Oriented dilation of undirected graphs

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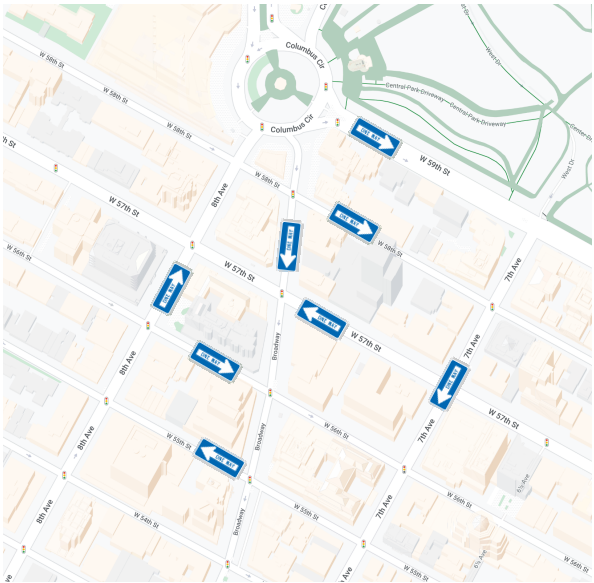
Motivation and model



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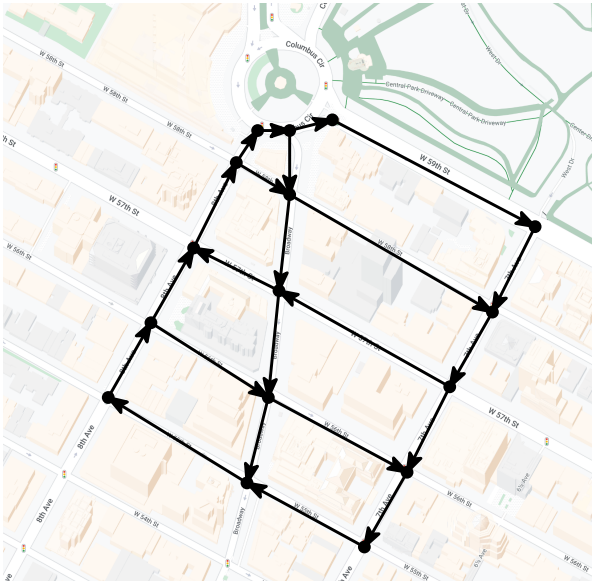

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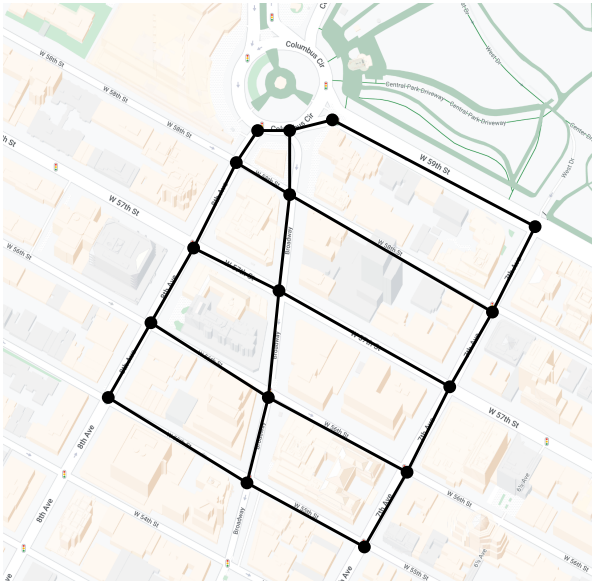

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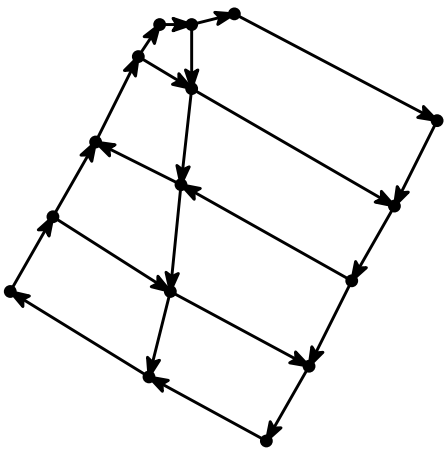

Oriented dilation:

$$\max_{p,p' \in P} \frac{\left|\begin{array}{c}\text{shortest cycle with } p \text{ and } p' \\ \text{in oriented graph}\end{array}\right|}{\left|\begin{array}{c}\text{smallest triangle with } p \text{ and } p' \\ \text{in point set } P\end{array}\right|}$$

