

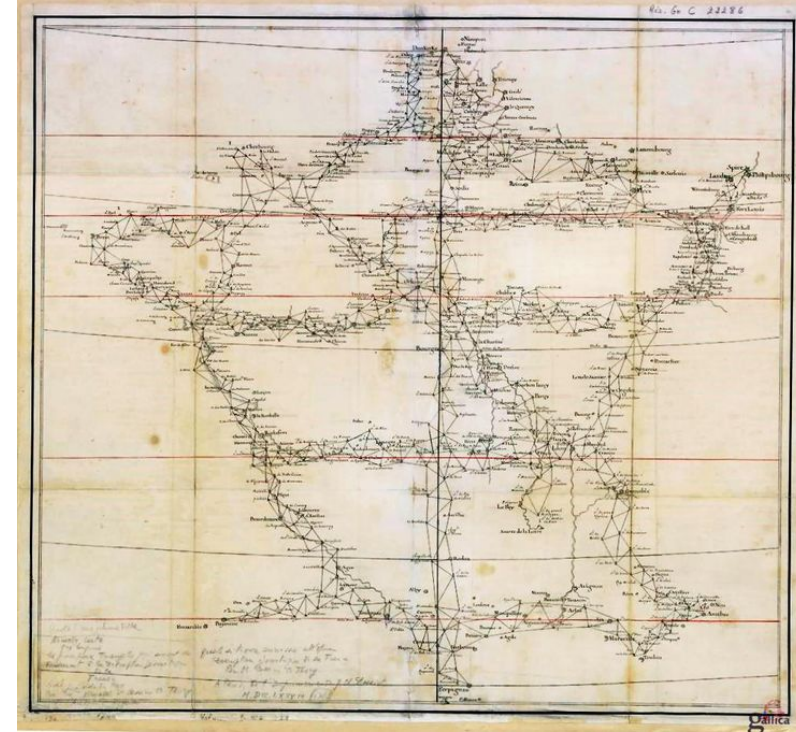
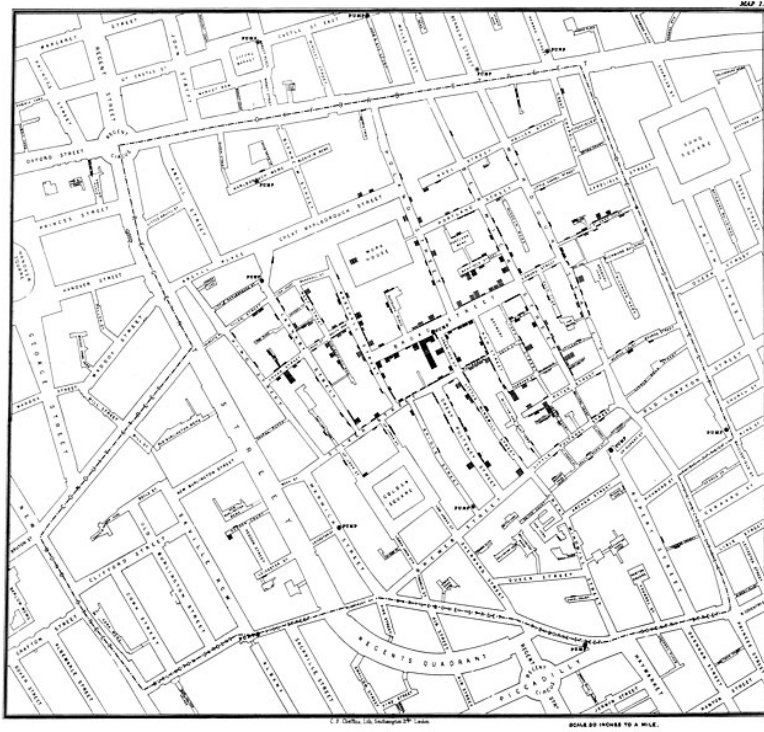
Orbital Labeling of Circular Maps

Annika Bonerath
University of Bonn

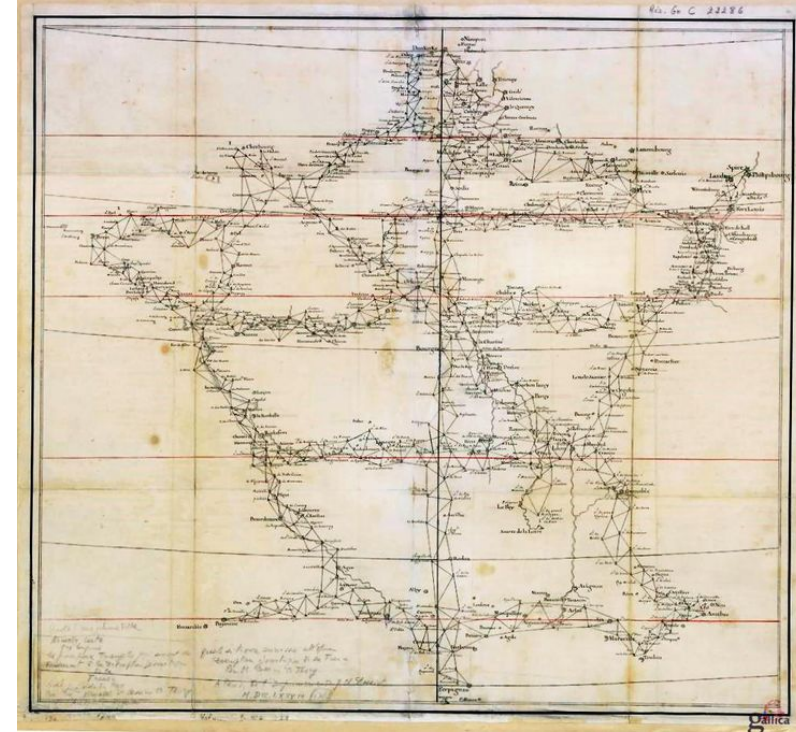
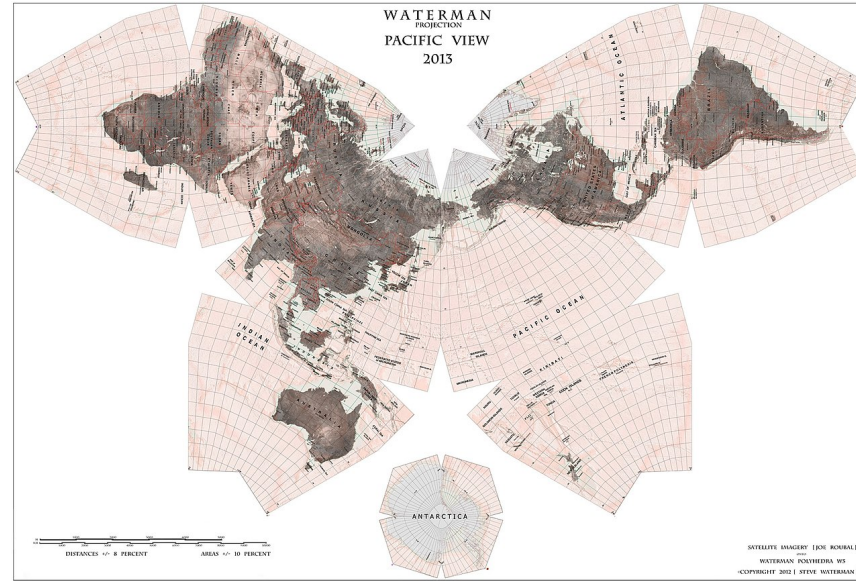
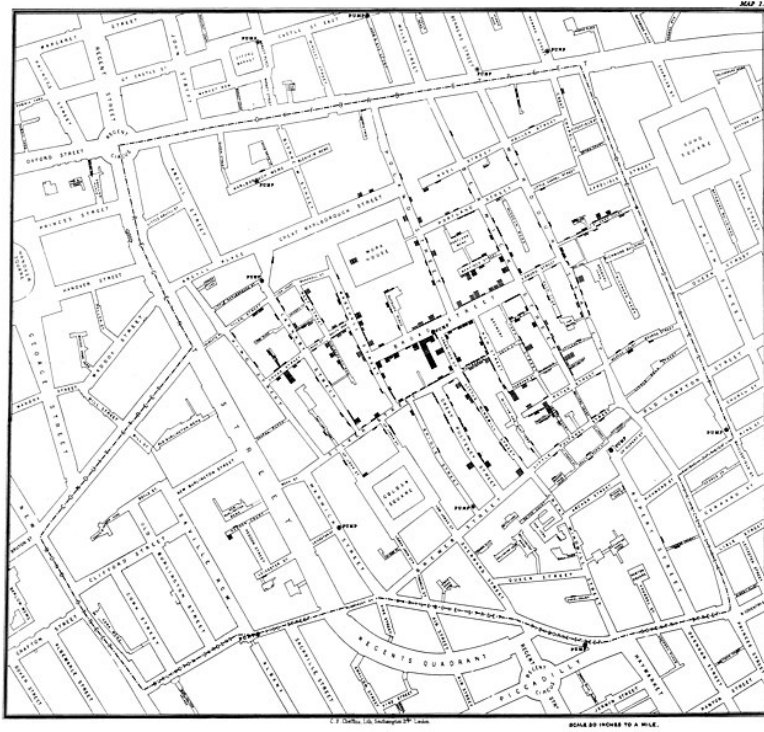
Martin Nöllenburg
Markus Wallinger
TU Wien

Soeren Terziadis
Jules Wulms
TU Eindhoven

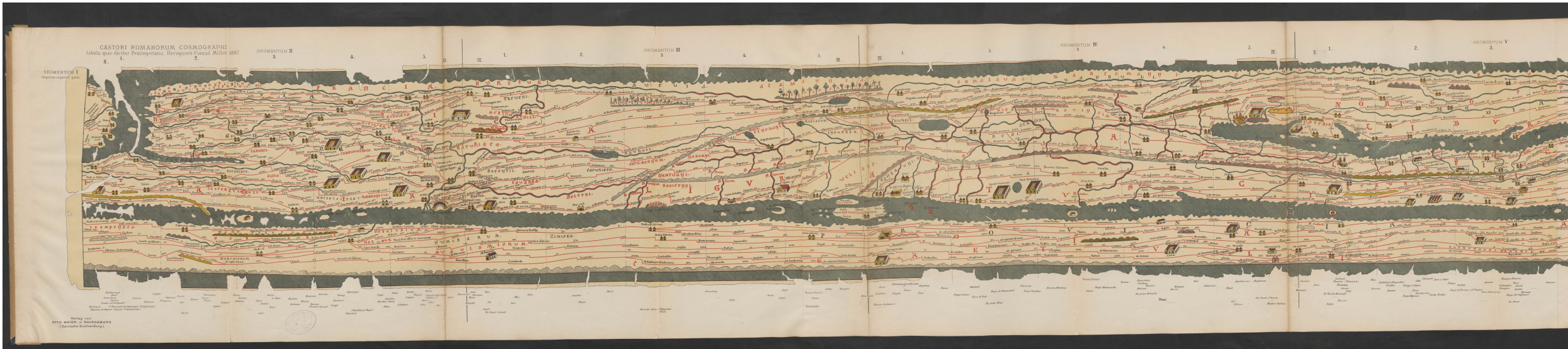
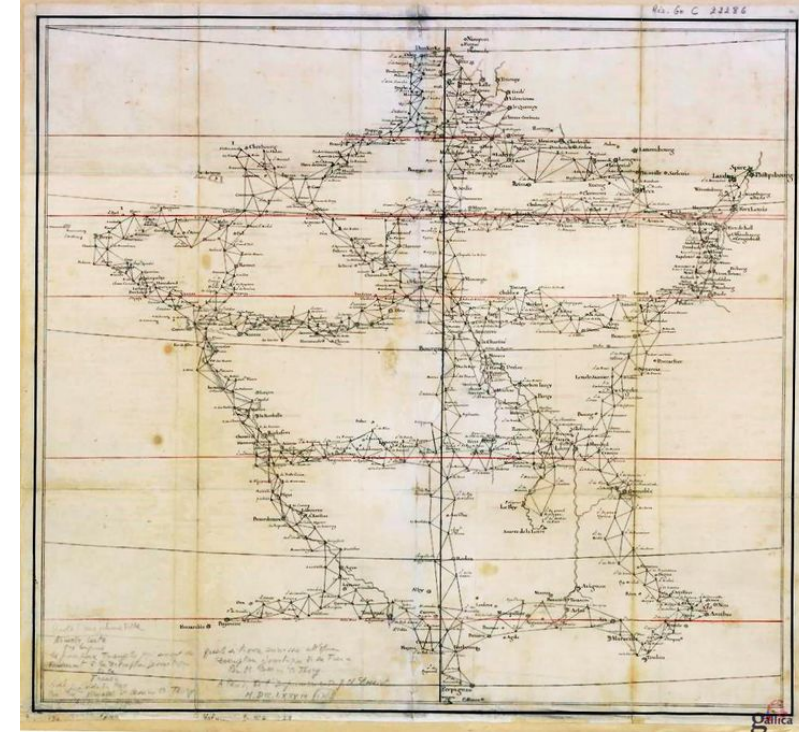
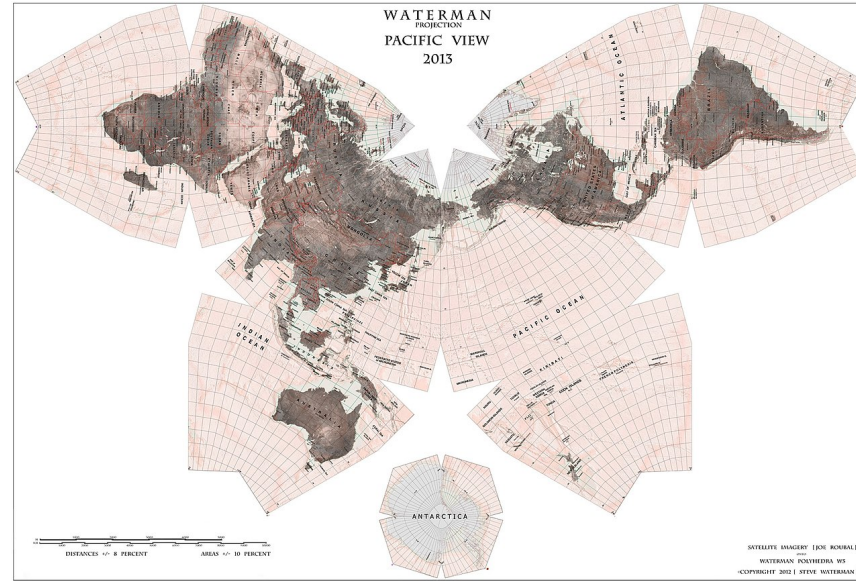
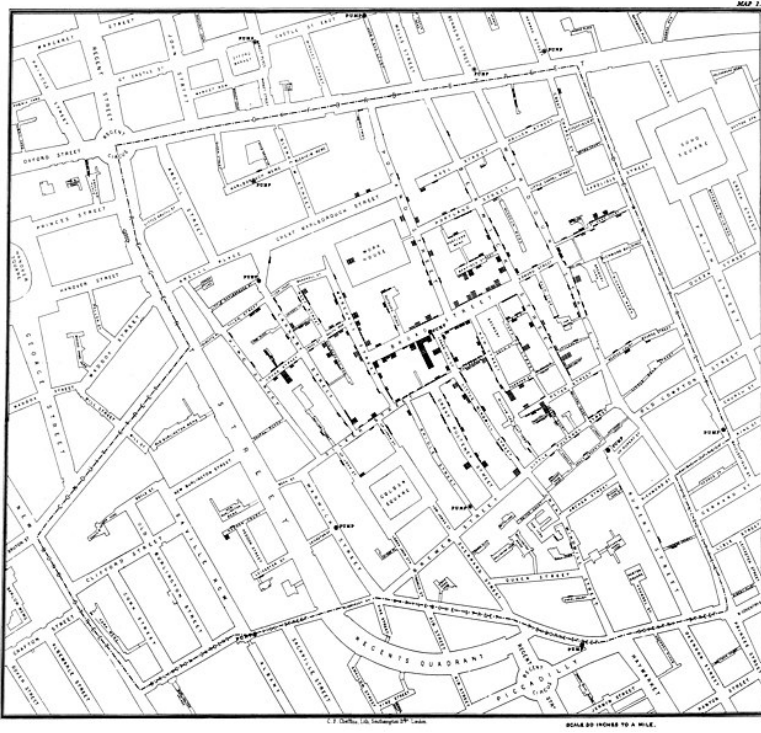
Maps



