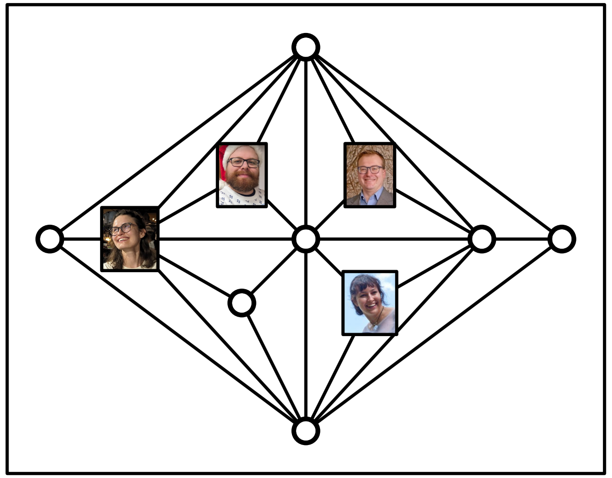
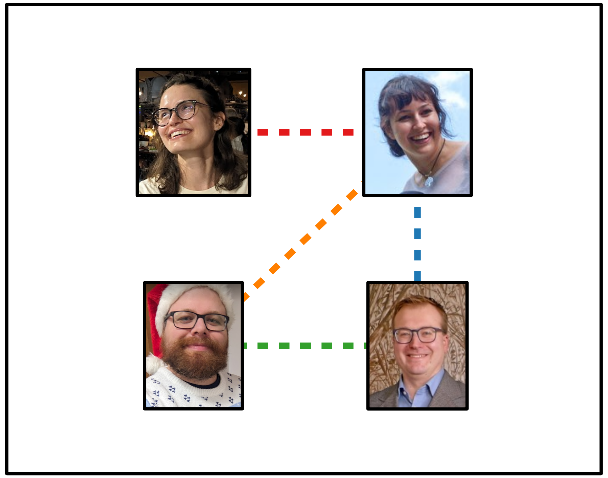
Inserting Edges Into a Plane Graph



plane graph G



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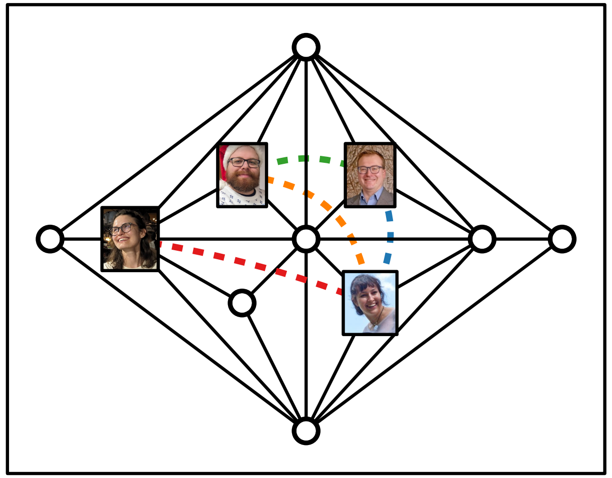

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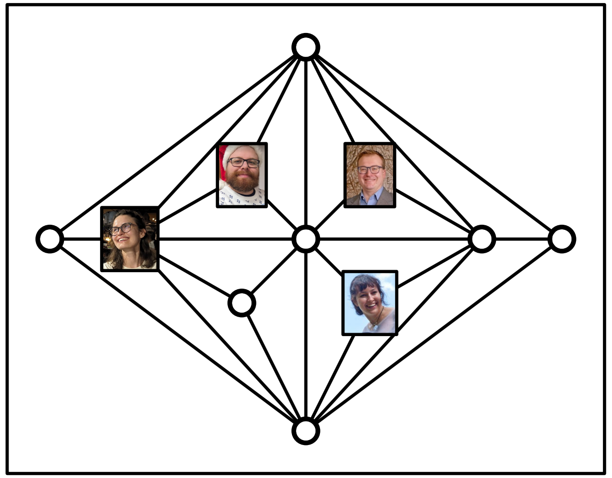
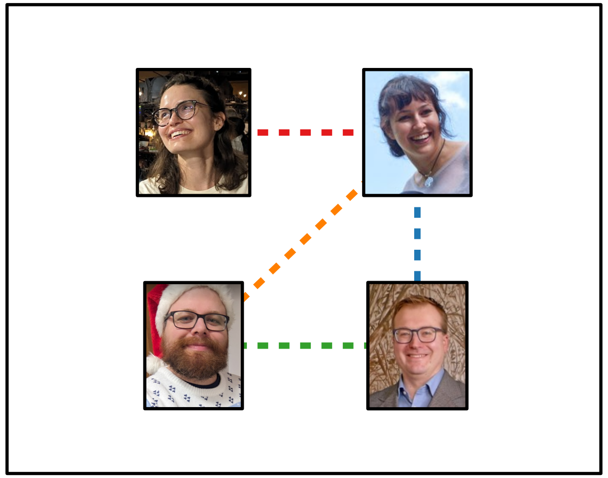
This problem is in FPT parameterized by #crossings. [Hamm & Hliněný '22]

Inserting **Edges** Into a Plane Graph



plane graph G



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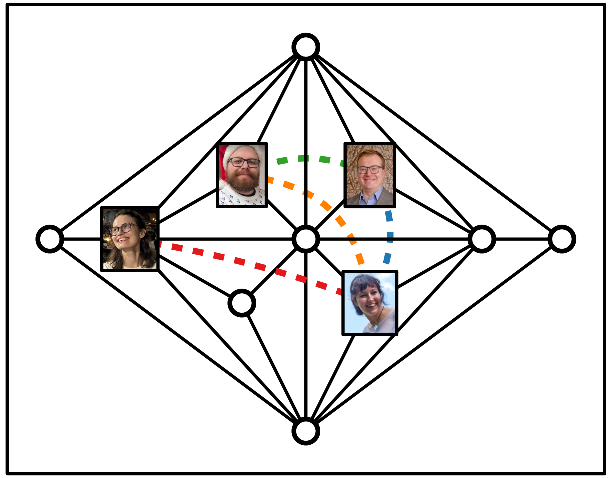

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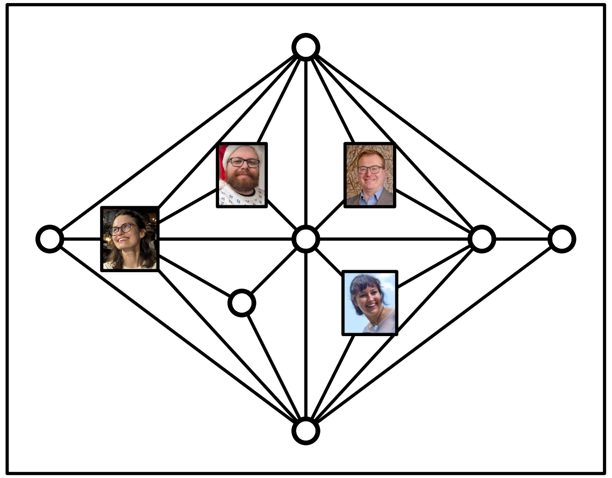
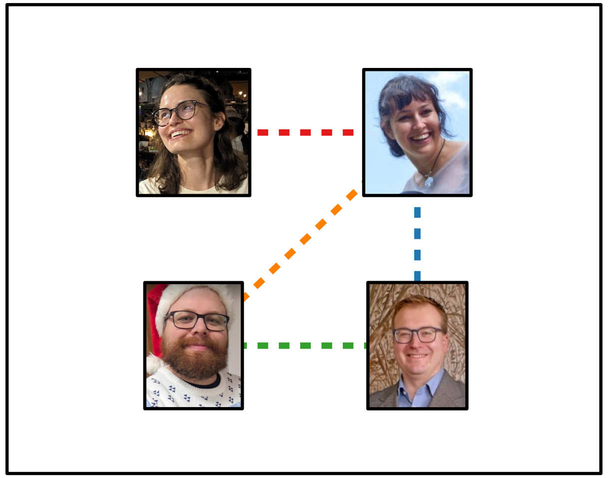
Inserting **Edges** Into a Plane Graph



plane graph G



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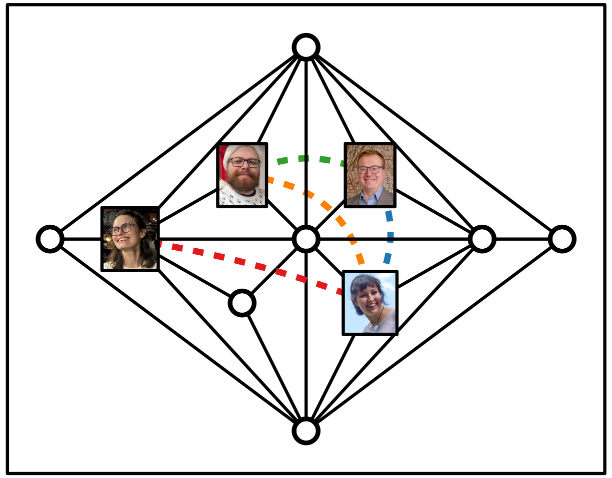

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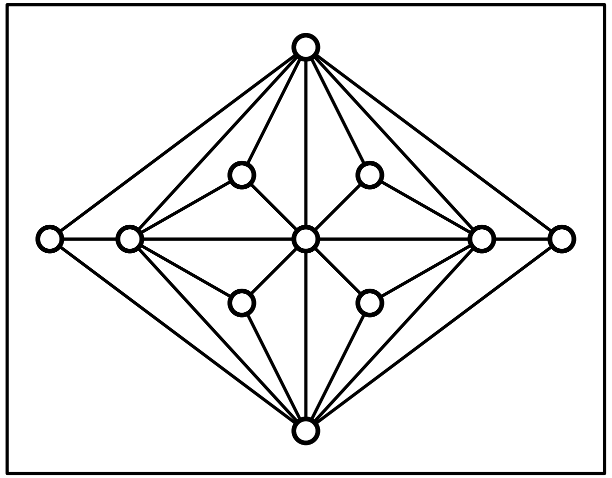
… even if G is non-planar (or drawn with crossings)

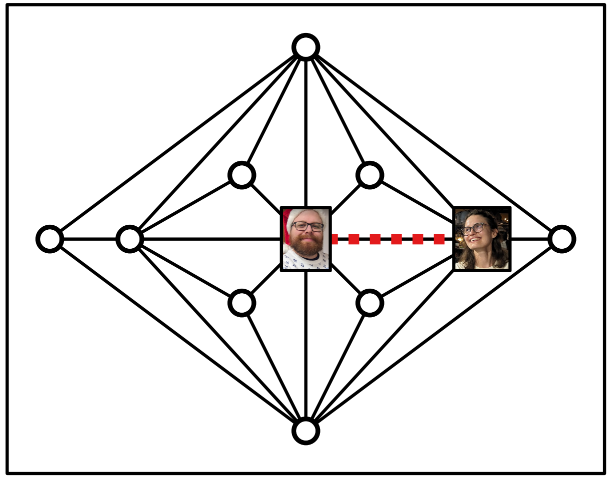
This problem is in FPT parameterized by $|E'|$ [Chimani & Hliněný '23]
if G is biconnected or all cutvertices have constant degree.

Partial Embedding – General Definition



graph G +
drawing style Φ
(e.g., straight-line planar)

Partial Embedding – General Definition



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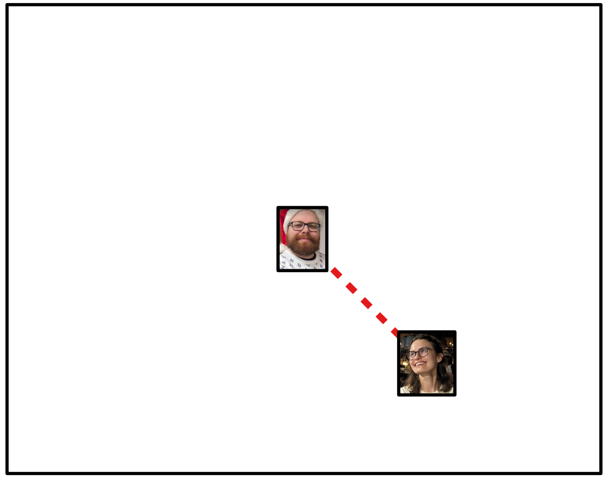
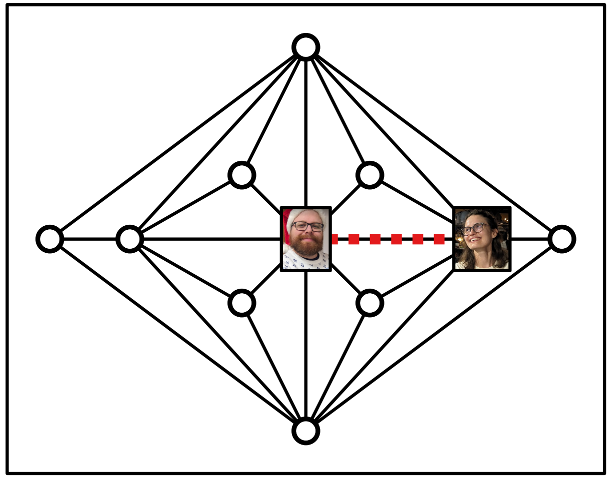


drawing with style Φ of
a subgraph $H \subseteq G$

Partial Embedding – General Definition



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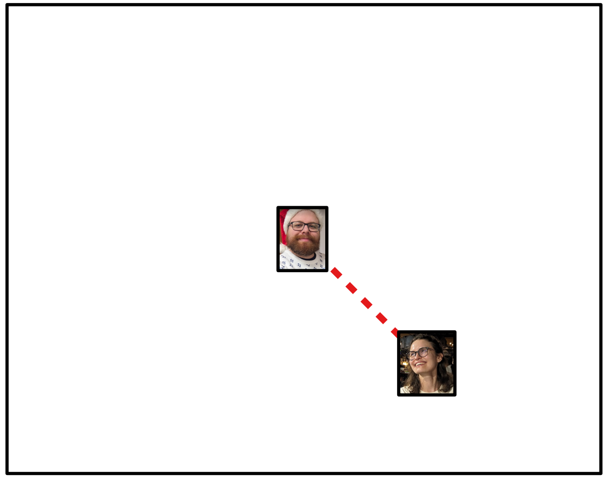

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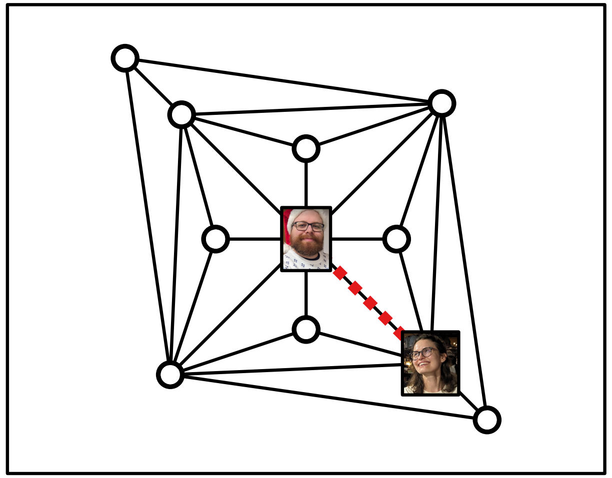
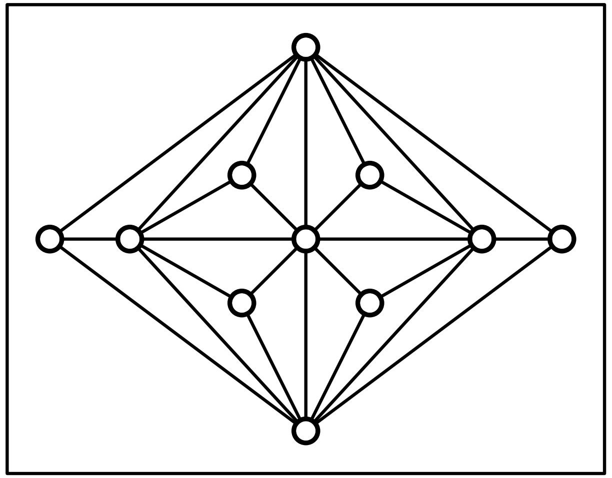


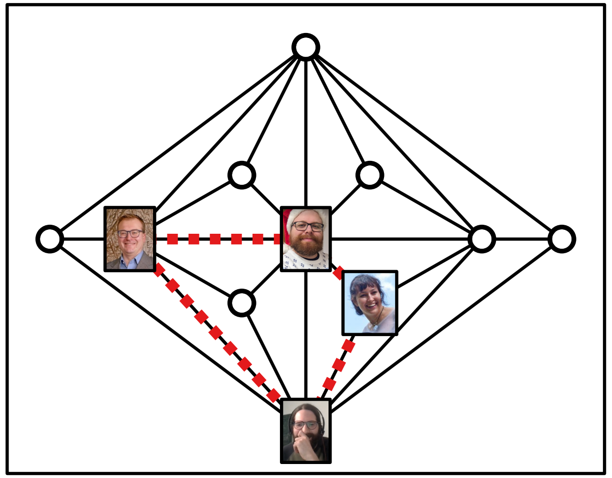
drawing with style Φ of G
s.t. H **keeps its drawing**

Partial Embedding of **Planar** Graphs



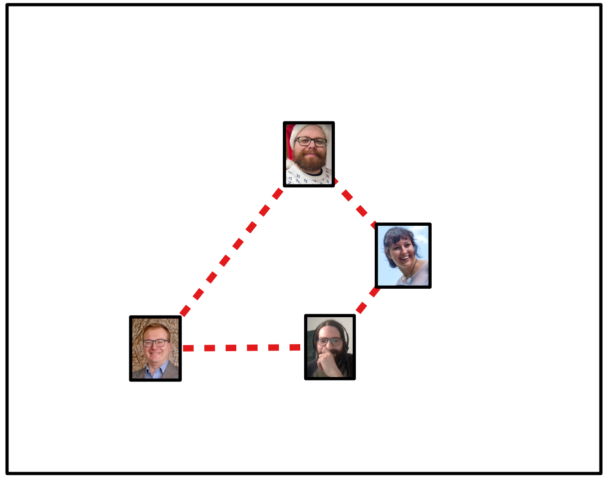
planar graph G

Partial Embedding of **Planar** Graphs



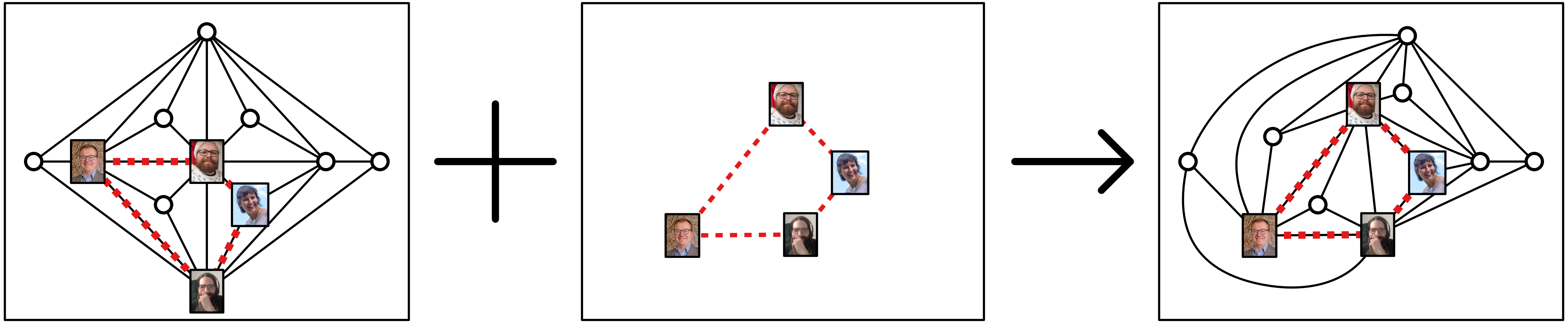
planar graph G

+



planar drawing of a subgraph $H \subseteq G$

Partial Embedding of **Planar** Graphs

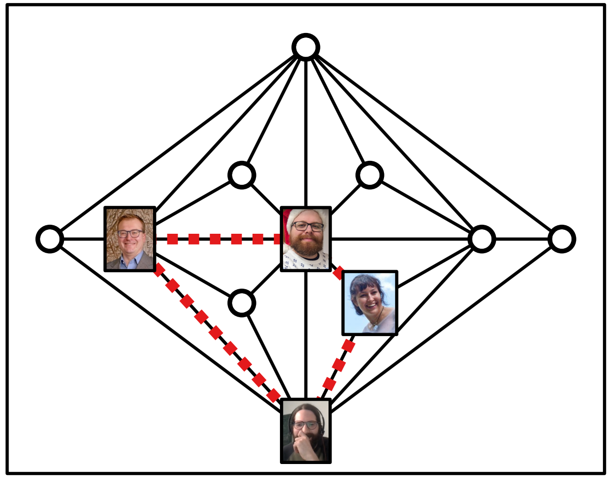


planar graph G

planar drawing of a subgraph $H \subseteq G$

planar drawing of G s.t. H keeps its drawing

Partial Embedding of **Planar** Graphs



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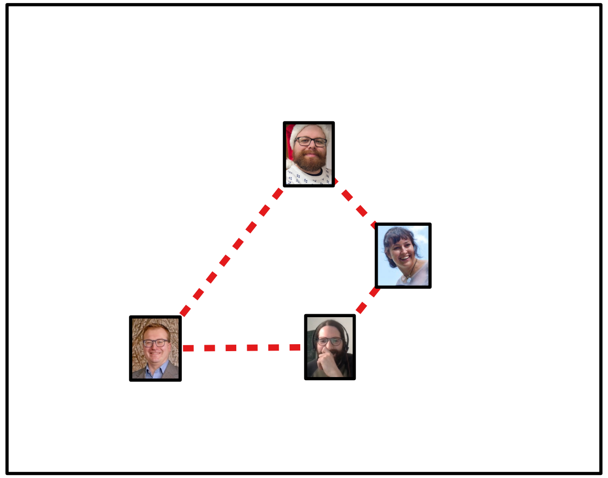

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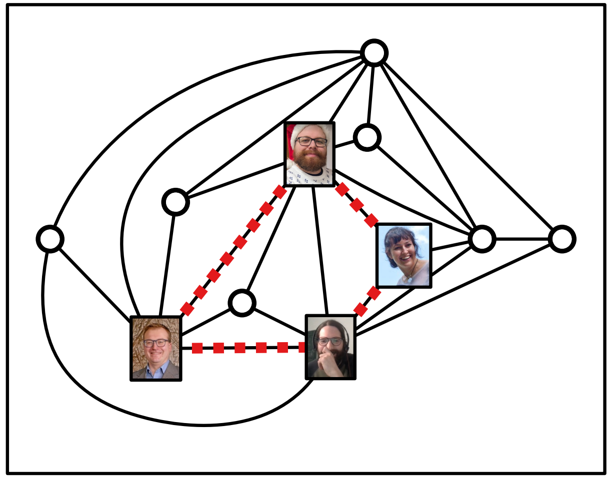
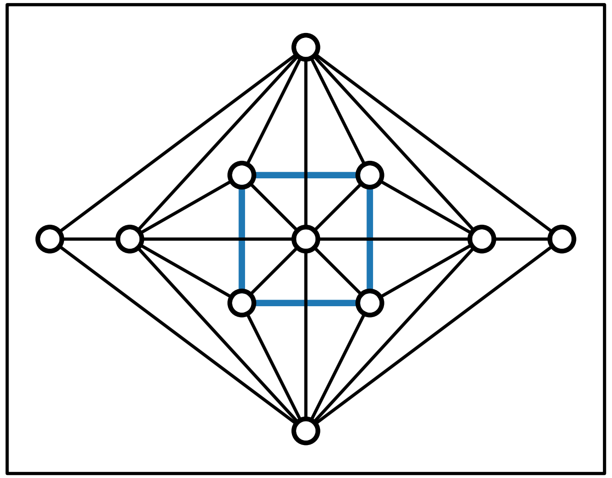


planar drawing of G s.t. H keeps its drawing

This problem can be solved in $\mathcal{O}(n)$ time.

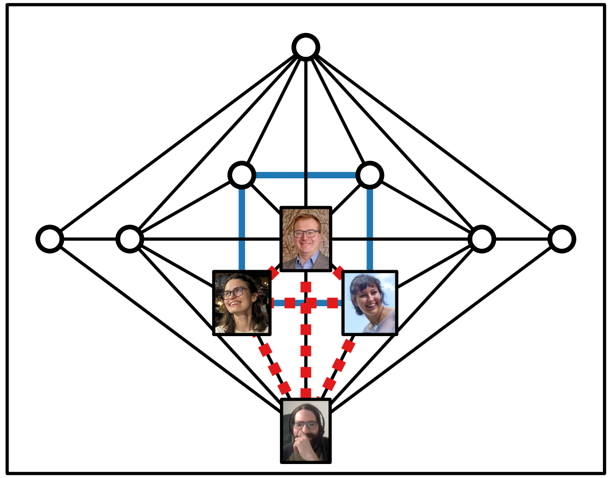
[Angelini, Di Battista, Frati, Jelínek, Kratochvíl, Patrignani, Rutter ’10]

Partial Embedding of 1-Planar Graphs



1-planar graph G
(can be drawn s.t.
every edge is crossed
at most once)

Partial Embedding of **1-Planar** Graphs



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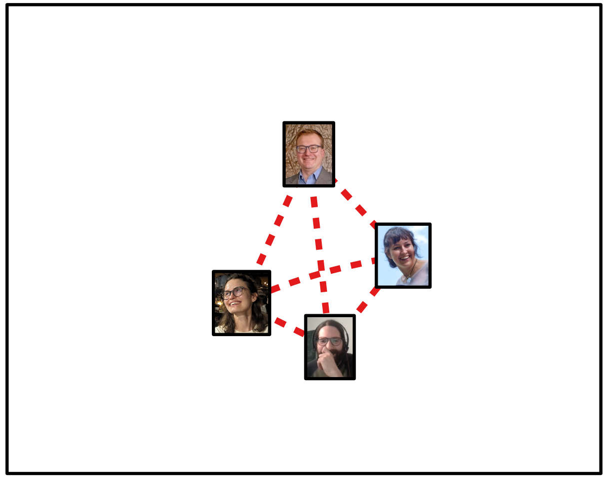
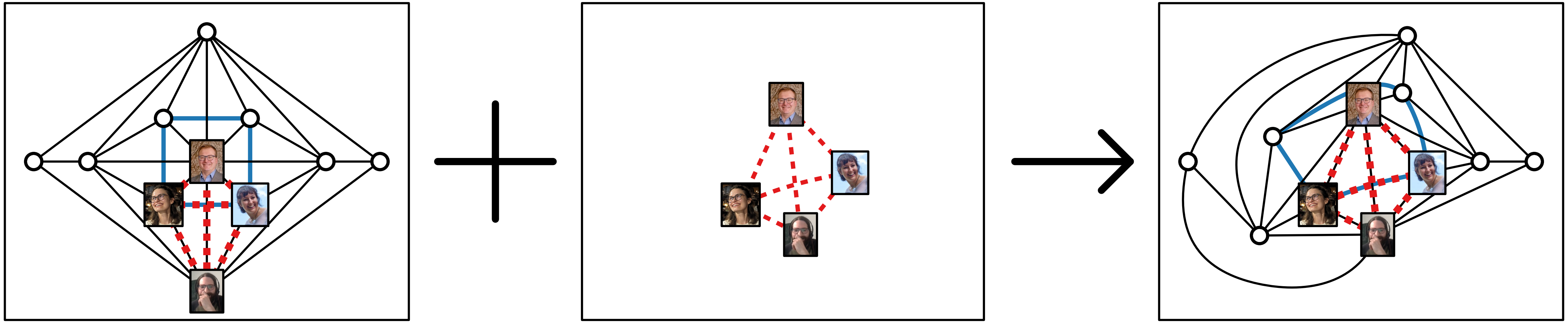
+



1-planar drawing of a subgraph $H \subseteq G$

Partial Embedding of **1-Planar** Graphs

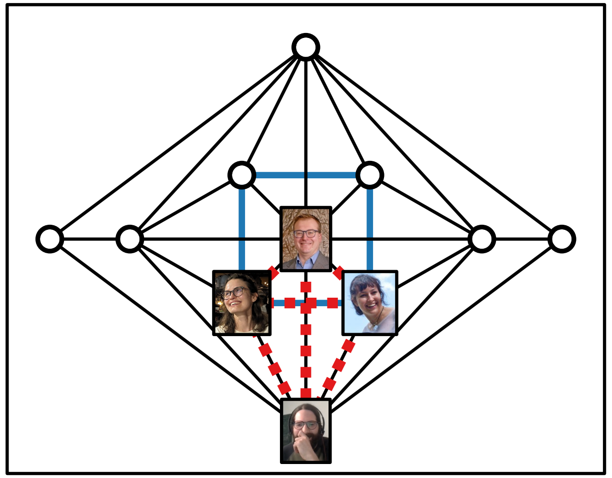


1-planar graph G (can be drawn s.t. every edge is crossed at most once)

1-planar drawing of a subgraph $H \subseteq G$

1-planar drawing of G s.t. H keeps its drawing

Partial Embedding of **1-Planar** Graphs



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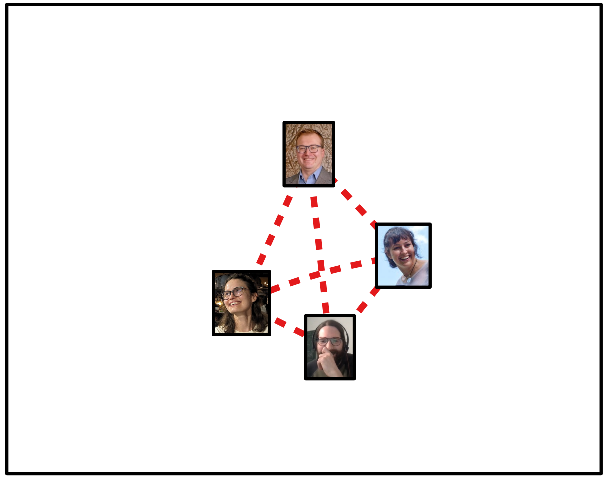
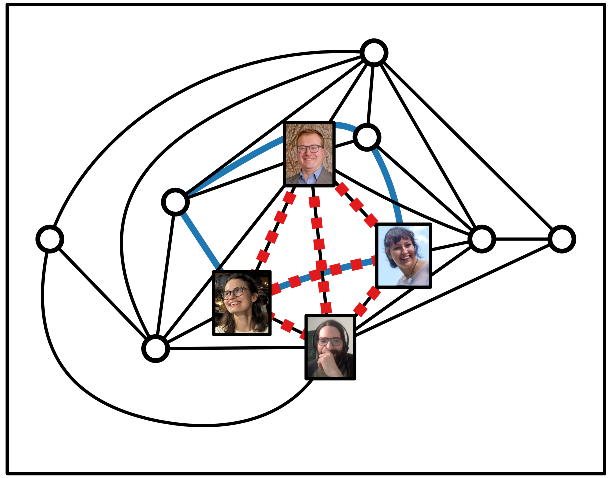


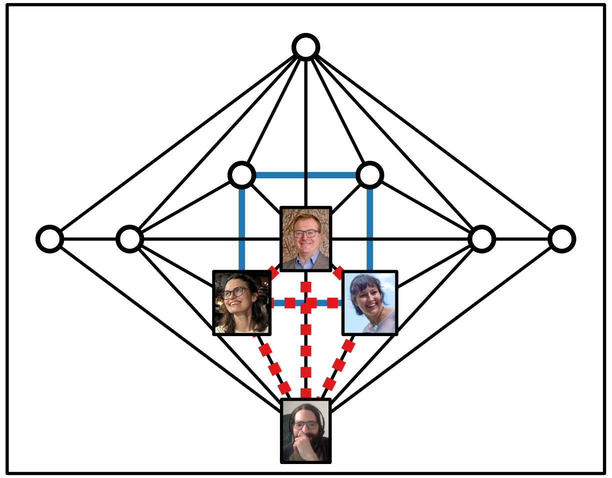
1-planar drawing of a subgraph $H \subseteq G$



1-planar drawing of G s.t. H keeps its drawing

This problem is NP-hard even if $H = \emptyset$. [Grigoriev & Bodlaender '07]

Partial Embedding of **1-Planar** Graphs



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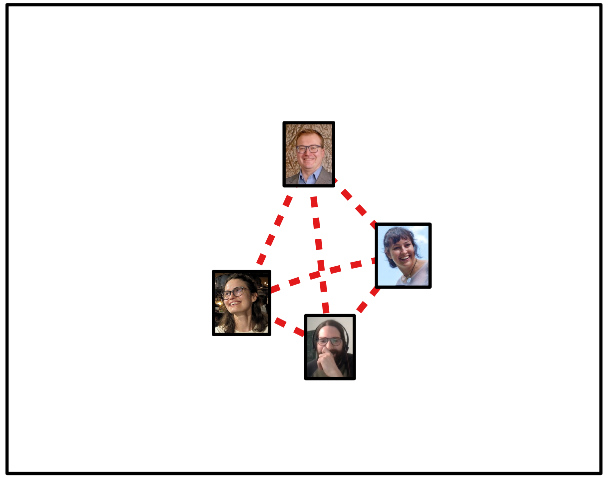

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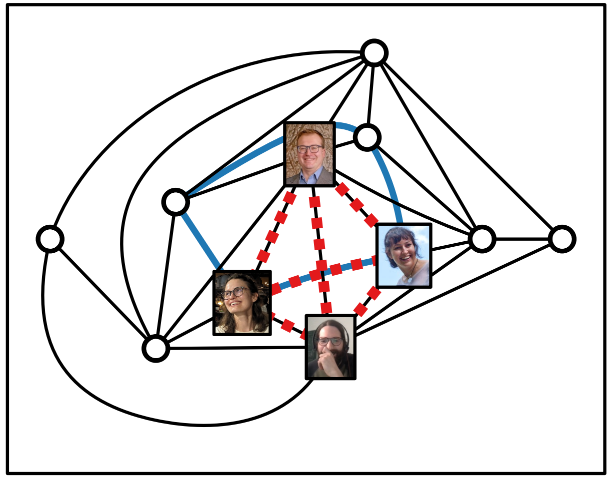

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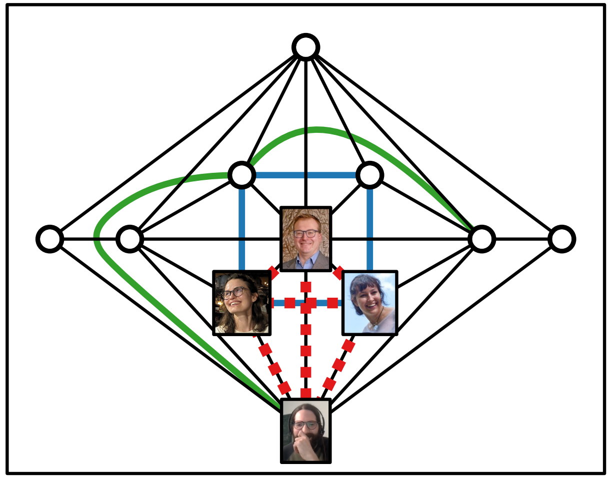
This problem is NP-hard even if $H = \emptyset$. [Grigoriev & Bodlaender '07]

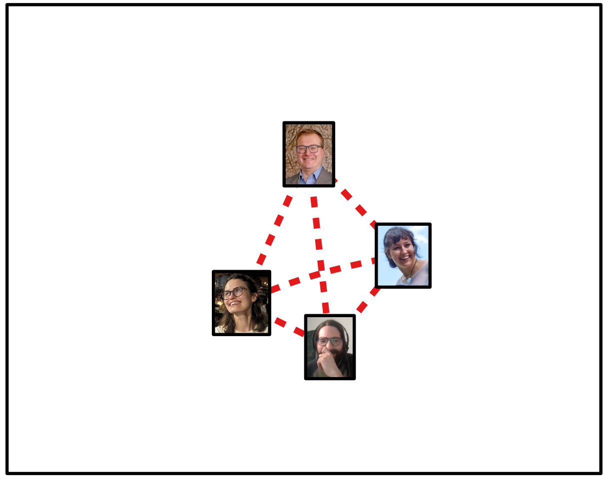
This problem is in FPT parameterized by the vertex+edge deletion distance between G and H. [Eiben, Ganian, Hamm, Klute & Nöllenburg '20]

Partial Embedding of *k*-**Planar** Graphs



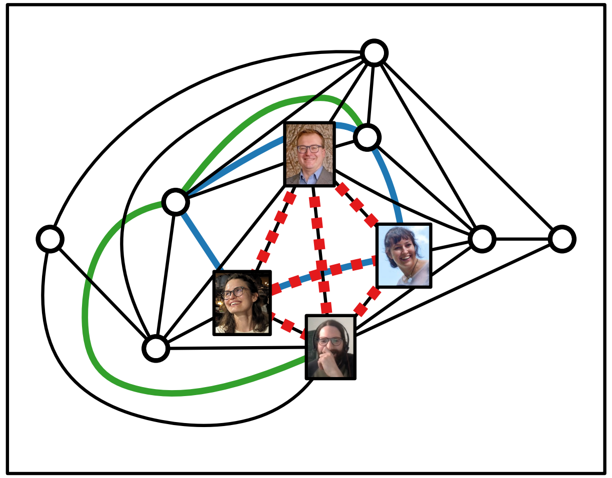
k-**planar** graph G
(can be drawn s.t. every edge is crossed at most k times)