[Buchin et al, ESA, 2023]

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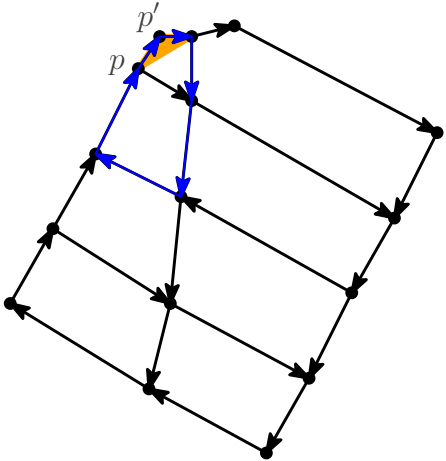

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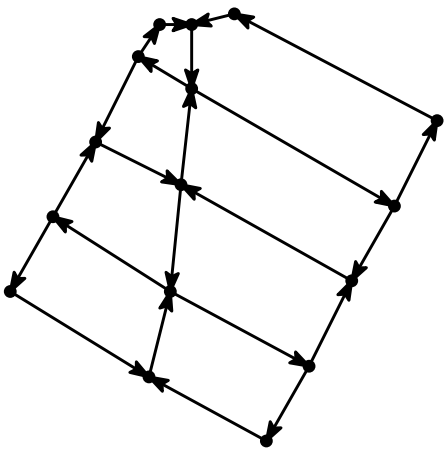

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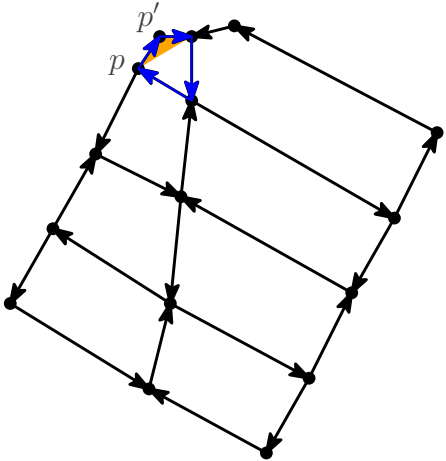

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New result

NP-hard decision

Given:
undirected geometric graph G,
parameter t

Is there an orientation of G
with oriented dilation $\leq t$?

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Proof: Reduction from
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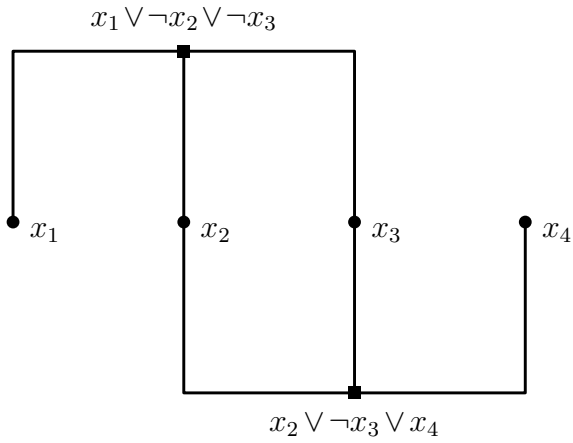

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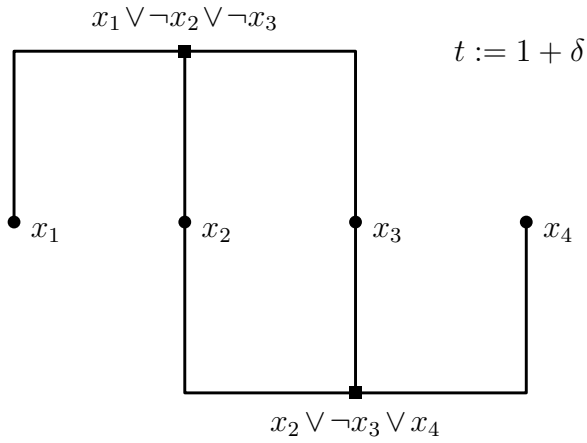
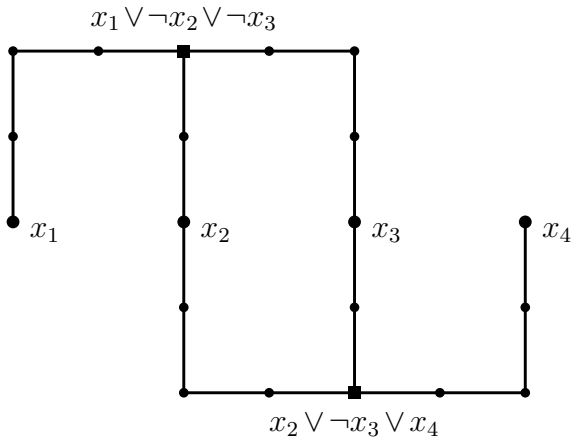


NP-hard: Orientation with dilation $\leq t$?