Maps



Maps





6.75m

Circular maps

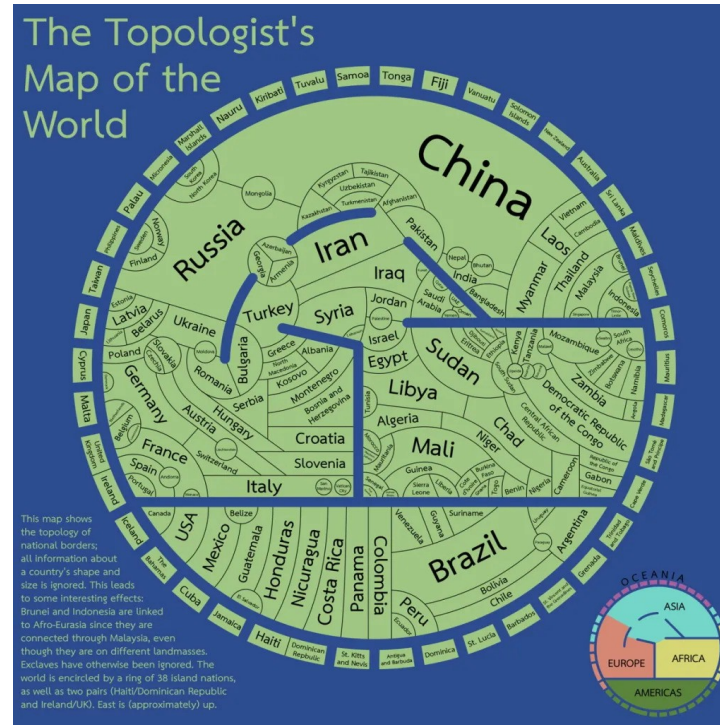


UNITED NATIONS

Apple Watch Series 6



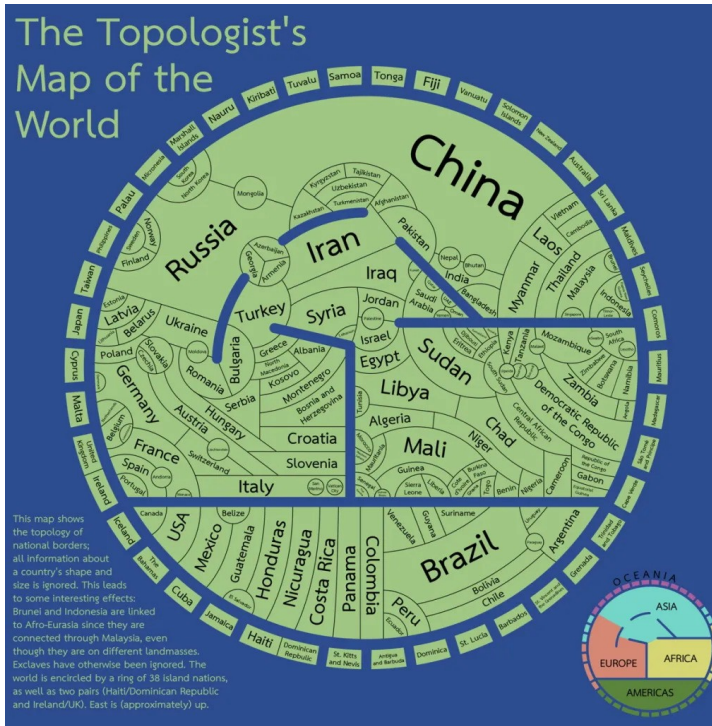
<https://www.dimensions.com/collection/smart-watches>