+



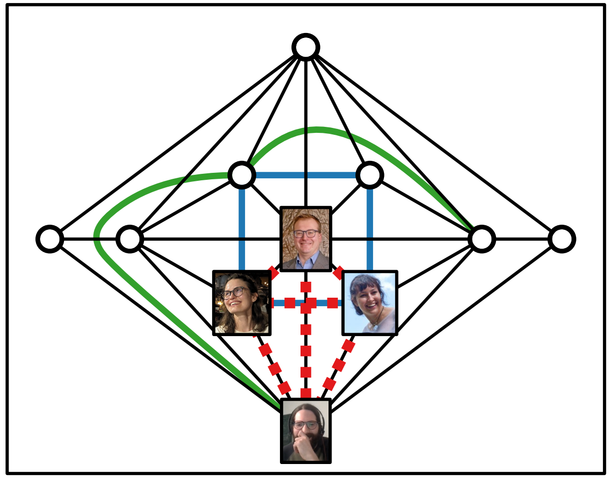
k-**planar** drawing of a subgraph $H \subseteq G$

→

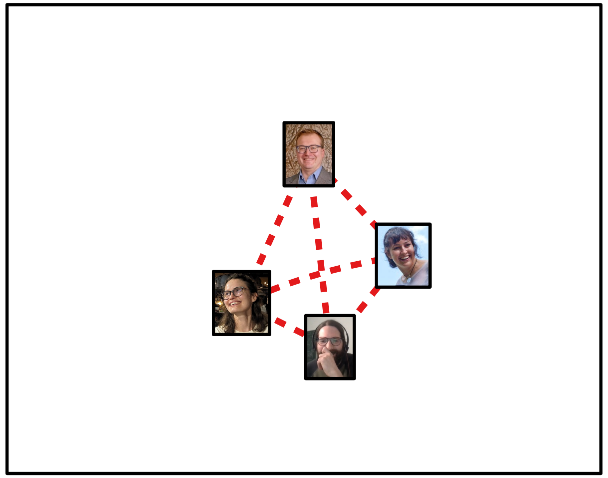


k-**planar** drawing of G s.t. H keeps its drawing

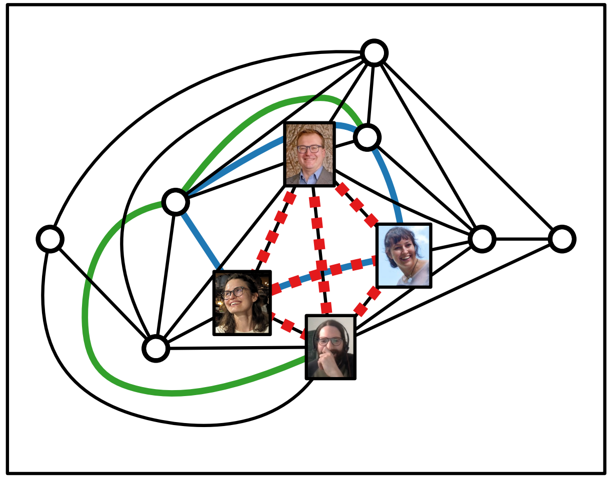
Partial Embedding of *k*-**Planar** Graphs



k-**planar** graph G (can be drawn s.t. every edge is crossed at most k times)



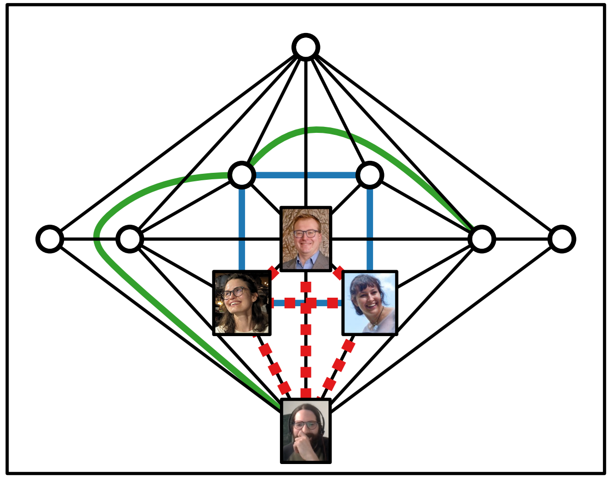
k-**planar** drawing of a subgraph $H \subseteq G$



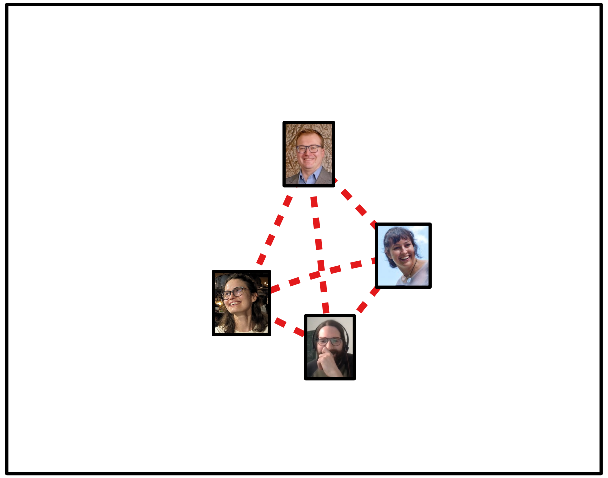
k-**planar** drawing of G s.t. H keeps its drawing

This problem is NP-hard for any constant k even if $H = \emptyset$. [Urschel & Wellens '21]

Partial Embedding of *k*-**Planar** Graphs



k-**planar** graph G (can be drawn s.t. every edge is crossed at most k times)



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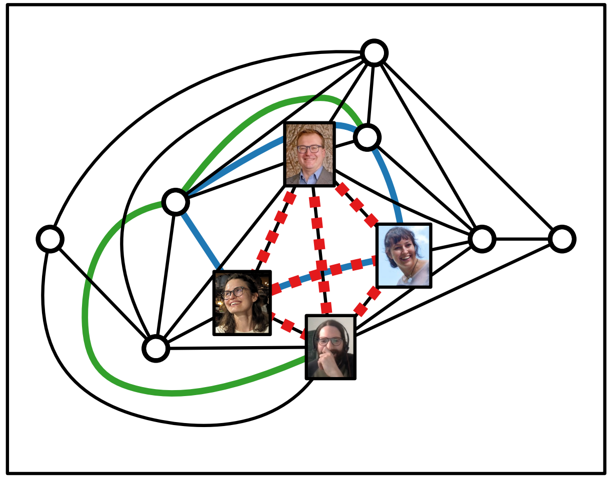
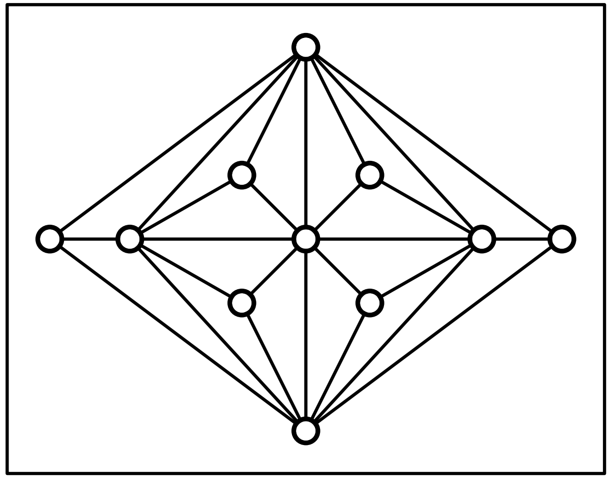


k-**planar** drawing of G s.t. H keeps its drawing

This problem is NP-hard for any constant k even if $H = \emptyset$. [Urschel & Wellens '21]

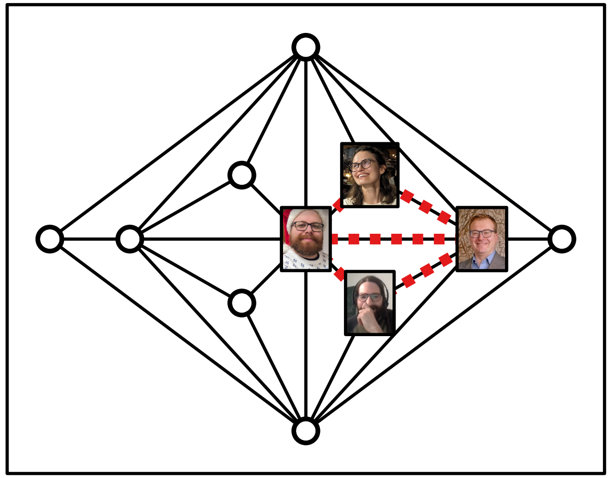
This problem is in FPT parameterized by k + #edges in $G - H$.
[Ganian, Hamm, Klute, Parada & Vogtenhuber '21]

Generalization of Partial Embedding



Graph G +
drawing style Φ
(e.g., planar)

Generalization of Partial Embedding



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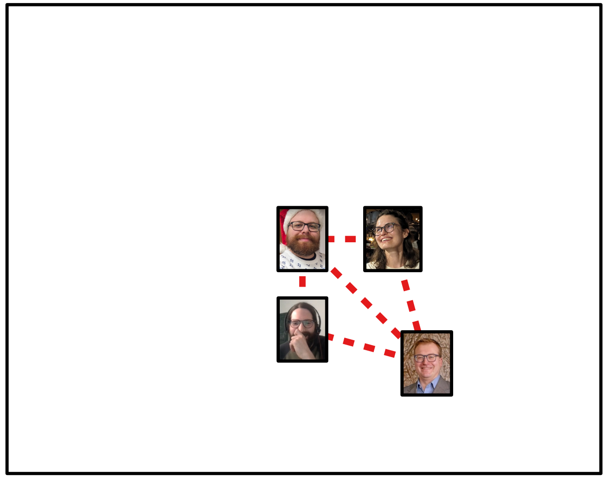

Drawing with style
$\Phi' \subseteq \Phi$
(e.g., straight-line planar)
of a subgraph $H \subseteq G$

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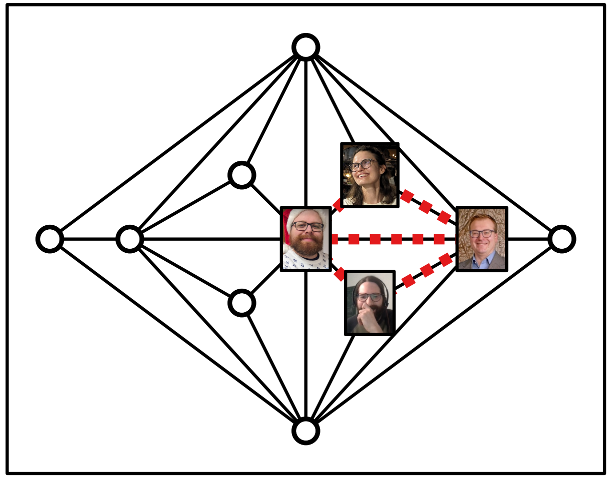

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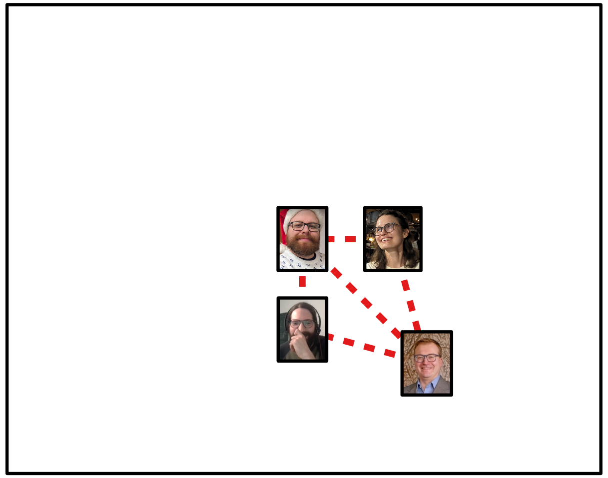

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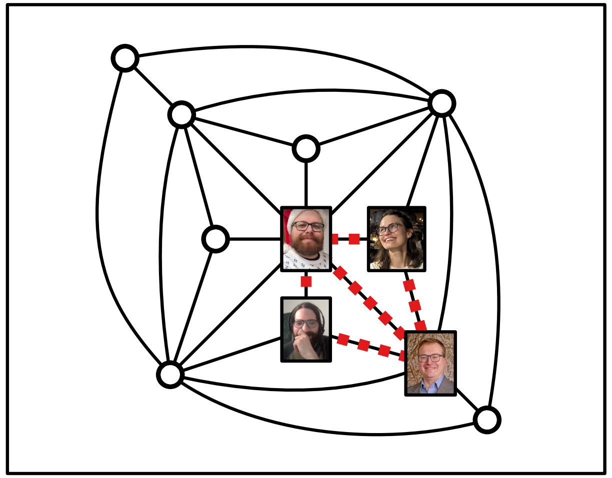
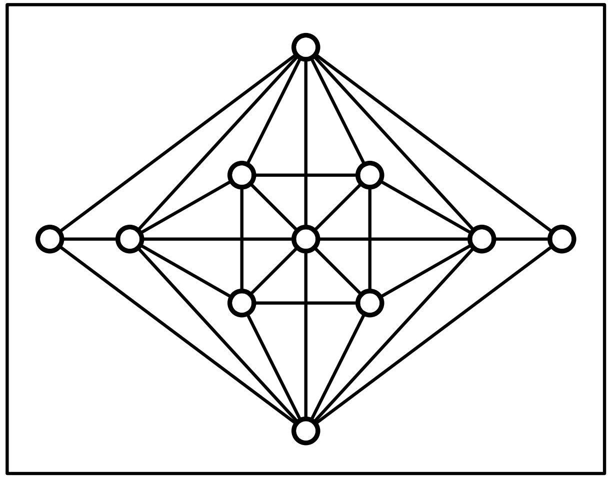


Drawing with style Φ of G
s.t. H **keeps its drawing**

1-Plane Insertion Into a Plane Graph



1-planar graph G

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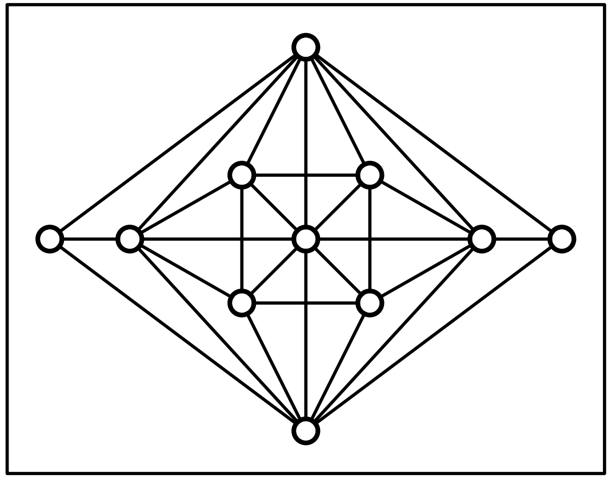

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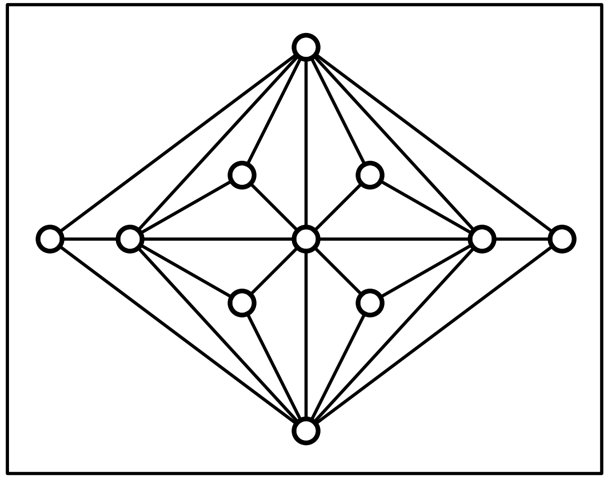

planar drawing of a
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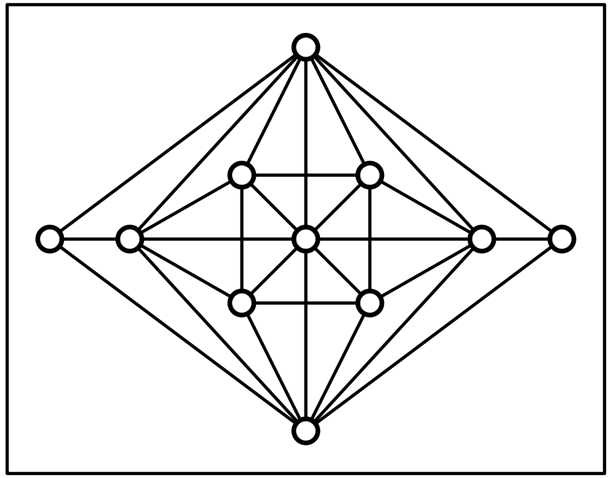

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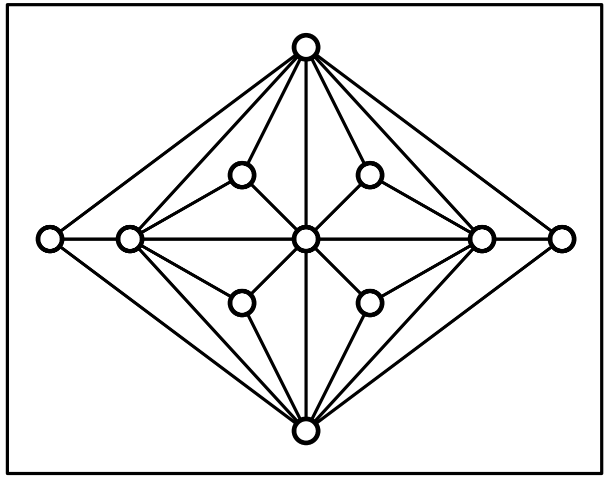
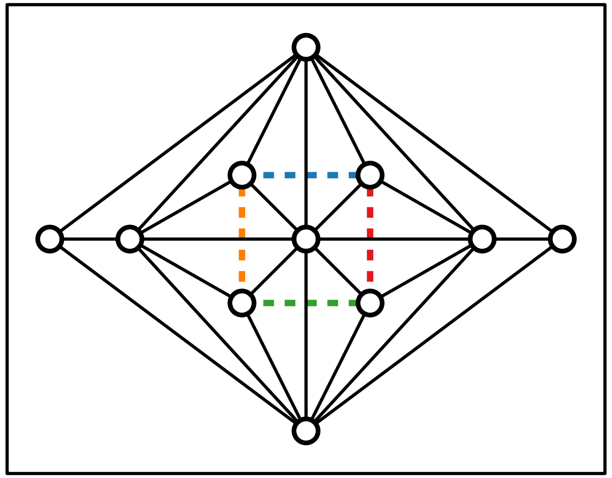
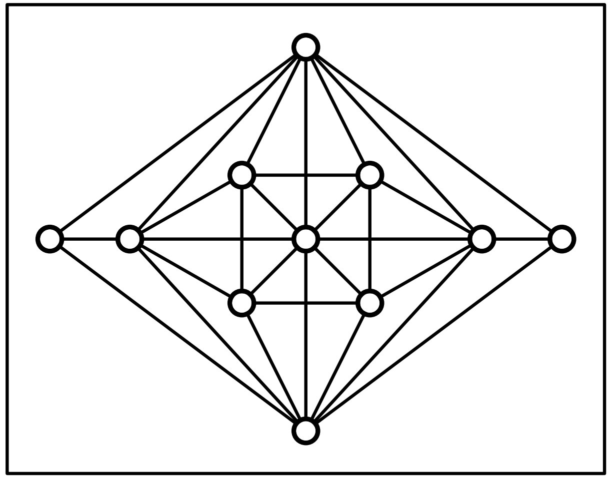


planar drawing of a **spanning** subgraph $H \subseteq G$



1-planar drawing of G that keeps the drawing of H

1-Plane Insertion Into a Plane Graph



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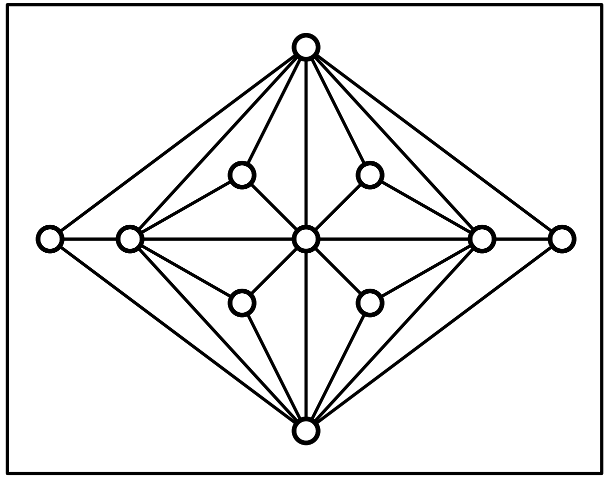
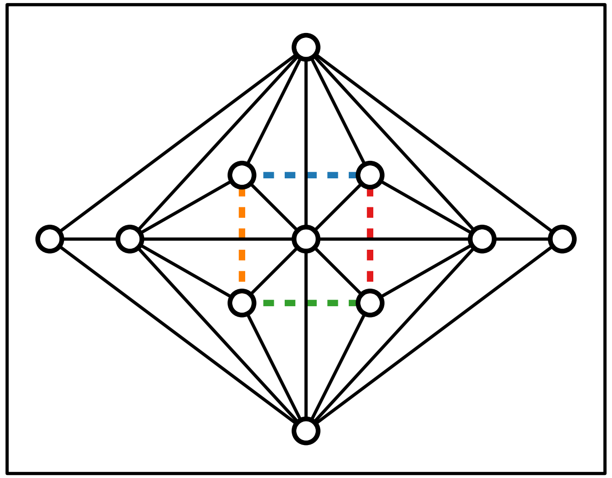


planar drawing of a spanning subgraph $H \subseteq G$



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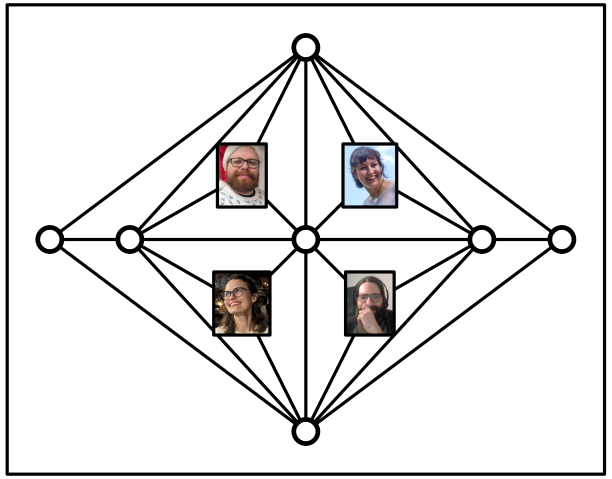
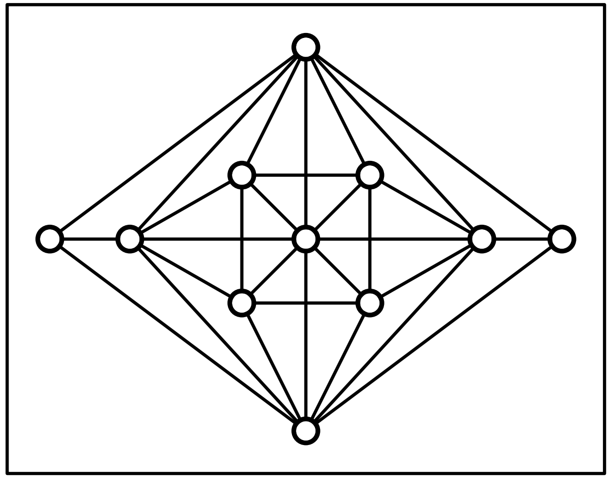
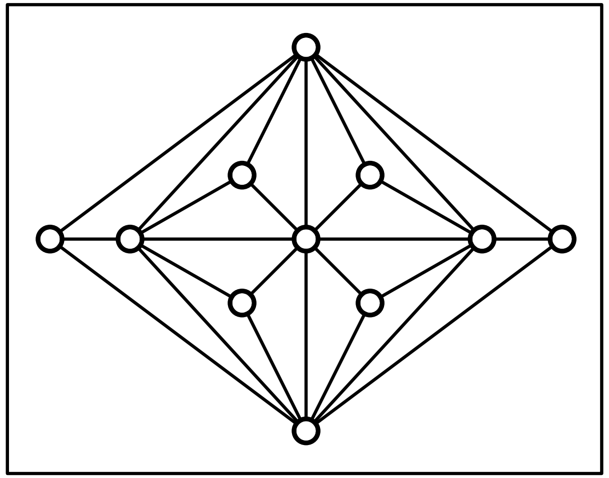


plane graph G

1-Plane Insertion Into a Plane Graph



1-planar graph G



planar drawing of a
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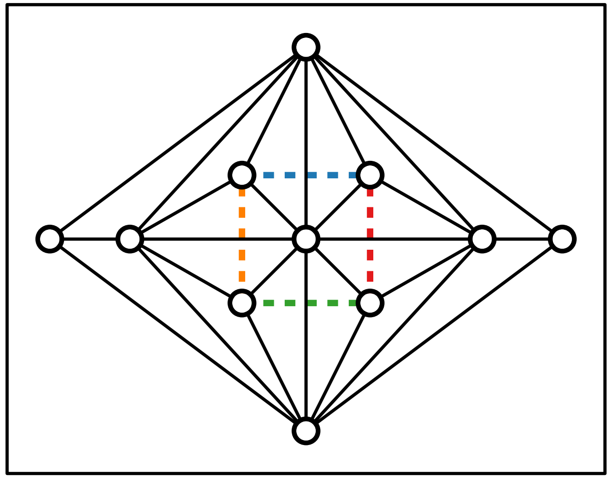

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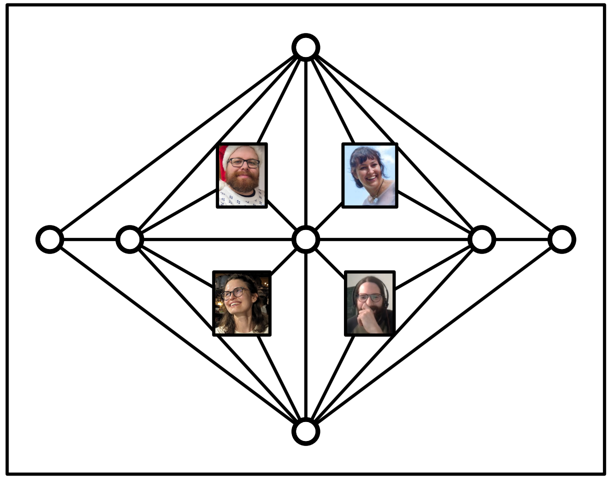
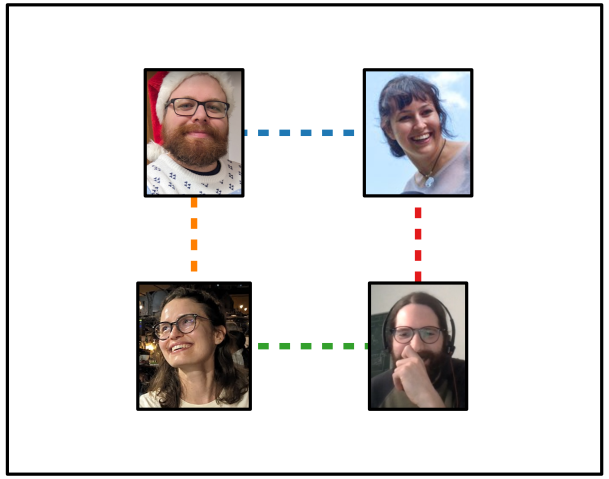
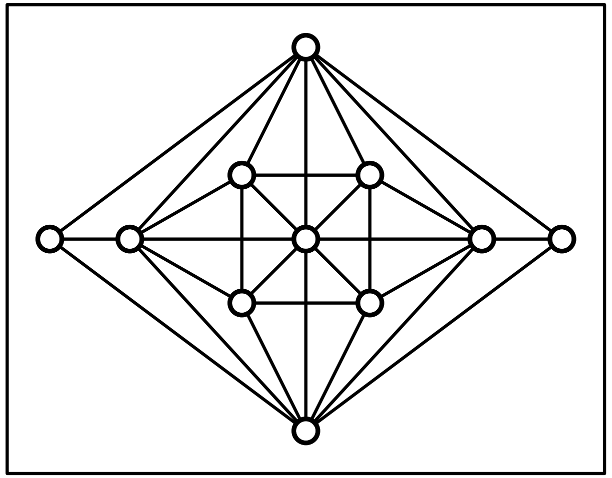


plane graph G

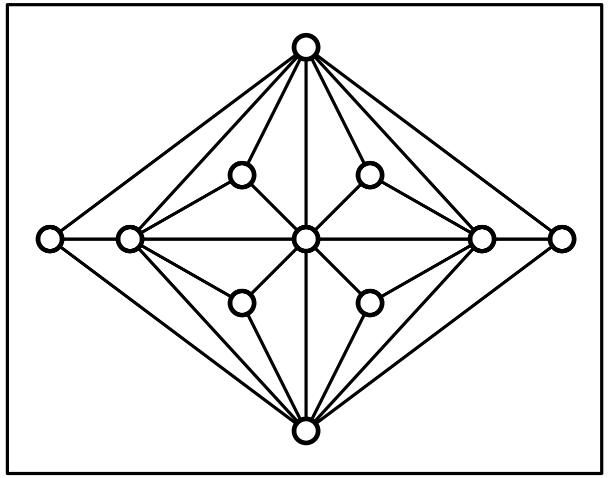


edges E' btw. vtcs in G

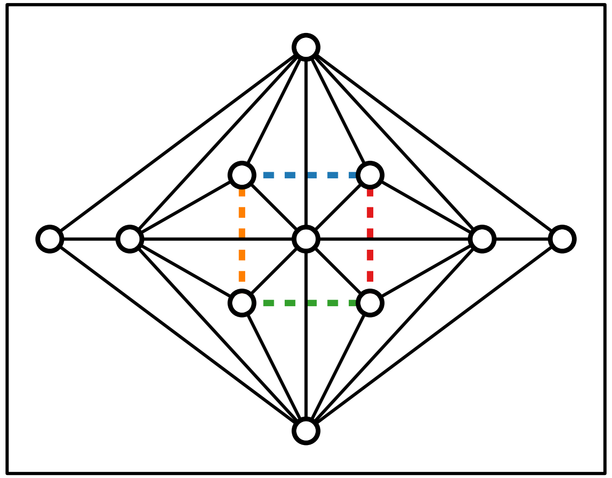
1-Plane Insertion Into a **Plane** Graph



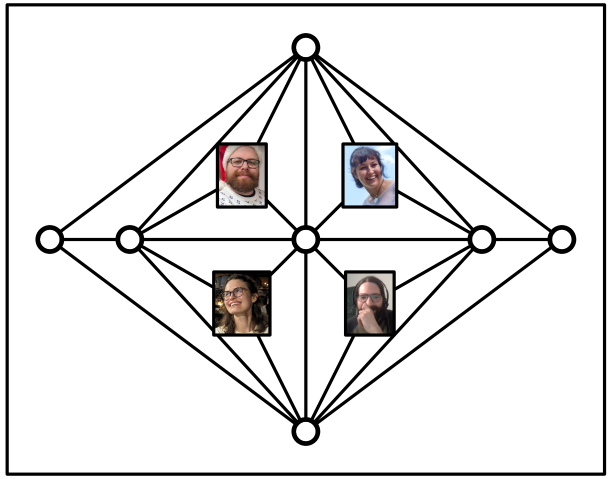
1-planar graph G



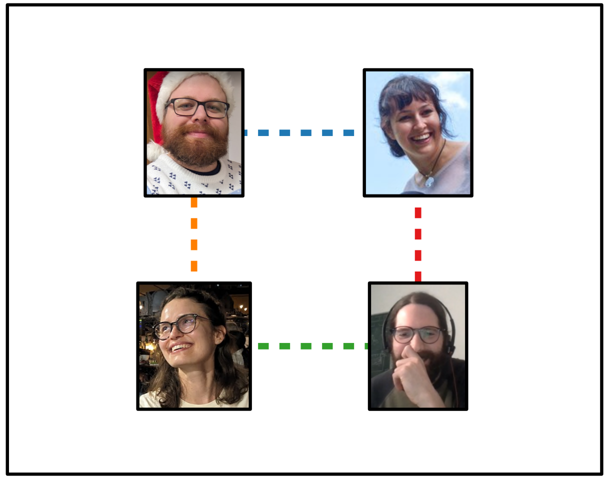
planar drawing of a **spanning** subgraph $H \subseteq G$



1-planar drawing of G that keeps the drawing of H



plane graph G



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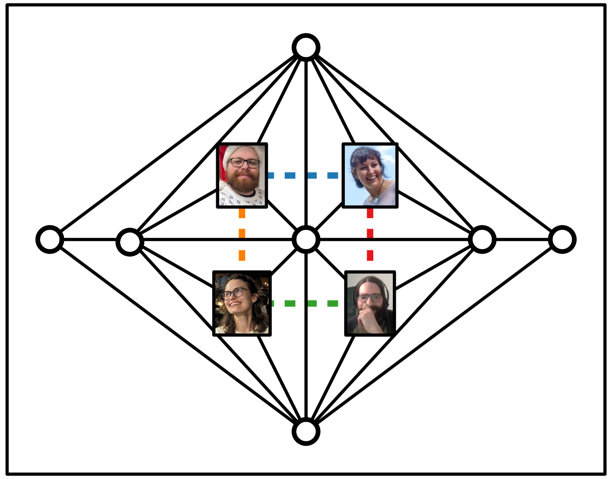
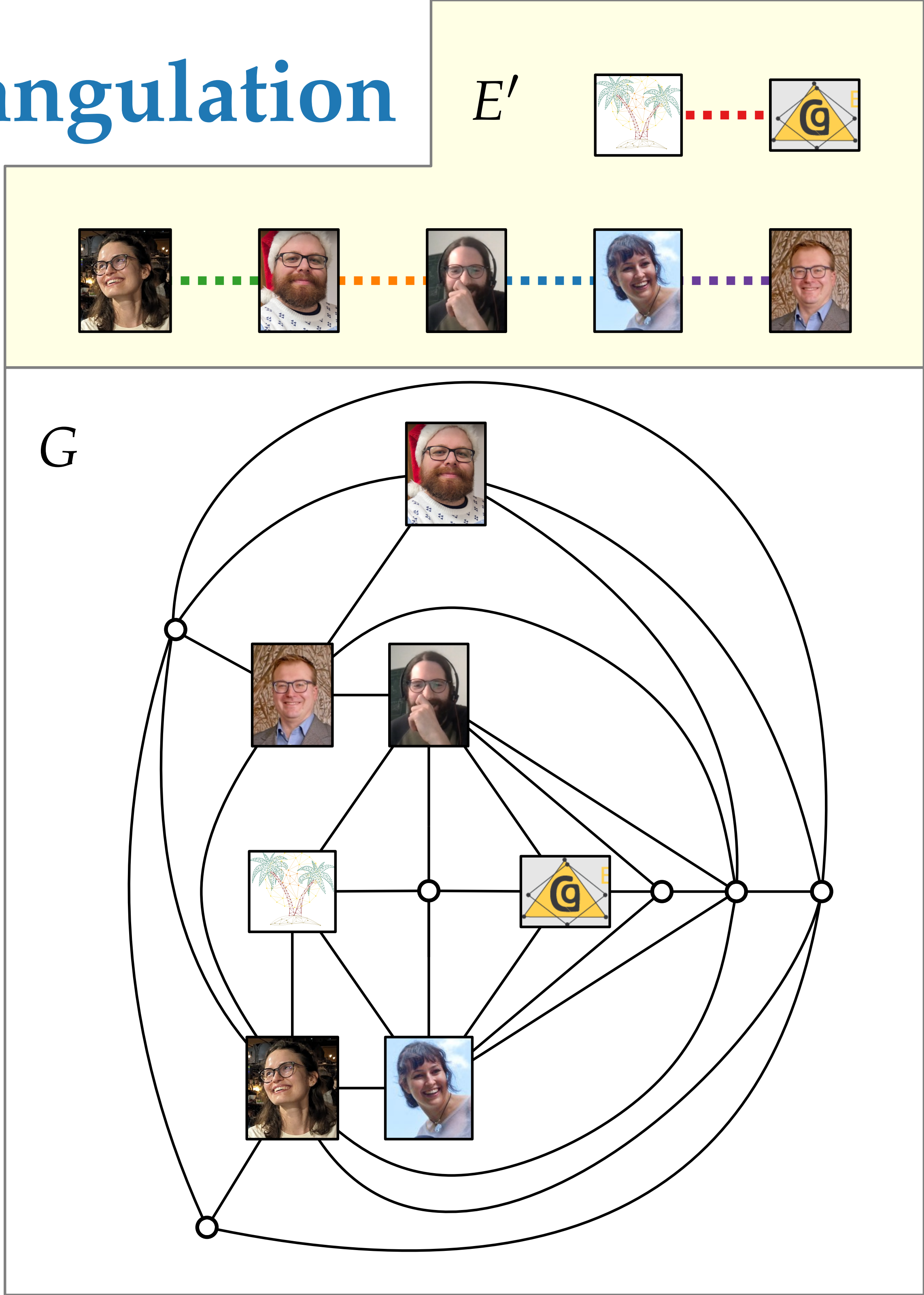