Neighbours have distance 1



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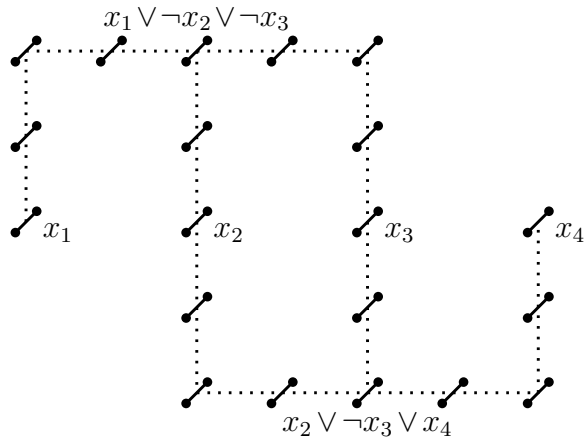
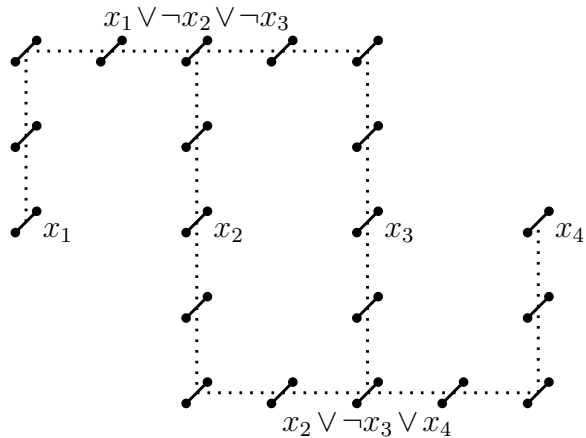
Points replaced by two ε-close points



NP-hard: Orientation with dilation $\leq t$?

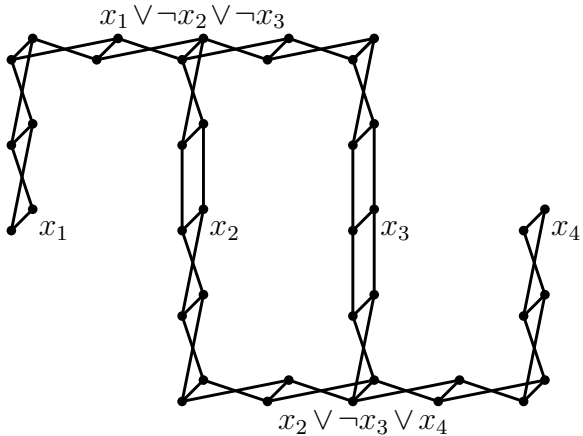
Points replaced by two ε-close points

true  false 



NP-hard: Orientation with dilation $\leq t$?