Circular maps



UNITED NATIONS



Apple Watch Series 6

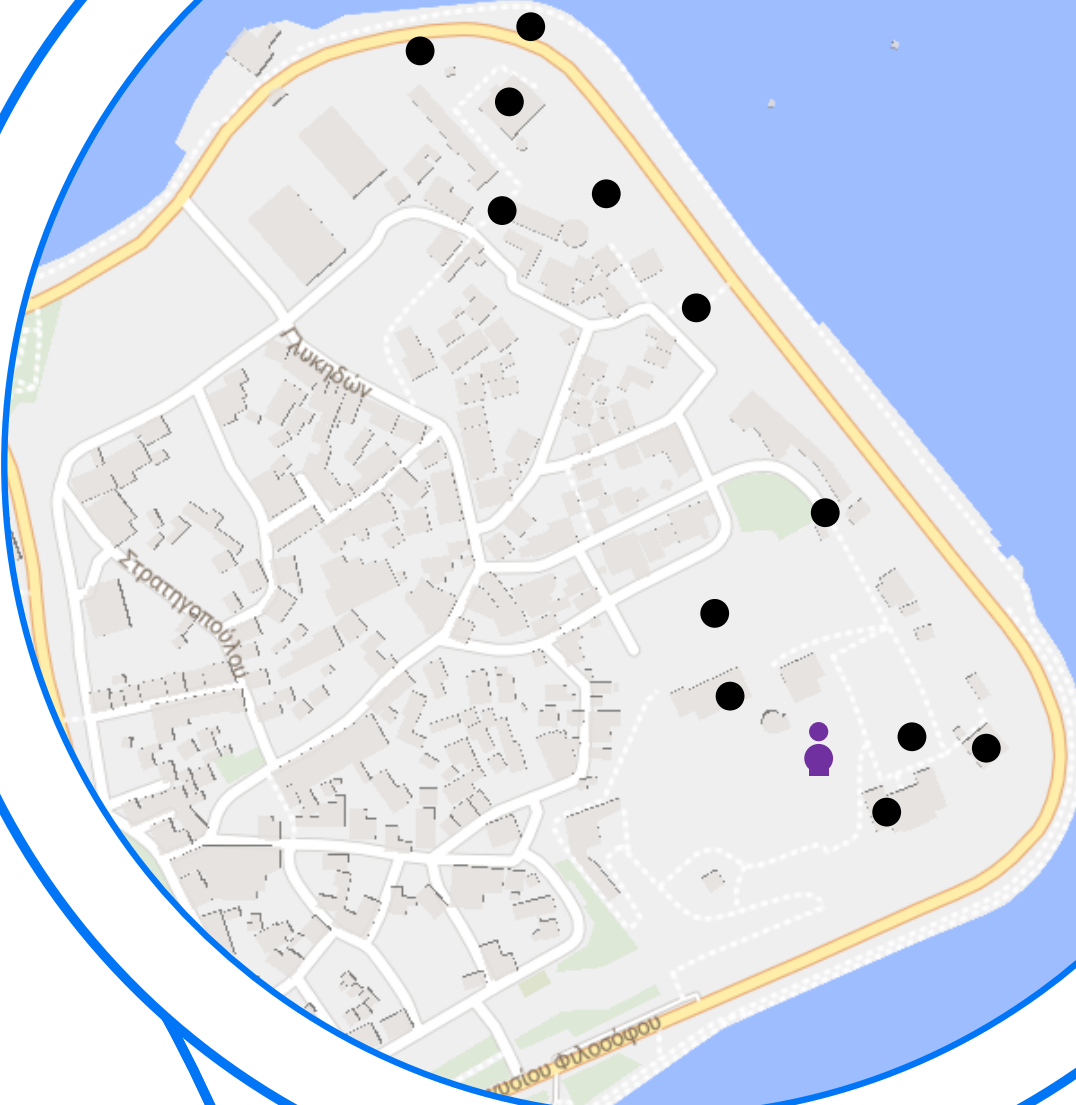
Google Pixel Watch

Huawei Watch GT 2

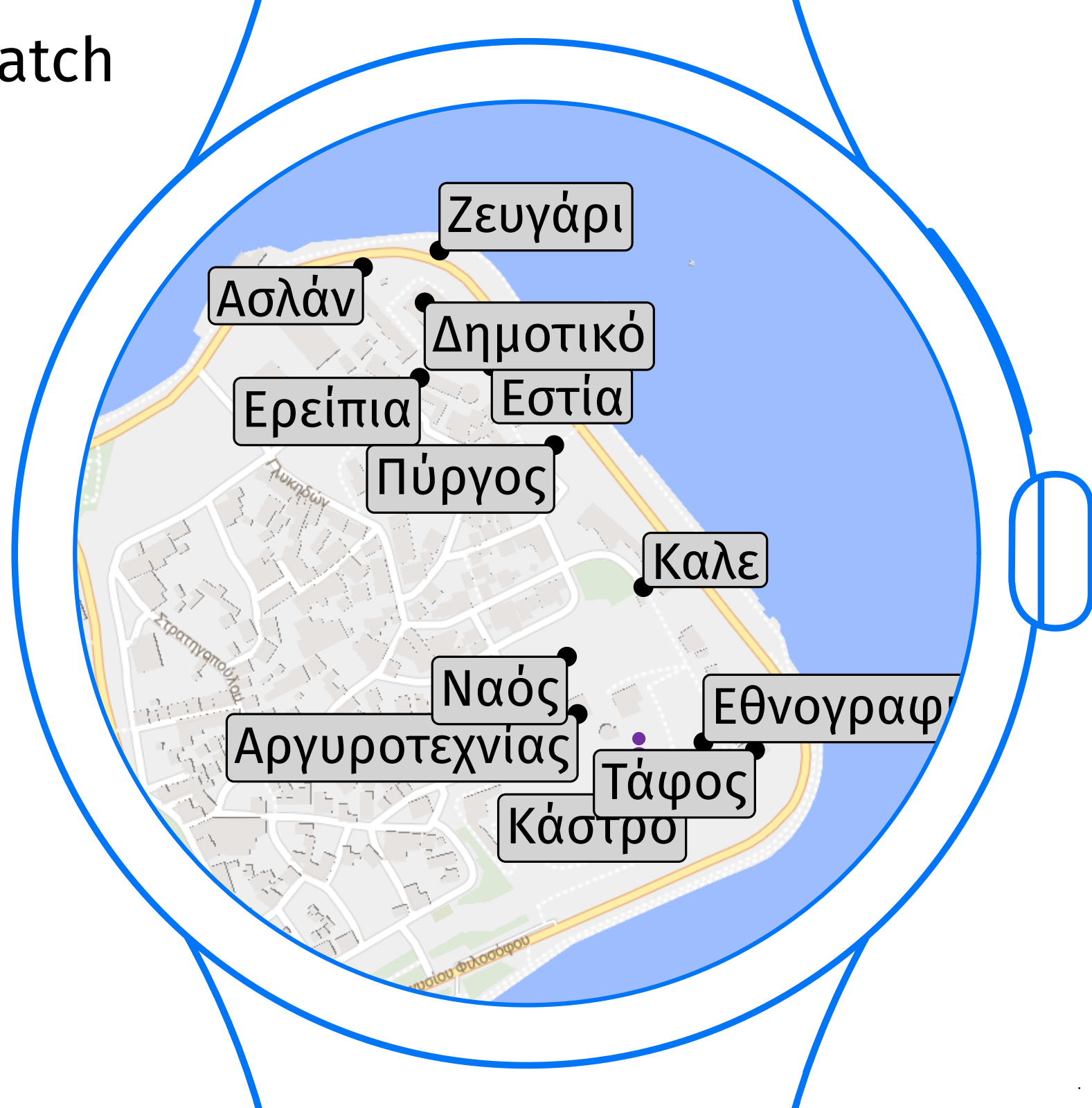
Samsung Galaxy Watch



Labeling on a smart watch

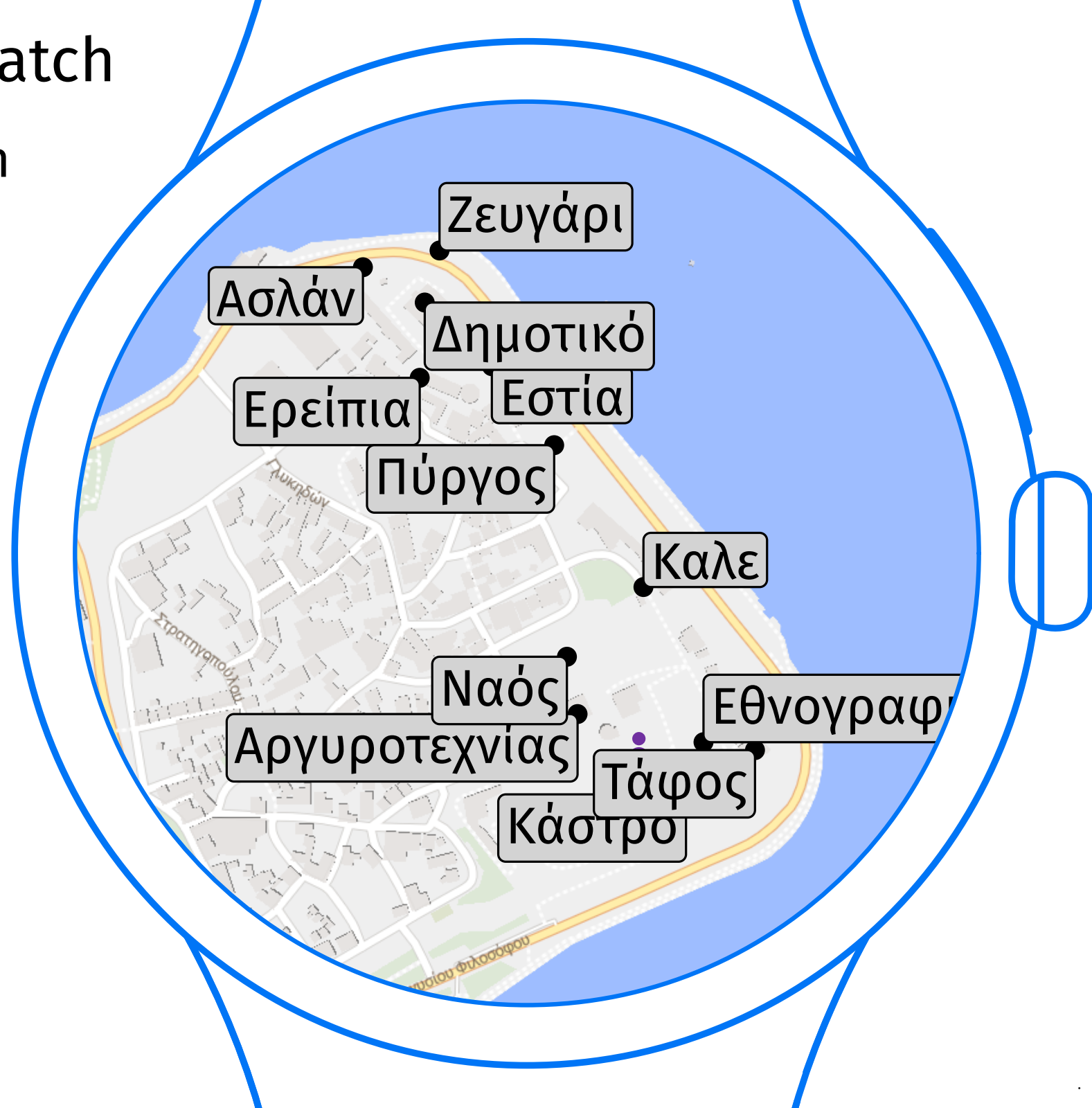


Labeling on a smart watch



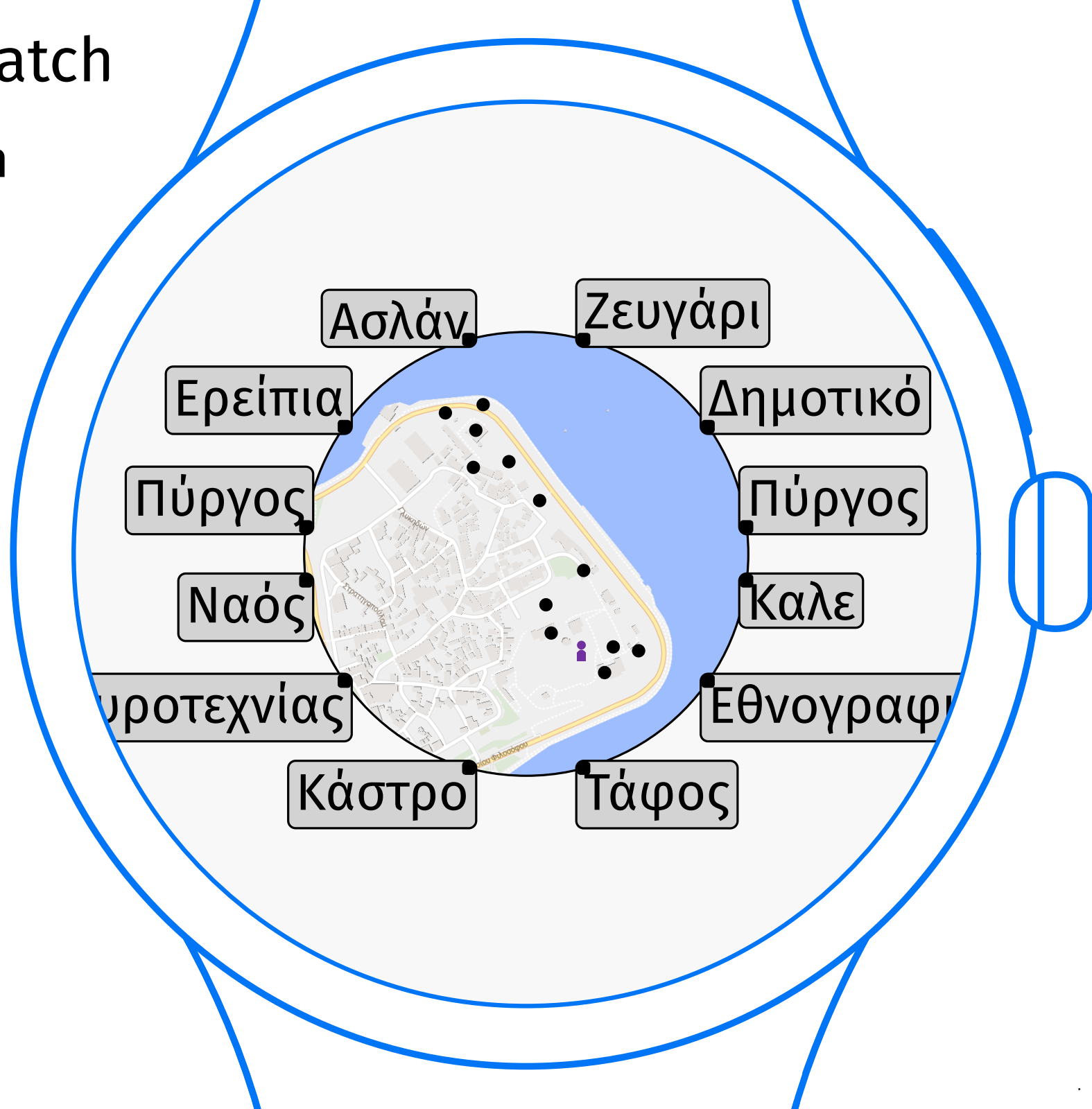
Labeling on a smart watch

- reduce map obstruction
- avoid label overlap



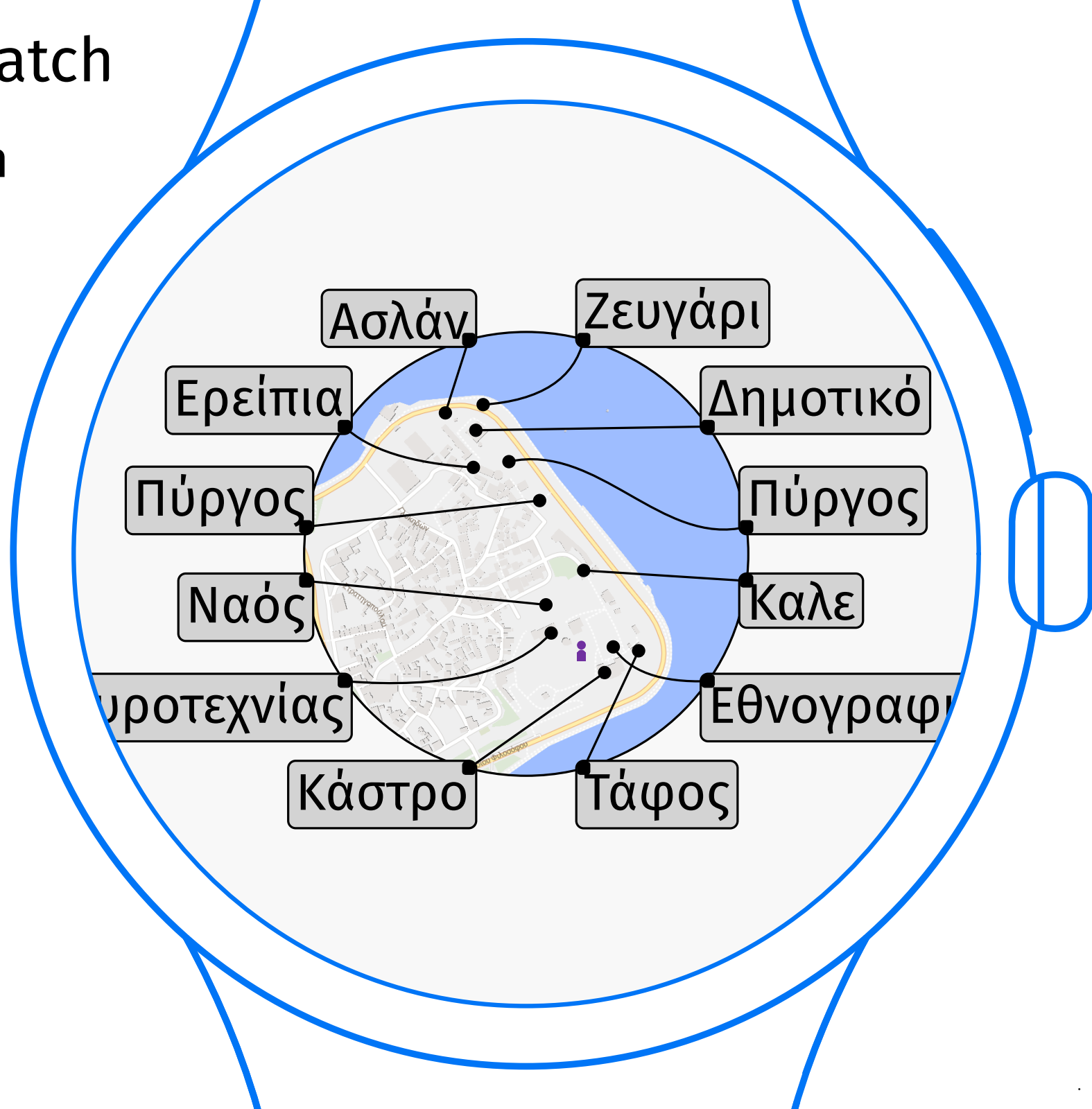
Labeling on a smart watch

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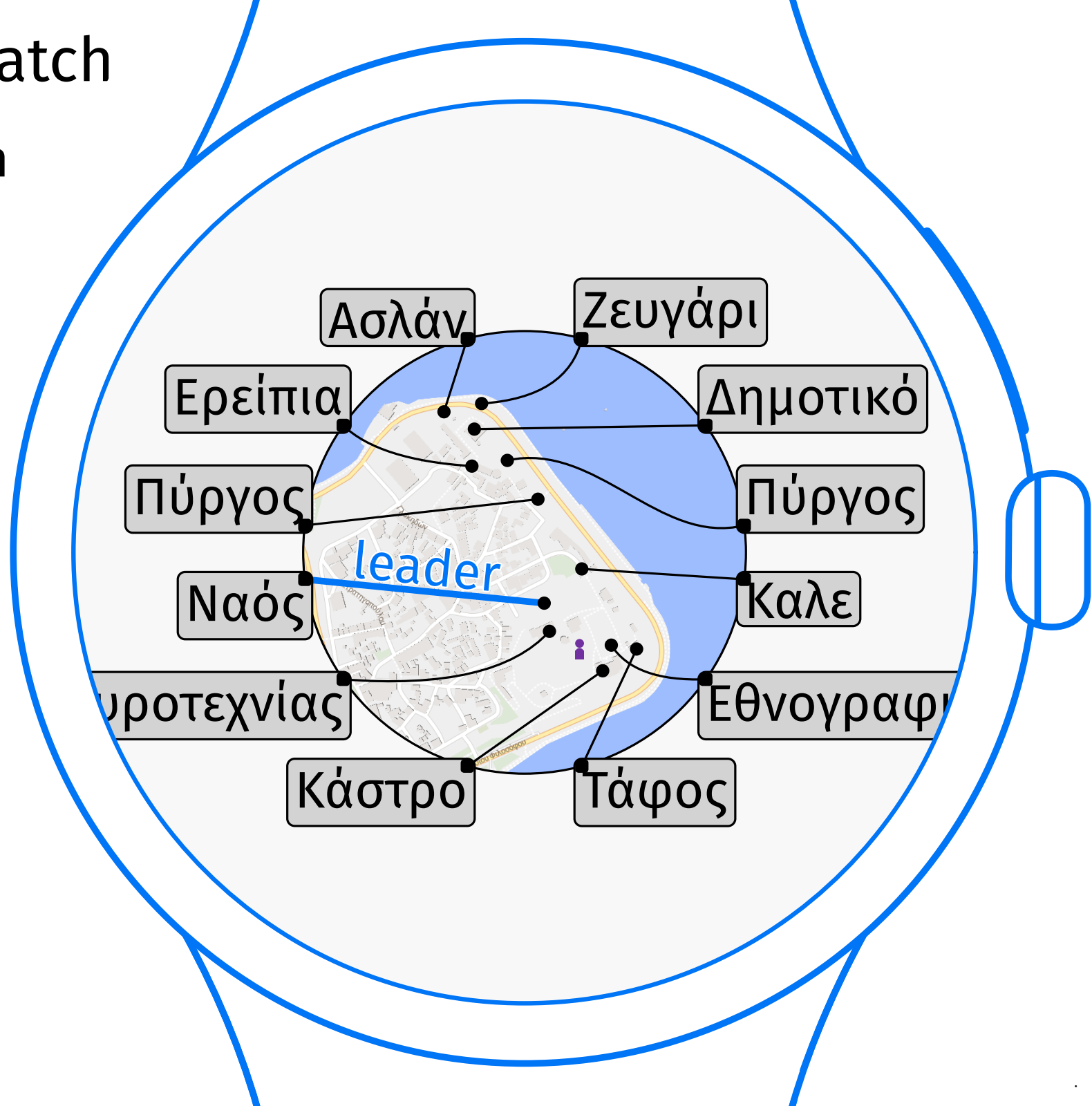
Labeling on a smart watch

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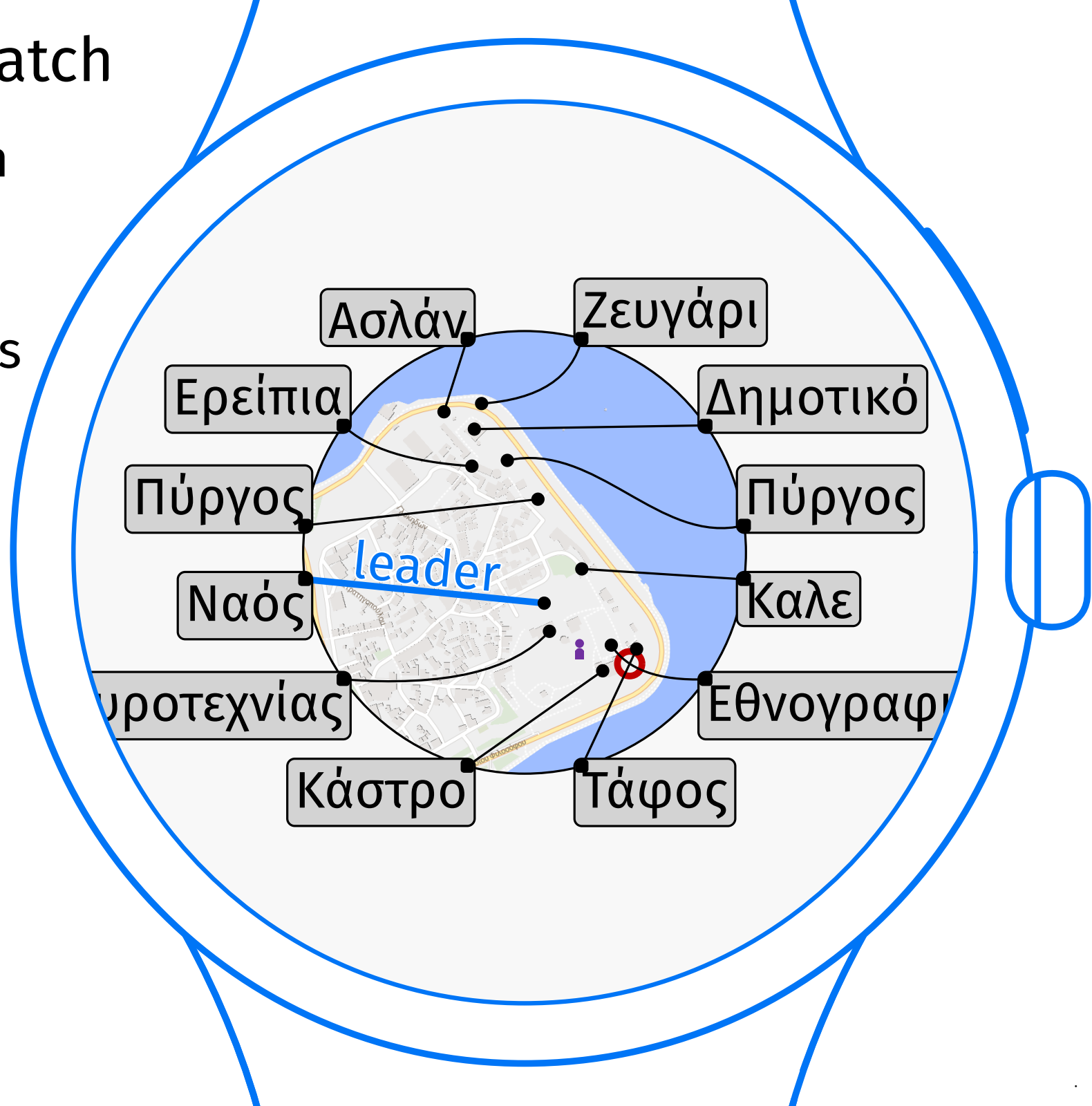
Labeling on a smart watch

- reduce map obstruction
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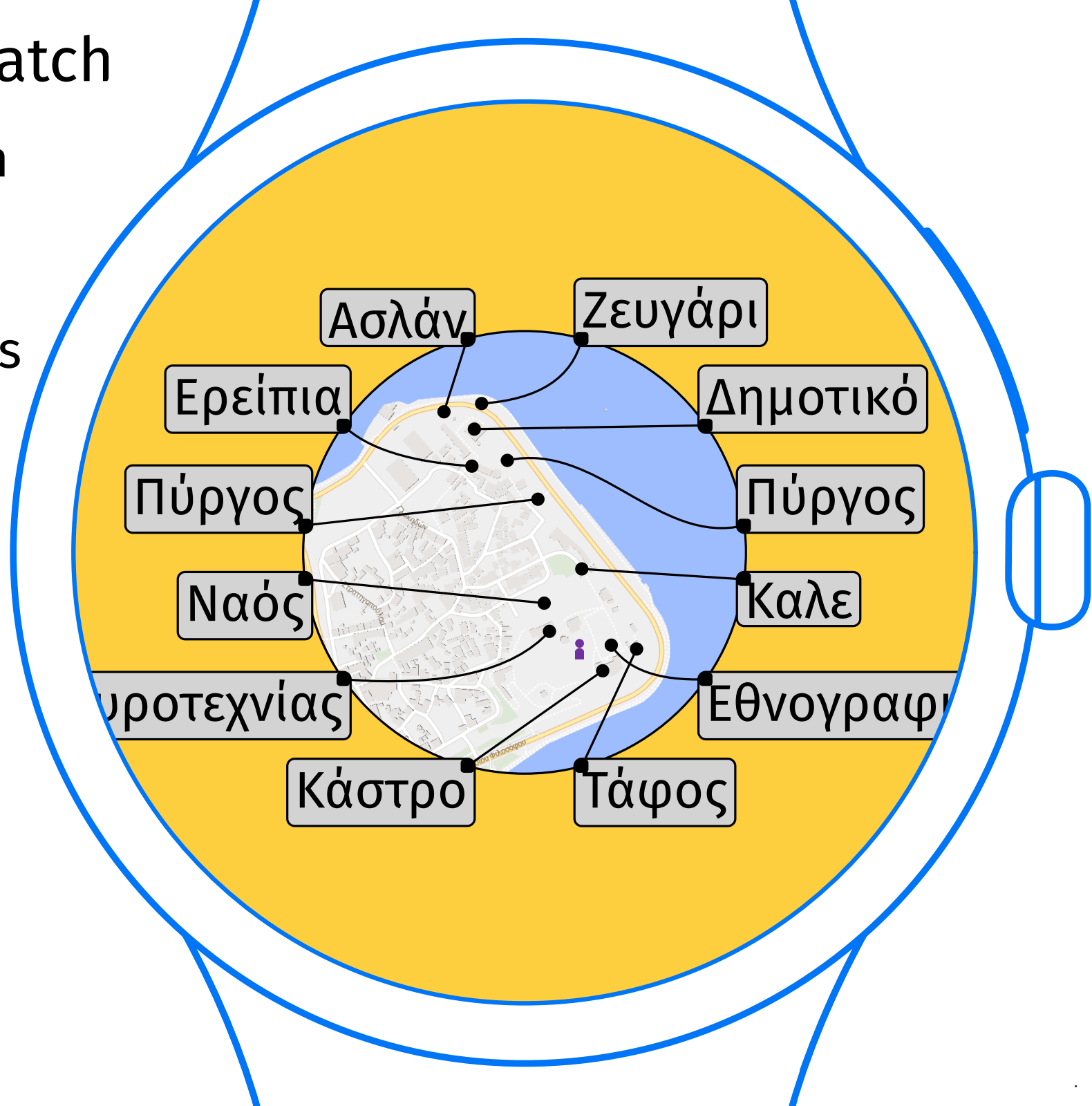
Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings



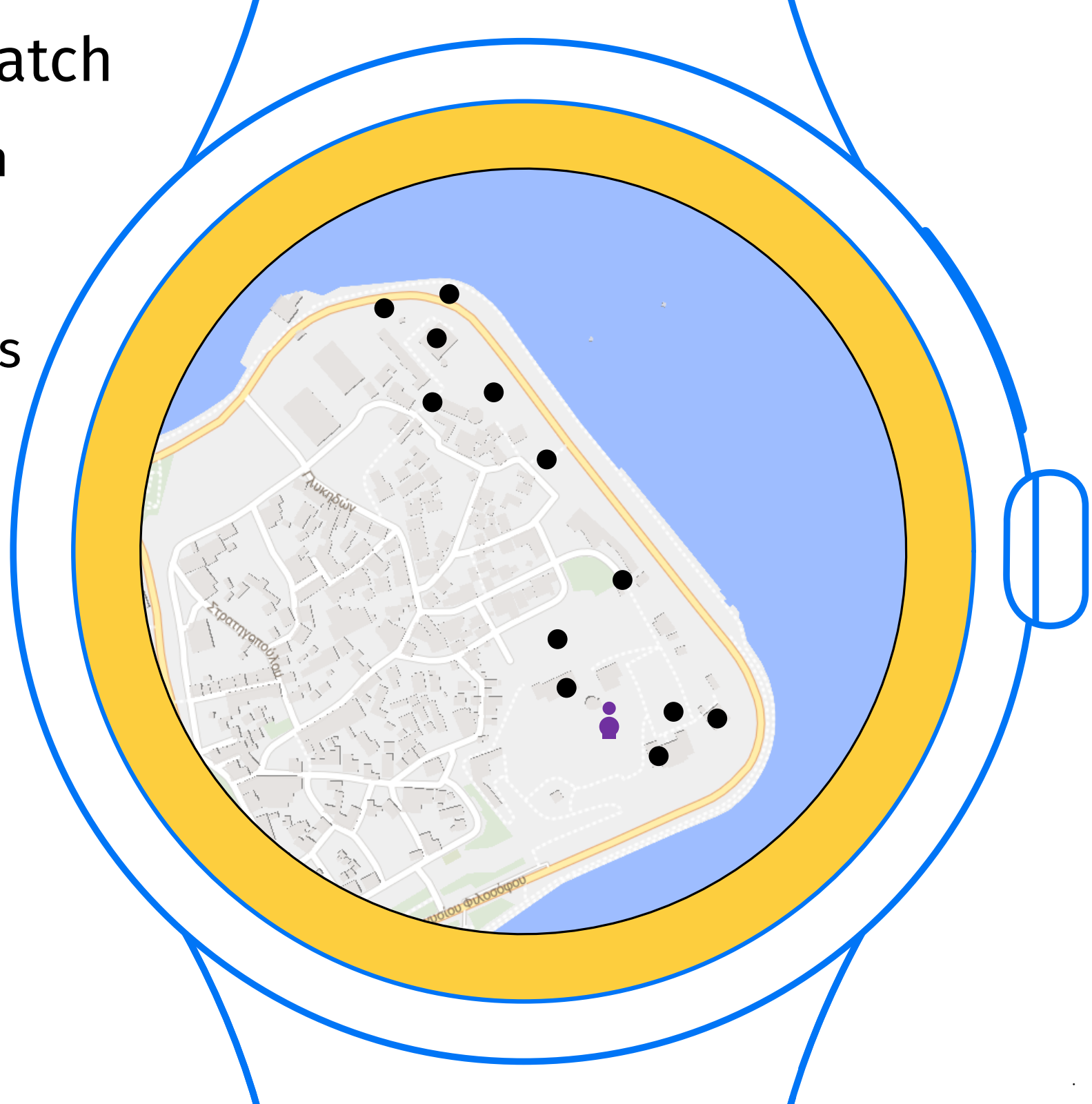
Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage



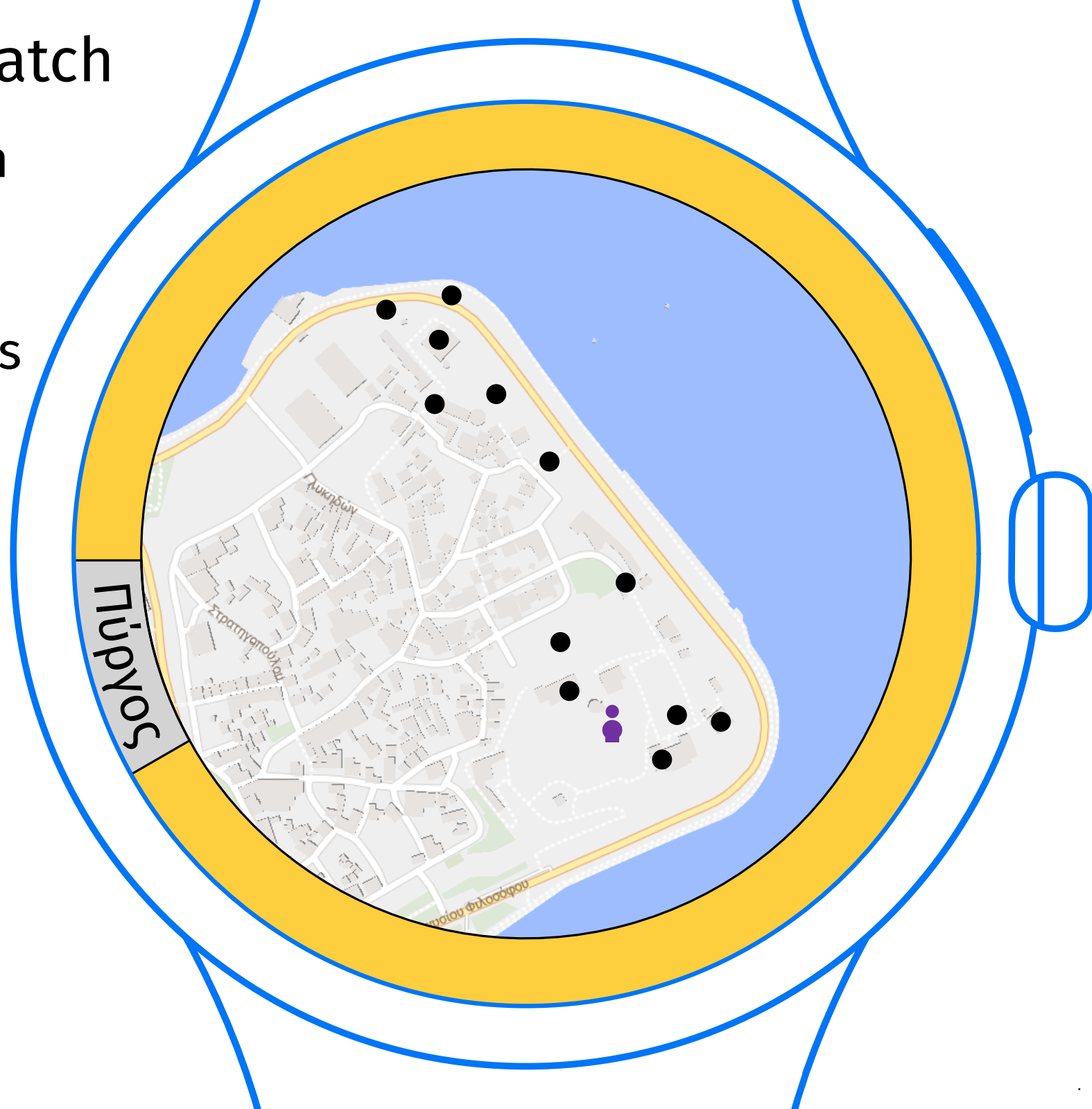
Labeling on a smart watch

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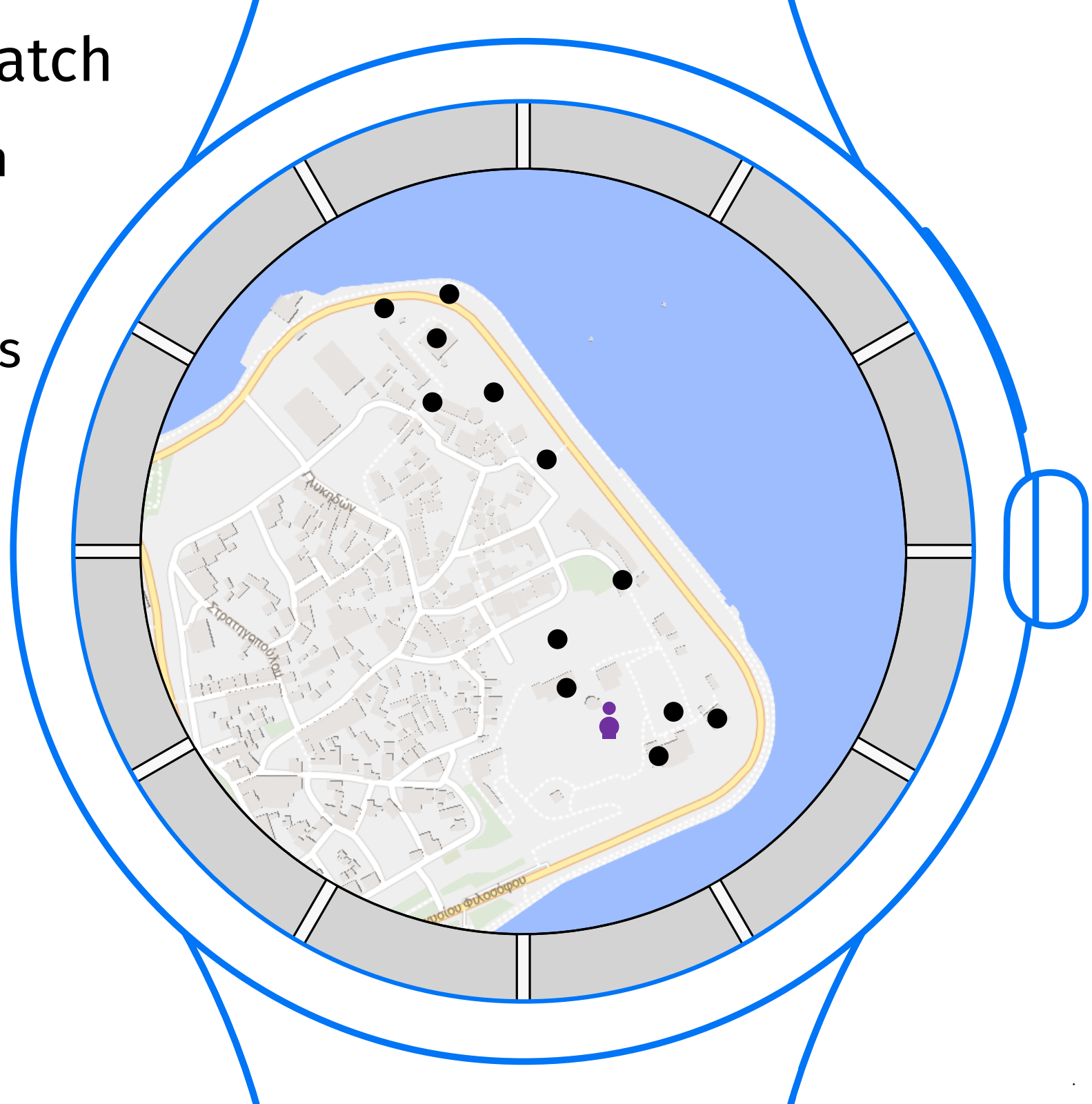
Labeling on a smart watch

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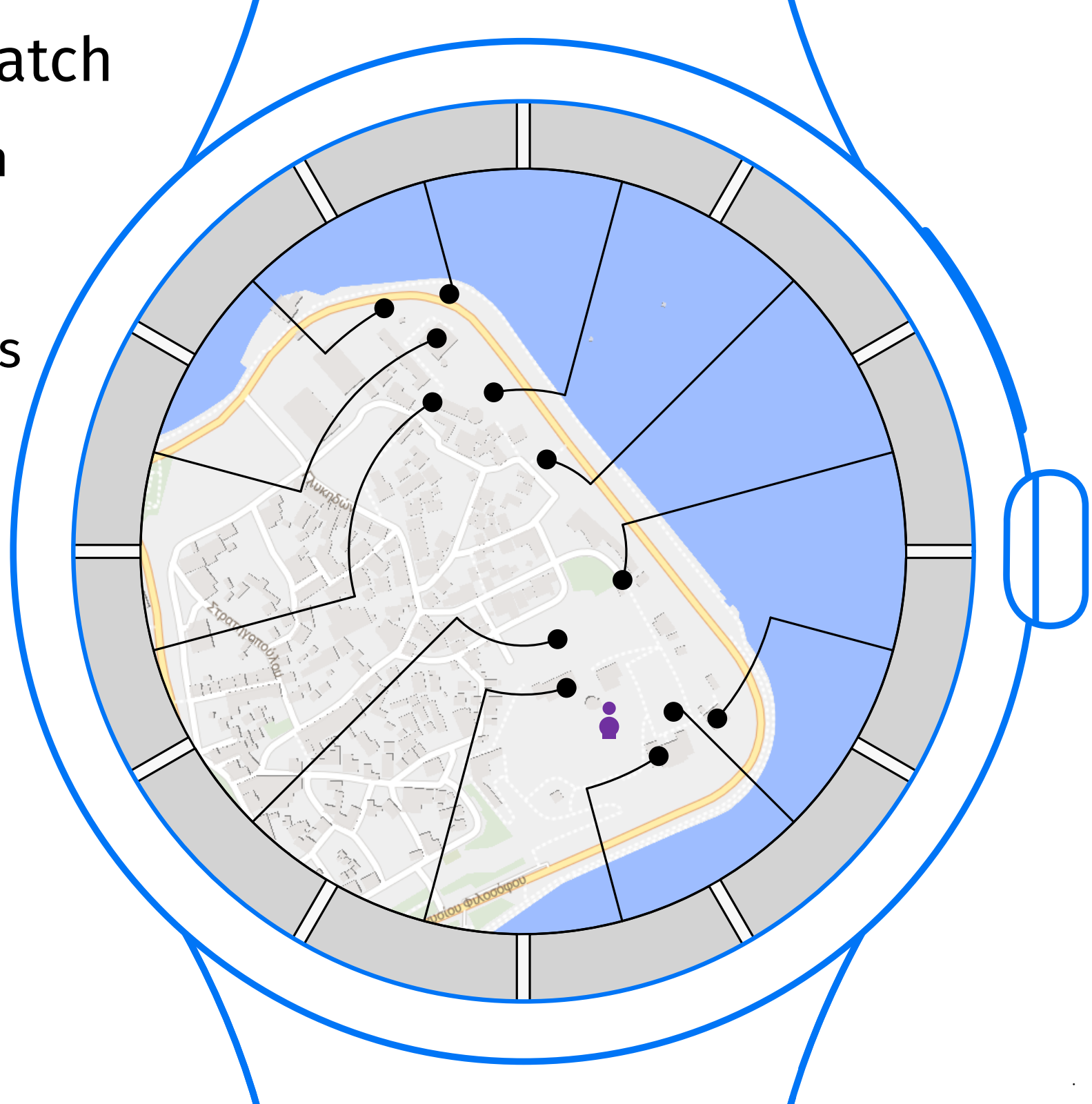
Labeling on a smart watch

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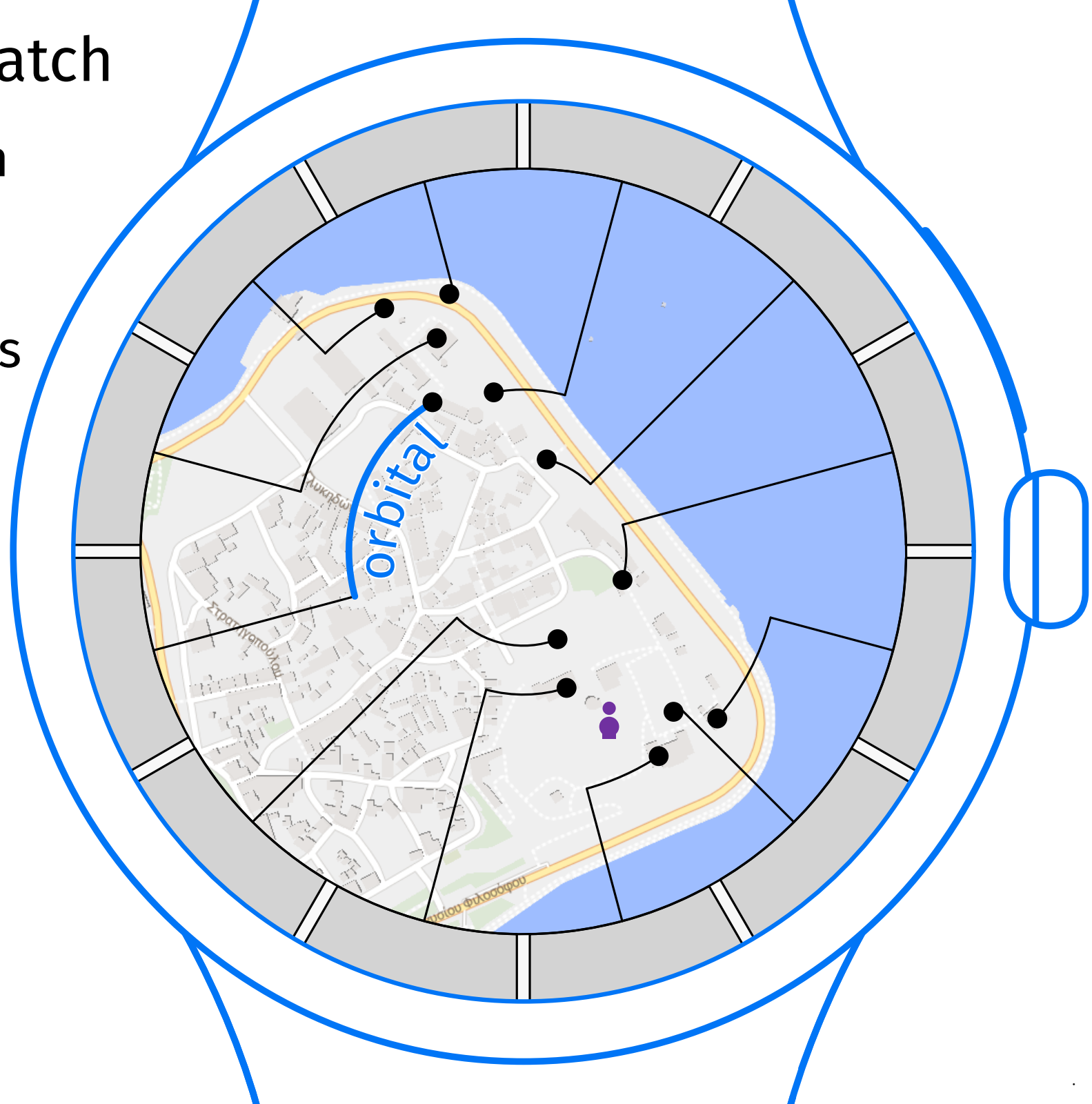
Labeling on a smart watch