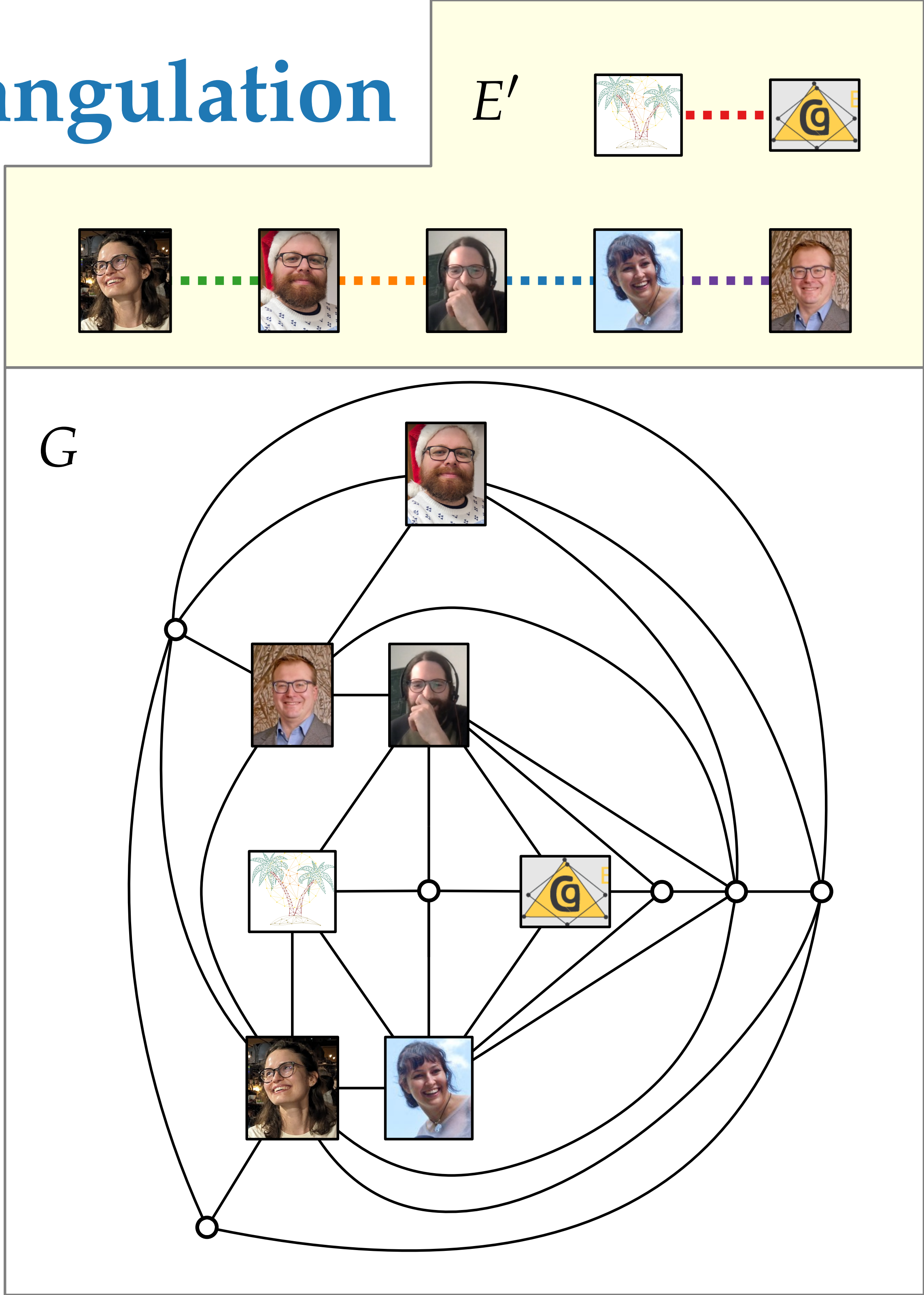
1-planar drawing of $G + E'$ that keeps the embedding of G

1-Plane Insertion Into a Plane Triangulation



1-Plane Insertion Into a Plane Triangulation

1. For each edge, find all possibilities to route it



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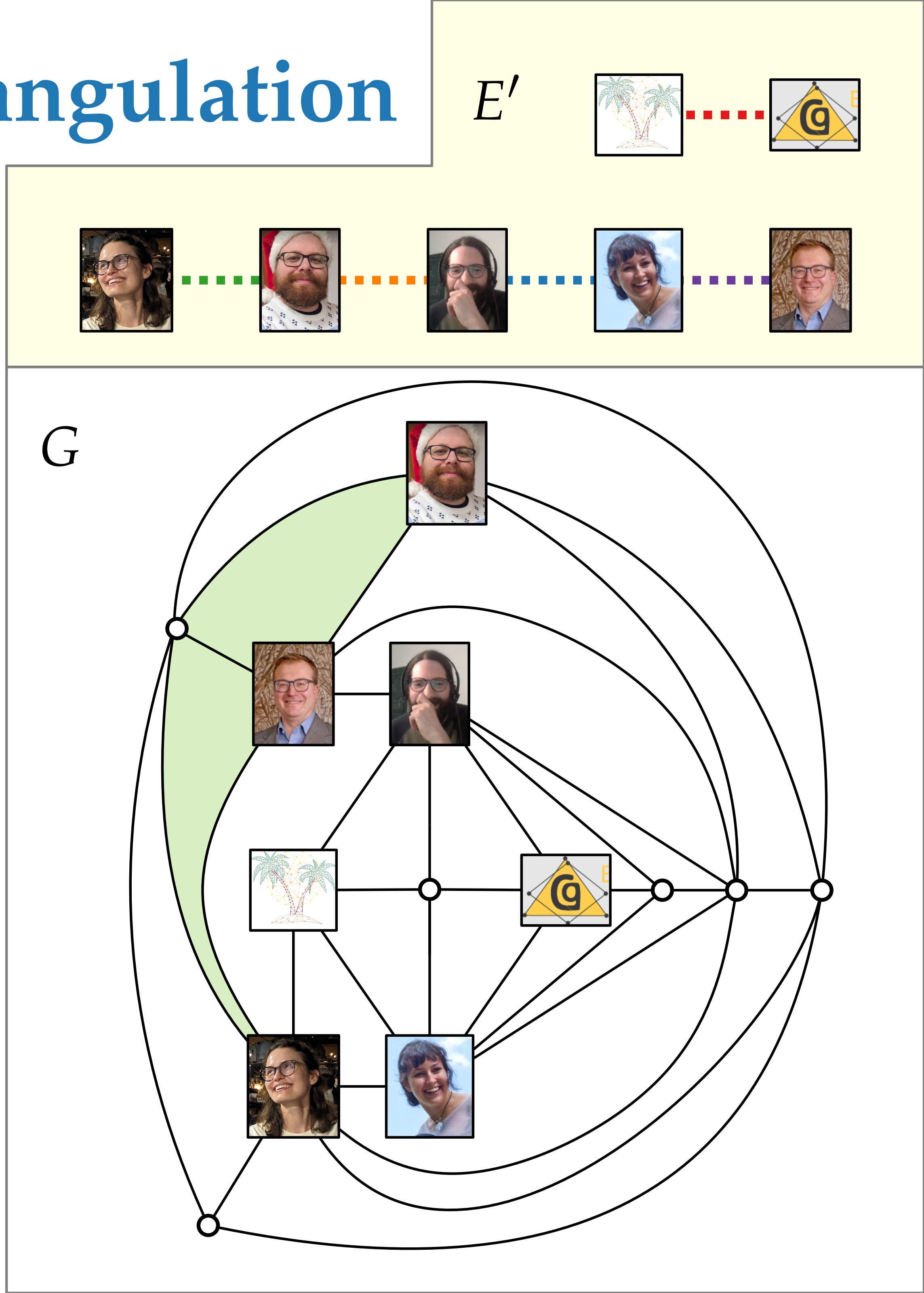

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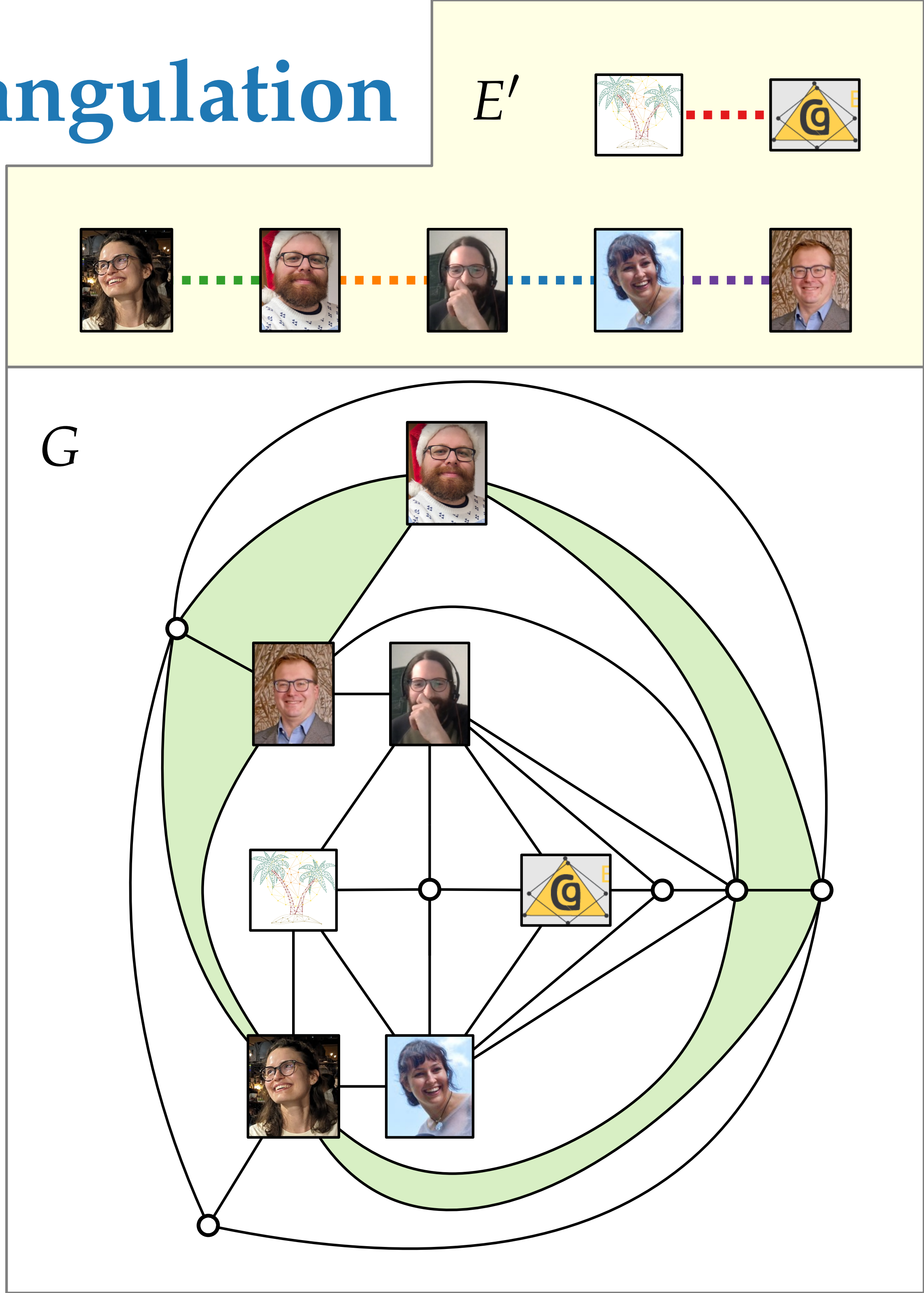

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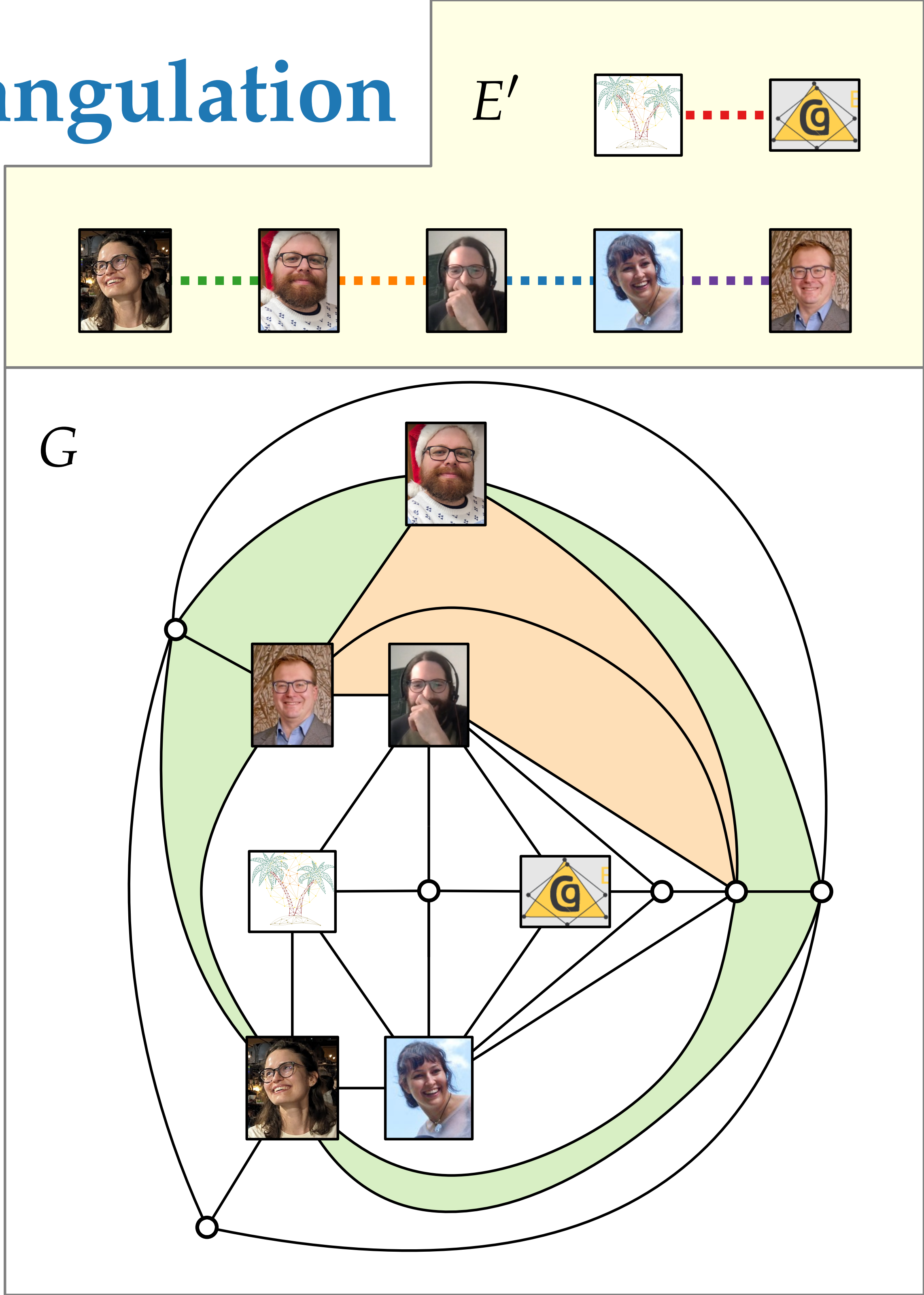

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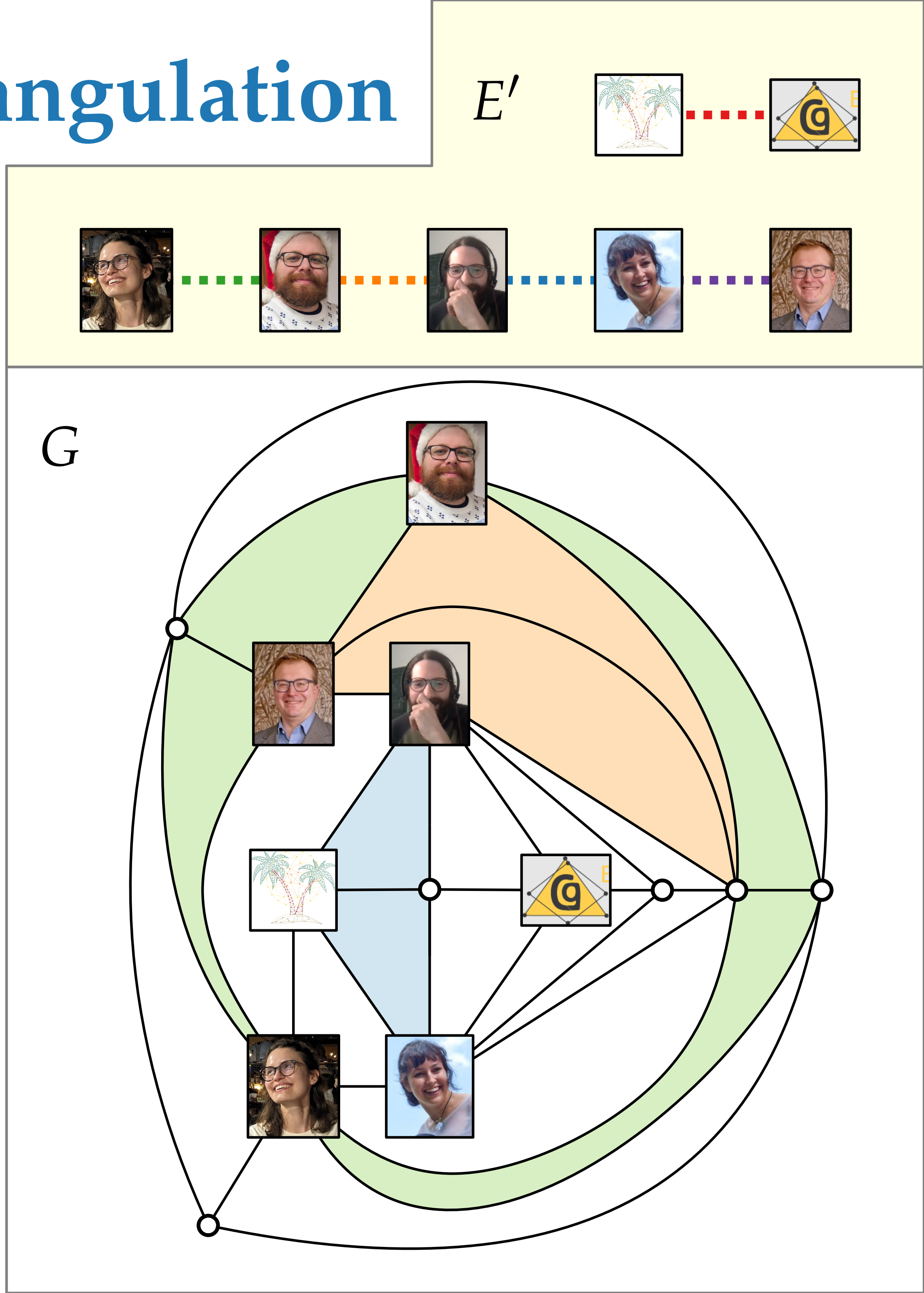

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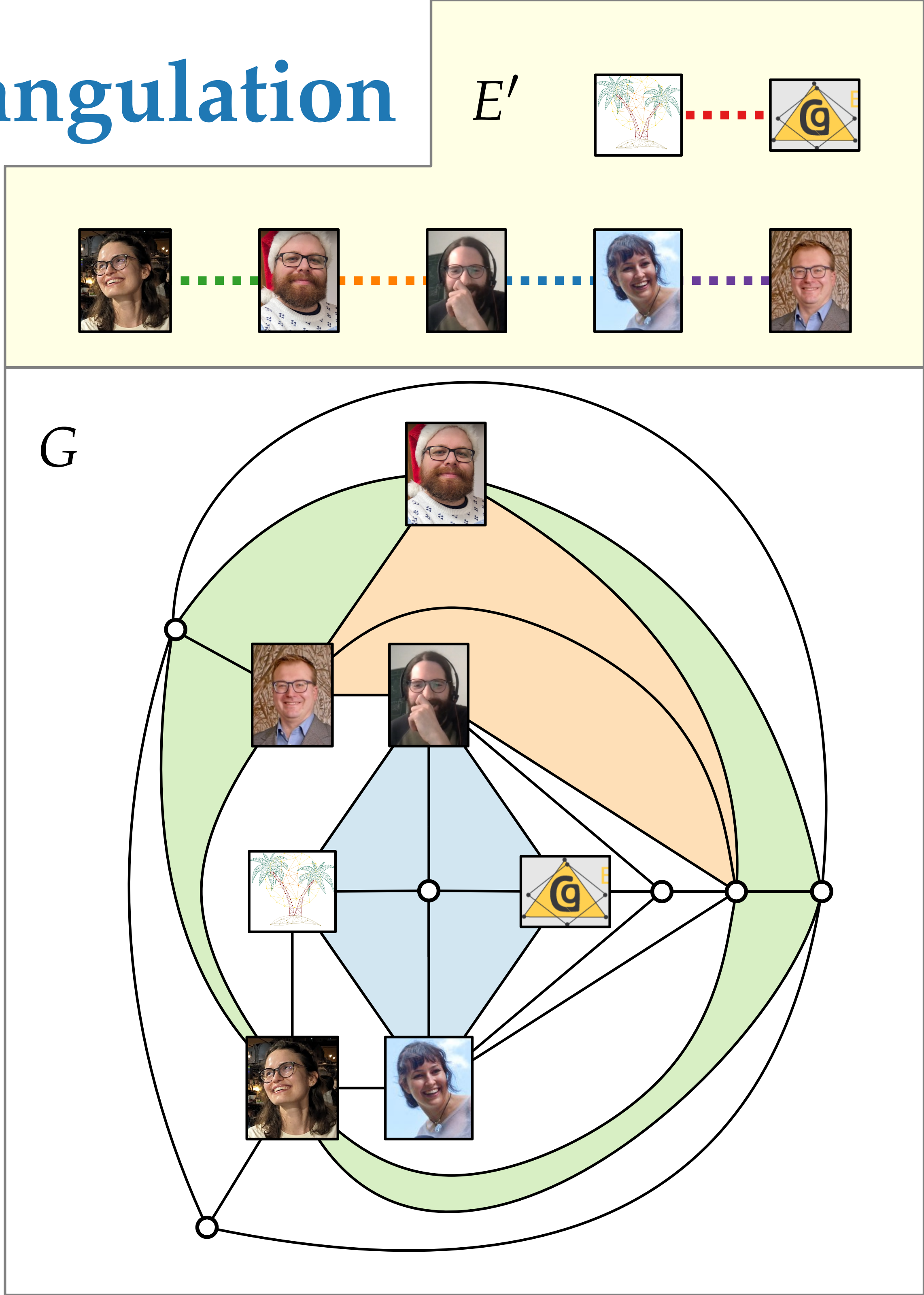

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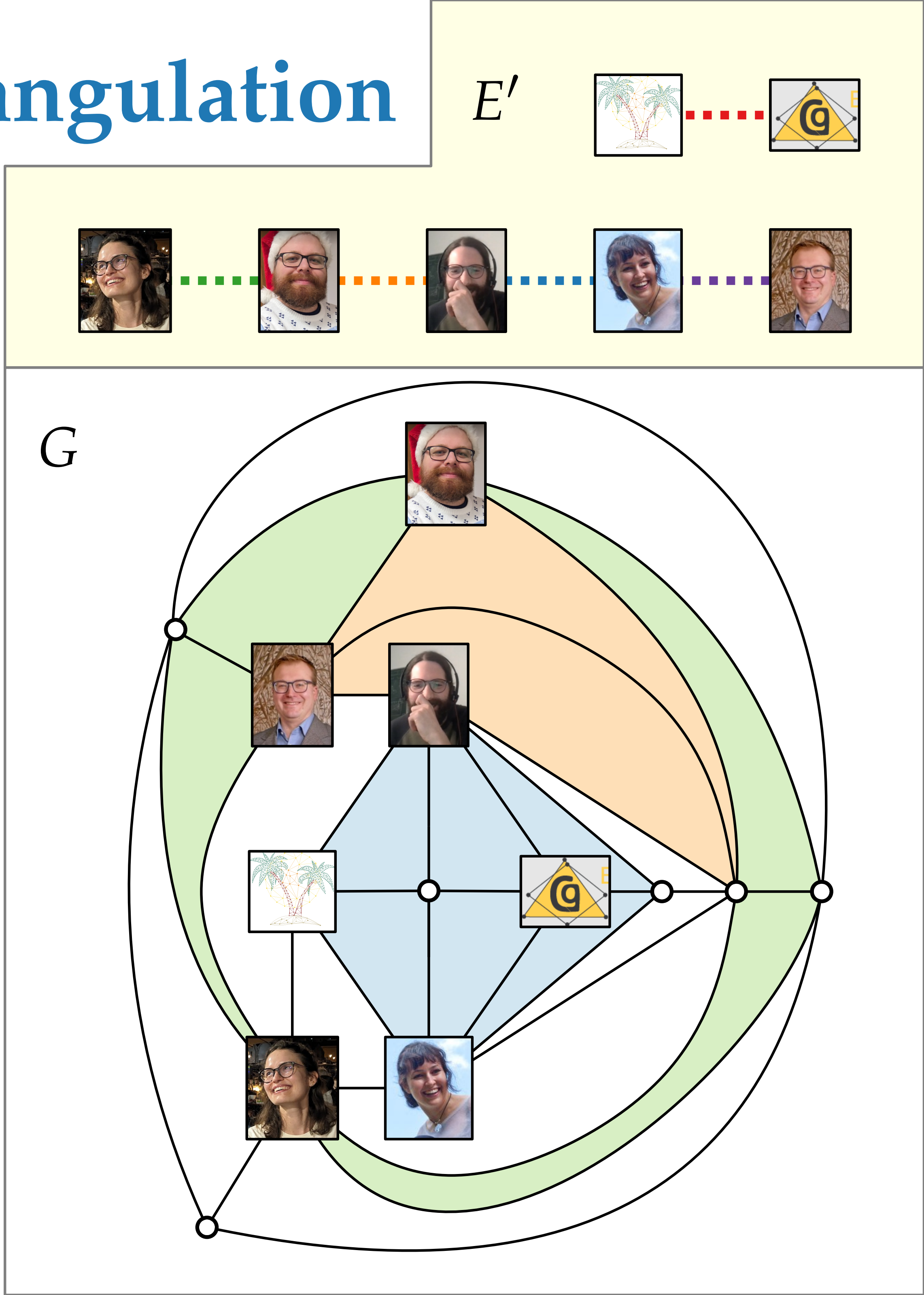

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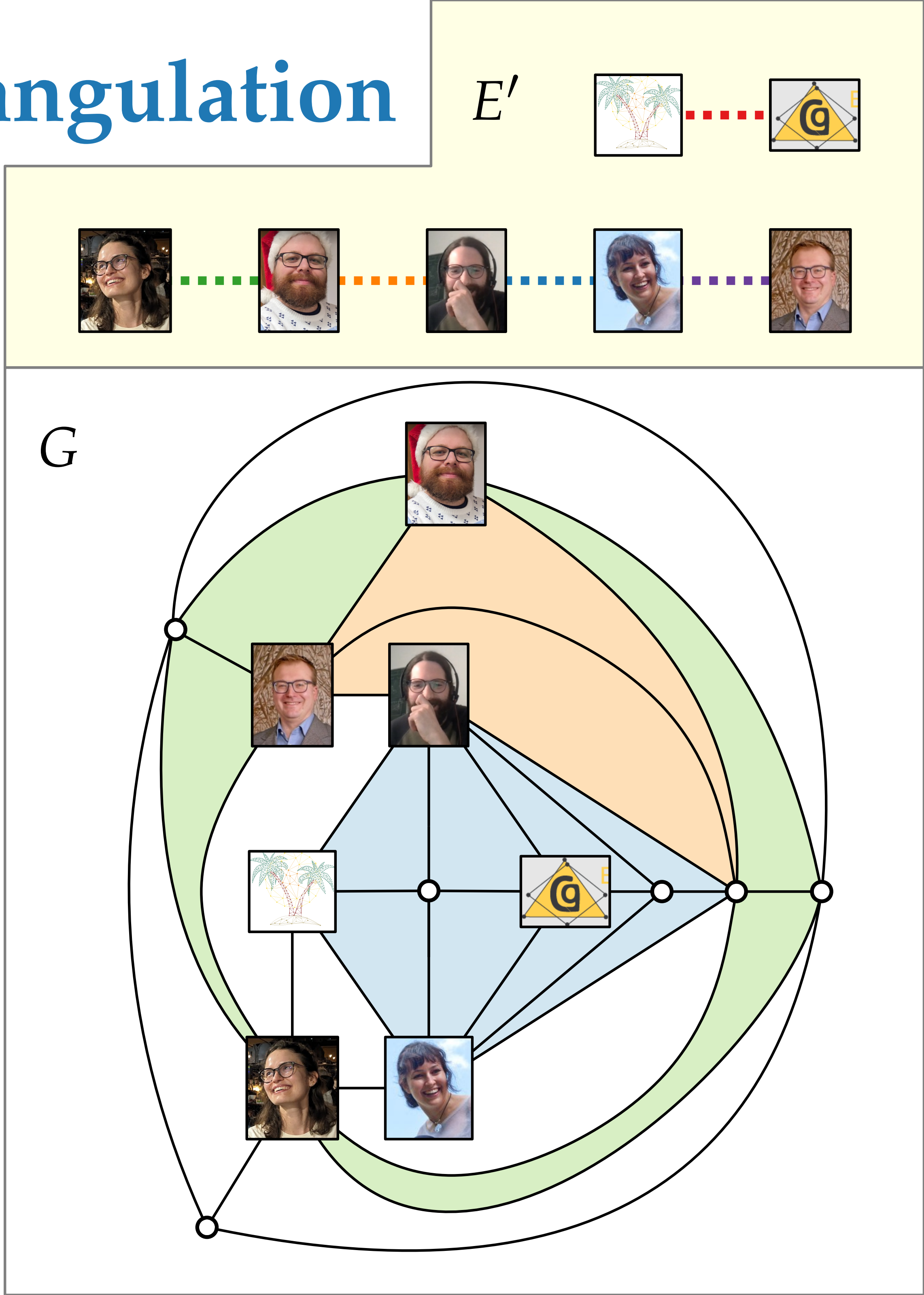

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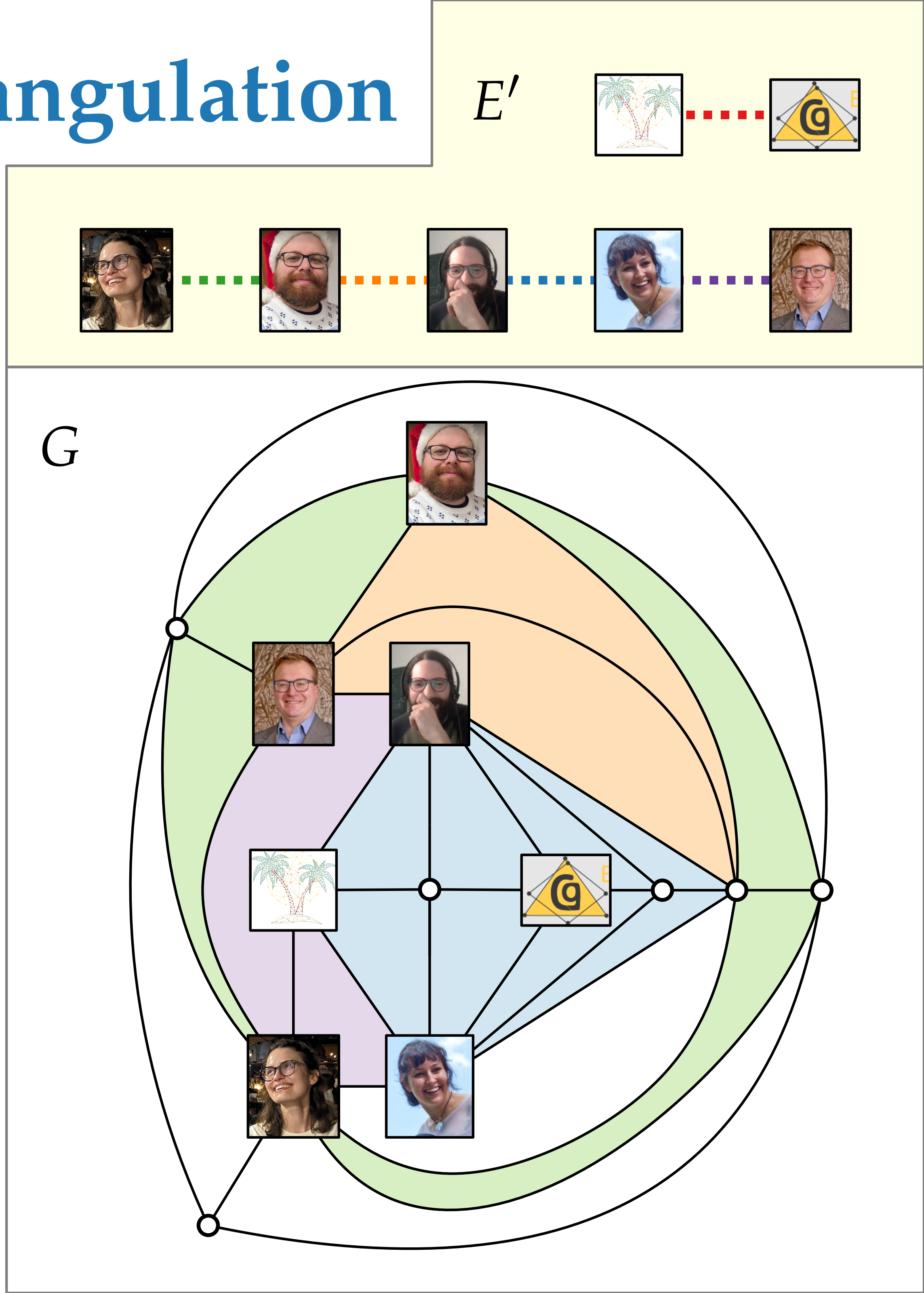

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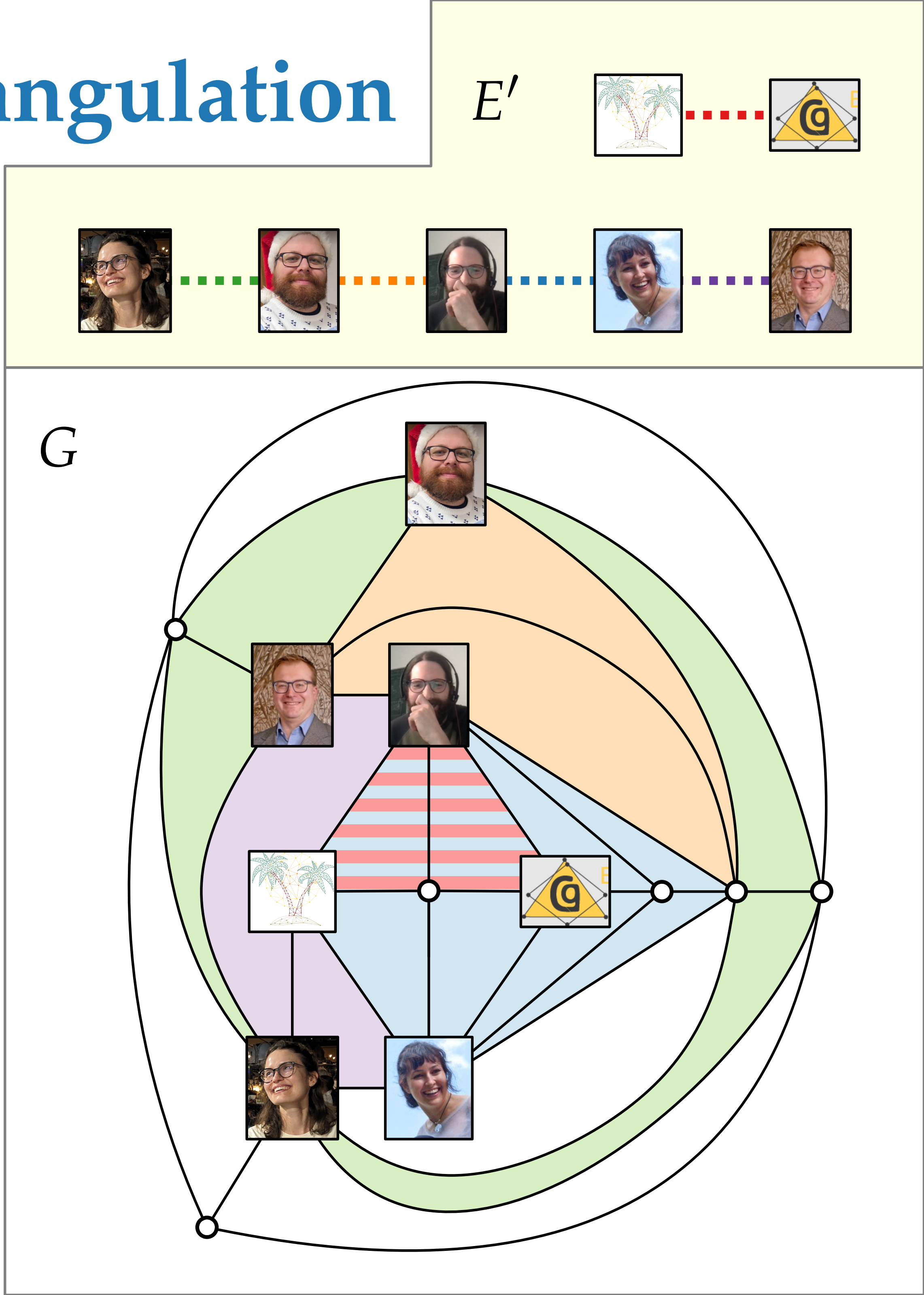