Wire gadget to propagate the orientation



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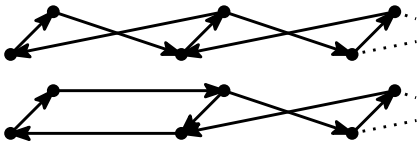
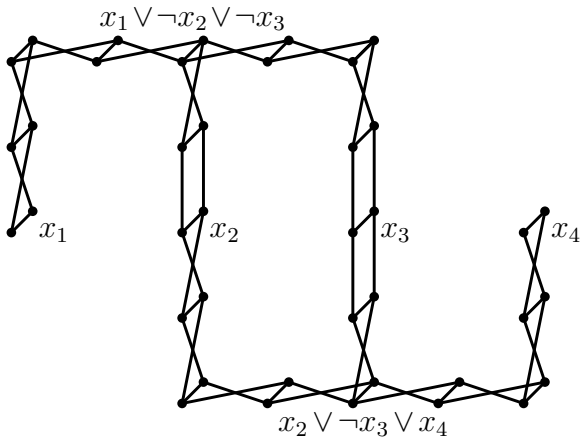
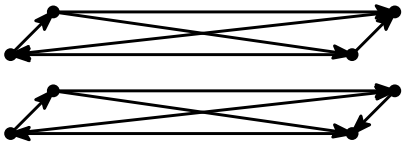
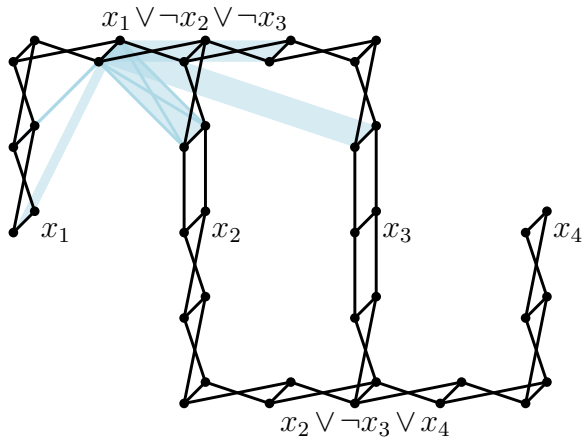




NP-hard: Orientation with dilation $\leq t$?

$K_{2,2}$ between non-neighbours





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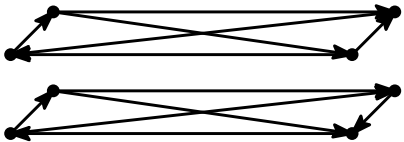

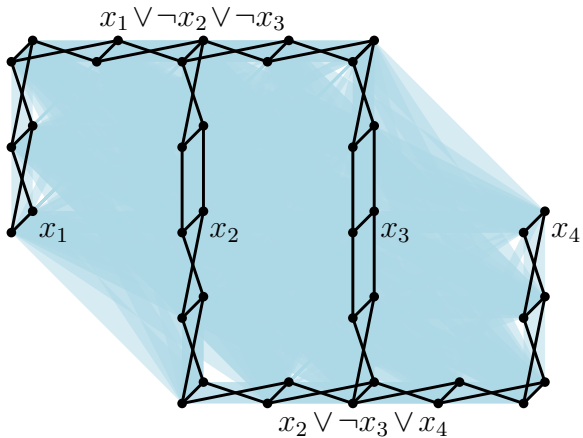

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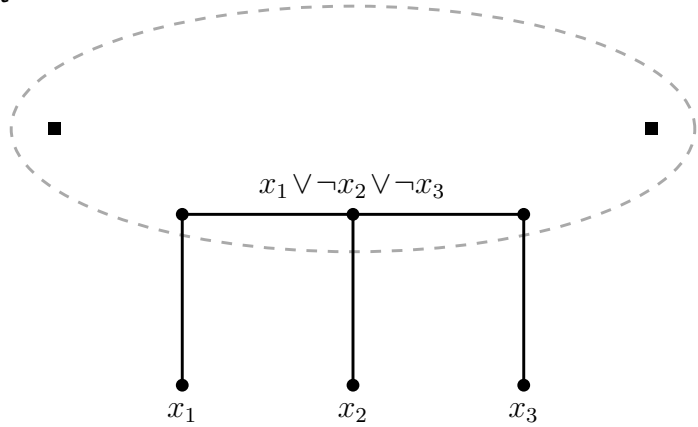
Clause gadget



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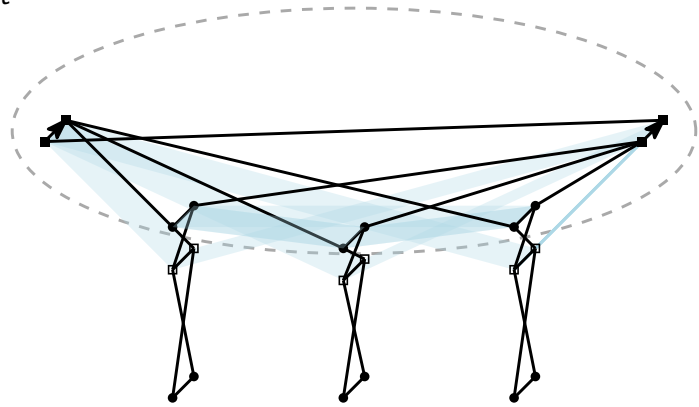