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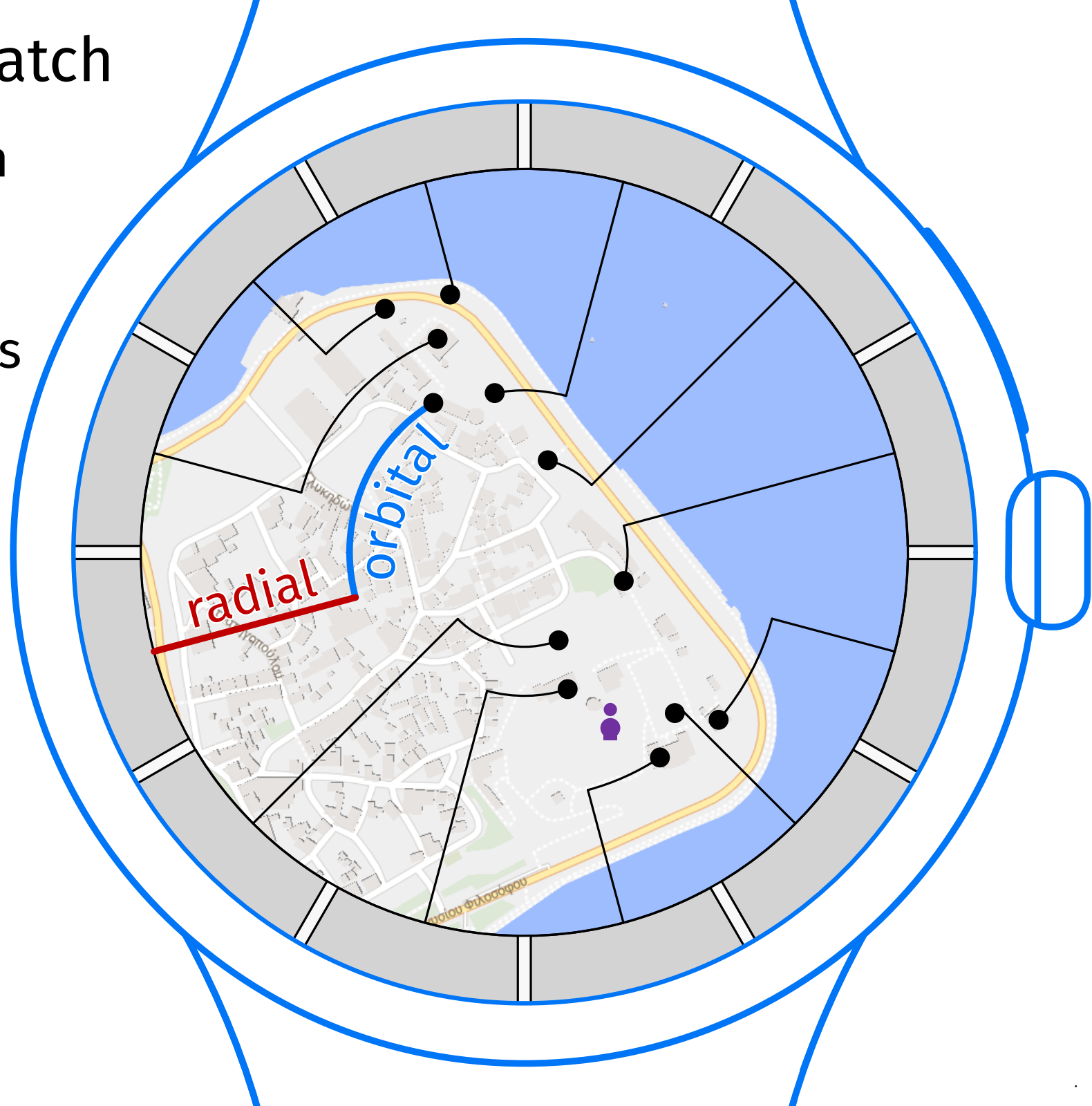
Labeling on a smart watch

- reduce map obstruction
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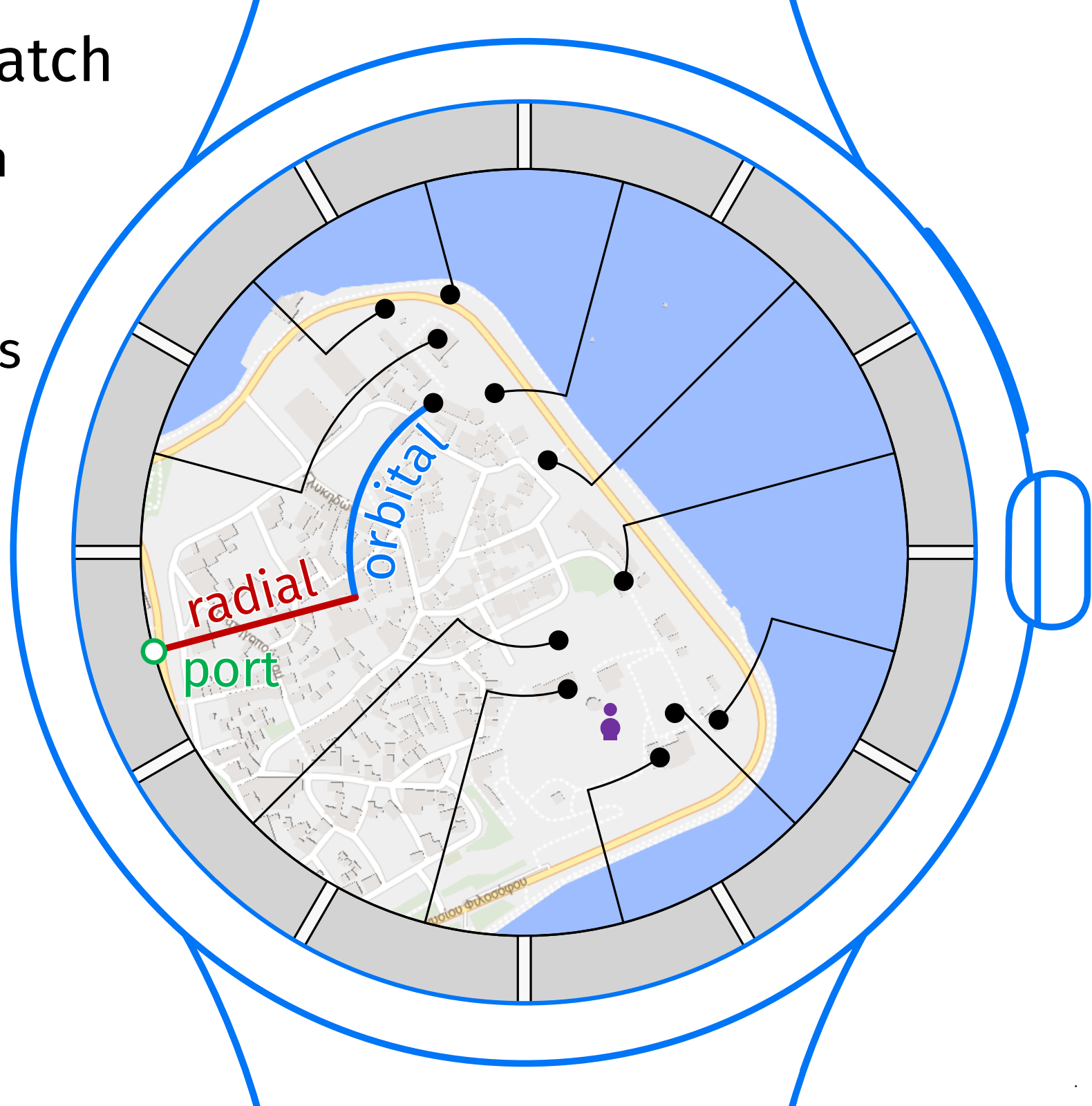
Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage



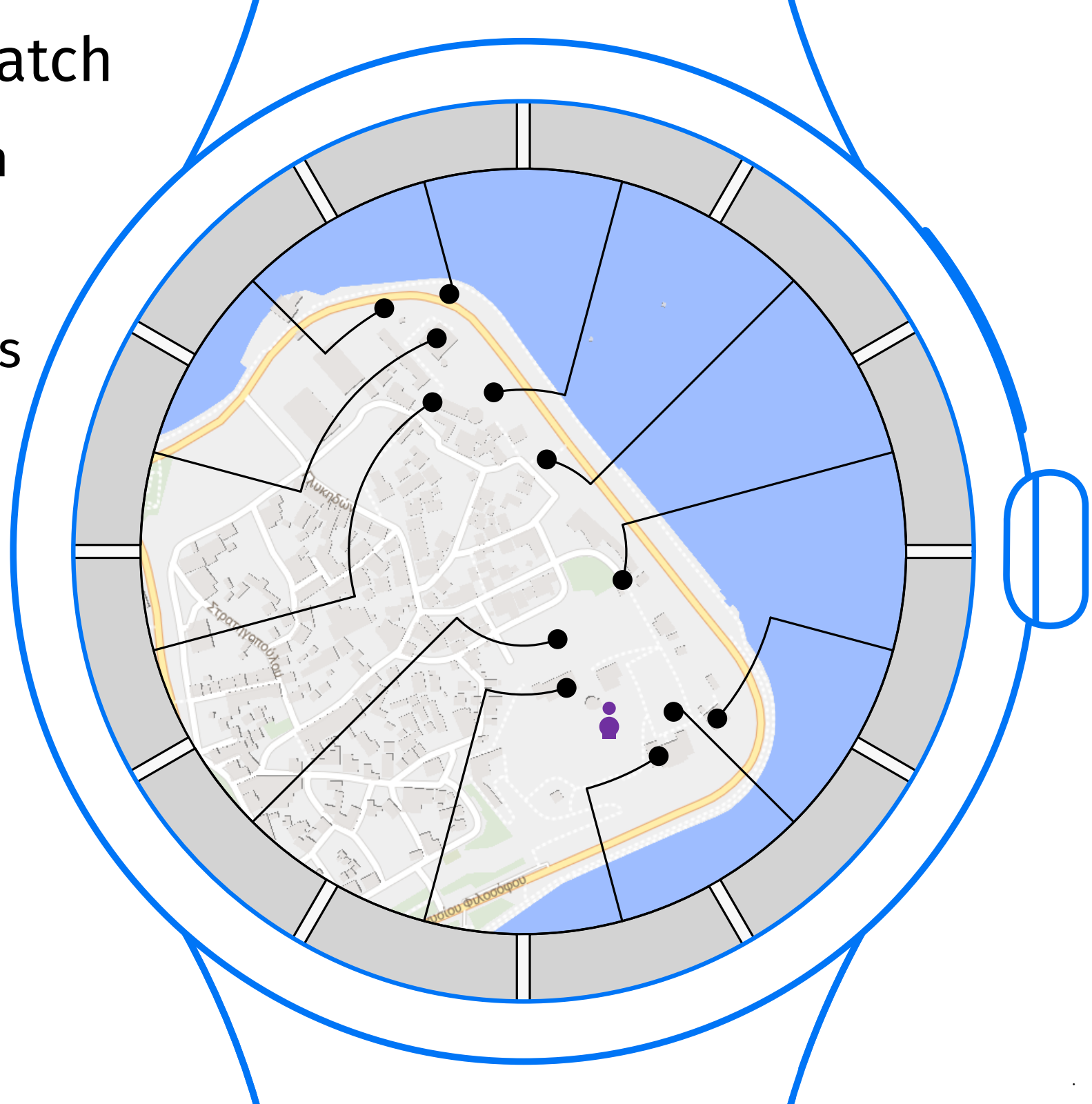
Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage



Labeling on a smart watch

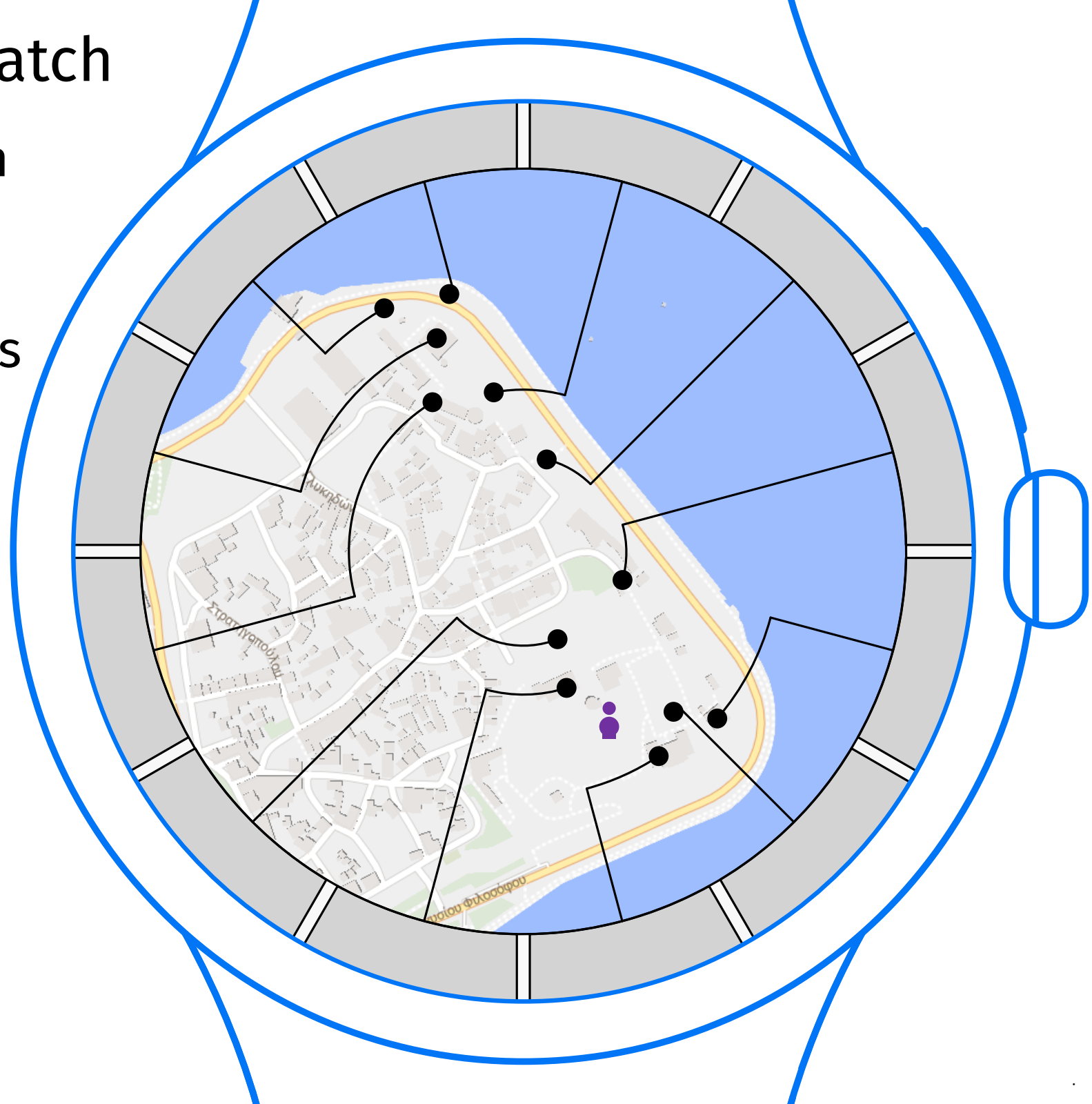
- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage



Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

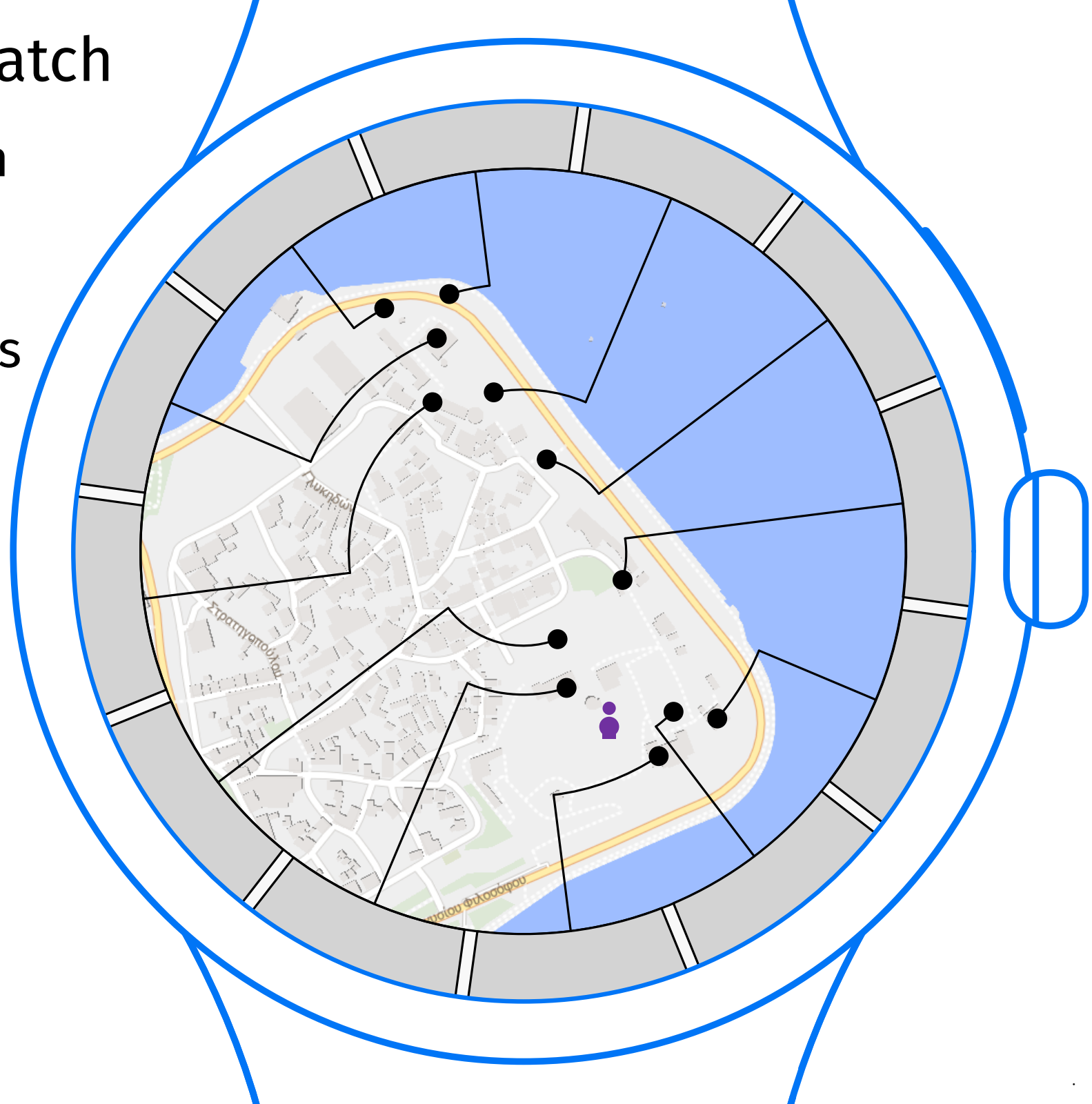
goal: minimize sum of leader lengths



Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

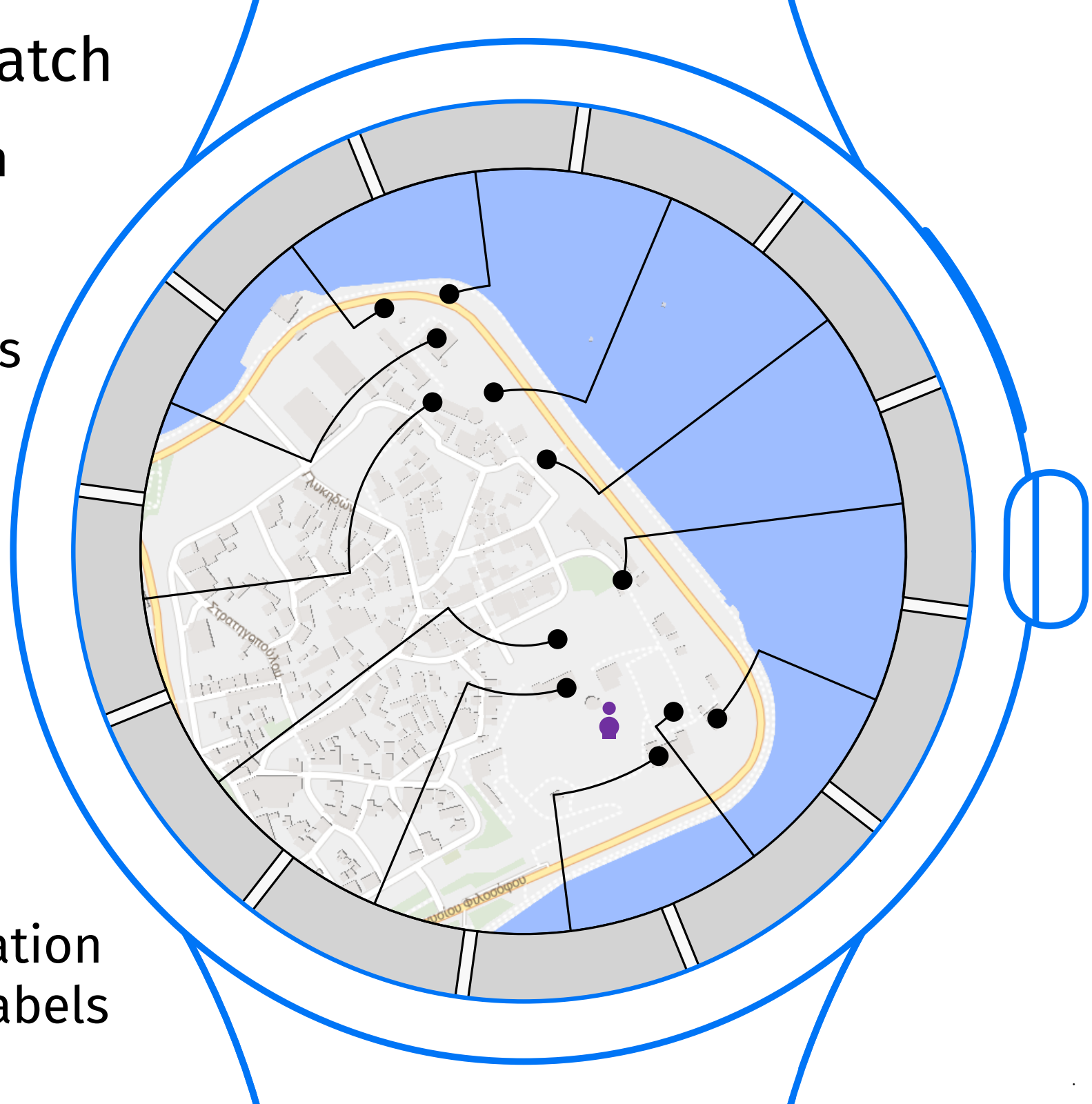


Labeling on a smart watch

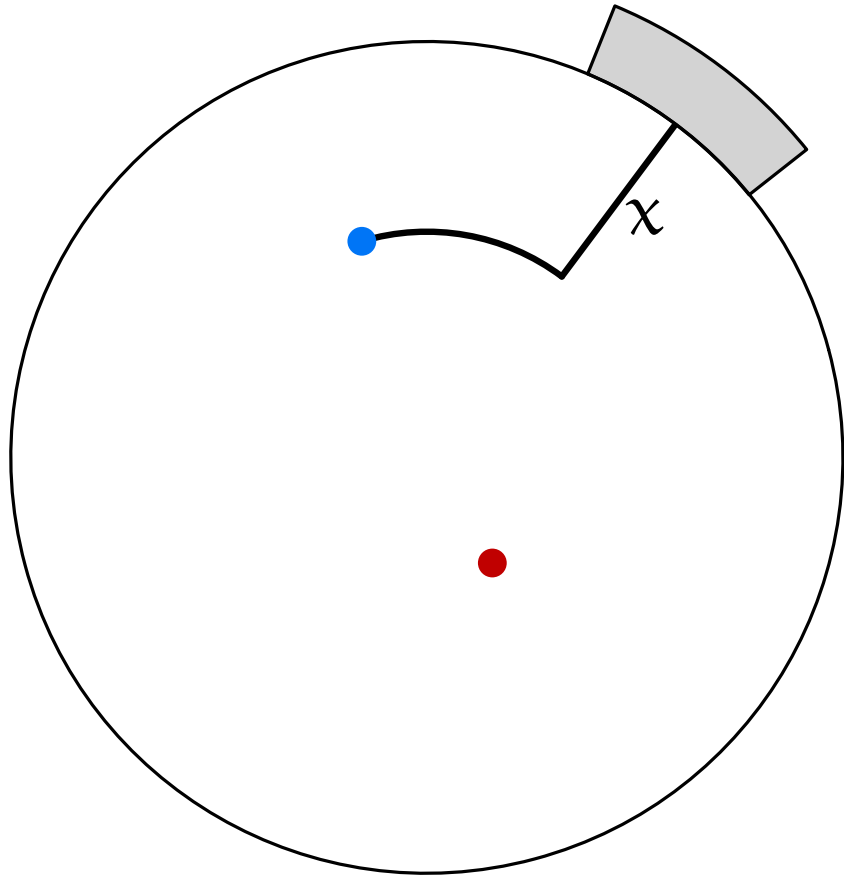
- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

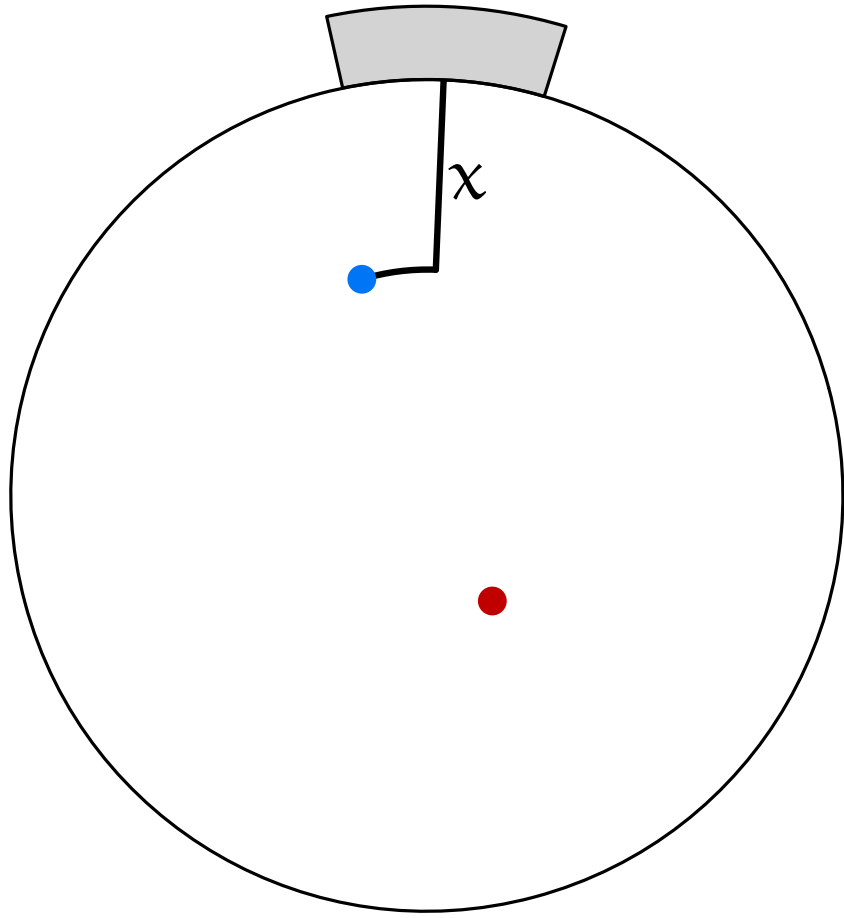
- under:**
- continuous rotation
 - fixed order of labels