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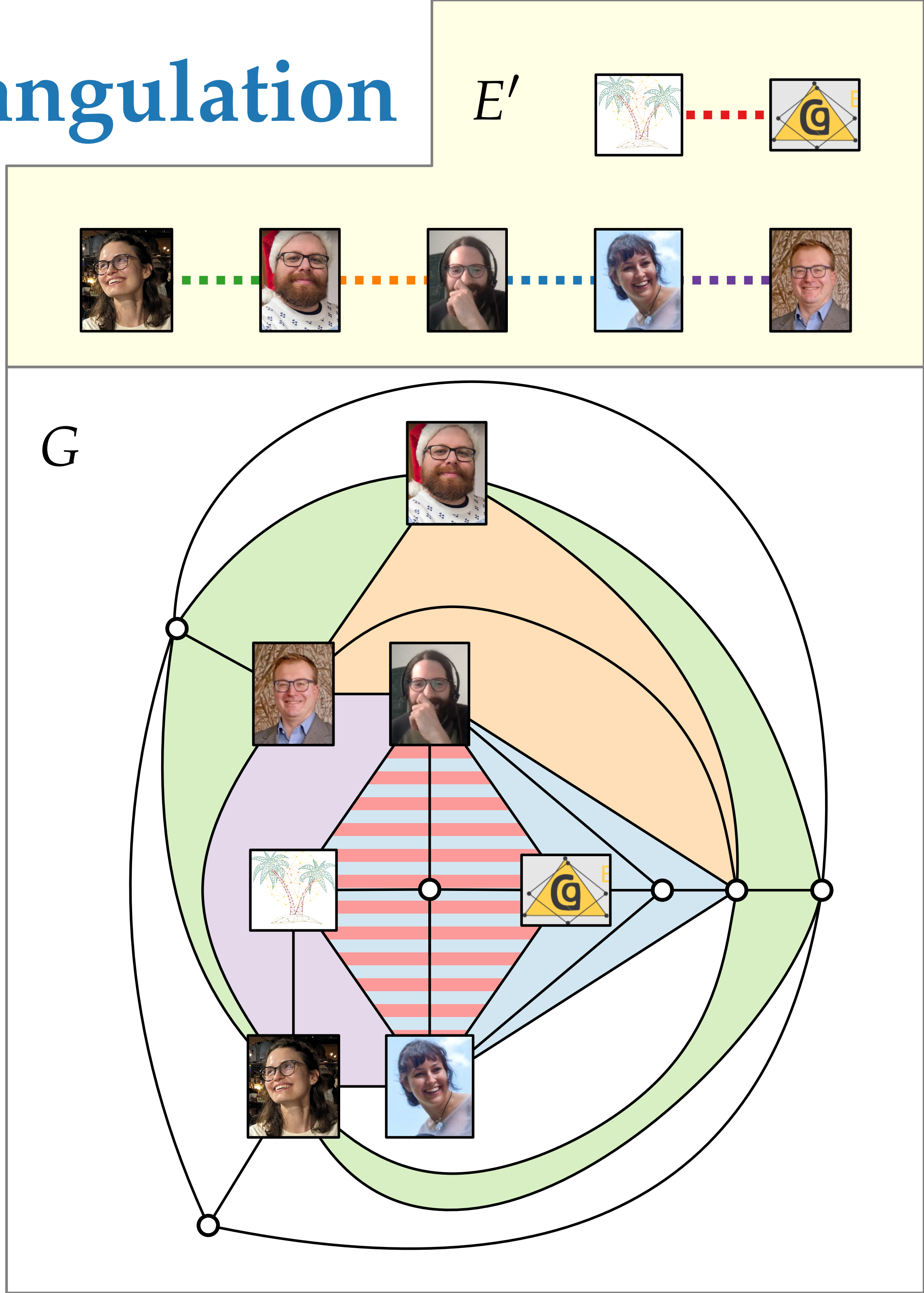

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1. For each edge, find all possibilities to route it
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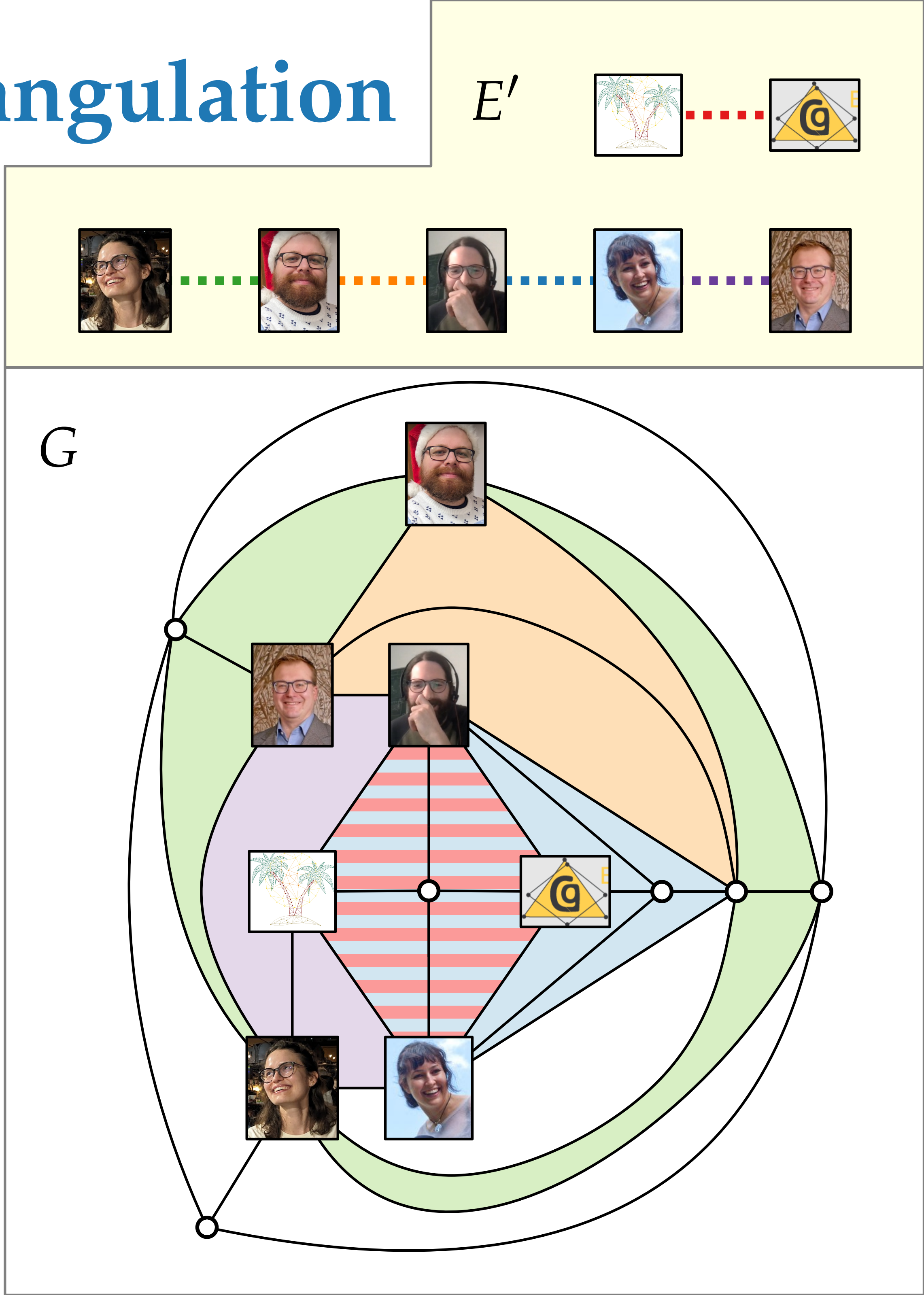

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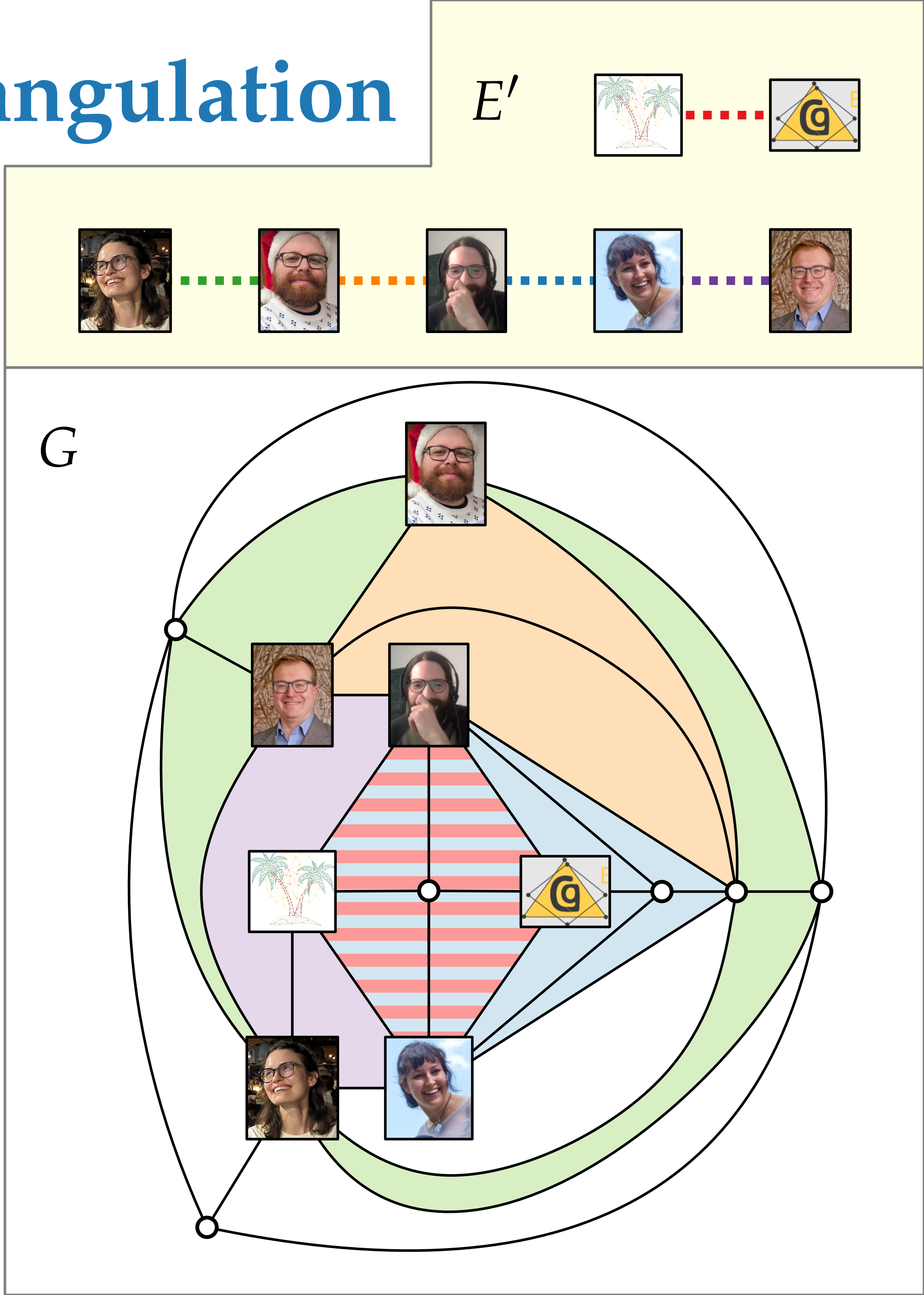

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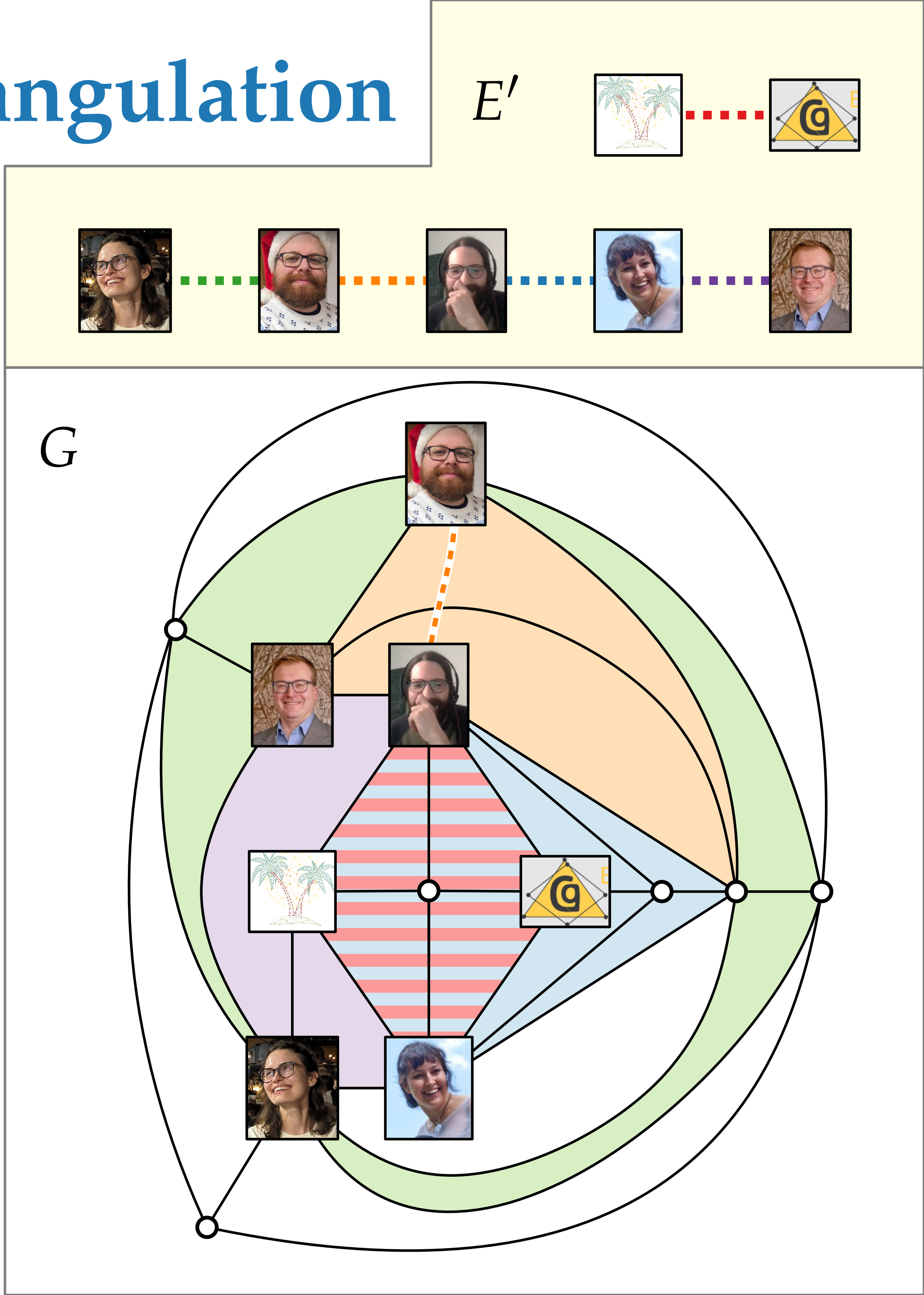

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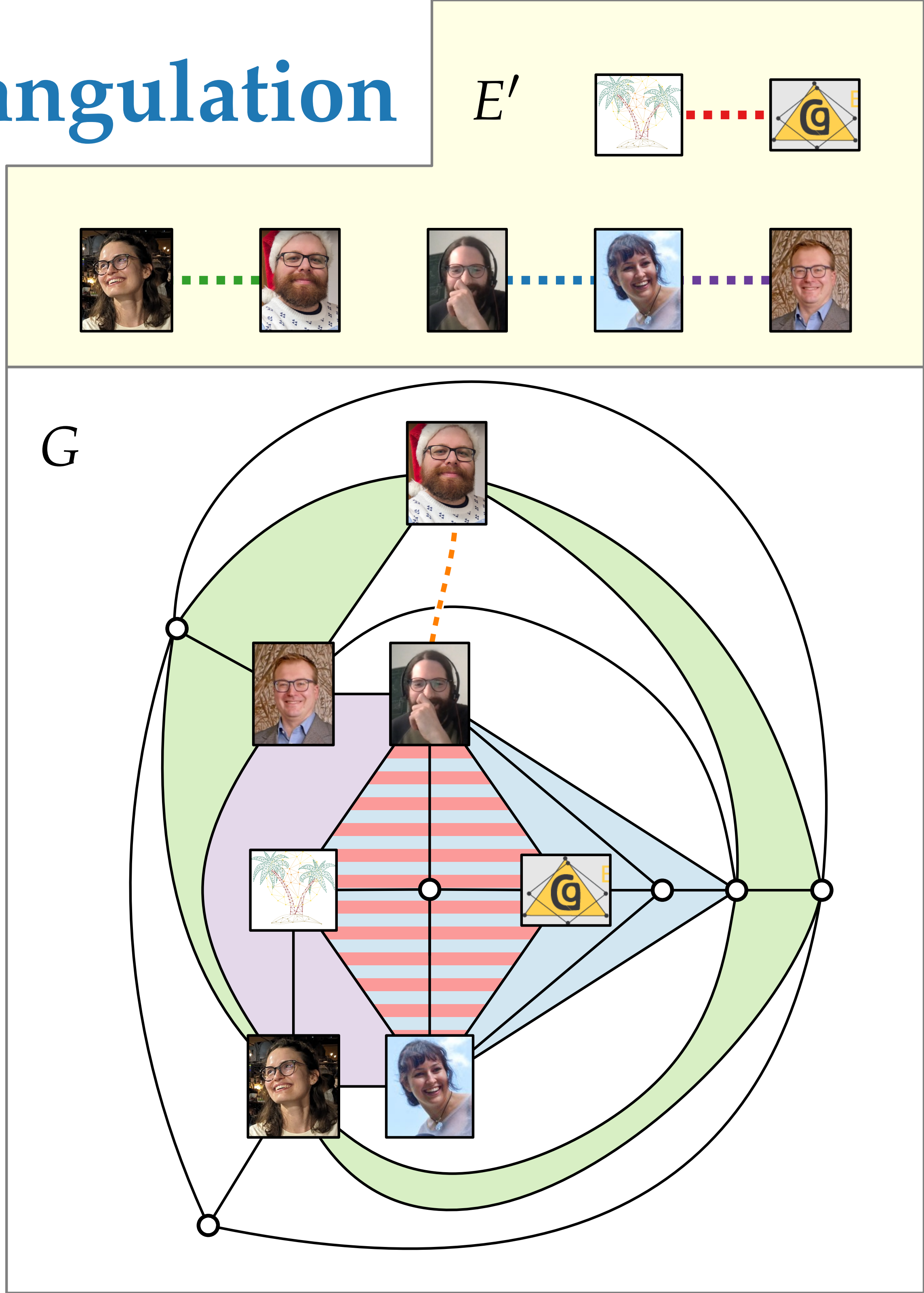

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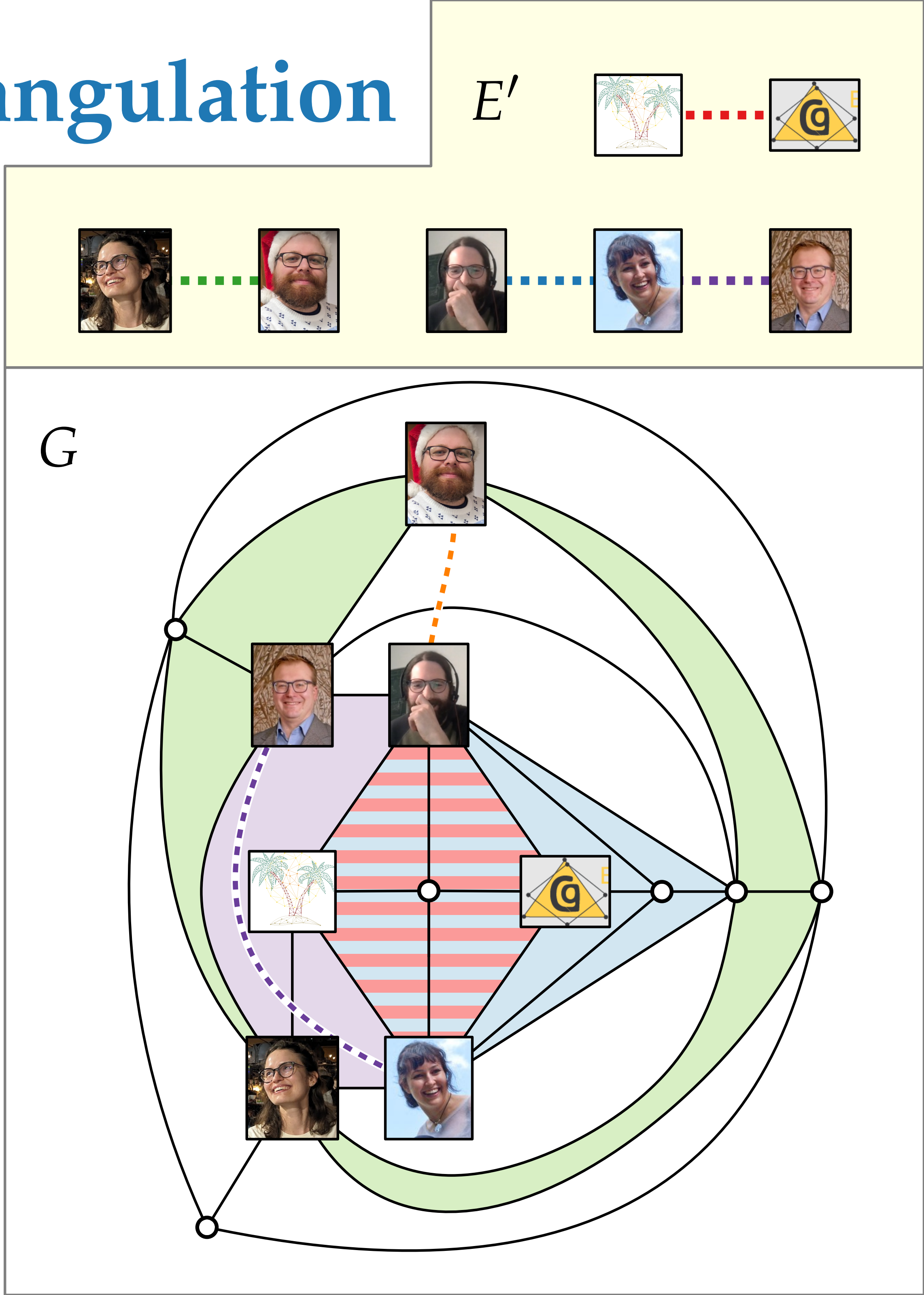

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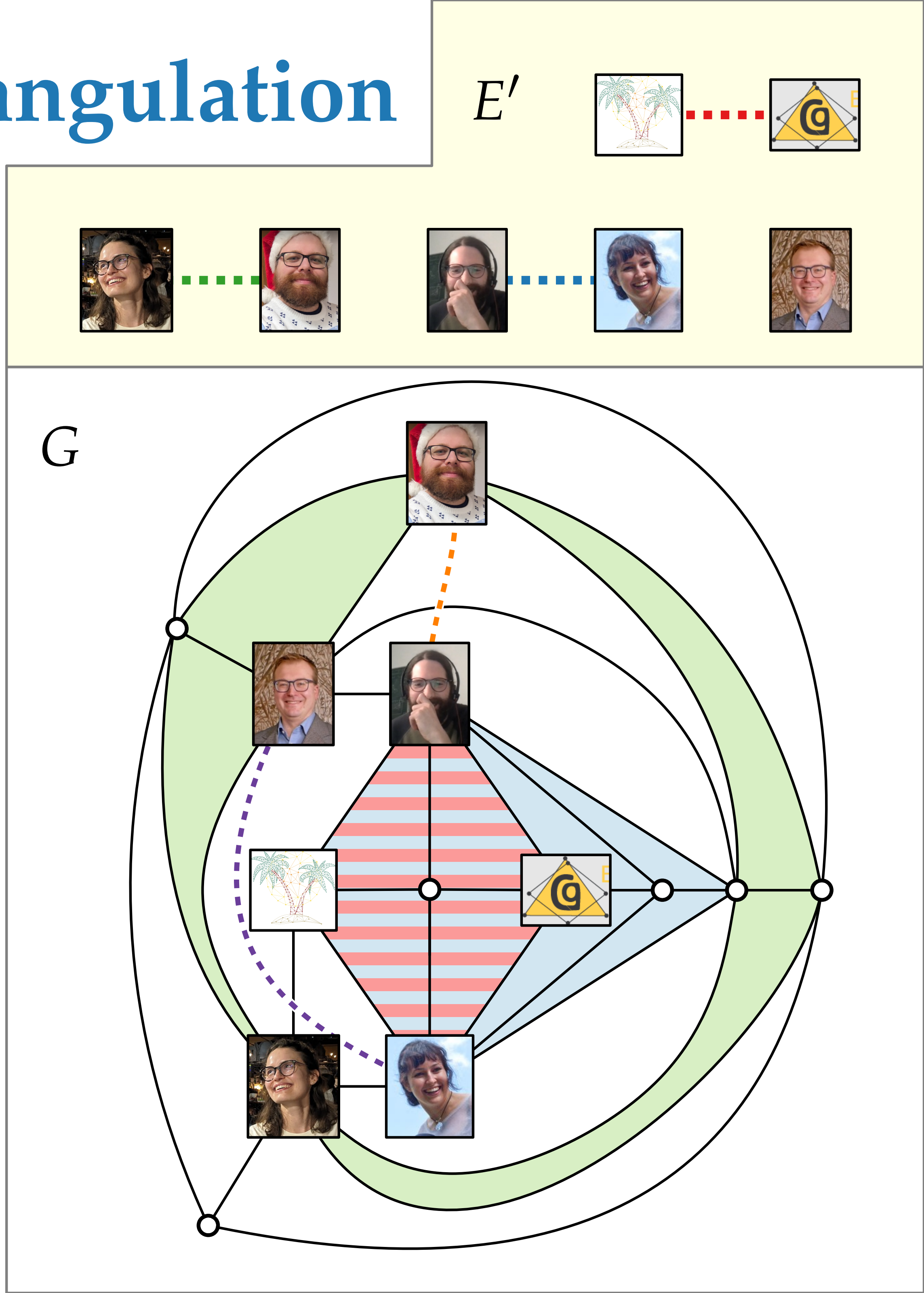

1-Plane Insertion Into a Plane **Triangulation**

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   ➡ there is always a **safe** or an **impossible** option
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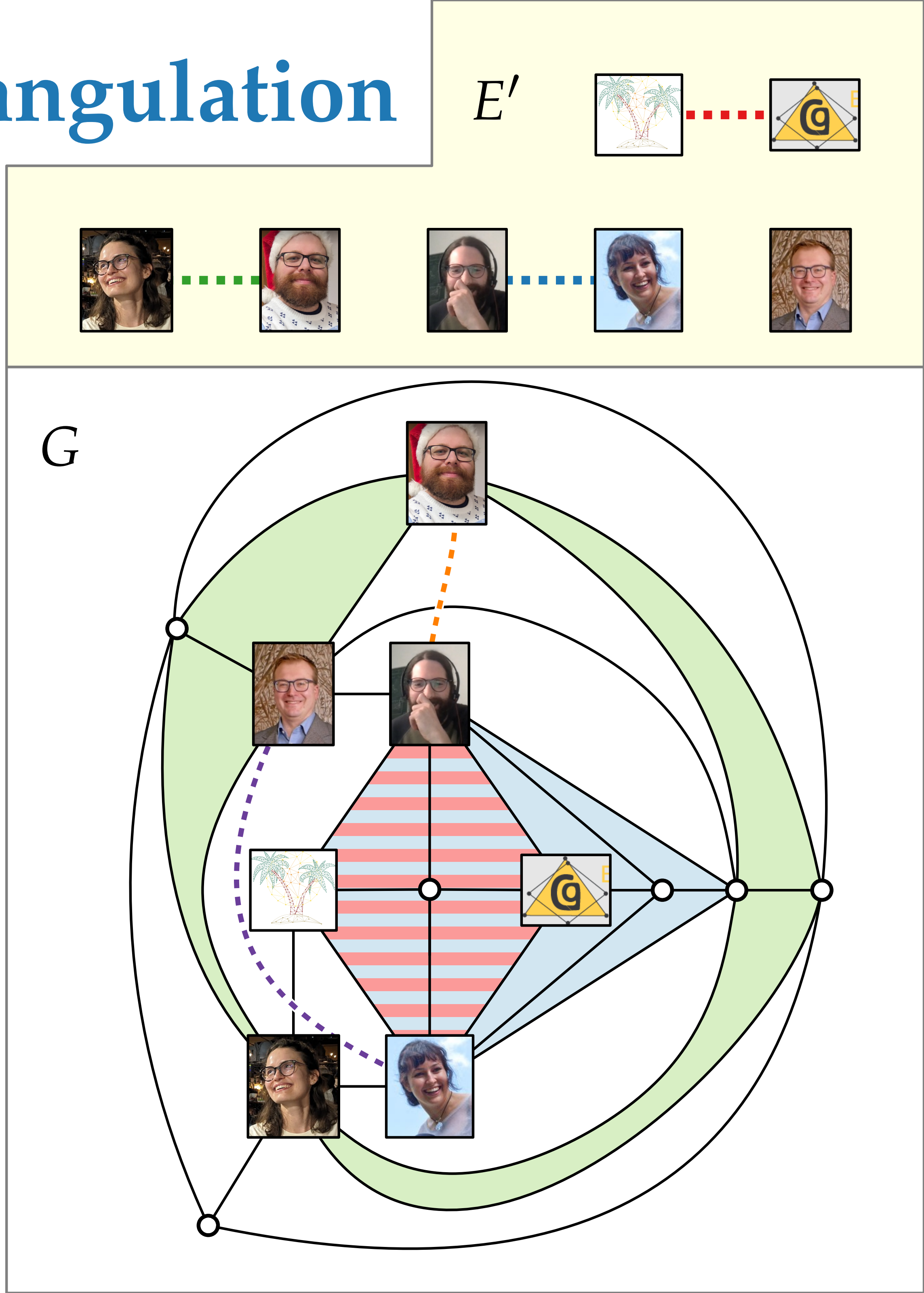

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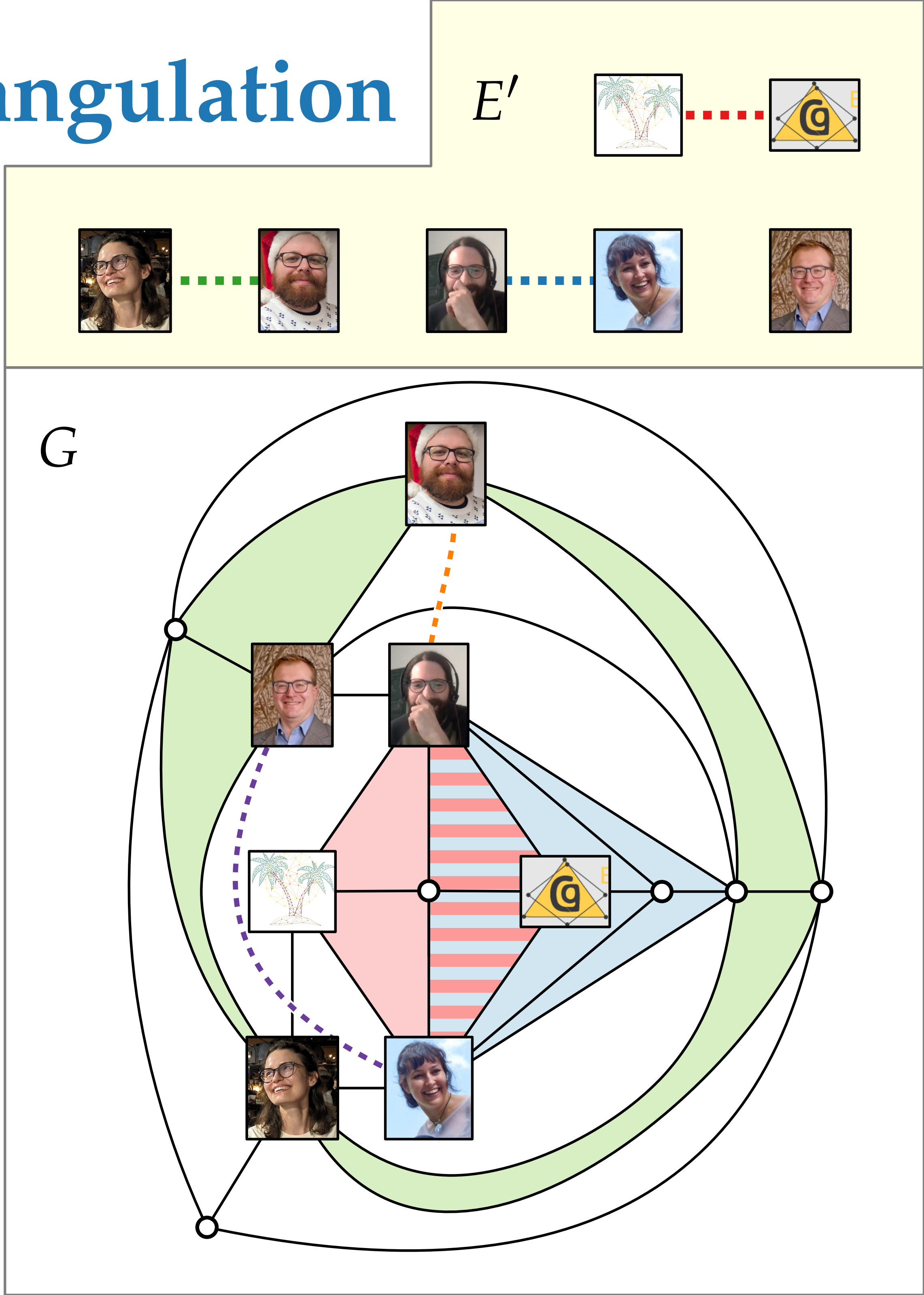

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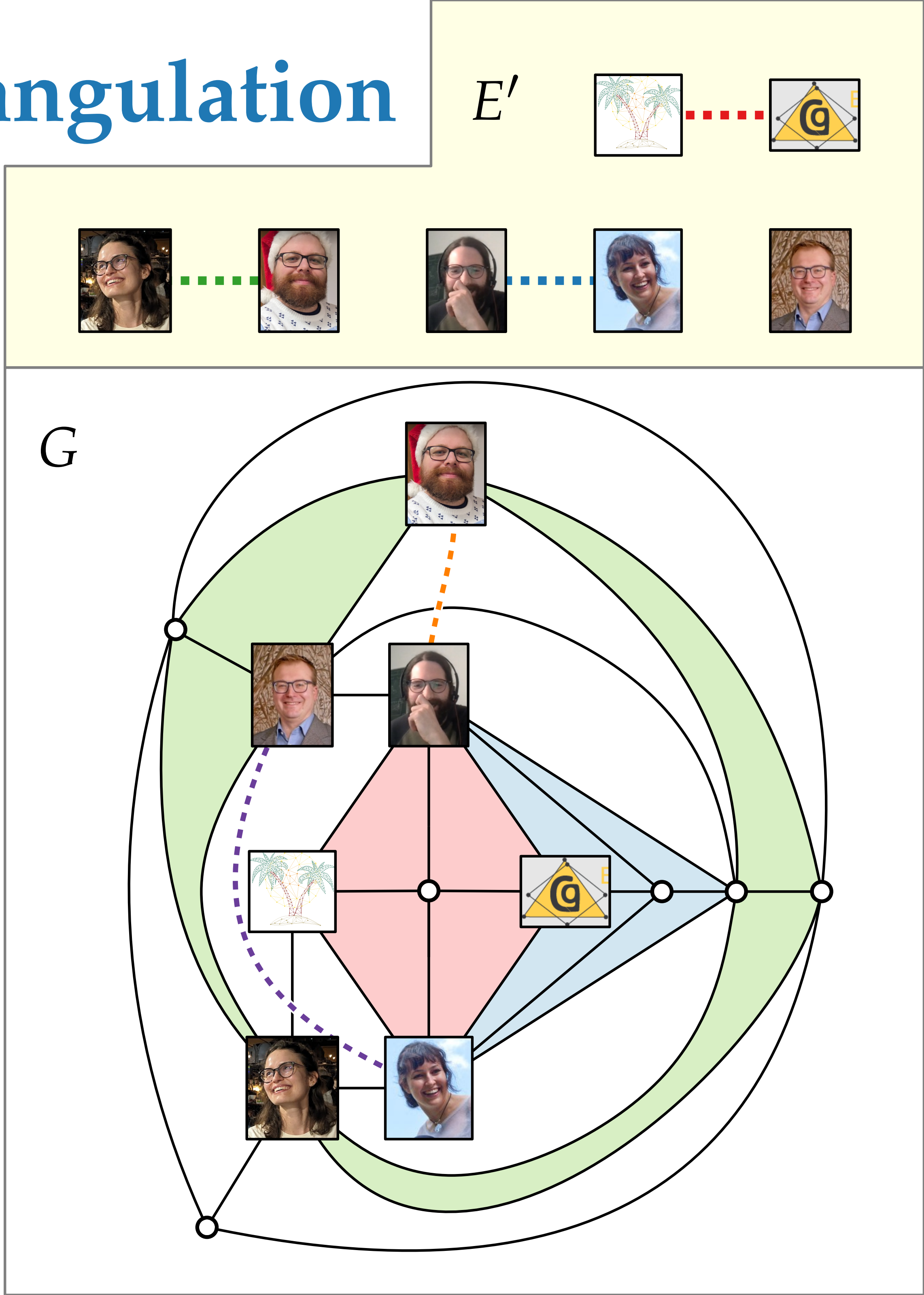