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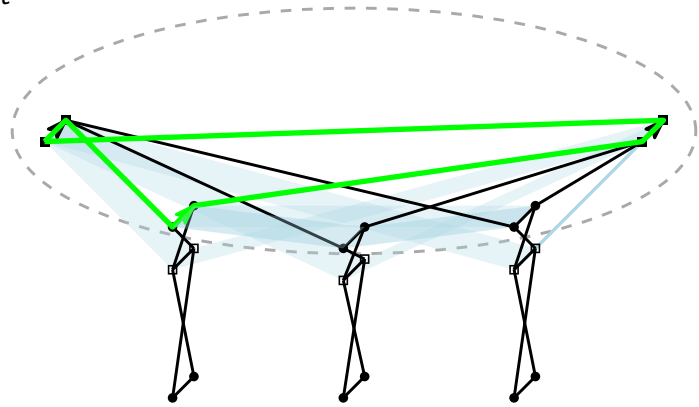

$t \leq 1 + \delta$

NP-hard: Orientation with dilation $\leq t$?

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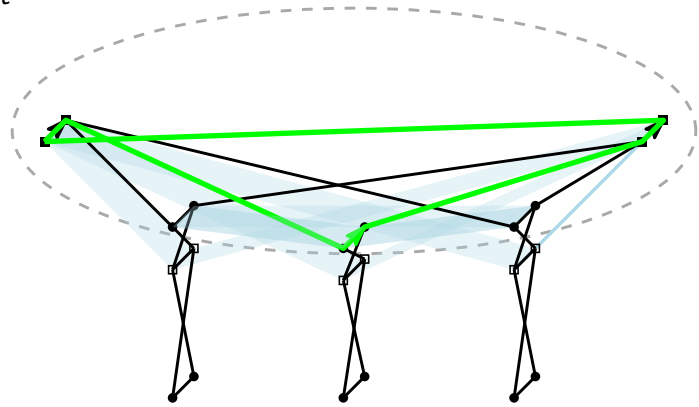

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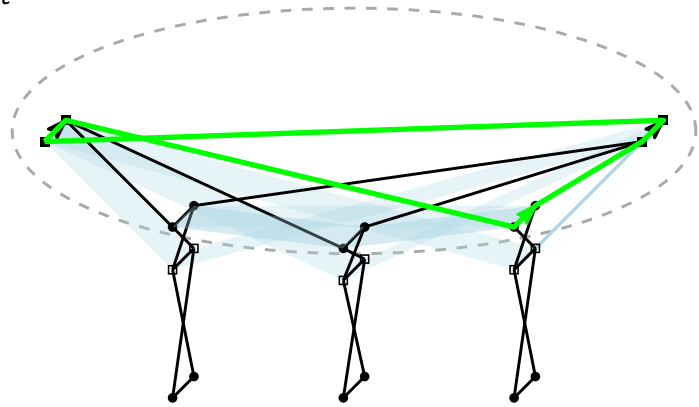

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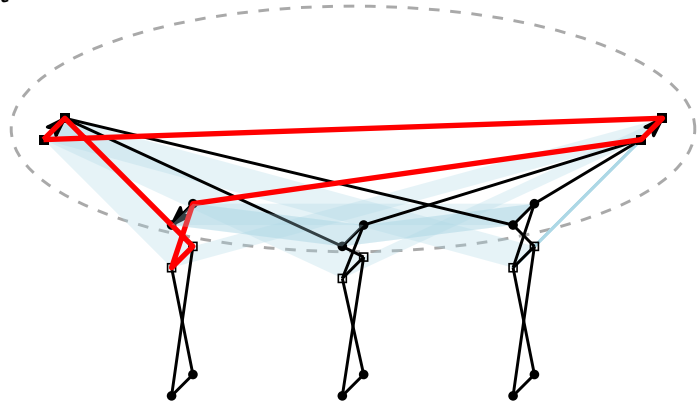

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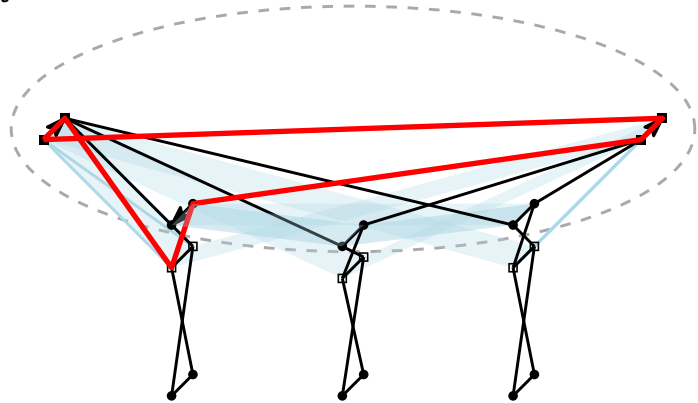