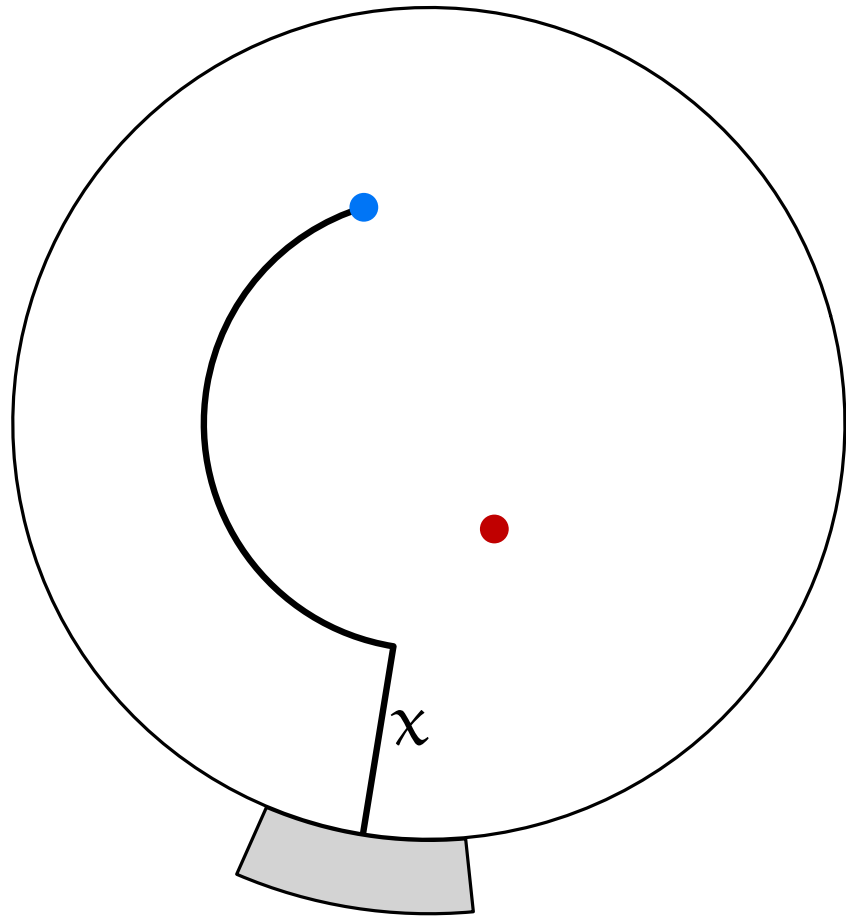
Rotating the labels



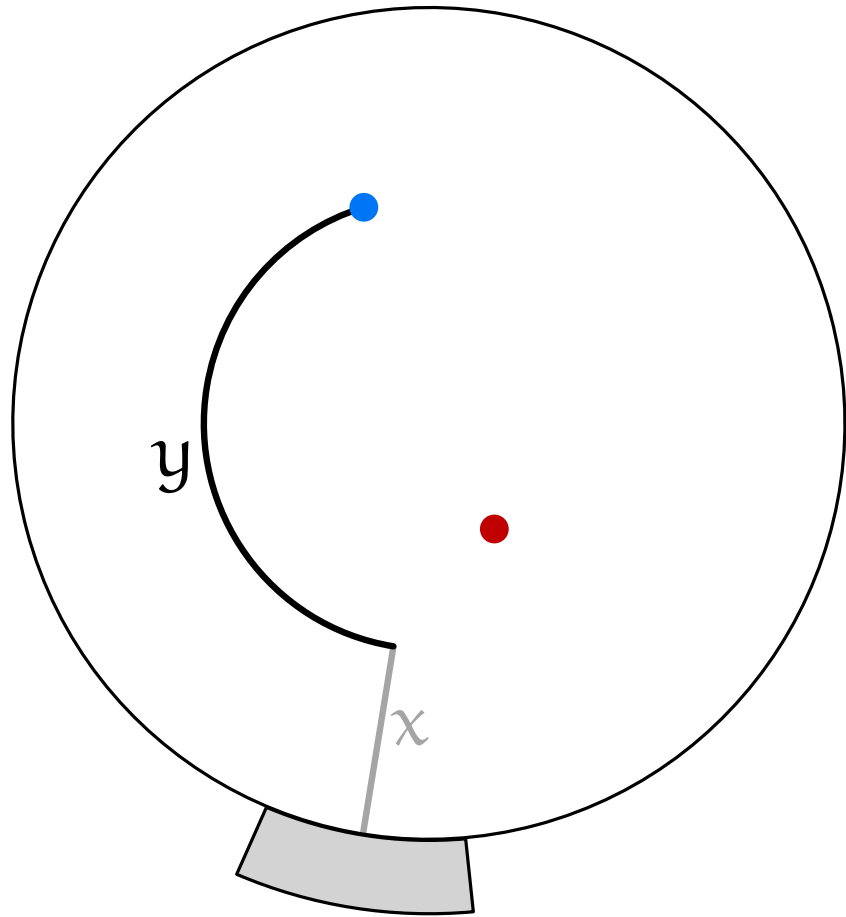
Rotating the labels



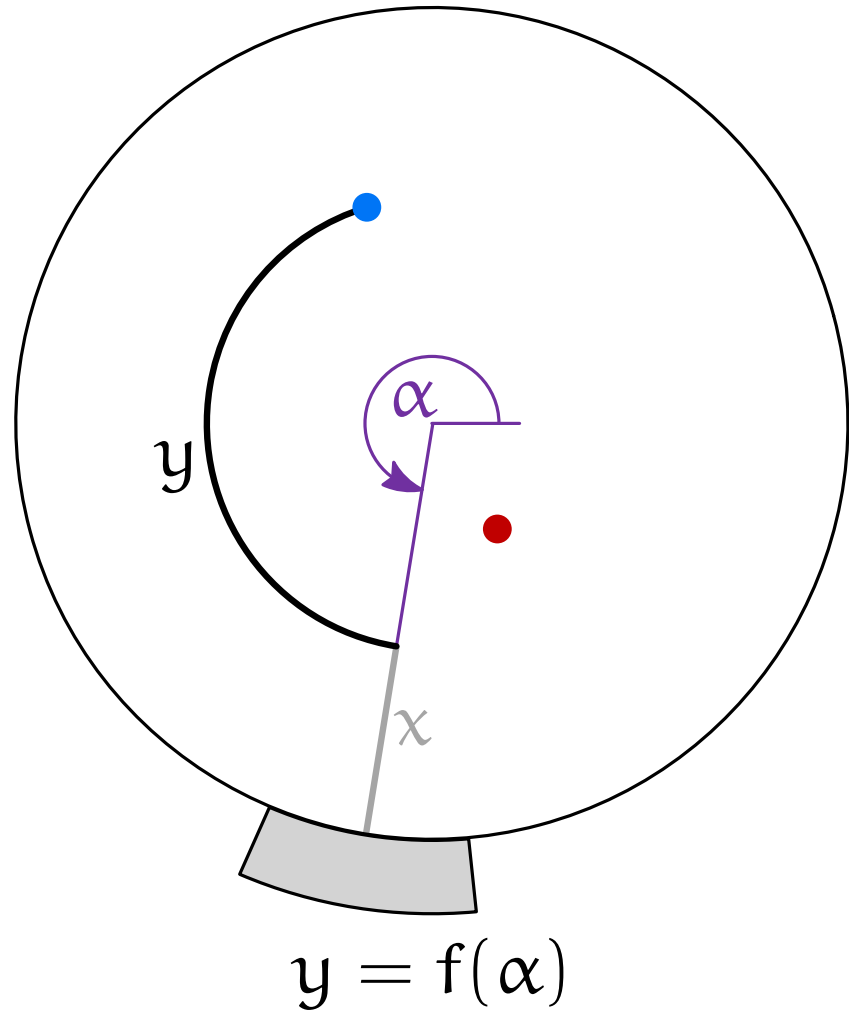
Rotating the labels



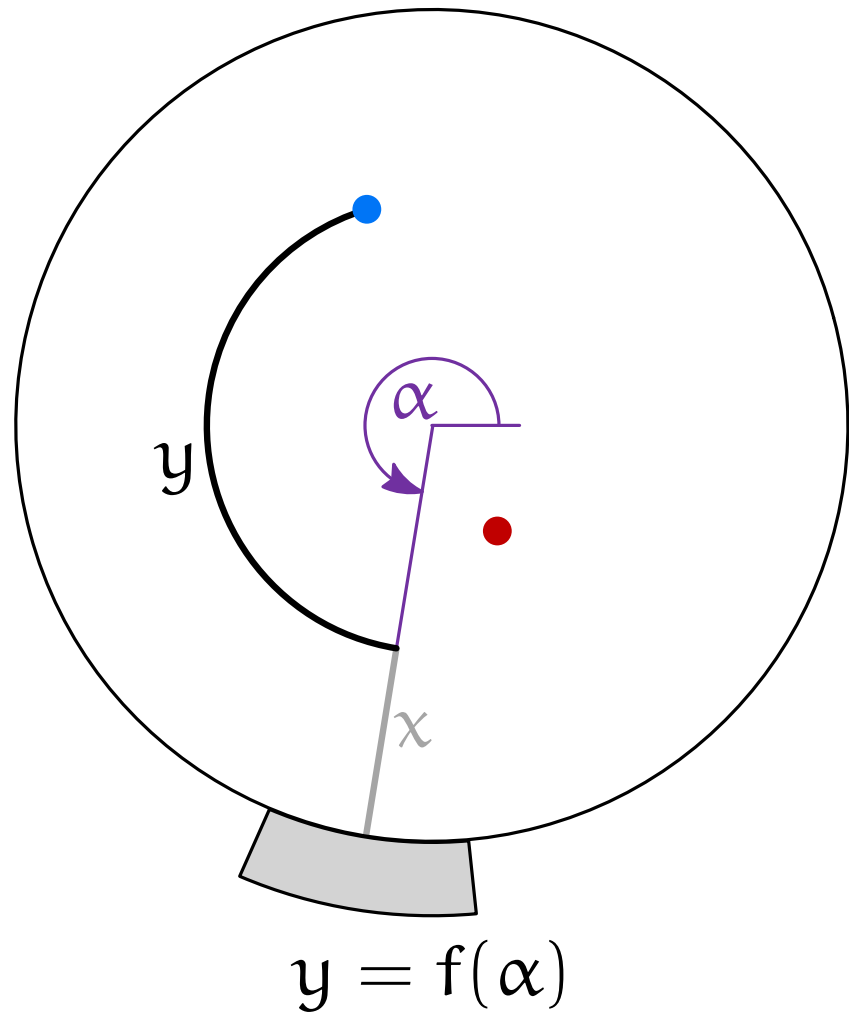
Rotating the labels



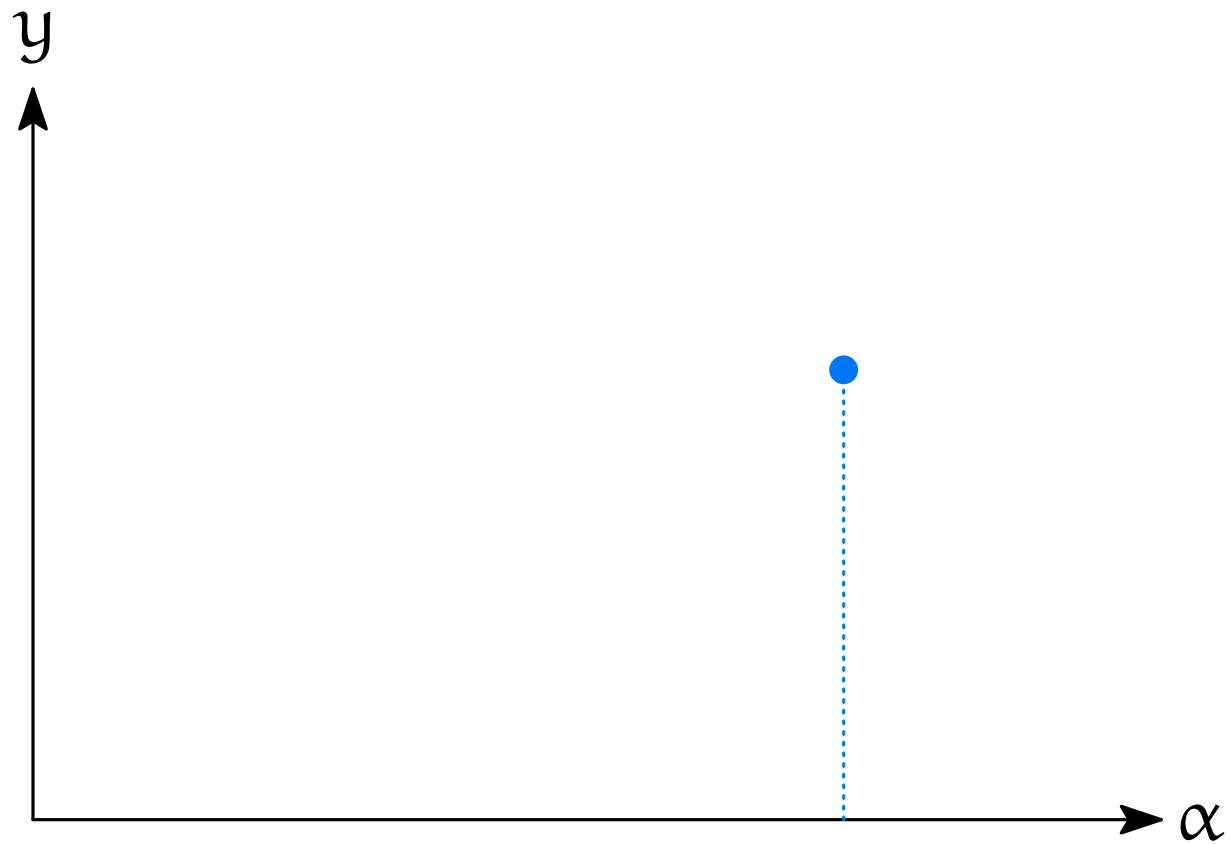
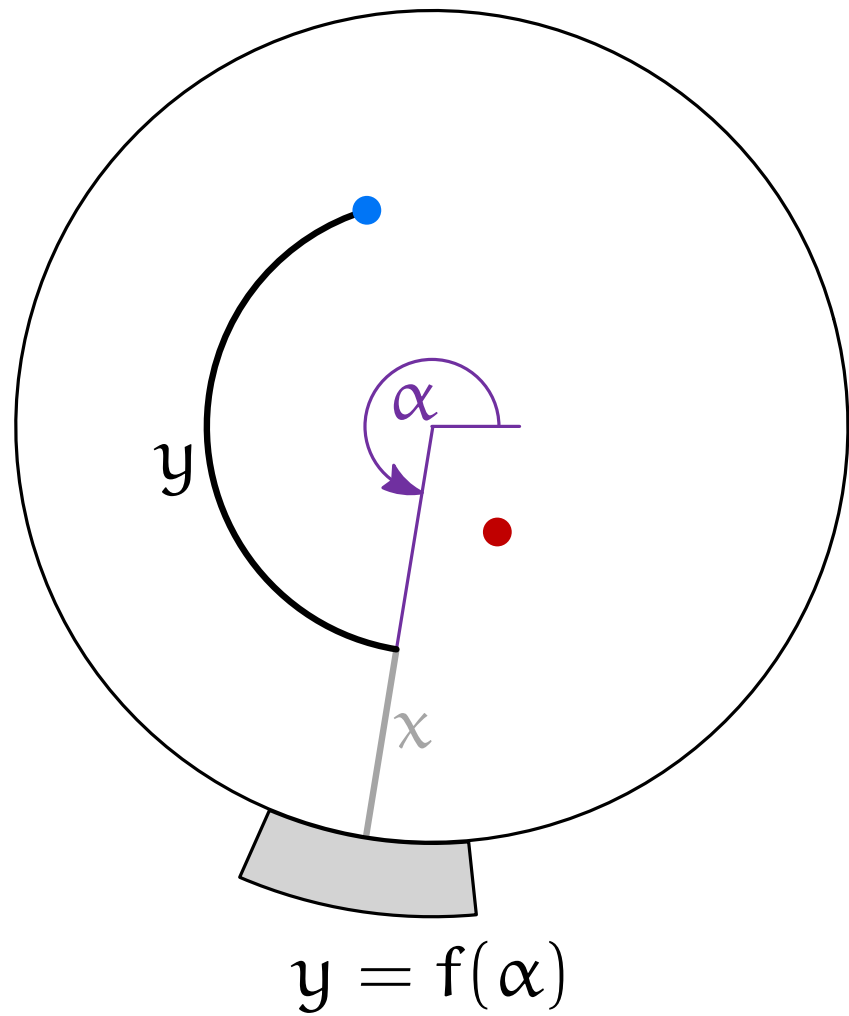
Rotating the labels



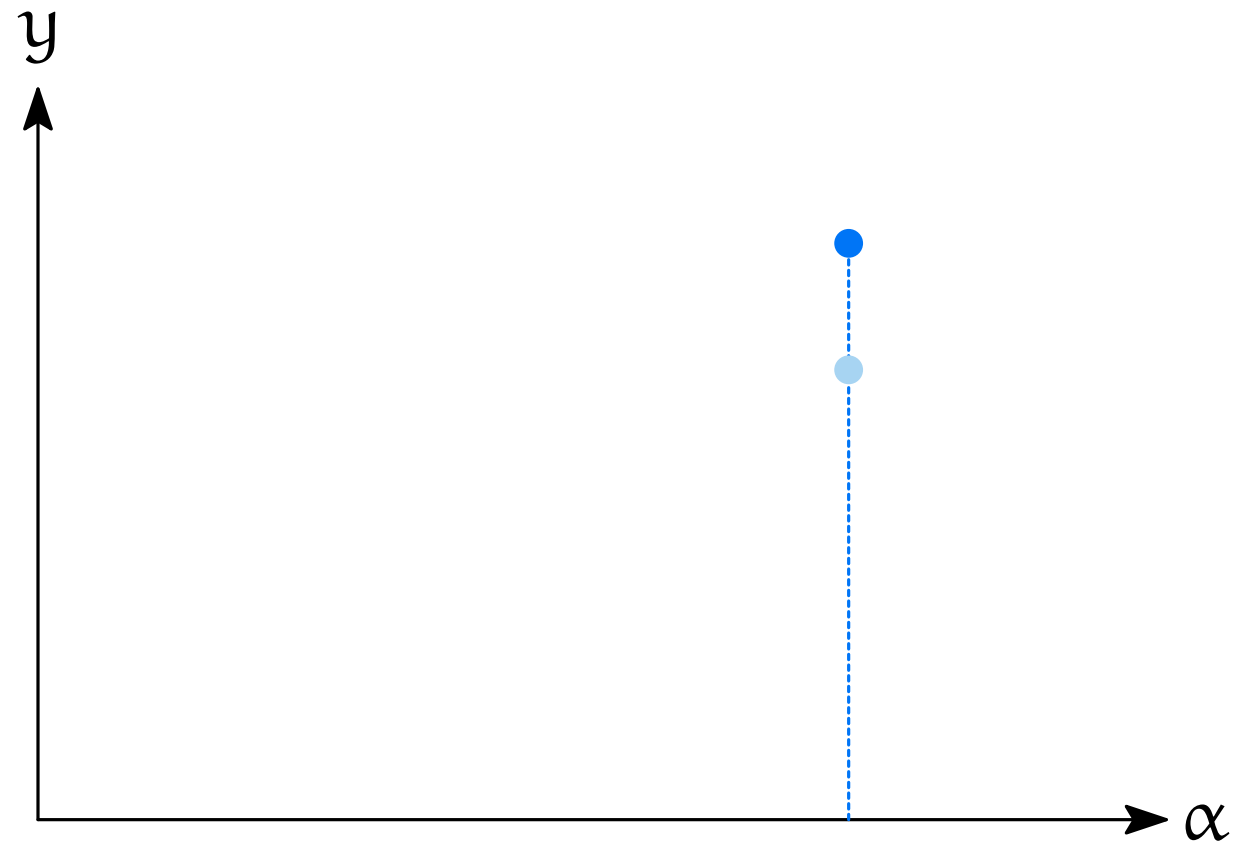
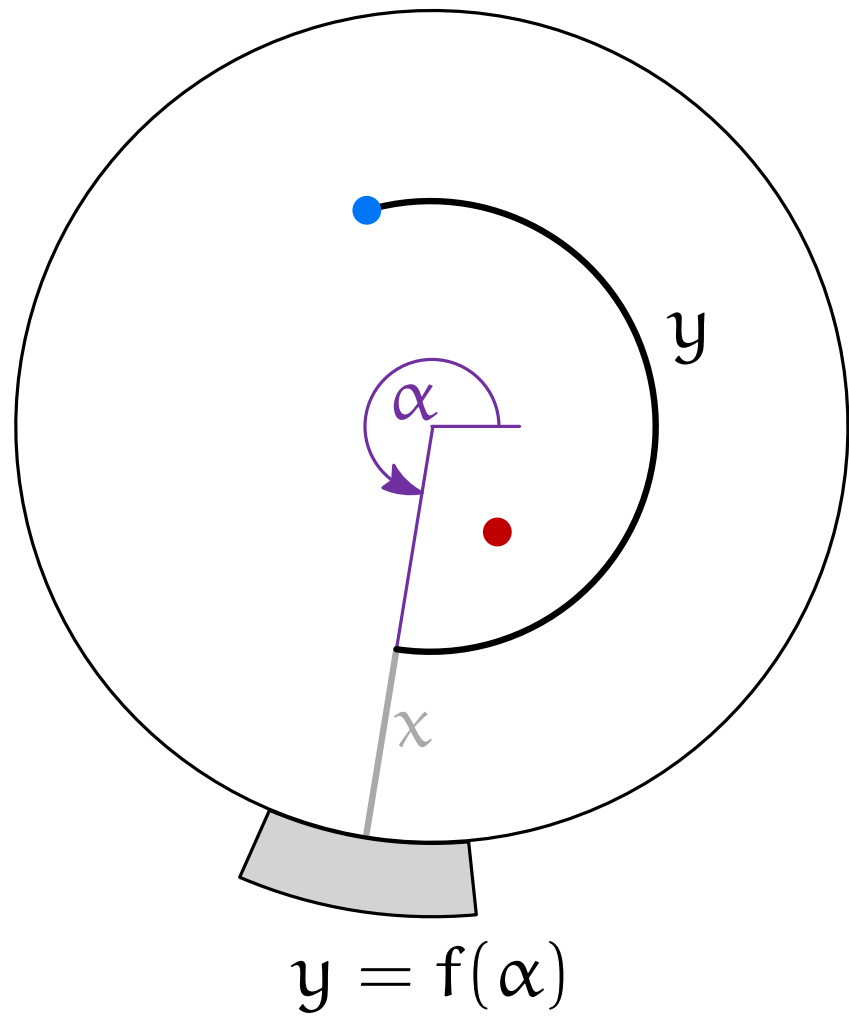
Rotating the labels