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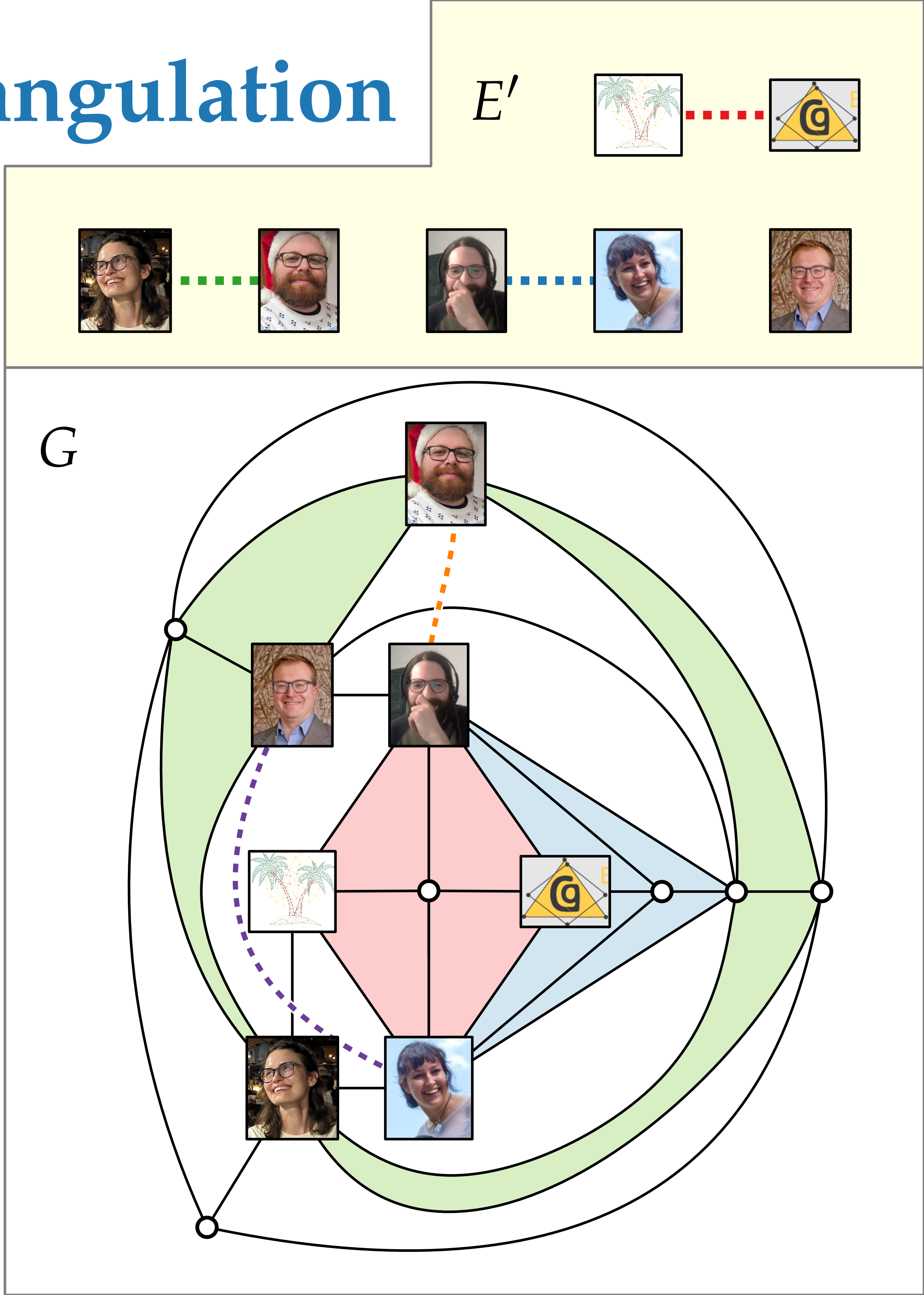
5. All edges have 2 options
   ➡ solve 2-SAT on conflict graph



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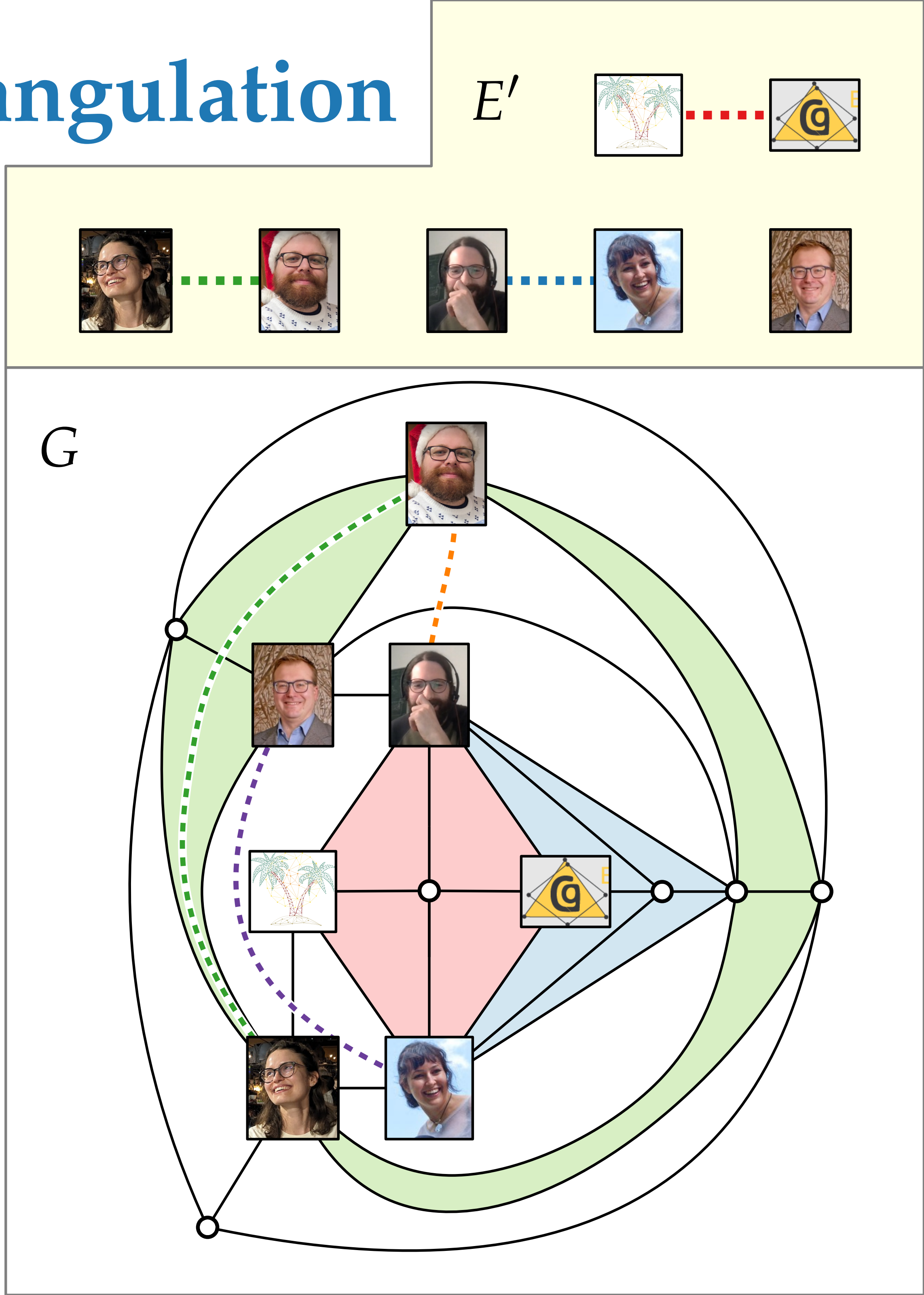

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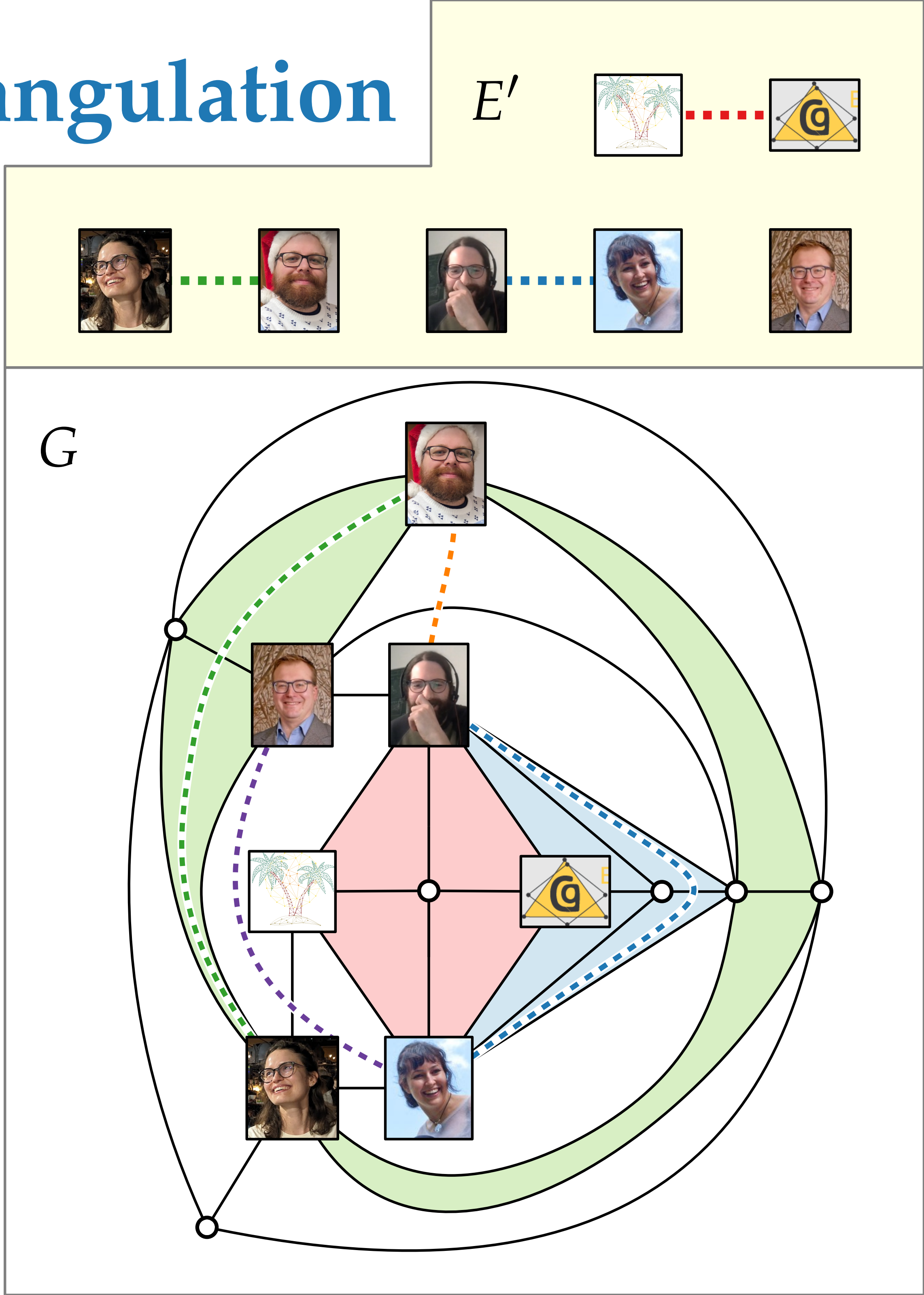

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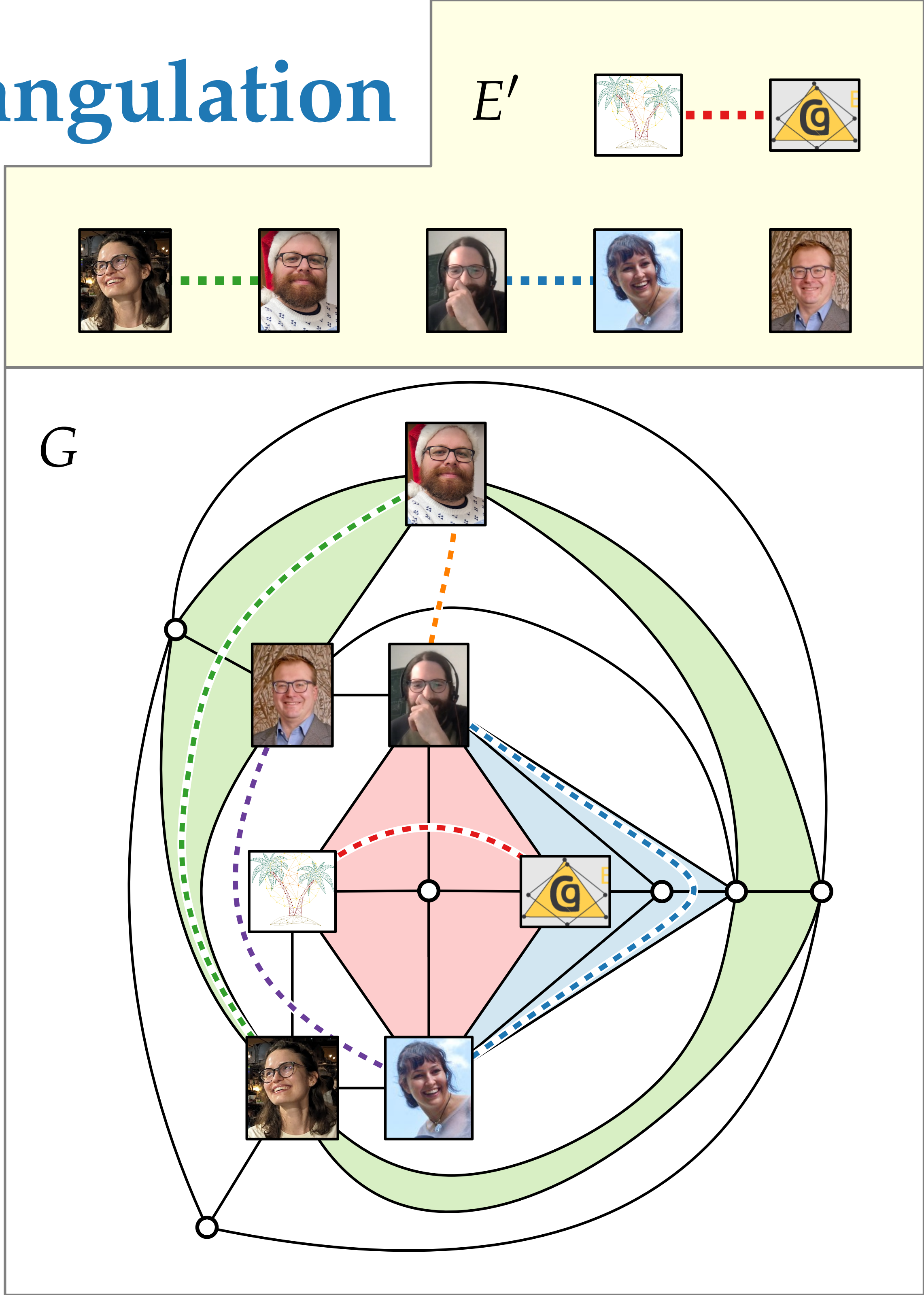

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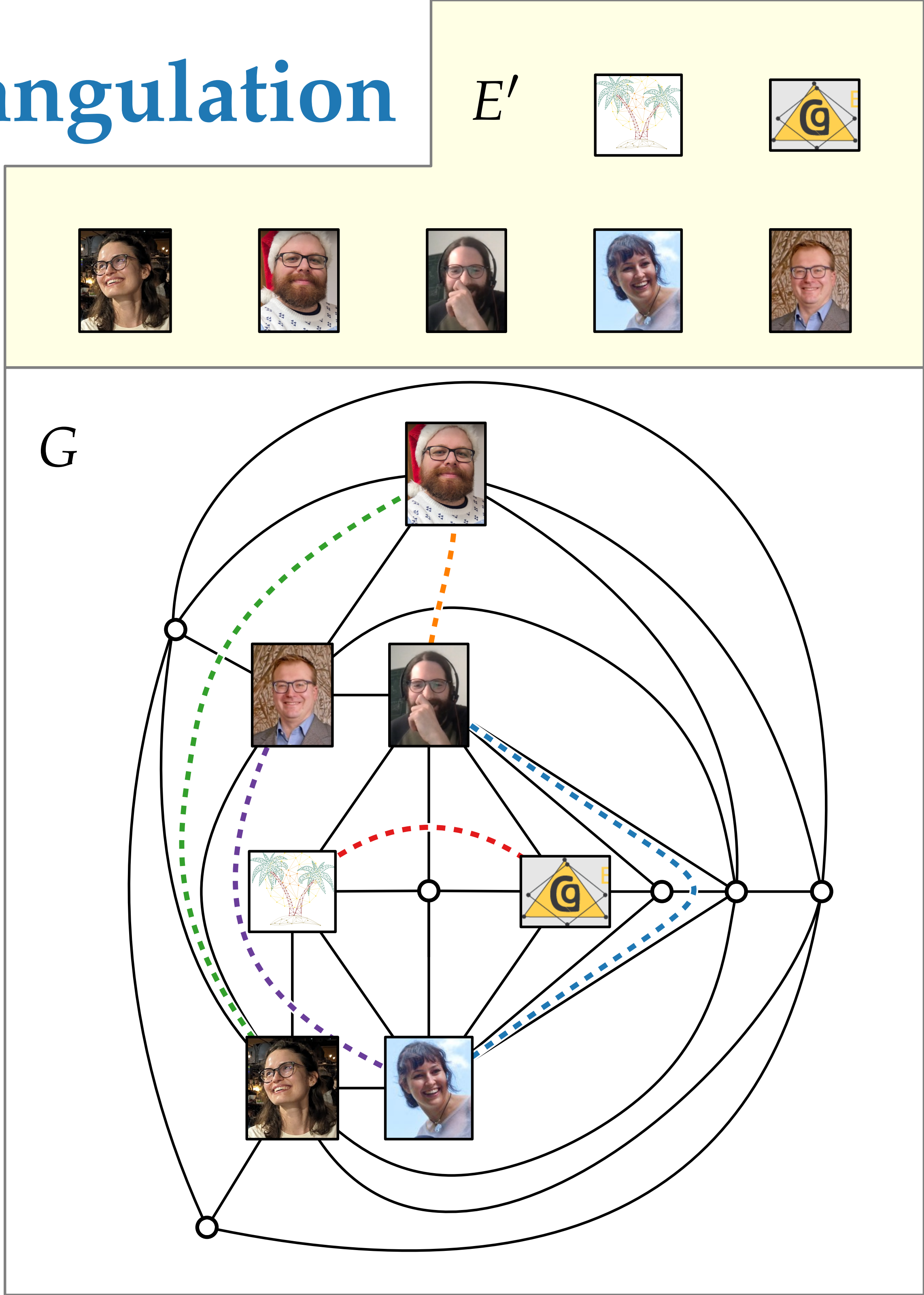

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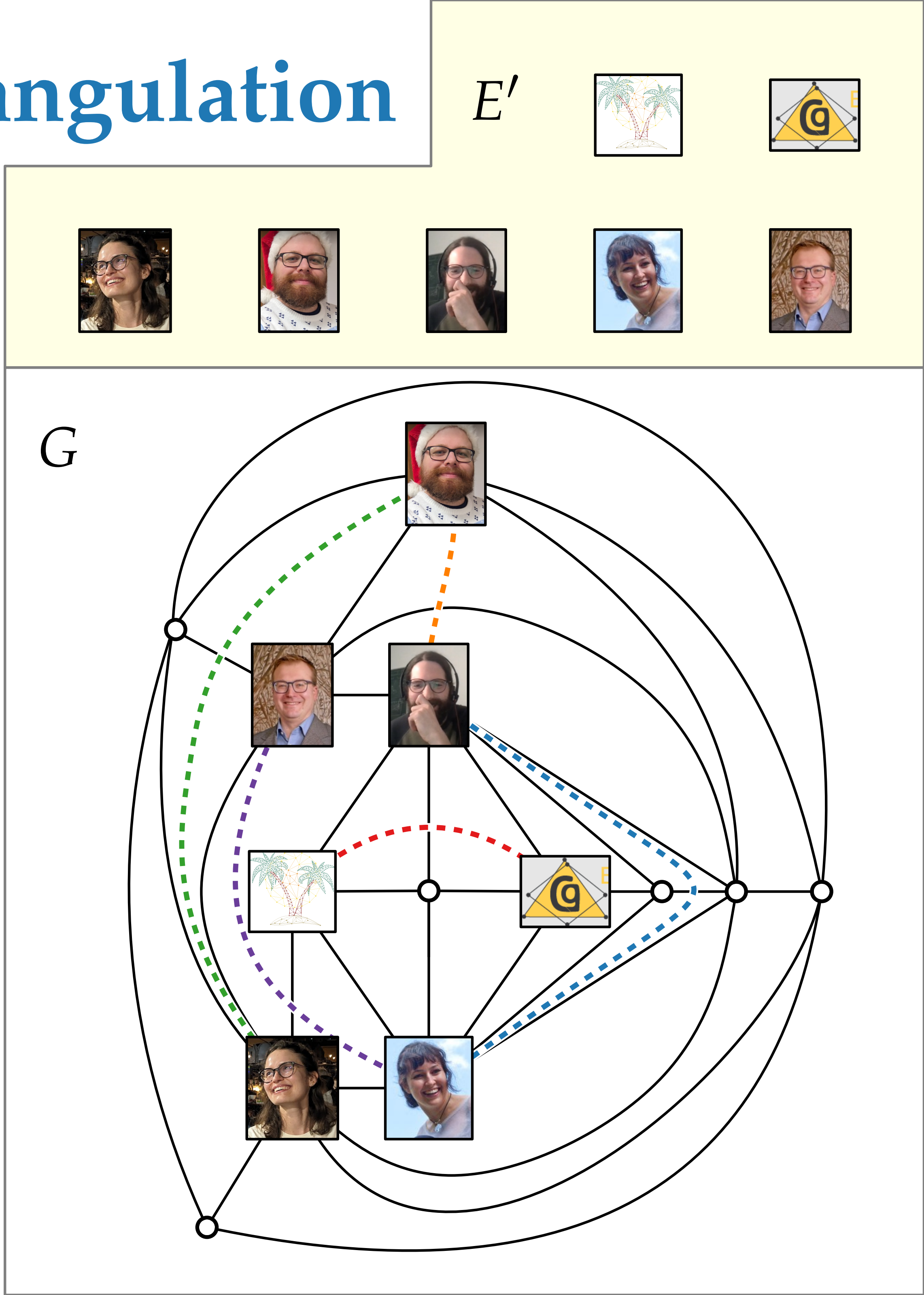

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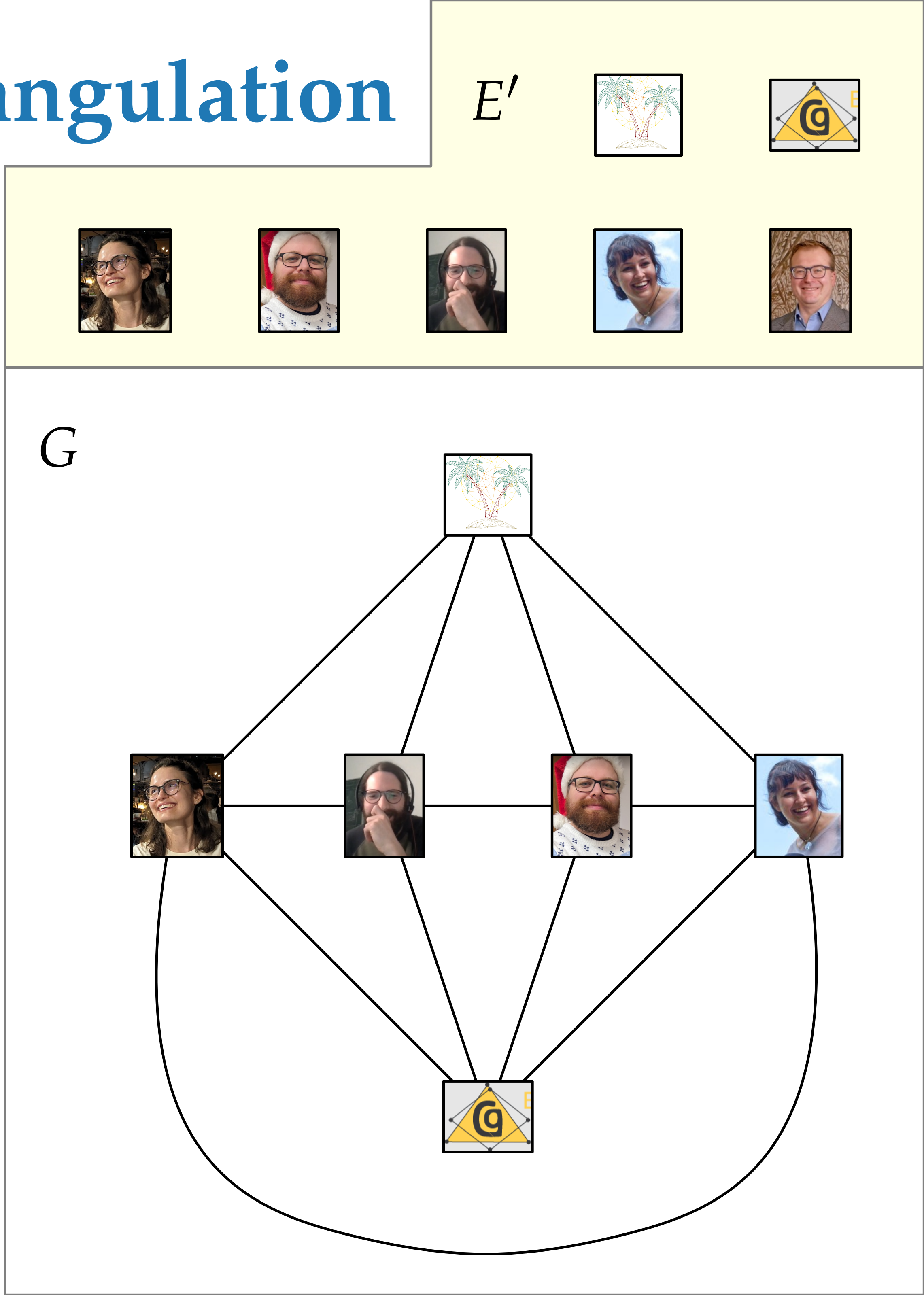

1-Plane Insertion Into a Plane **Triangulation**

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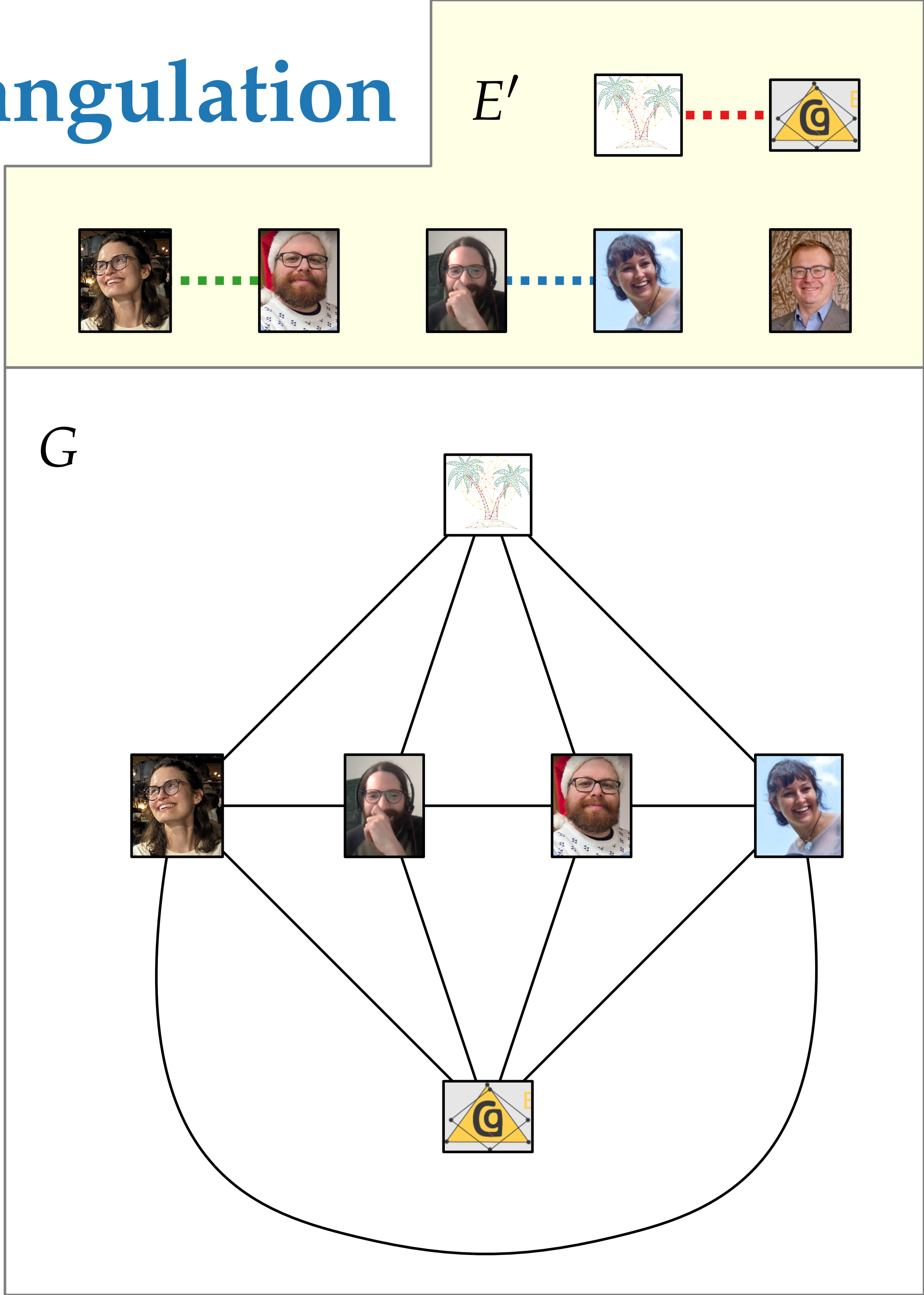

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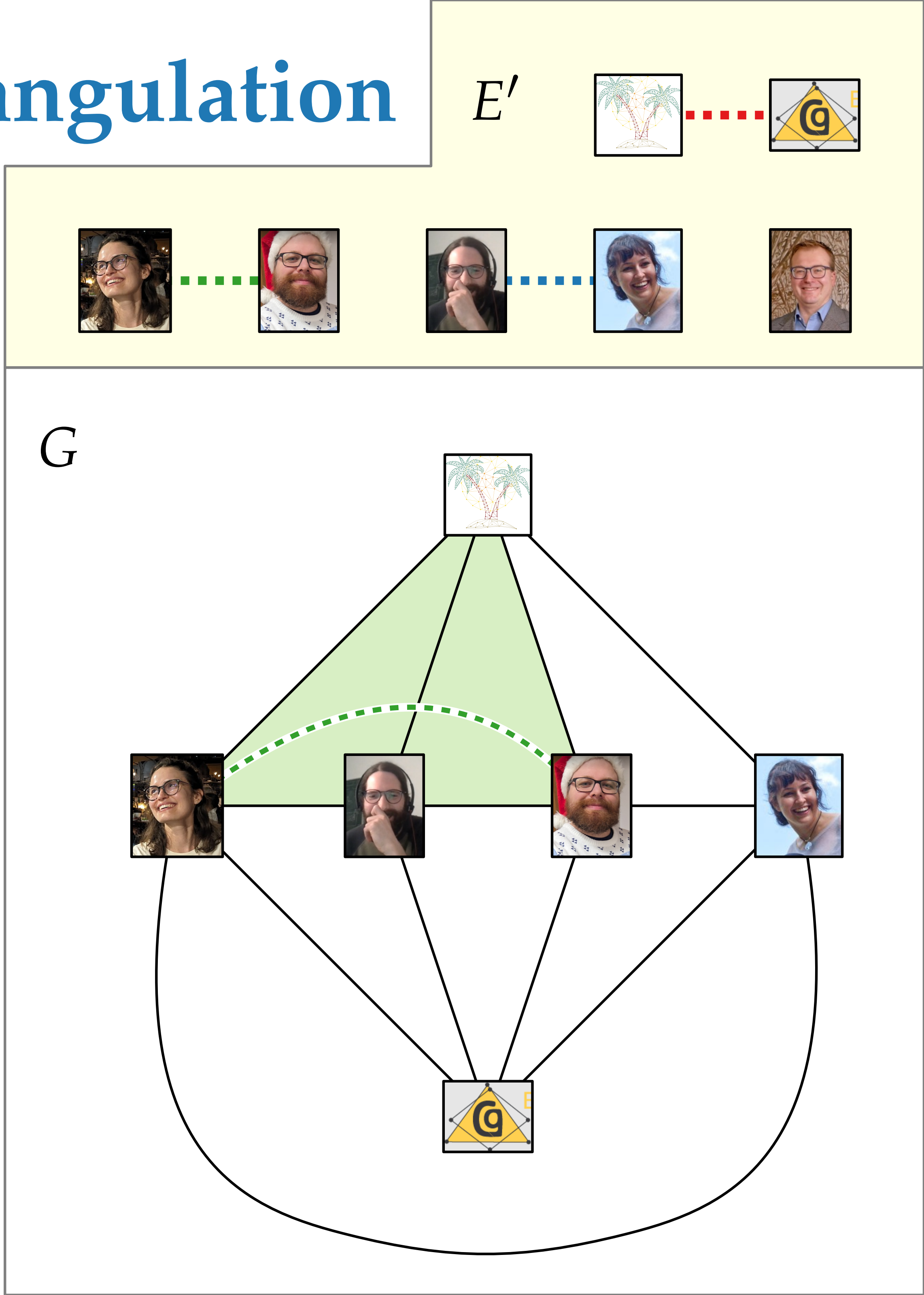

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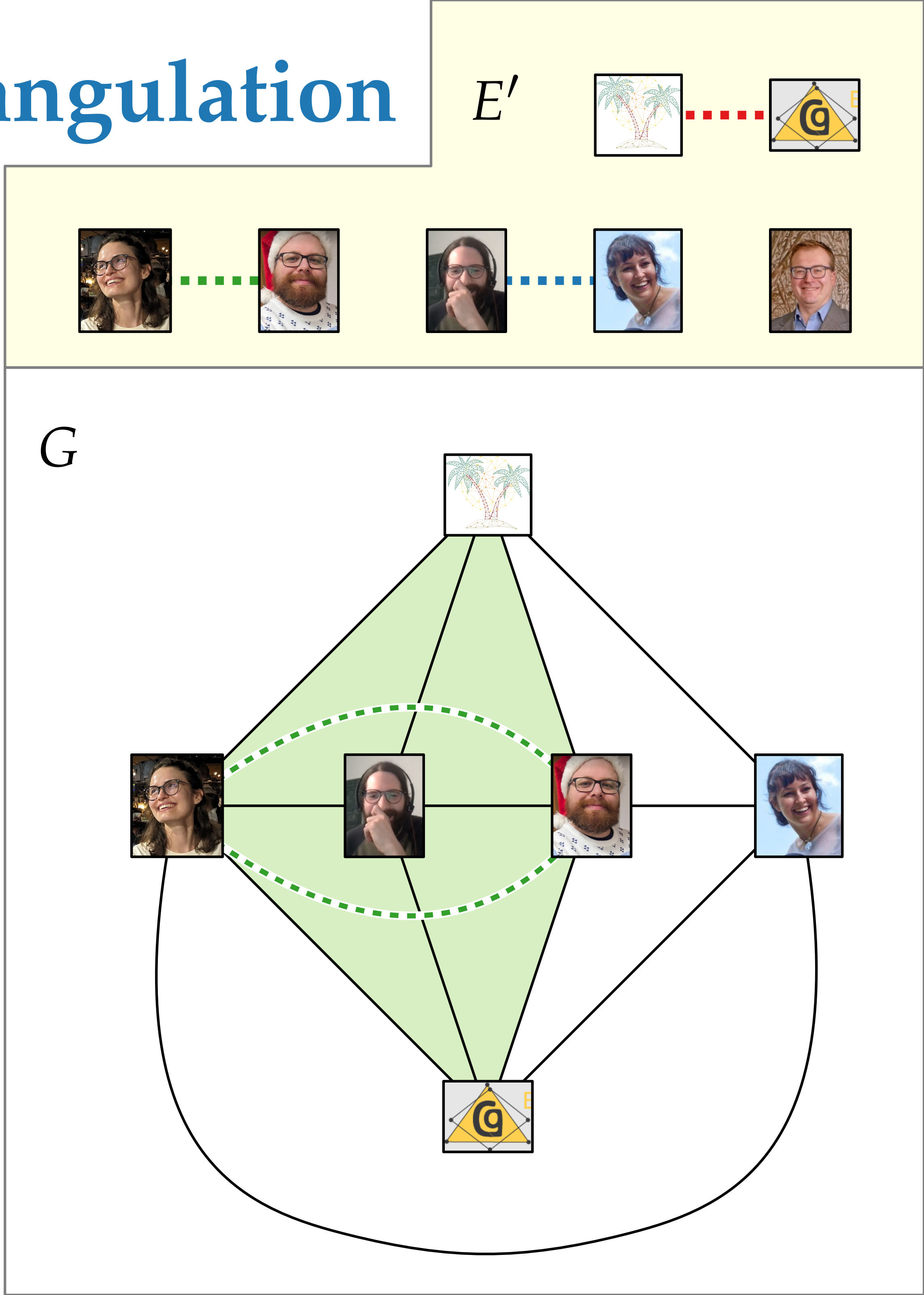