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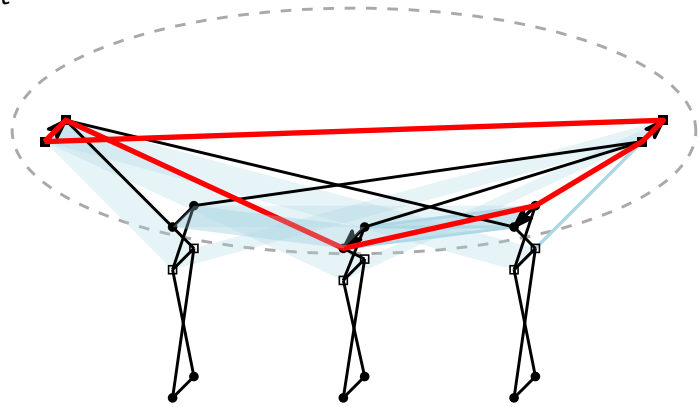

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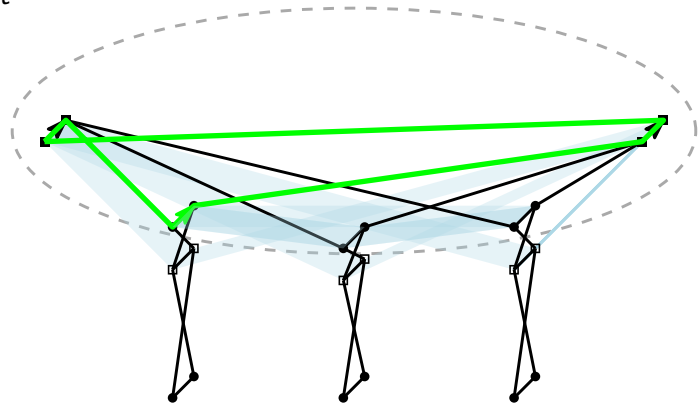

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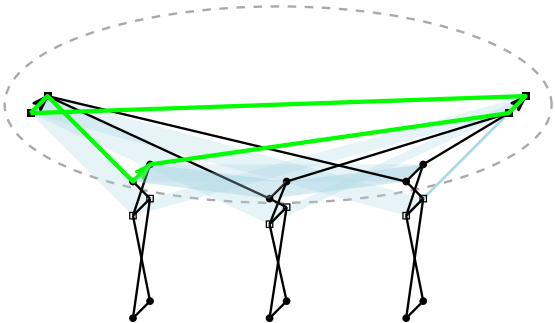
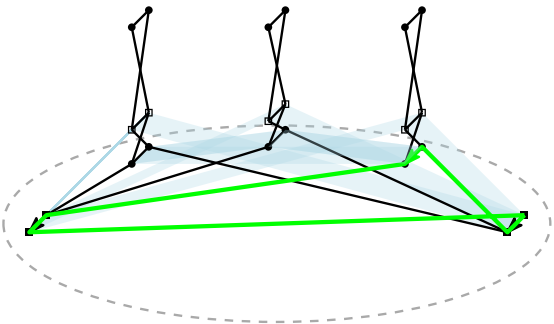
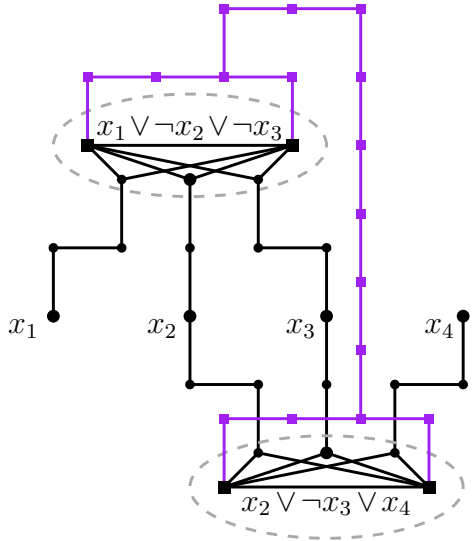




NP-hard: Orientation with dilation $\leq t$?

Tree of knowledge



Summary and open questions

NP-hard Decision

Given:
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parameter t

Is there an orientation of G with oriented dilation $\leq t$?

Open: Is optimal orienting easier for

- complete graphs?
- planar graphs?