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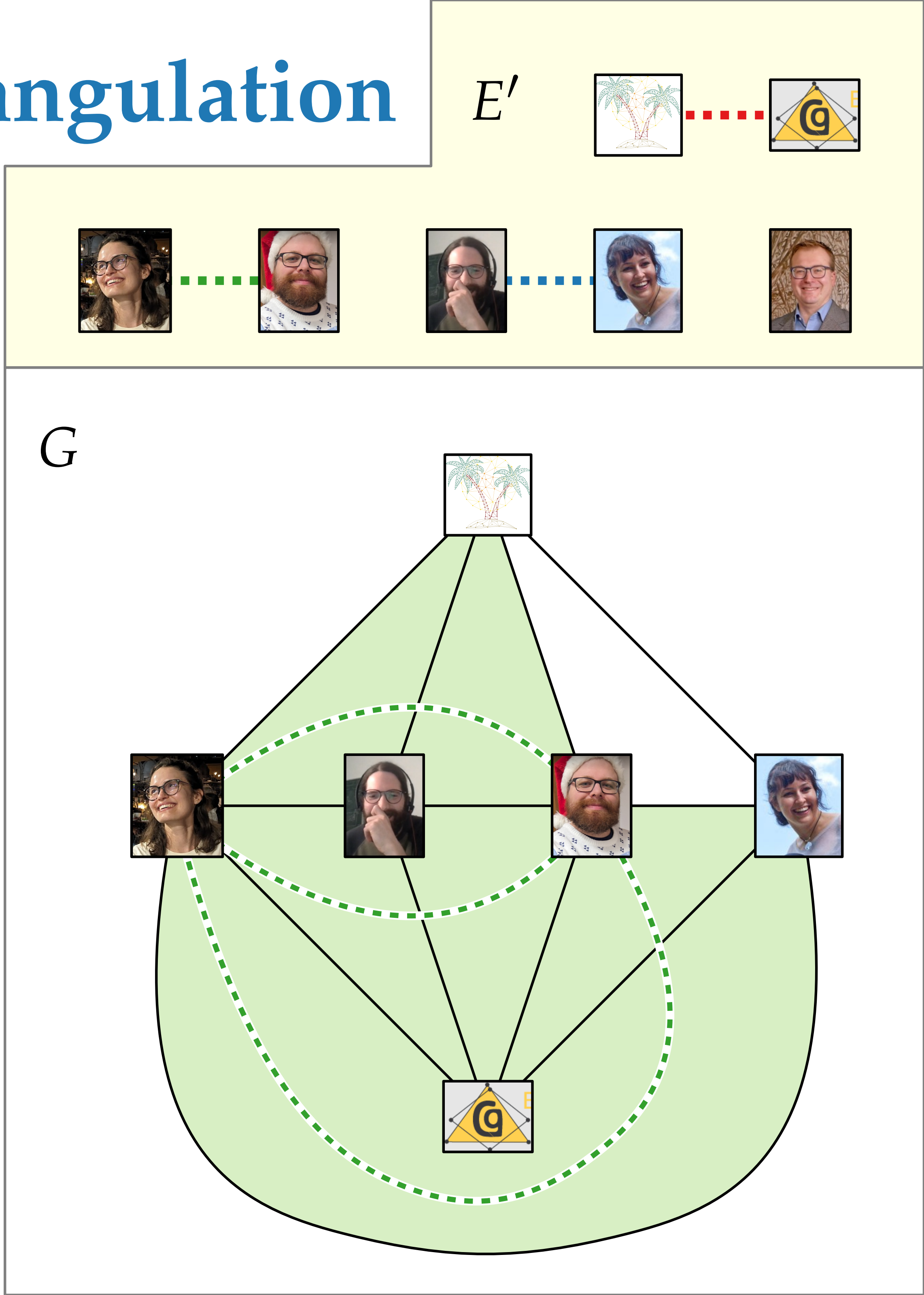

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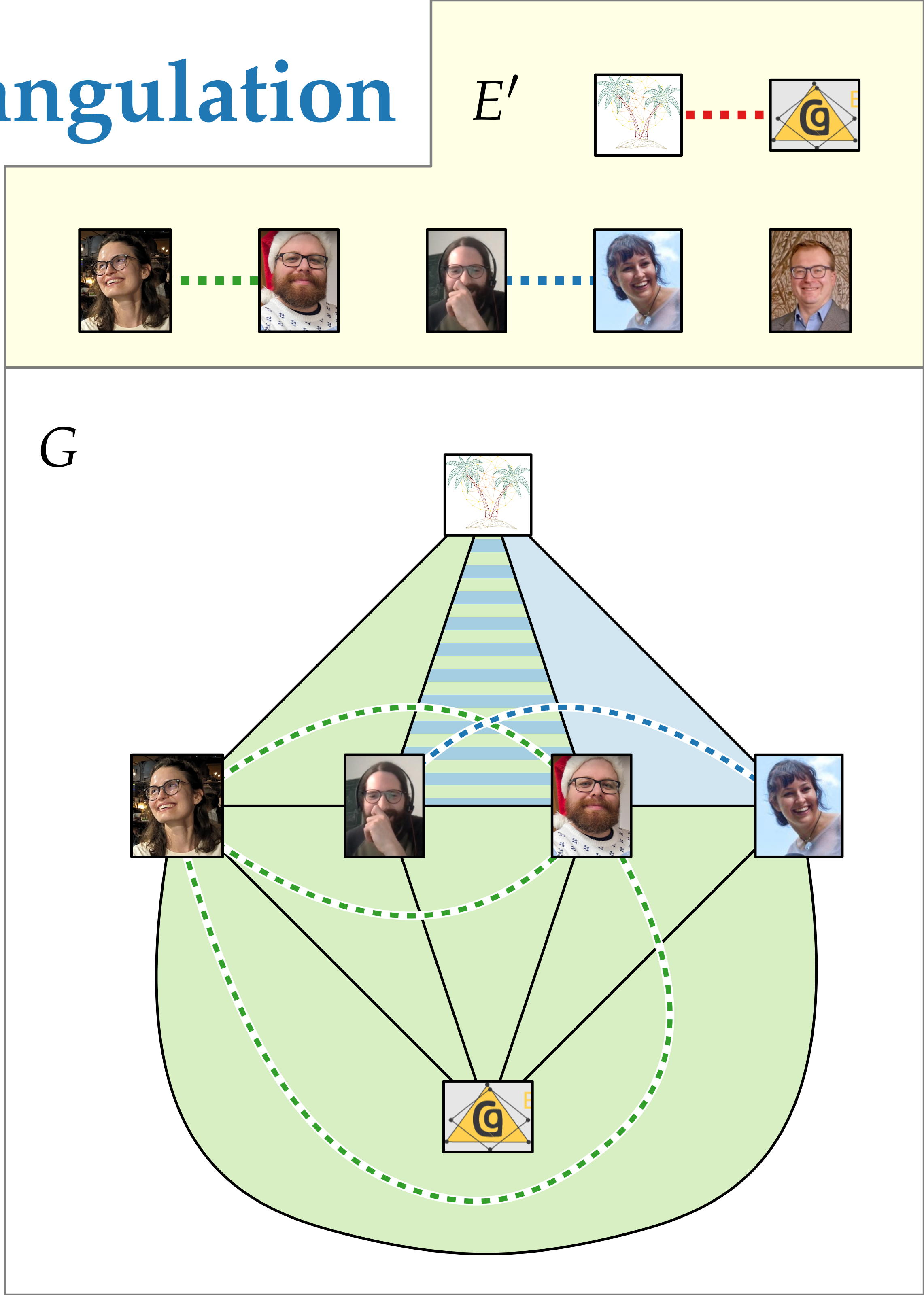

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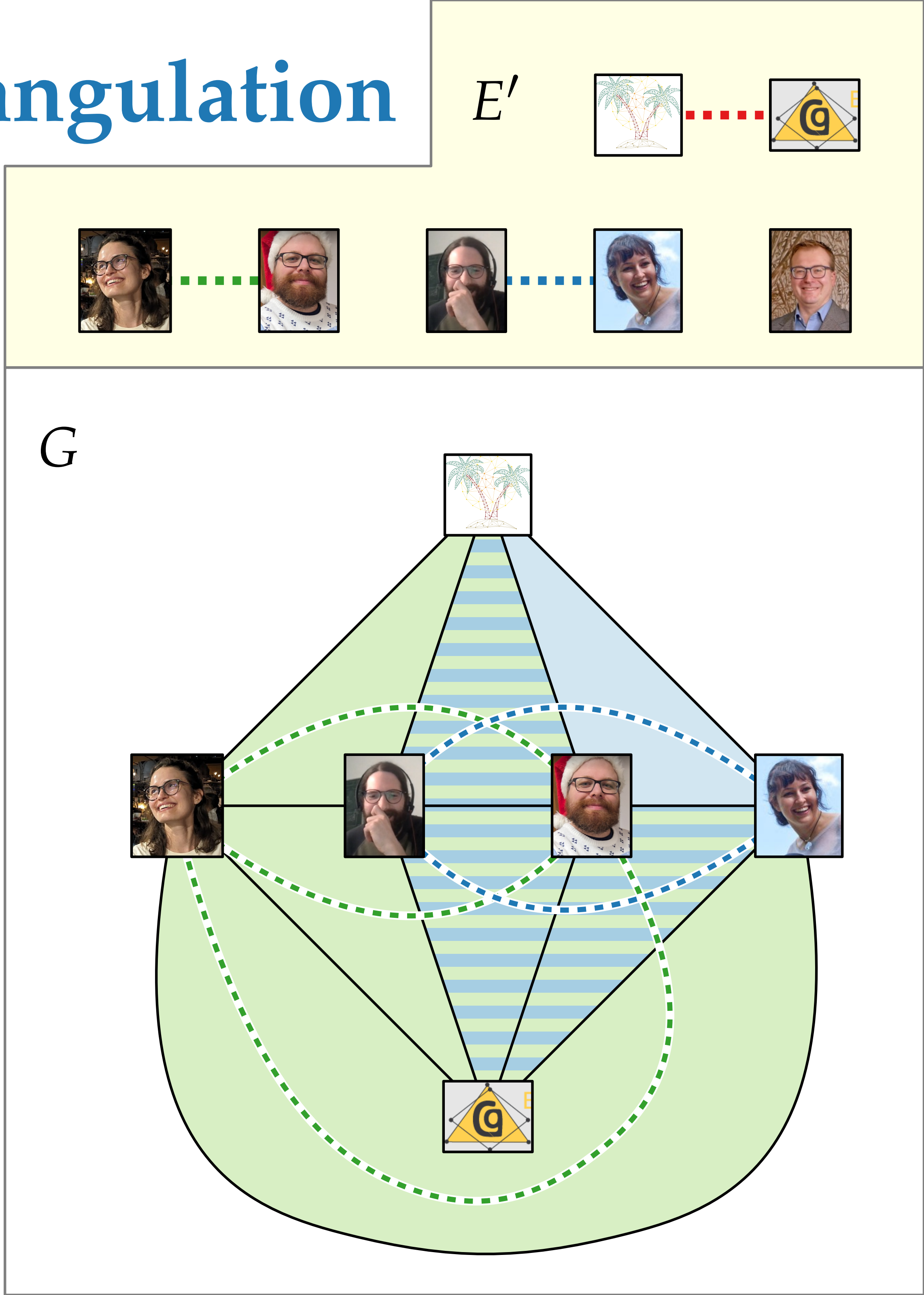

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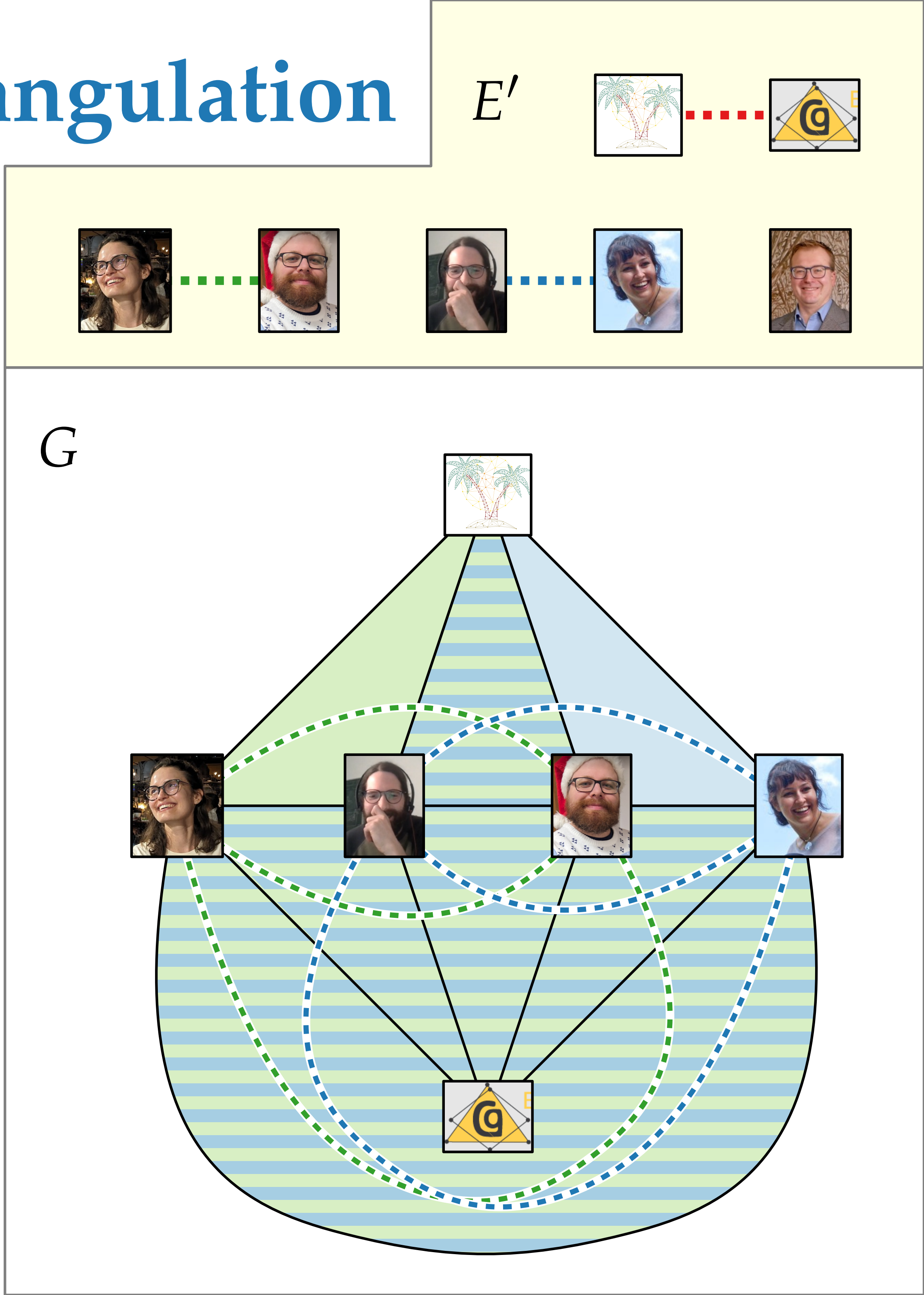

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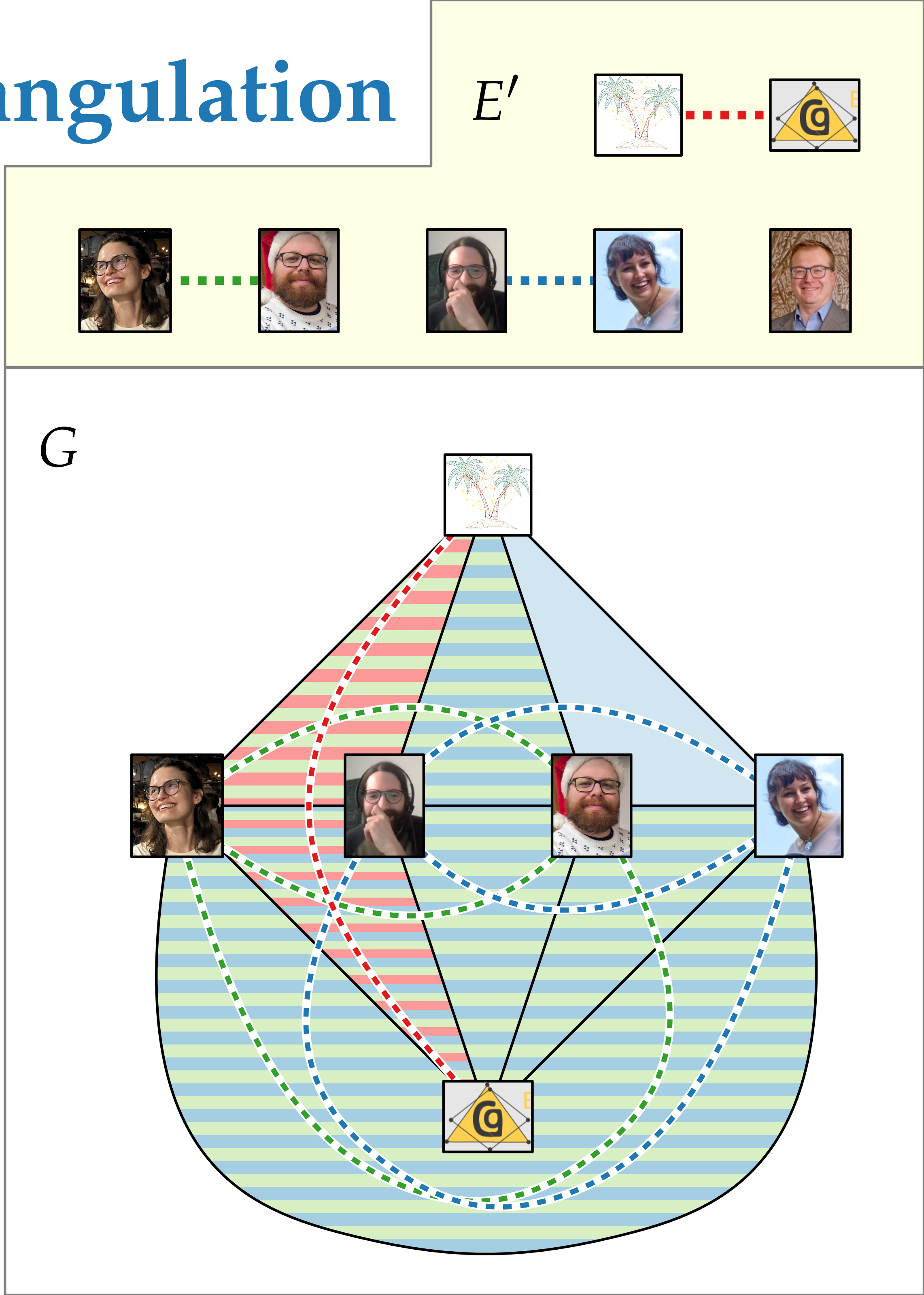

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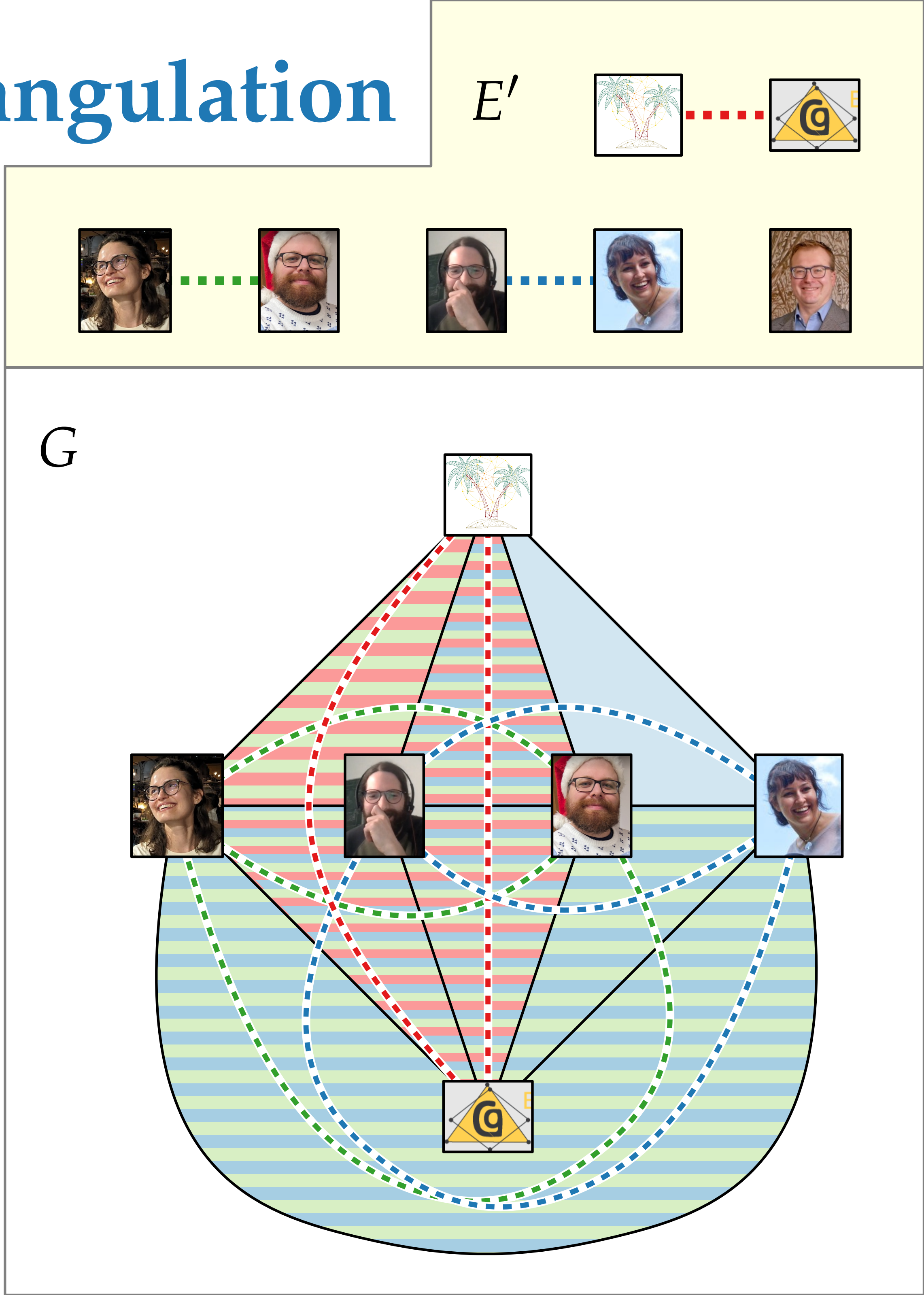

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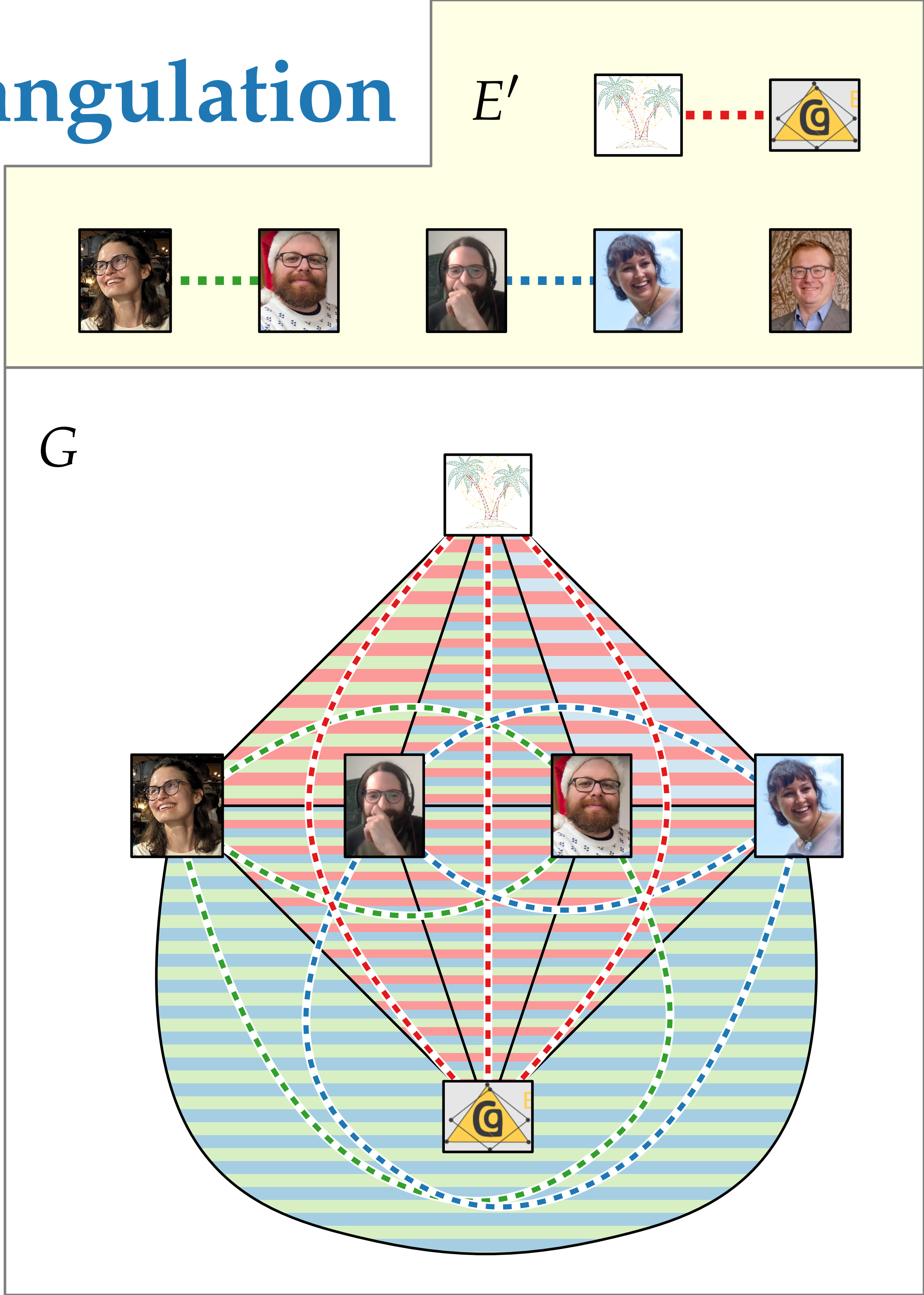

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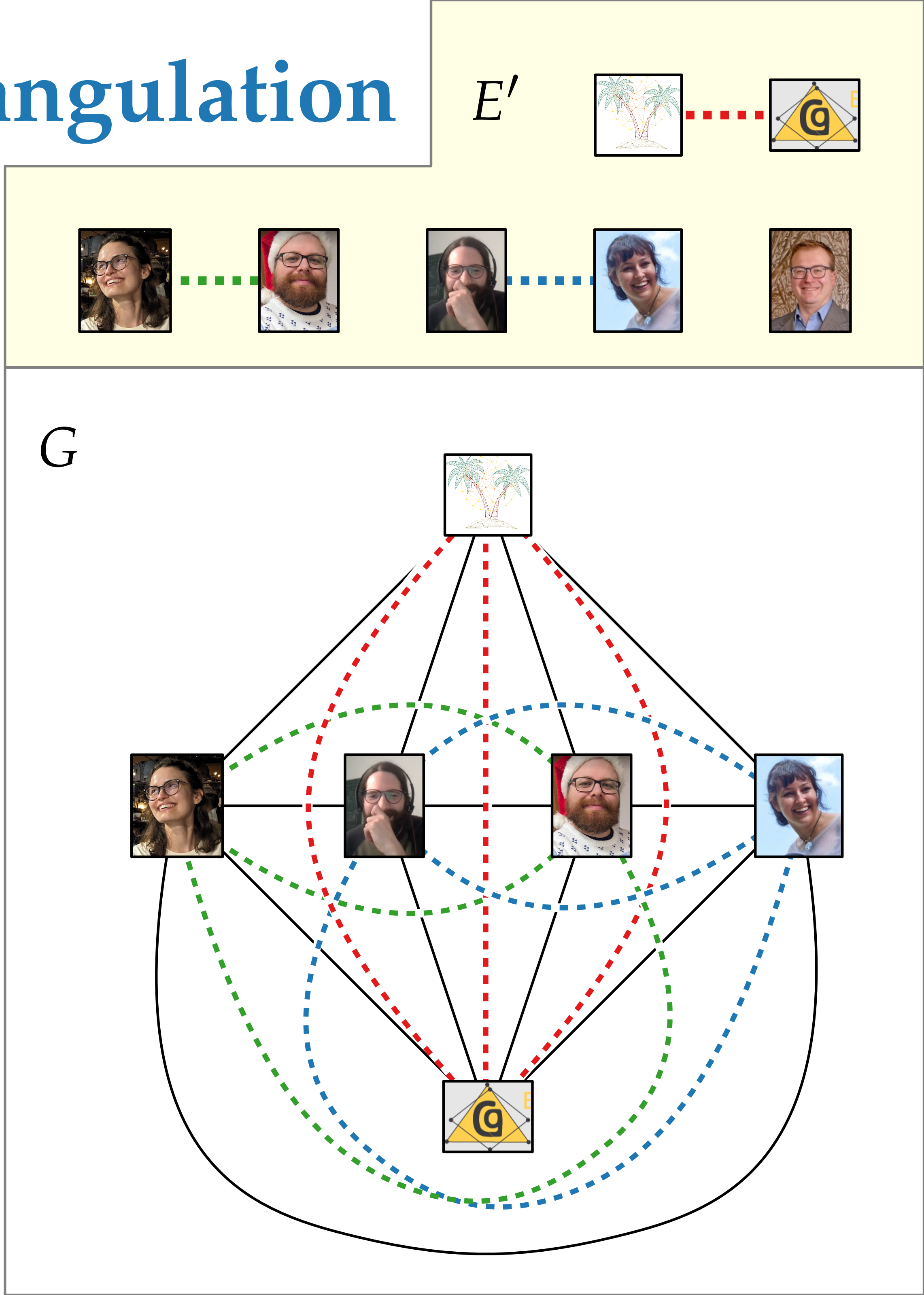

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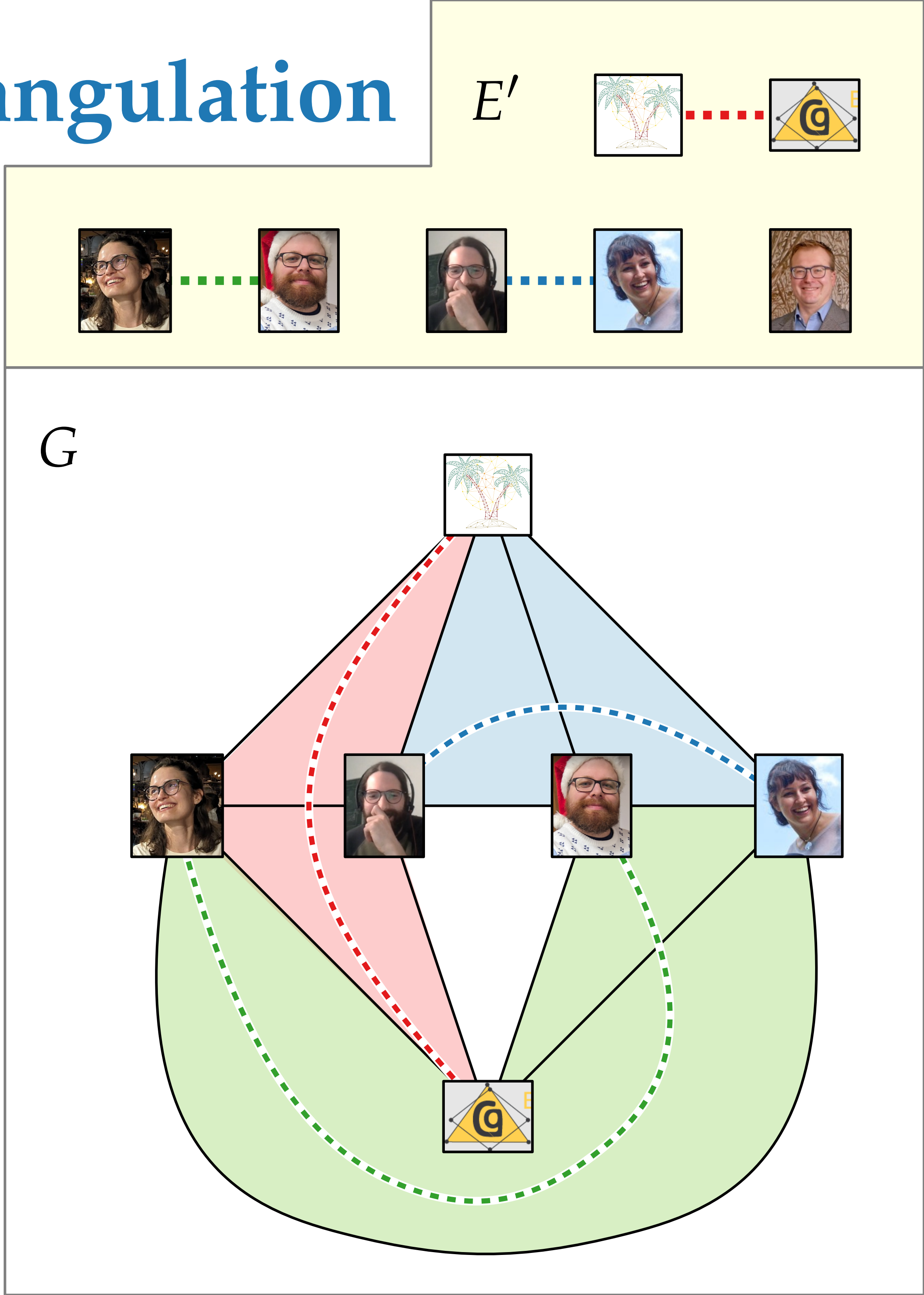

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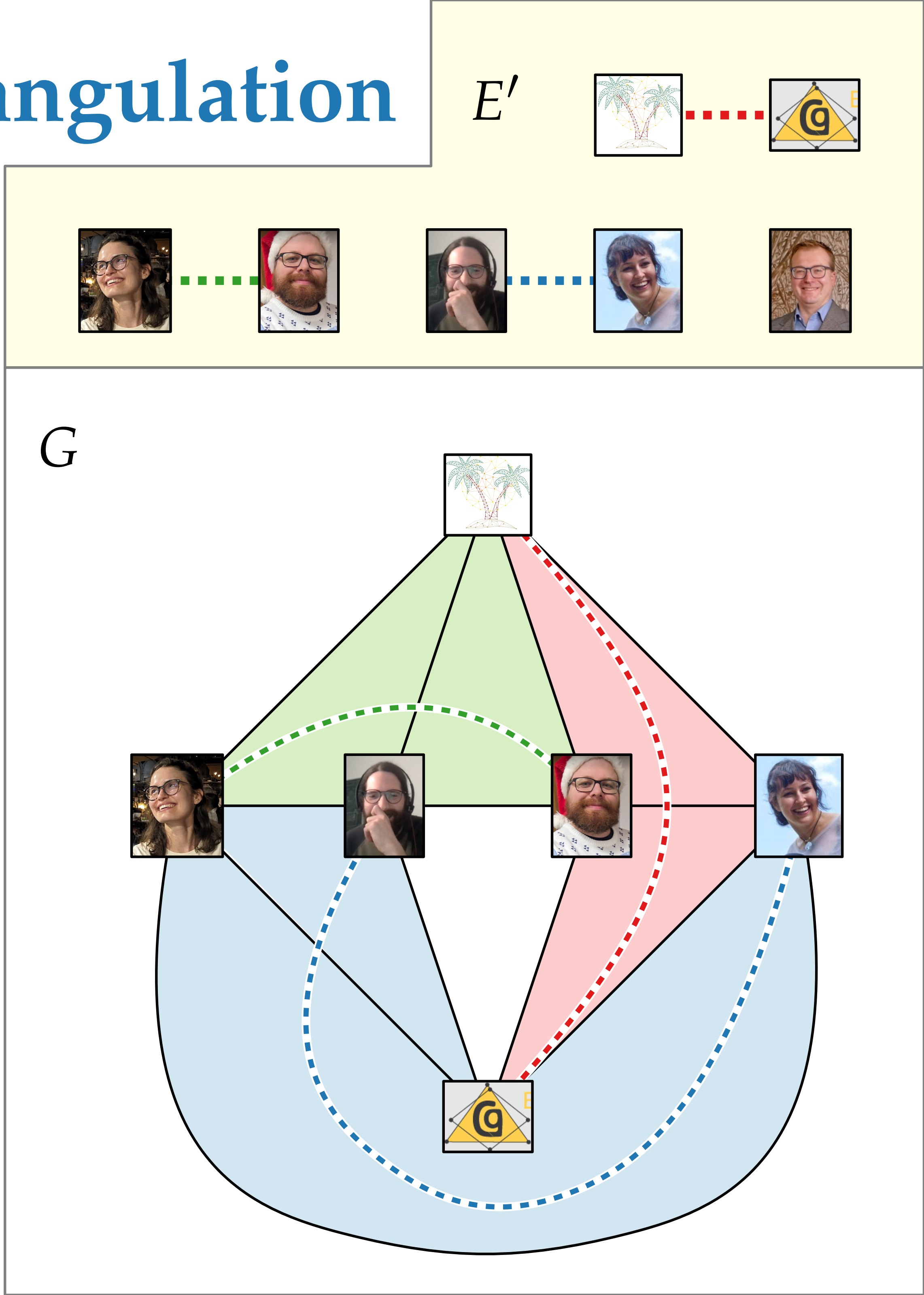

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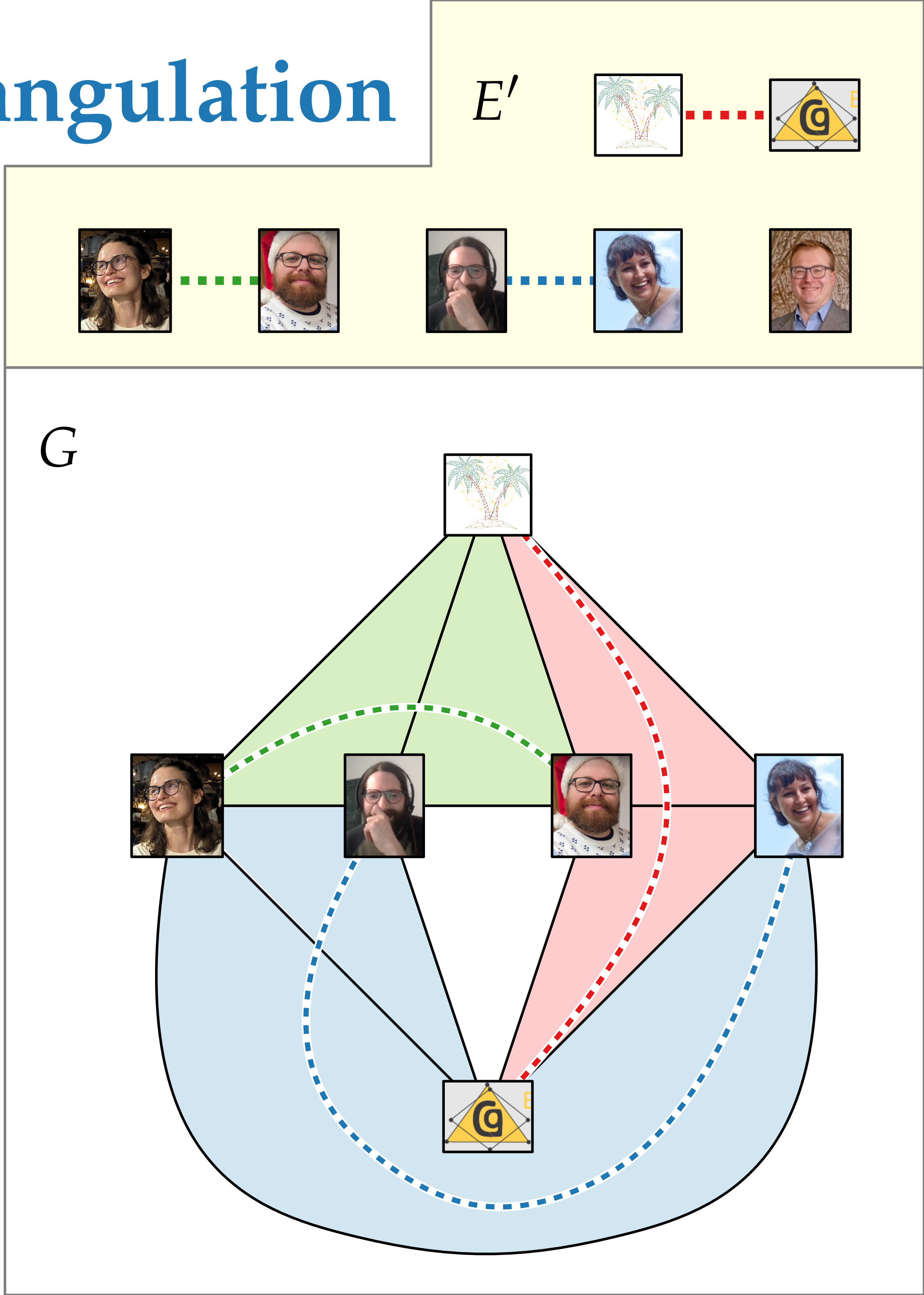
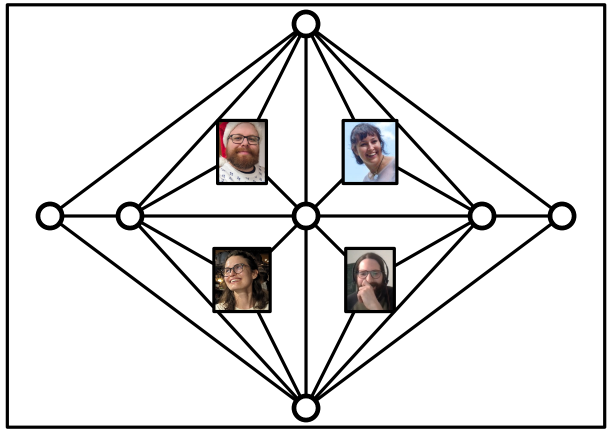
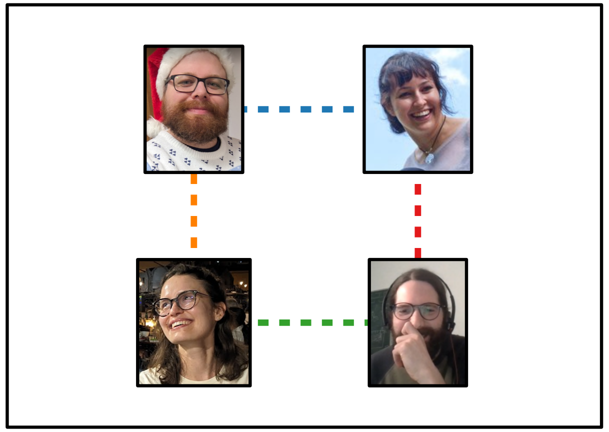
Theorem.
1-plane insertion into a plane triangulation can be solved in $O(n)$ time.



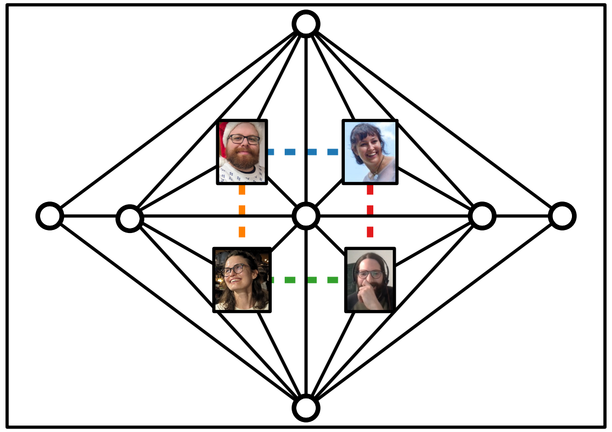
Conclusion



plane graph G

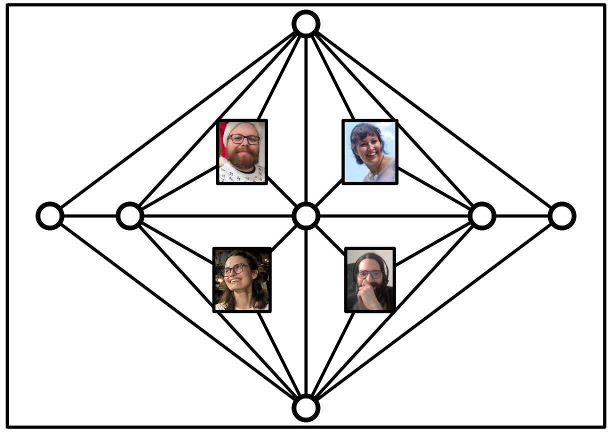


edges E' btw. vtcs in G



1-planar drawing of $G + E'$
that keeps the embedding of G

Conclusion



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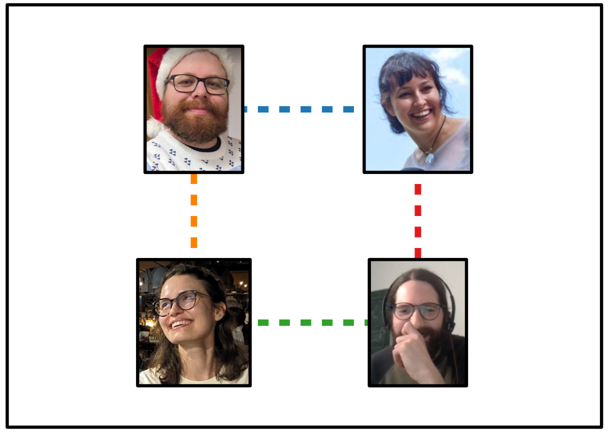

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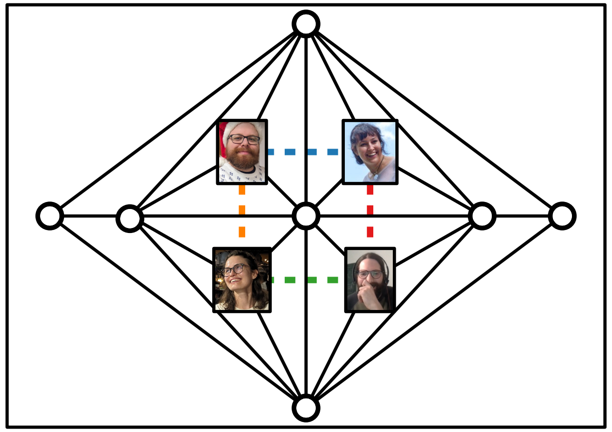
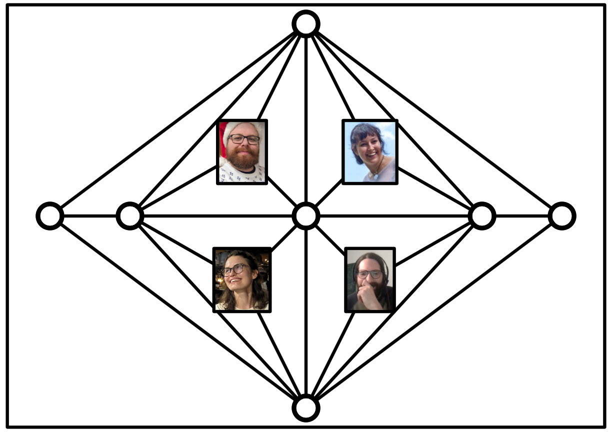


1-planar drawing of $G + E'$ that keeps the embedding of G

- G triangulated ➡ $\mathcal{O}(n)$ time

Conclusion



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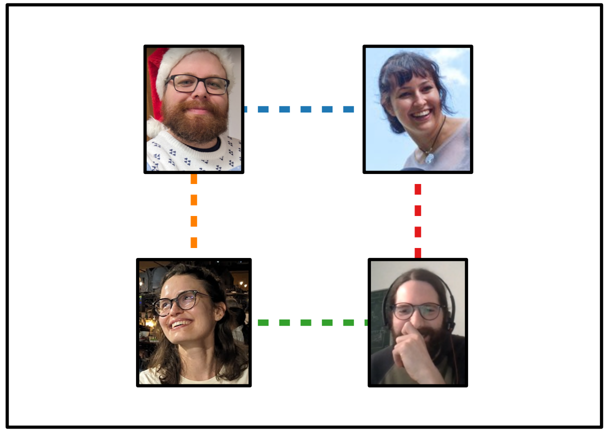

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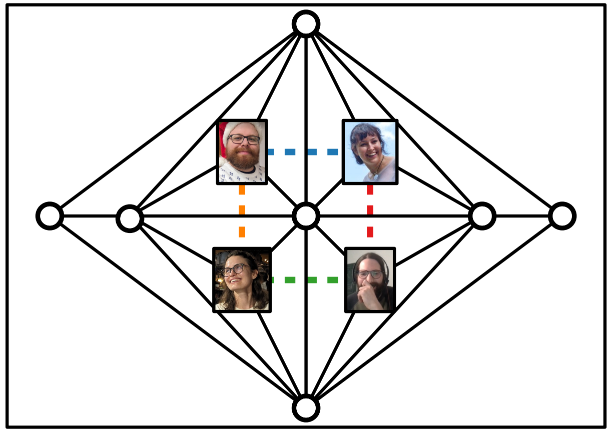
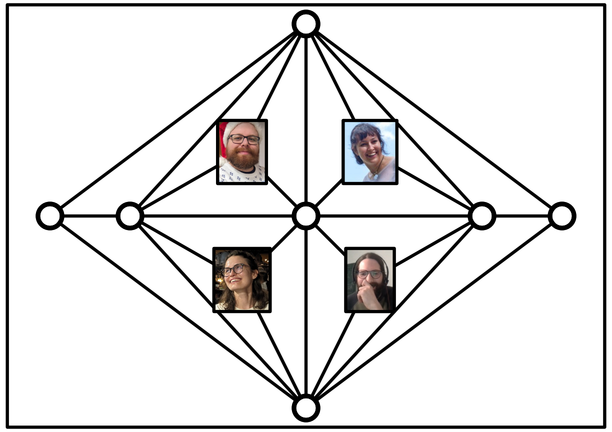


1-planar drawing of $G + E'$ that keeps the embedding of G

- G triangulated ➡ $\mathcal{O}(n)$ time
- G biconnected ➡ NP-complete

Conclusion



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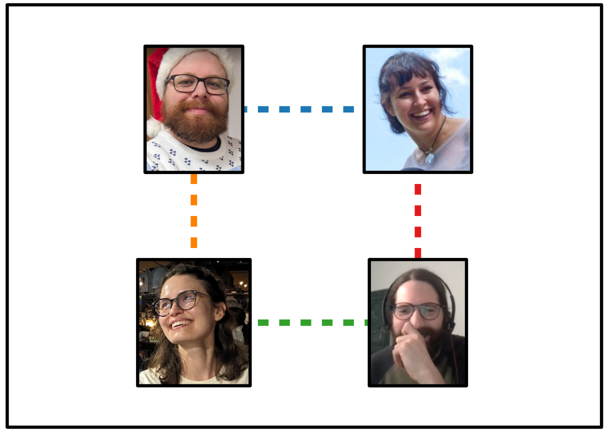

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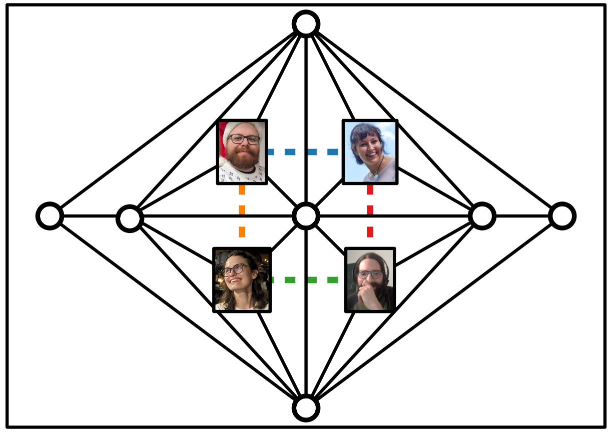

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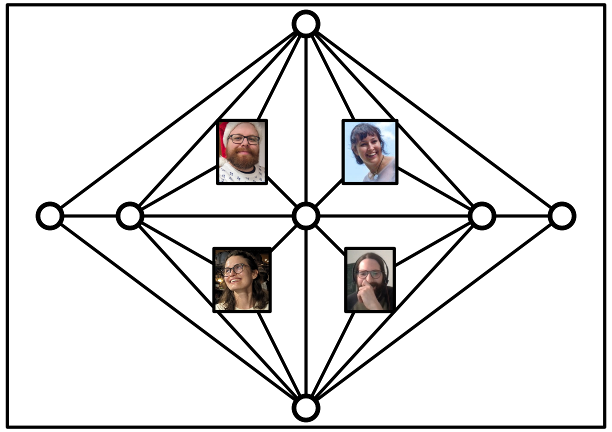
Open Problems

Conclusion



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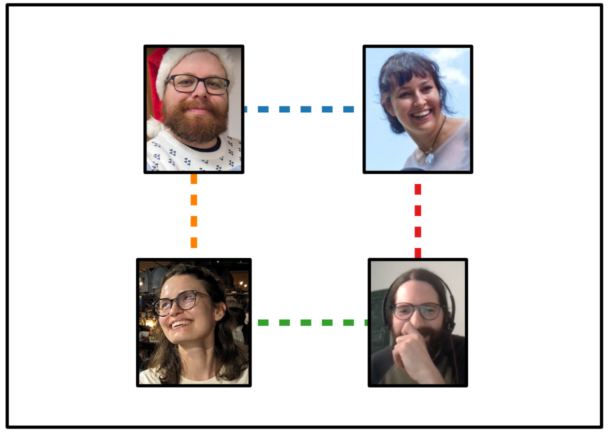

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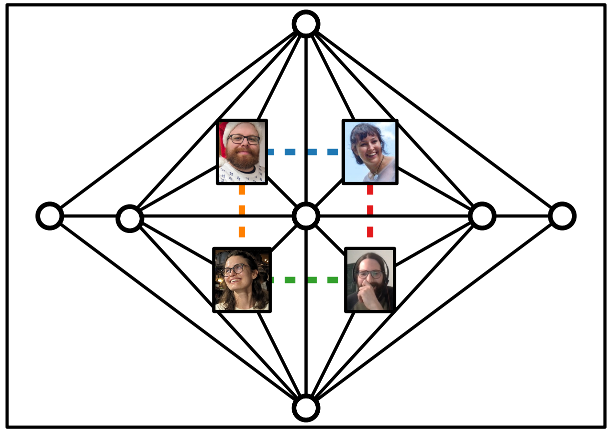

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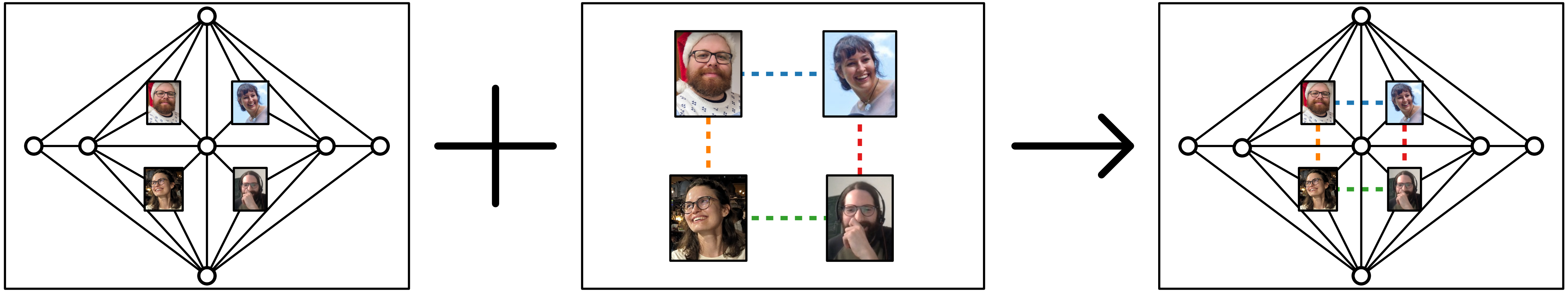
Open Problems

- G triconnected?

Conclusion



plane graph G edges E' btw. vtcs in G **1-planar** drawing of $G + E'$ that keeps the embedding of G

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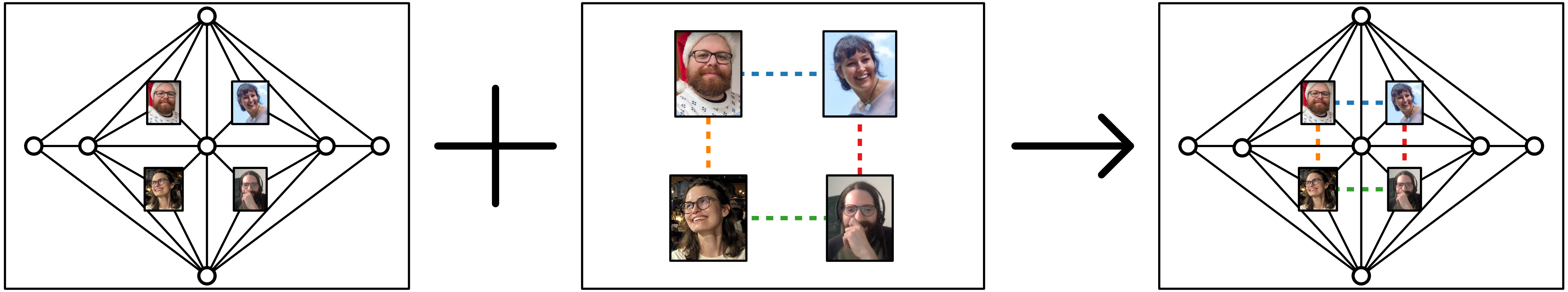
Open Problems

- G triconnected?
- Other drawing styles? For example

Conclusion



plane graph G — edges E' btw. vtcs in G — **1-planar** drawing of $G + E'$ that keeps the embedding of G

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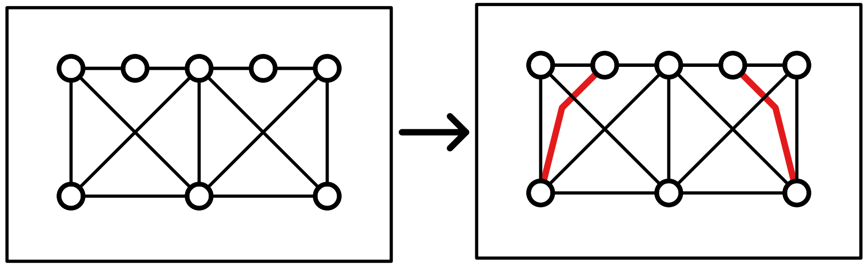
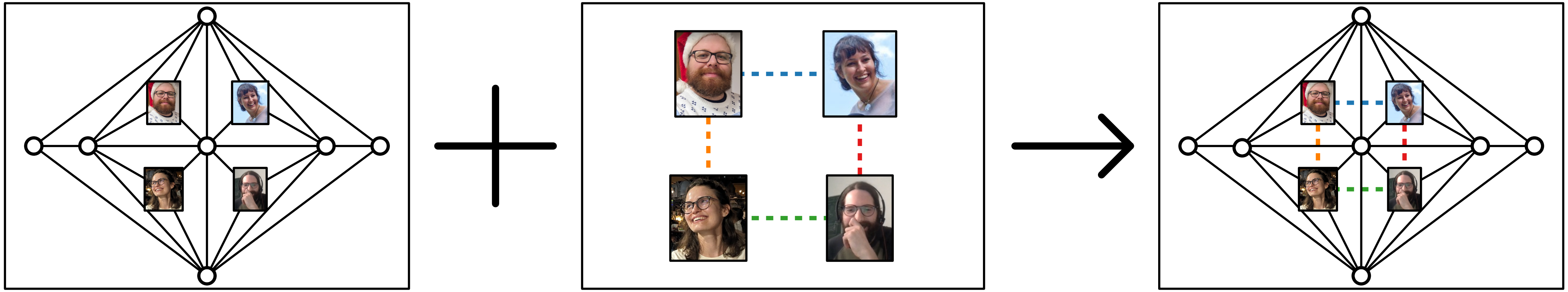


RAC → 1-bend RAC

Conclusion



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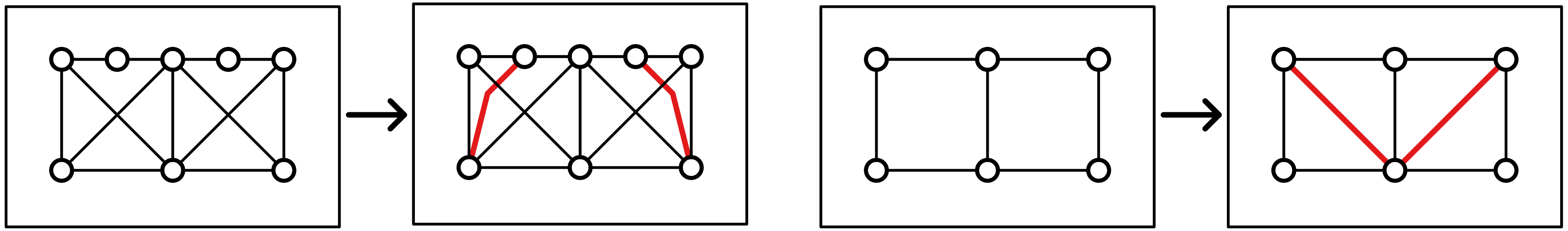
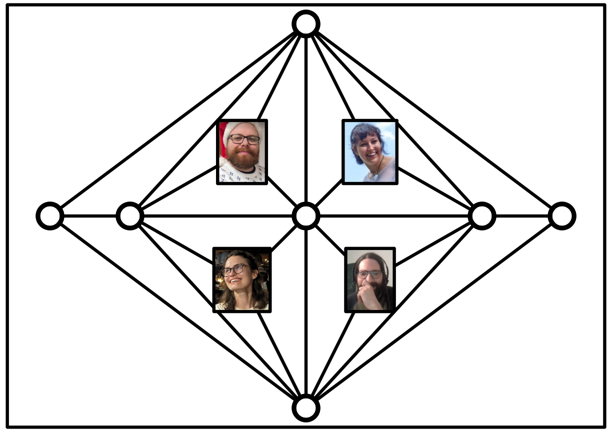
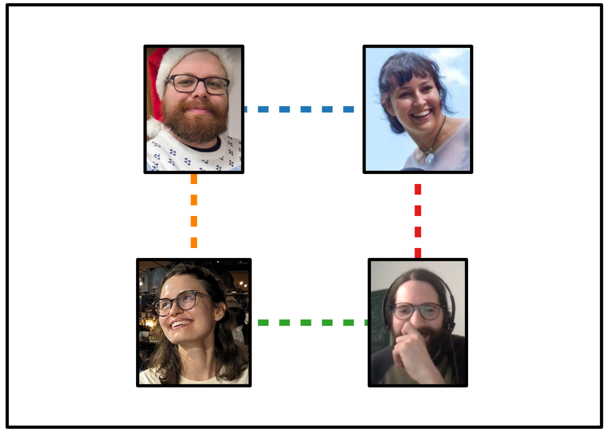


RAC → 1-bend RAC   orthogonal → octilinear

Conclusion



plane graph G



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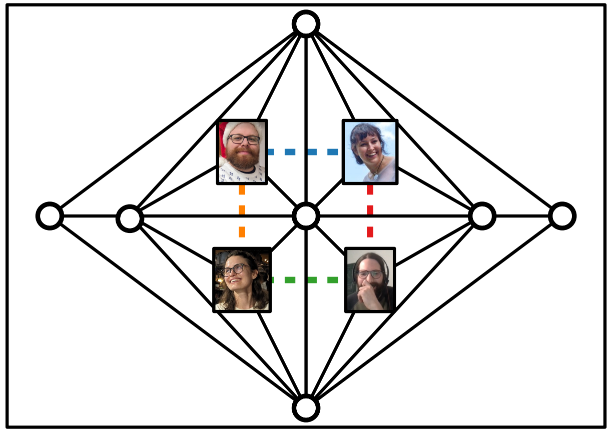

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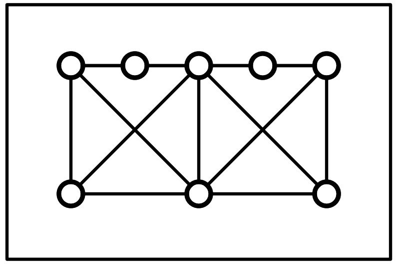
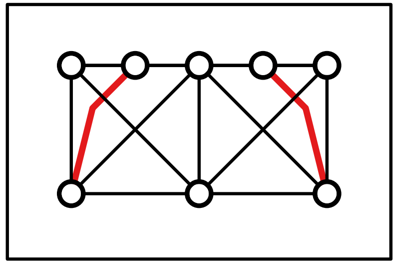
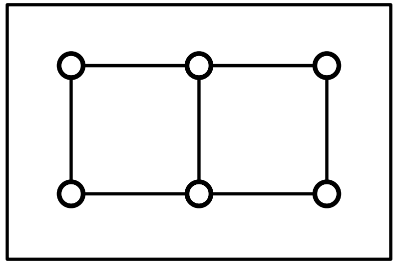


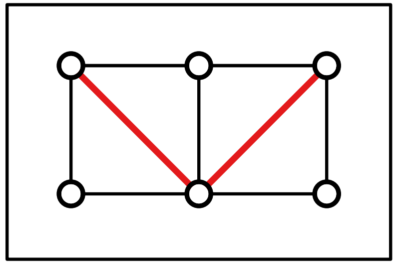
RAC



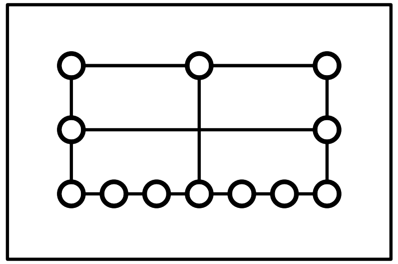
1-bend RAC



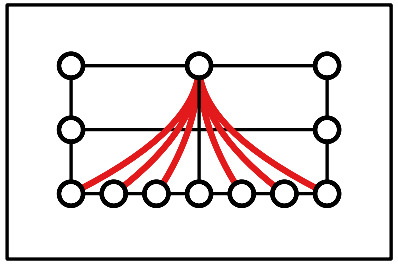
orthogonal



octilinear



1-planar



fan-planar

1-Plane Insertion Into a Plane **Biconnected Graph**

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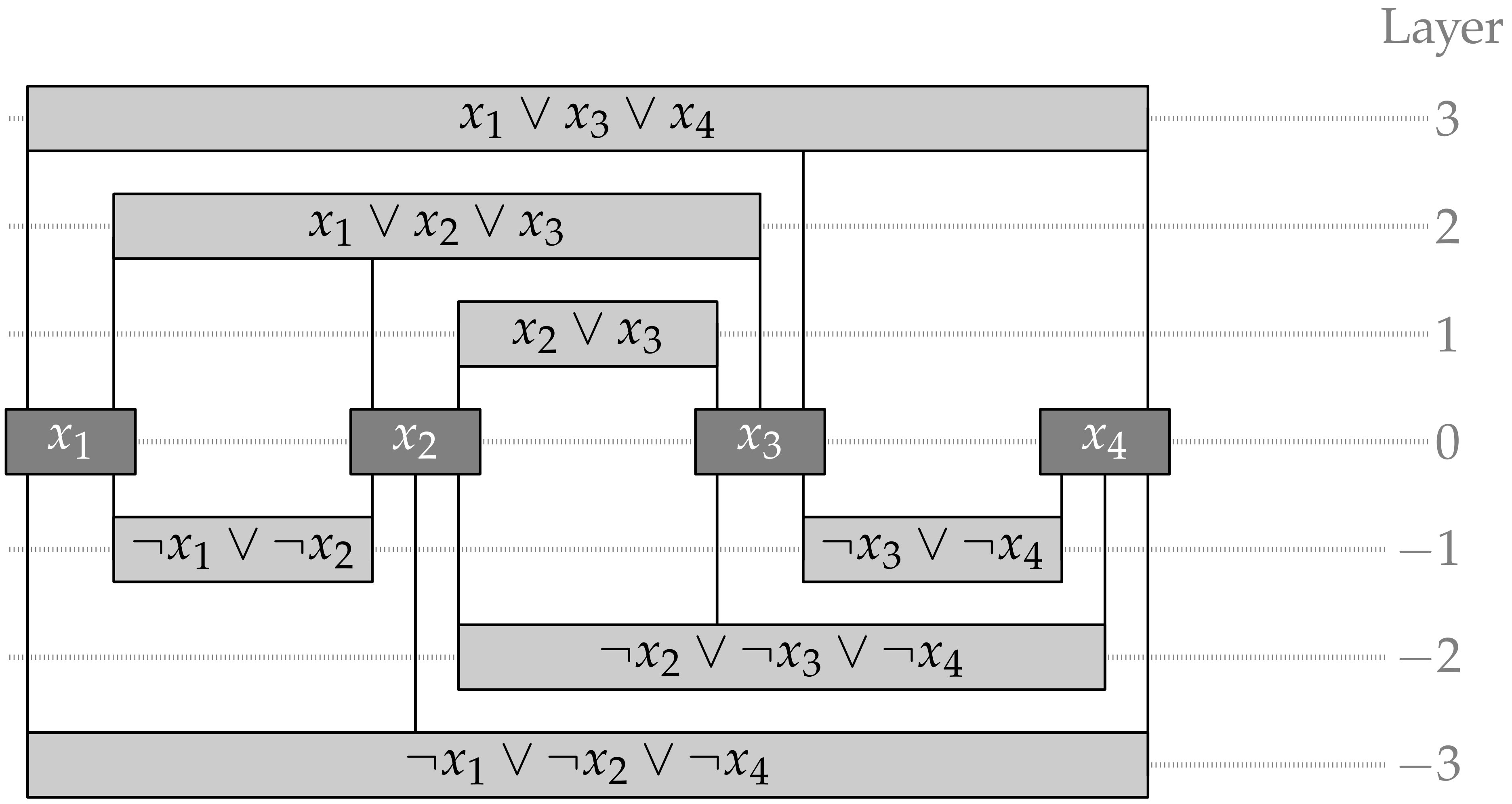
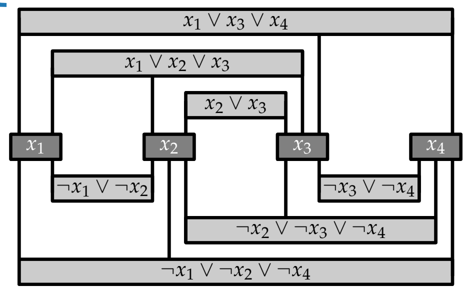


Planar Monotone 3-SAT

1-Plane Insertion Into a Plane **Biconnected Graph**



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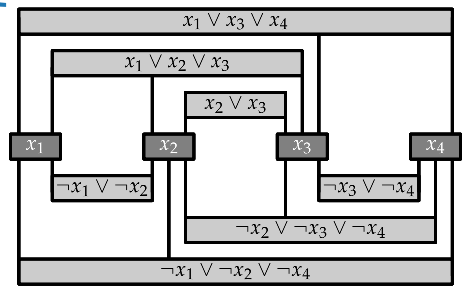
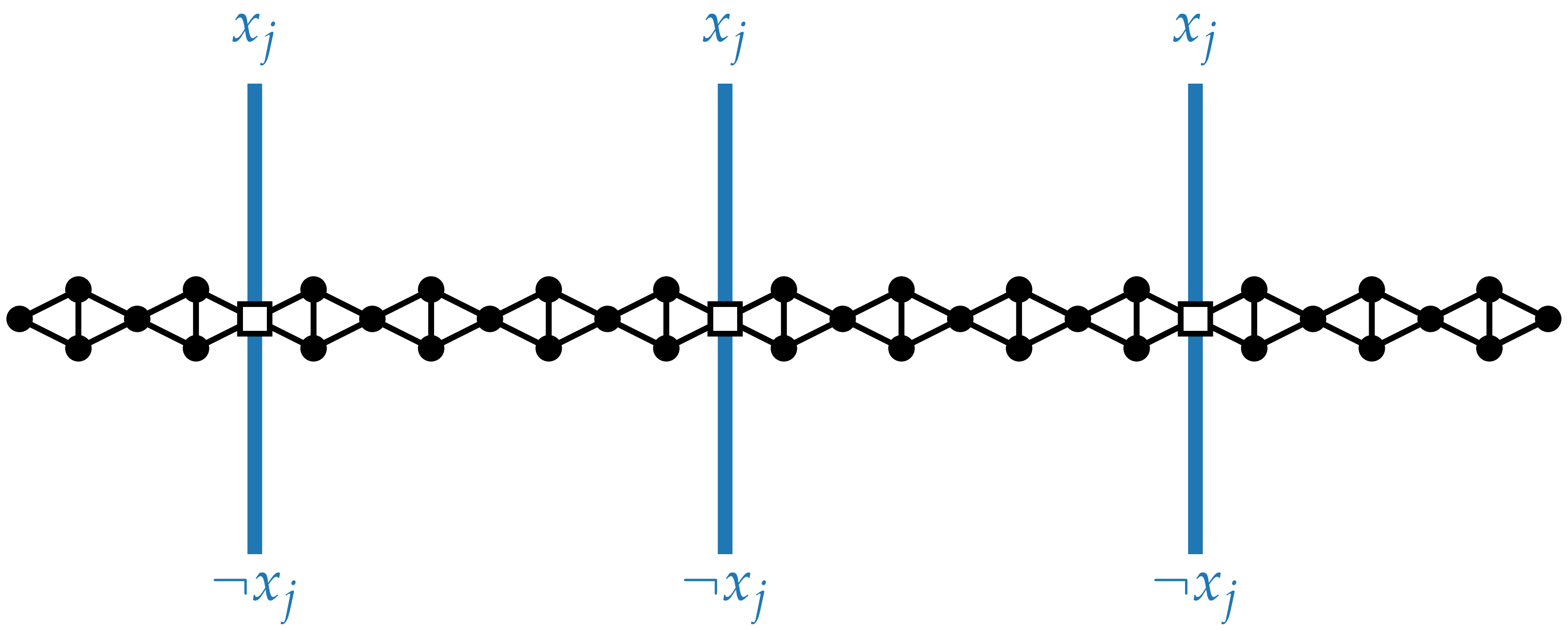


Variable Gadget.



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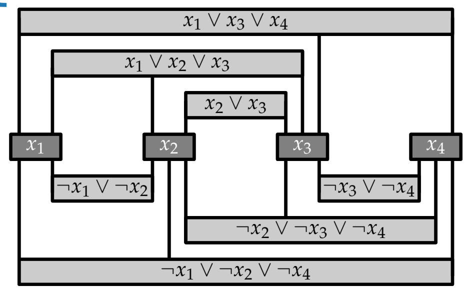

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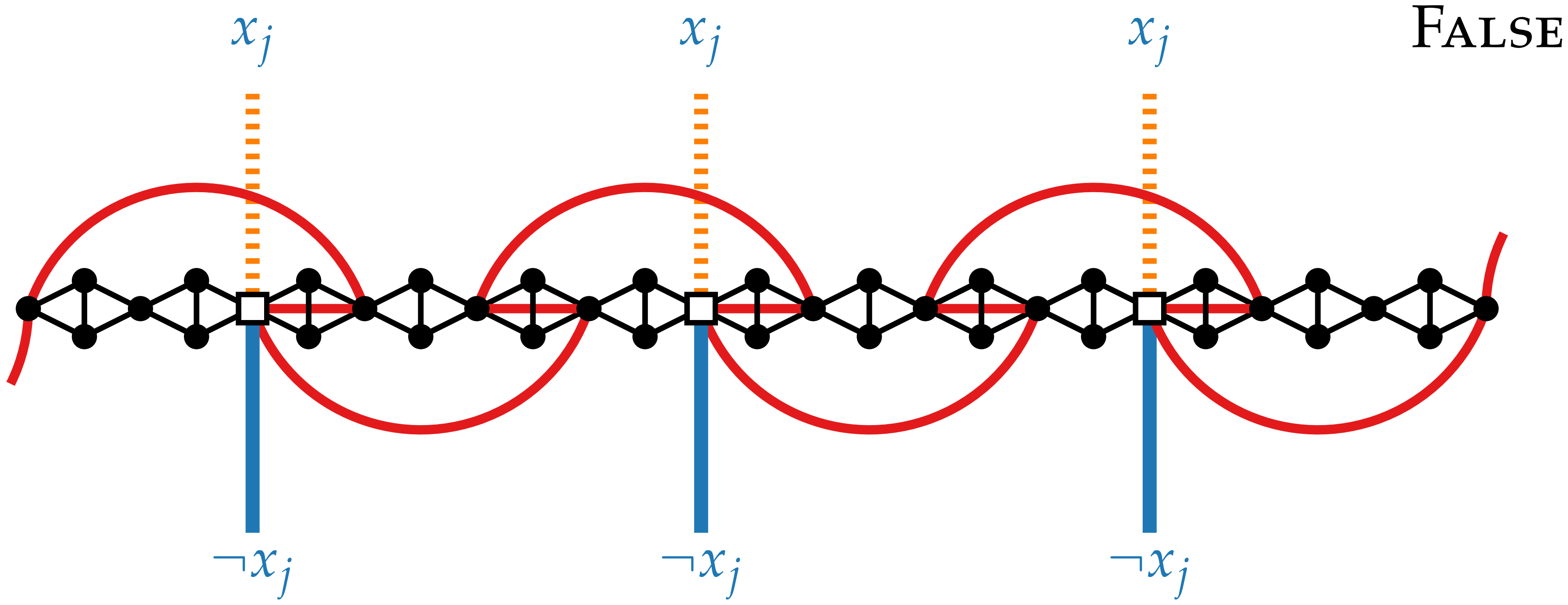

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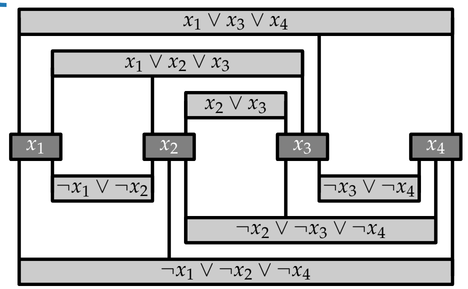

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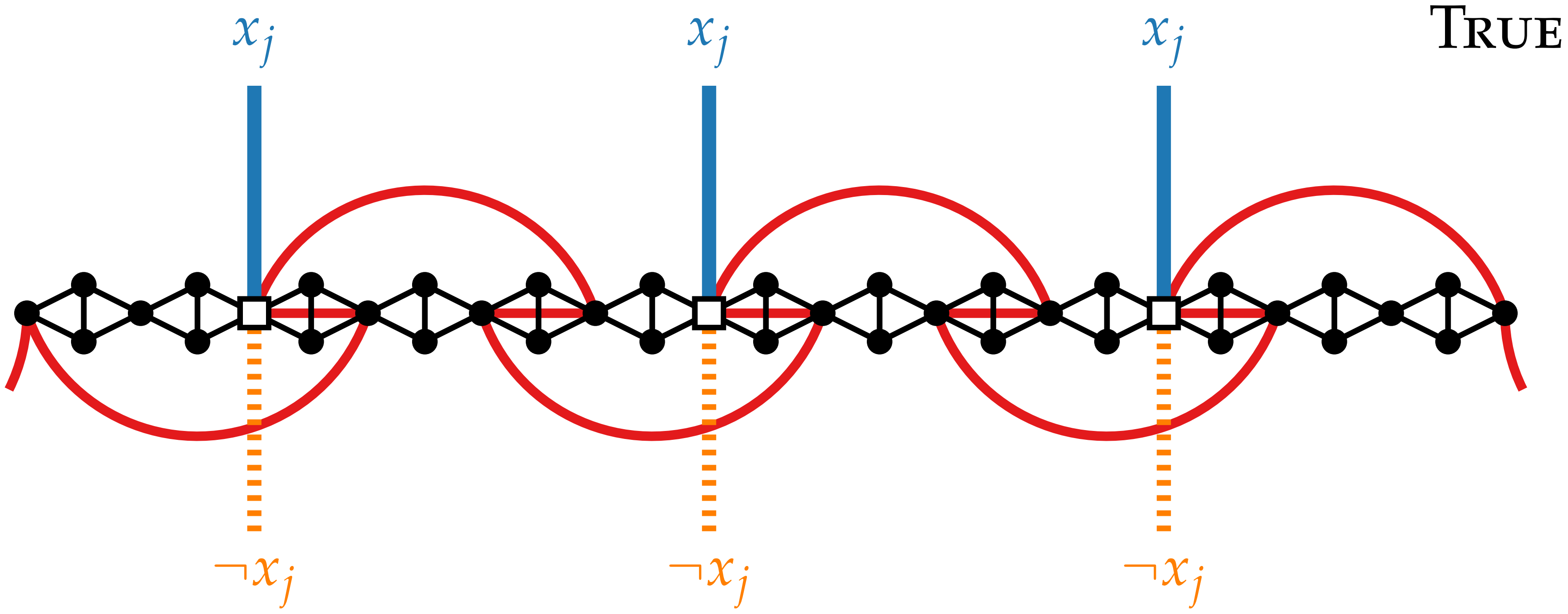

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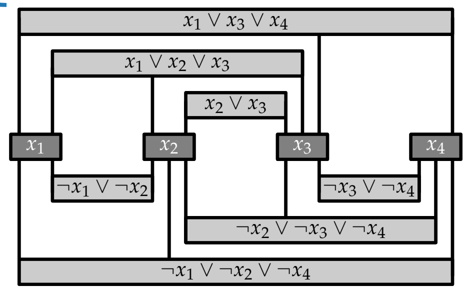

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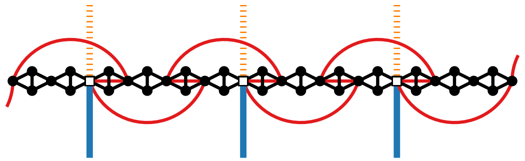
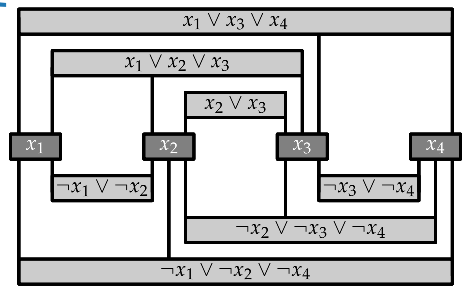


1-Plane Insertion Into a Plane **Biconnected Graph**

Clause Gadget.



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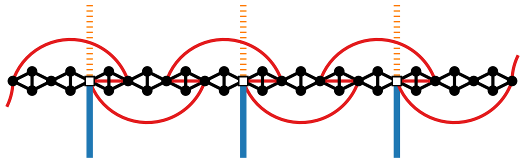

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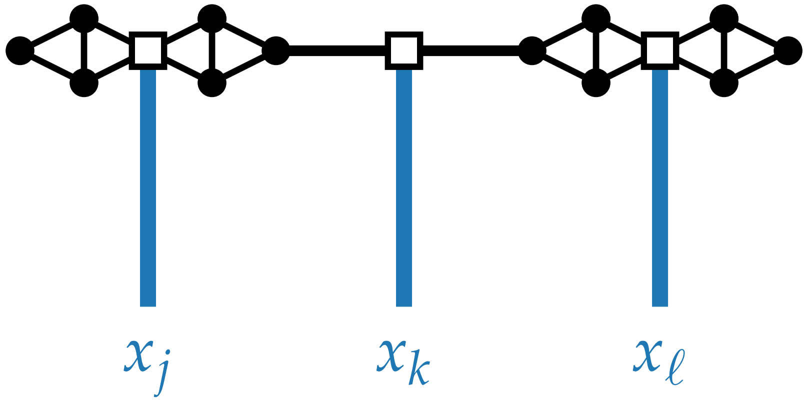

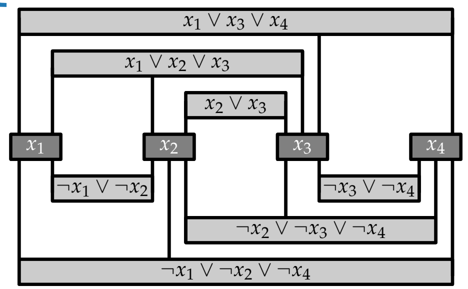

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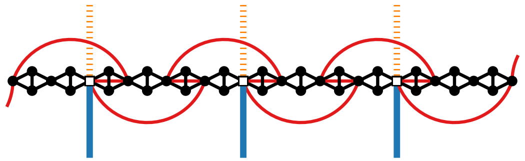

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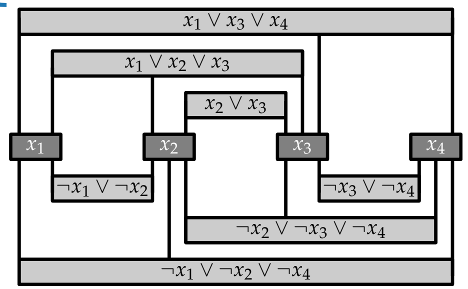

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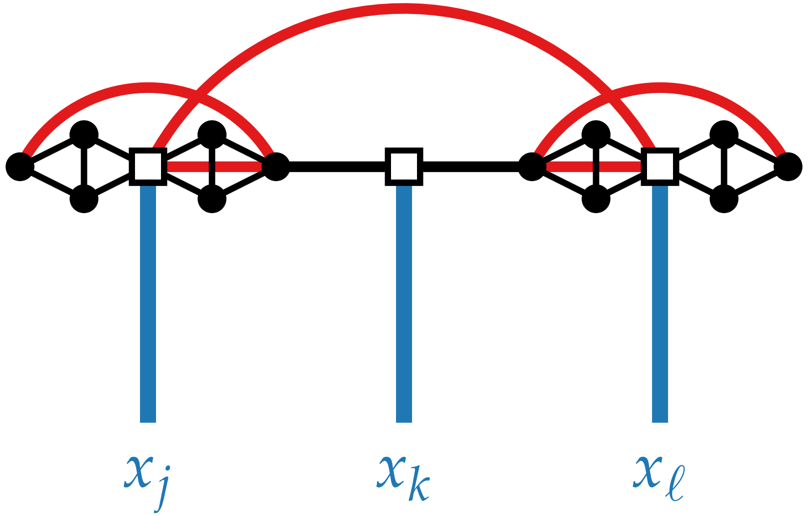

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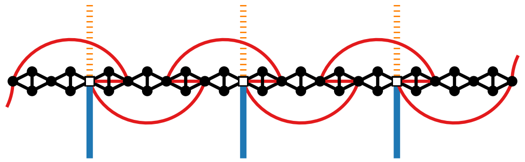

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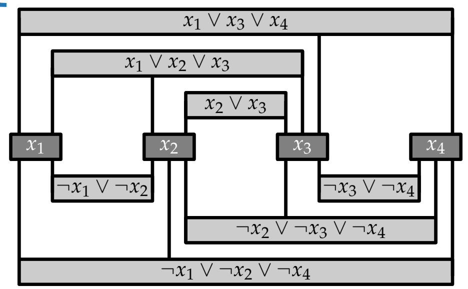

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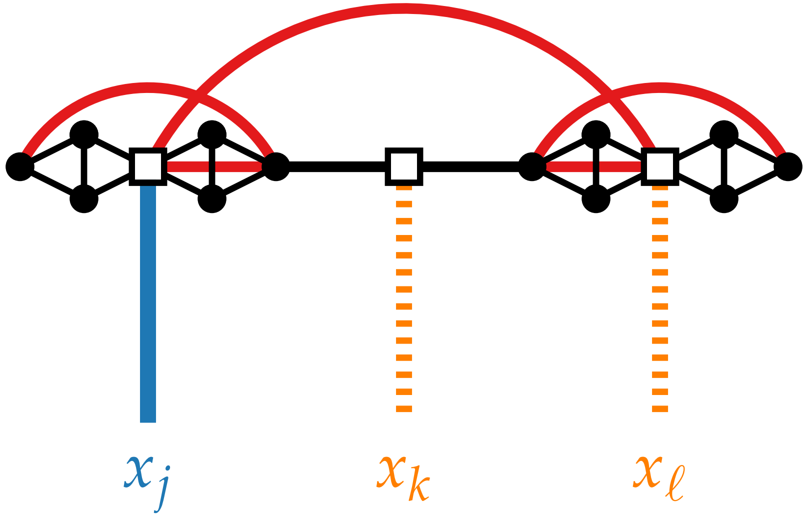

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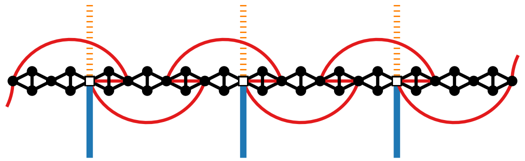

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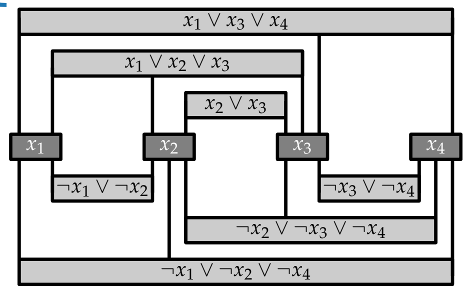

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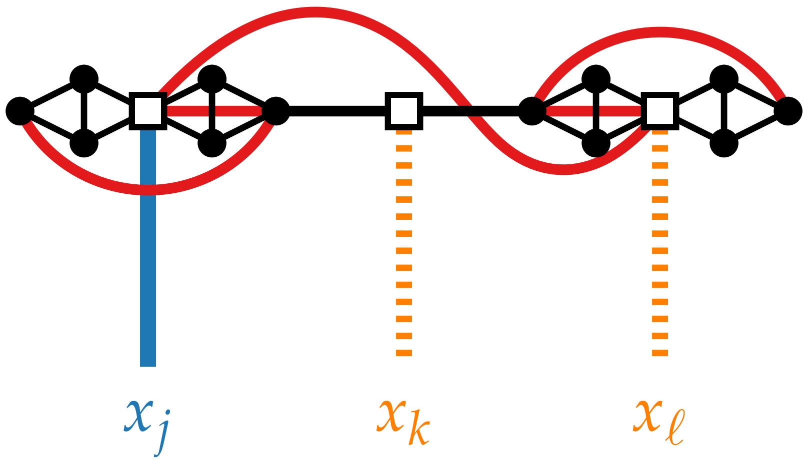

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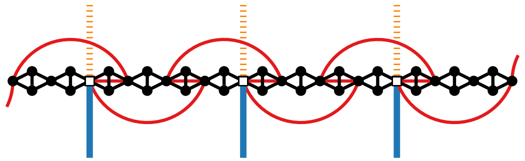

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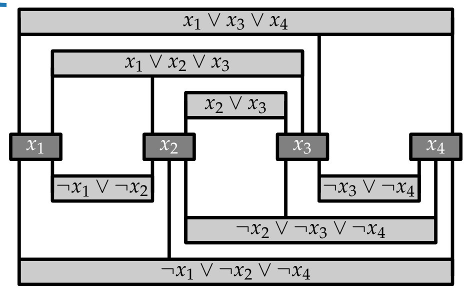

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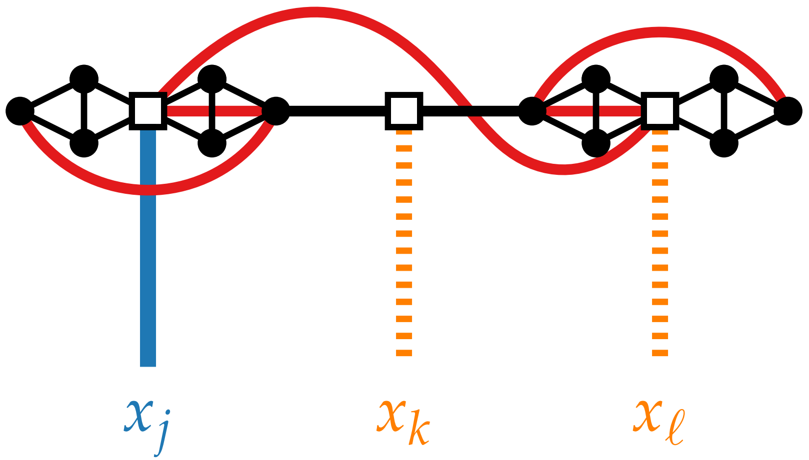

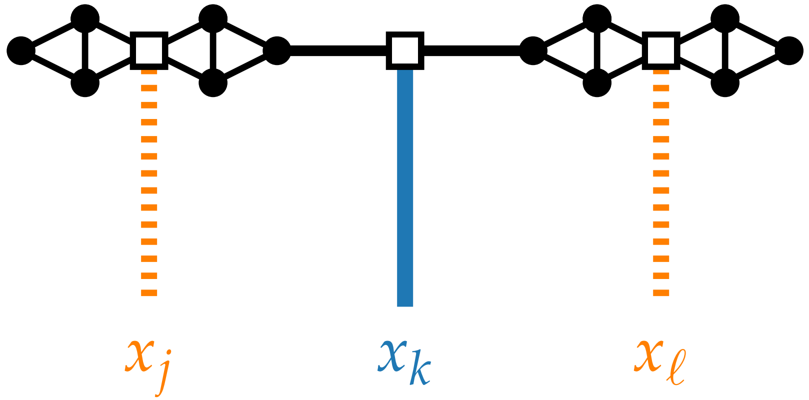

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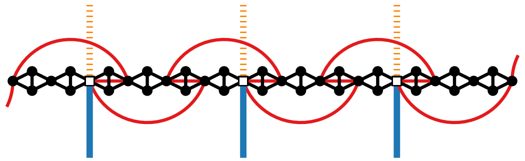

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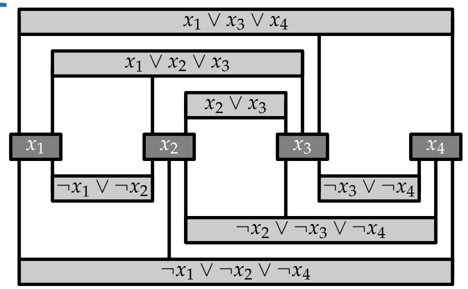

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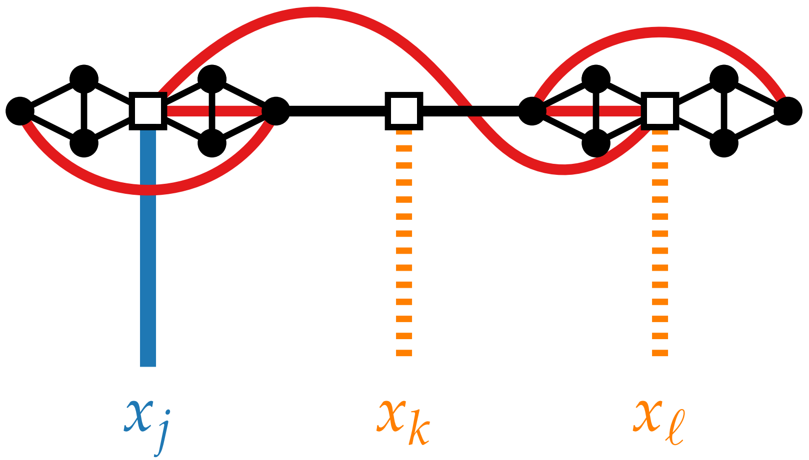

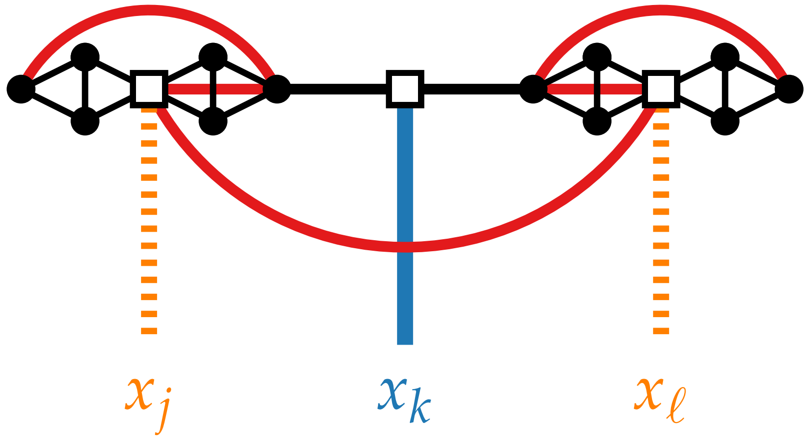

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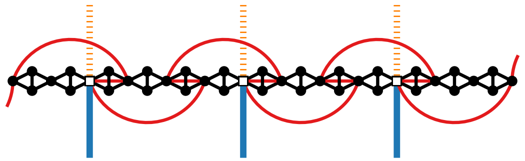

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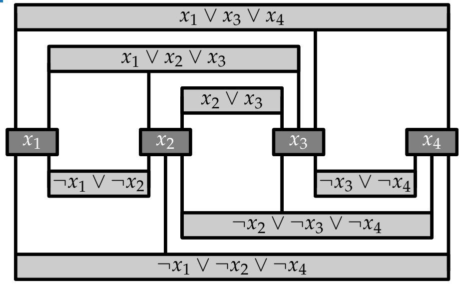

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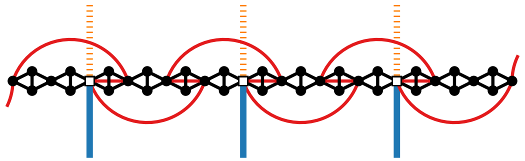

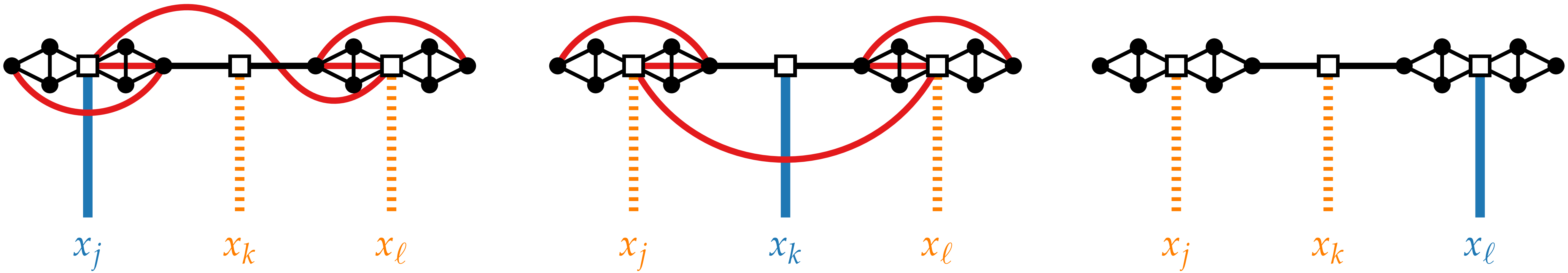

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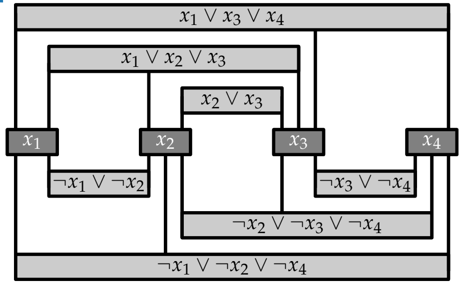

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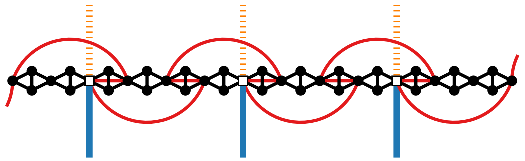

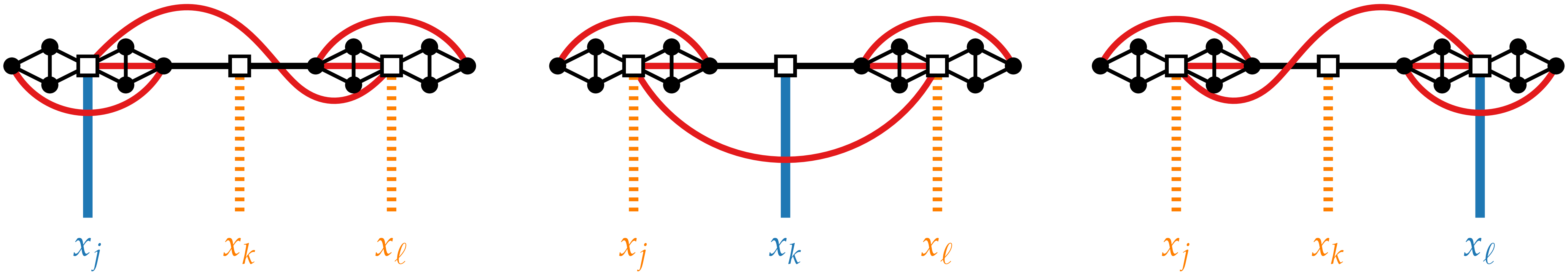

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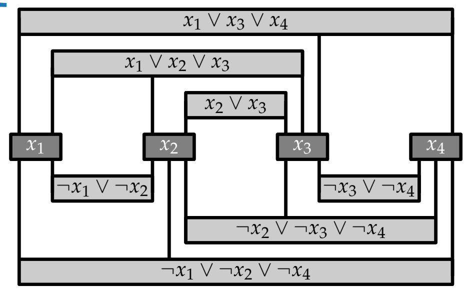

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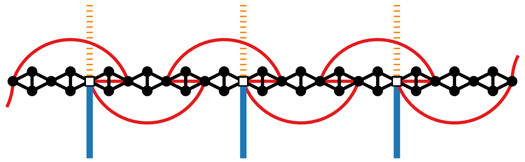

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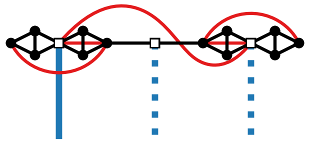

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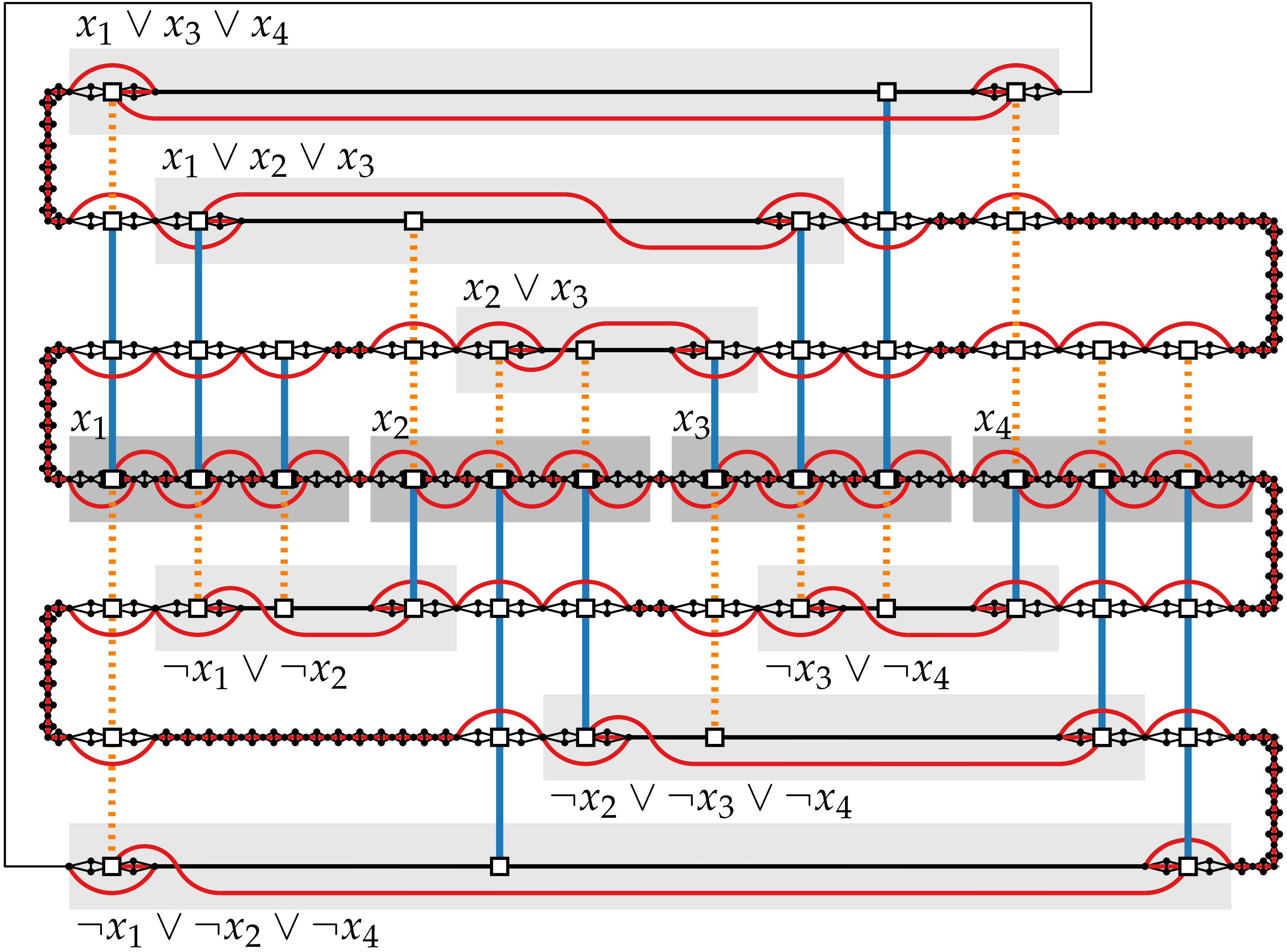

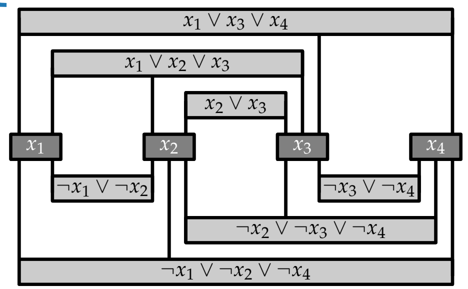

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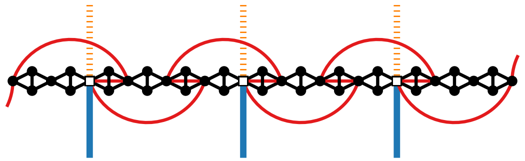

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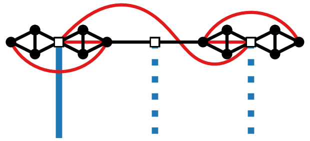

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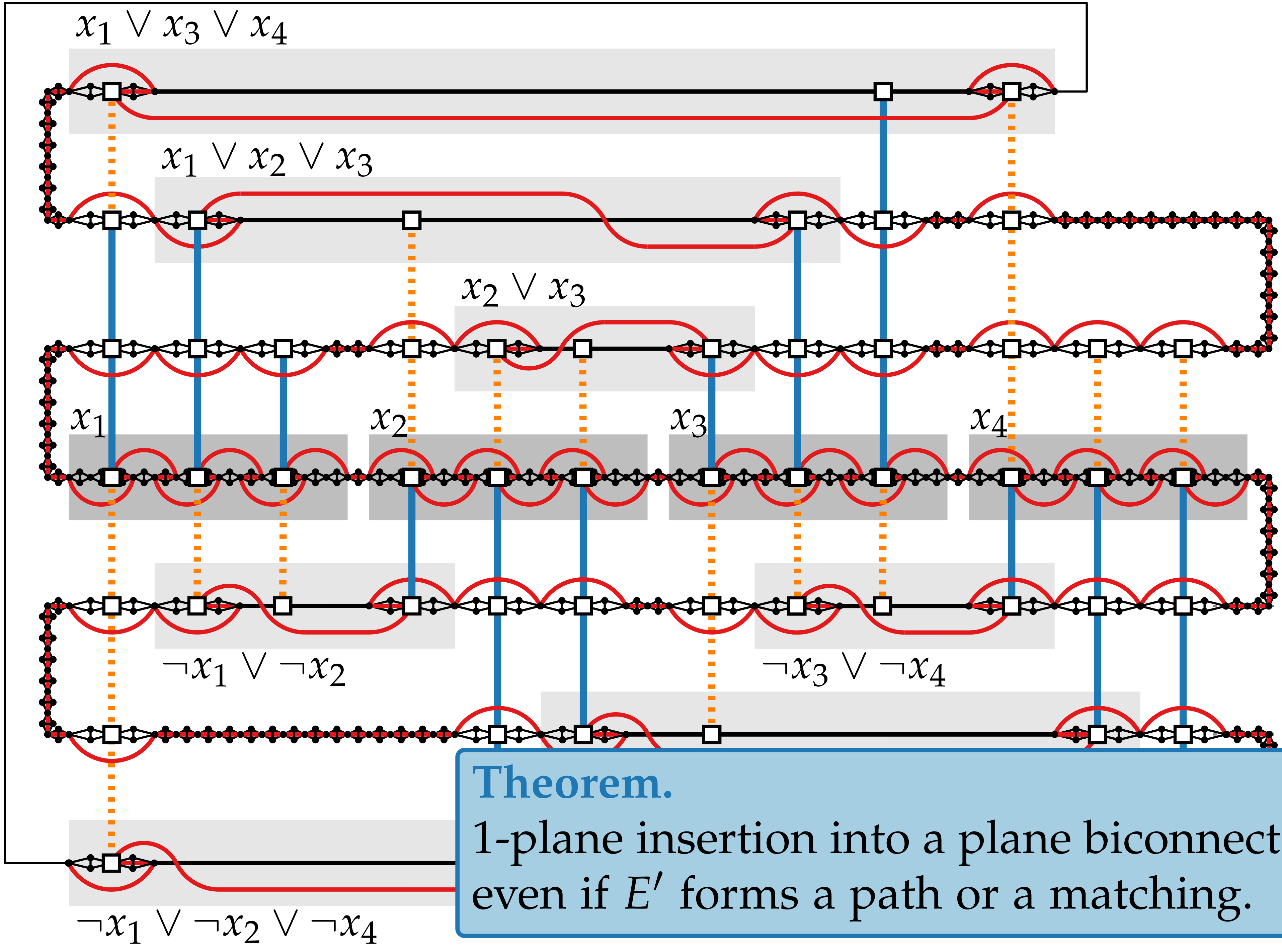

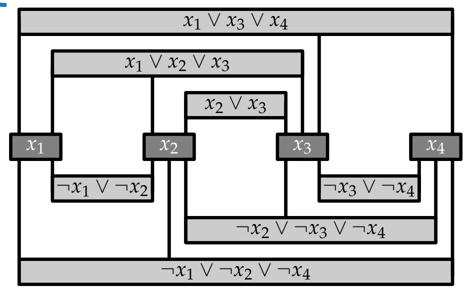

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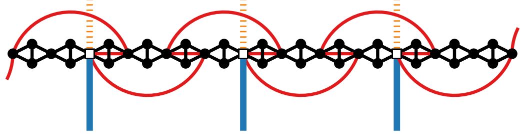

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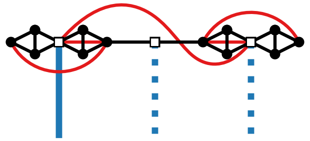

Theorem.
1-plane insertion into a plane biconnected graph is NP-complete, even if E' forms a path or a matching.