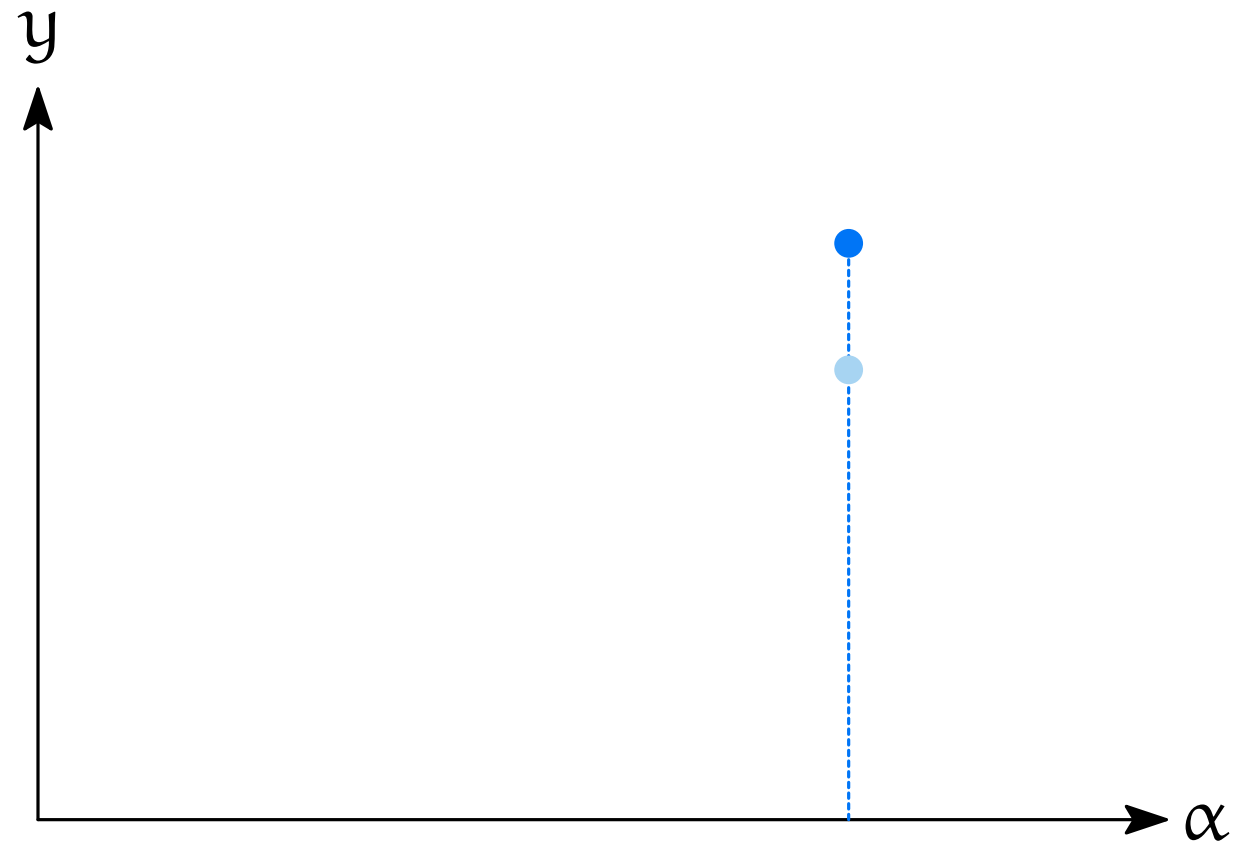
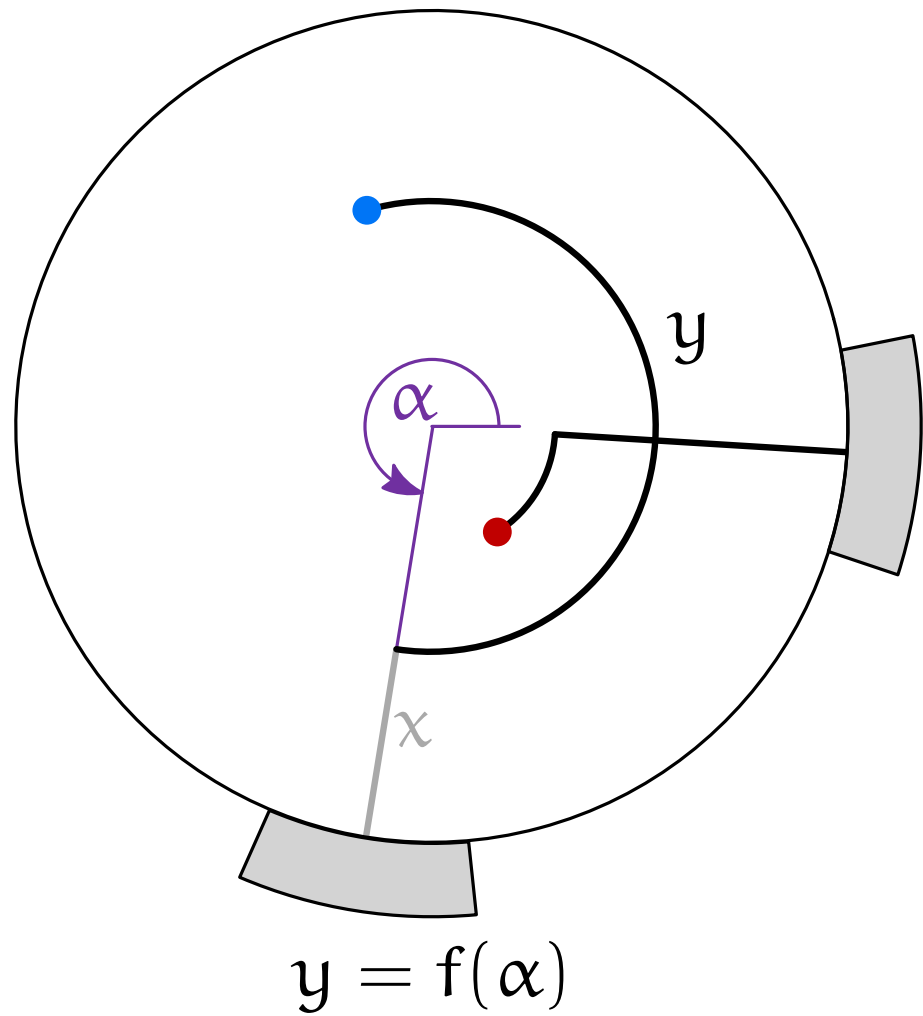
Rotating the labels



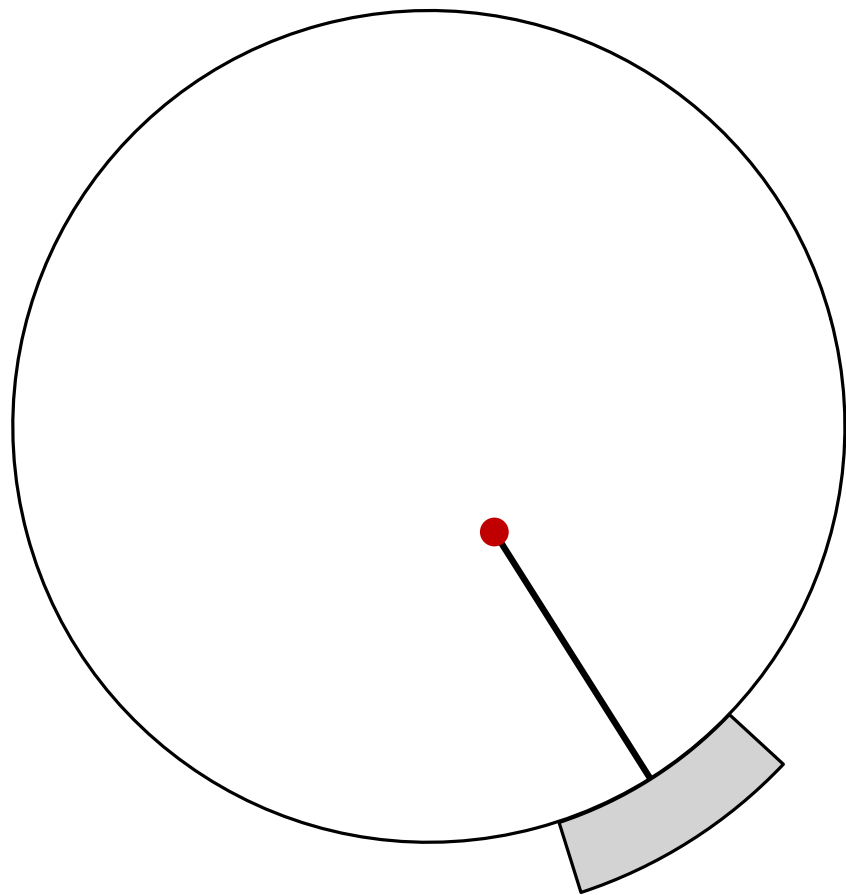
Rotating the labels



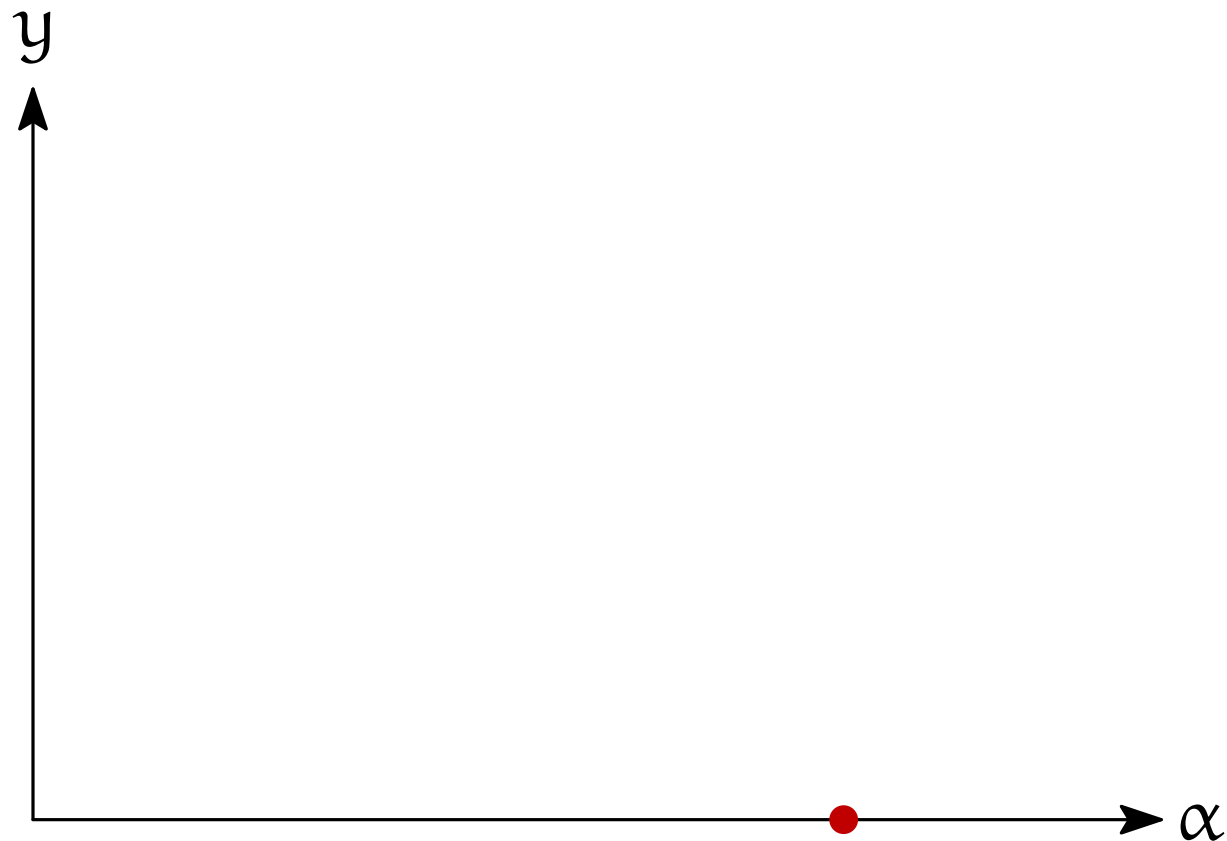
Rotating the labels



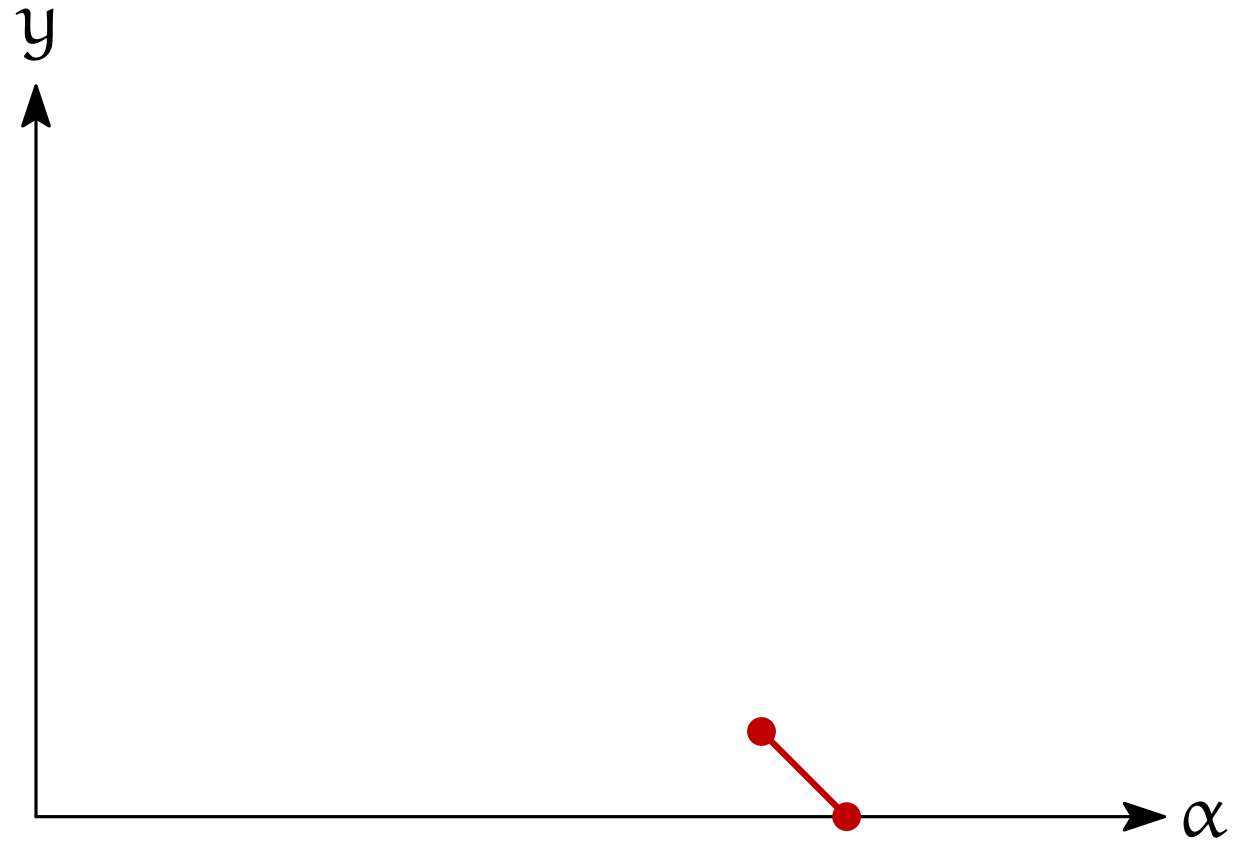
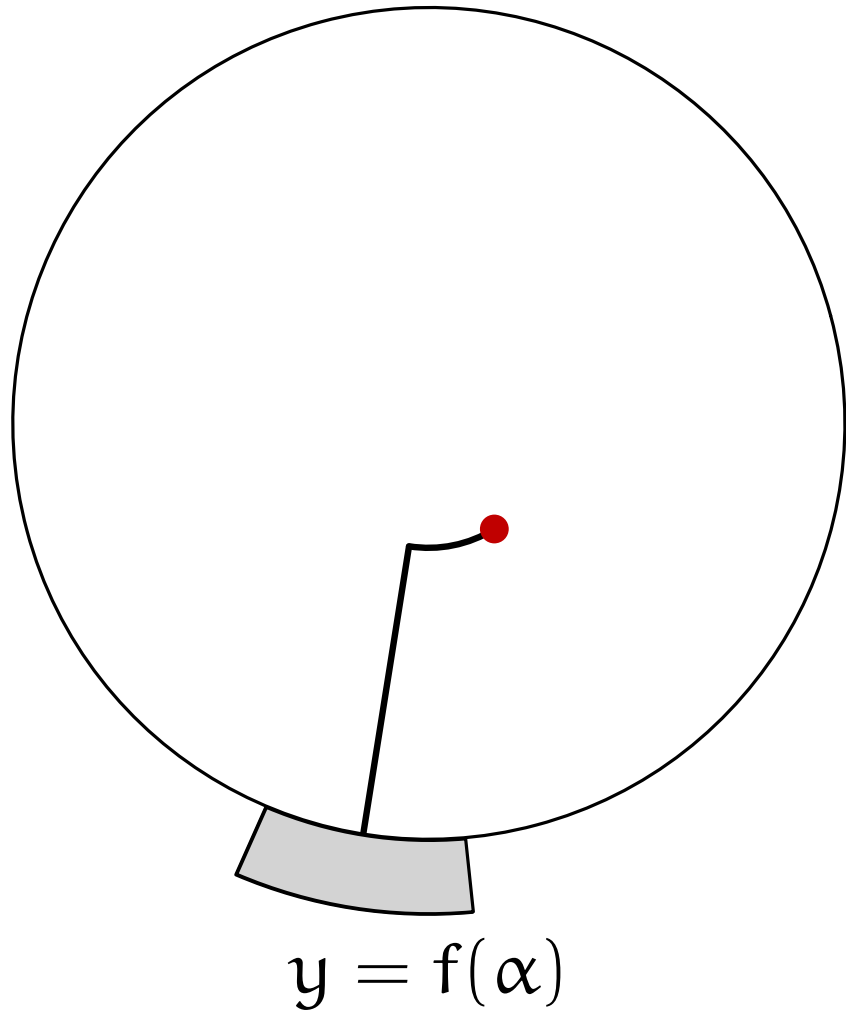
Rotating the labels



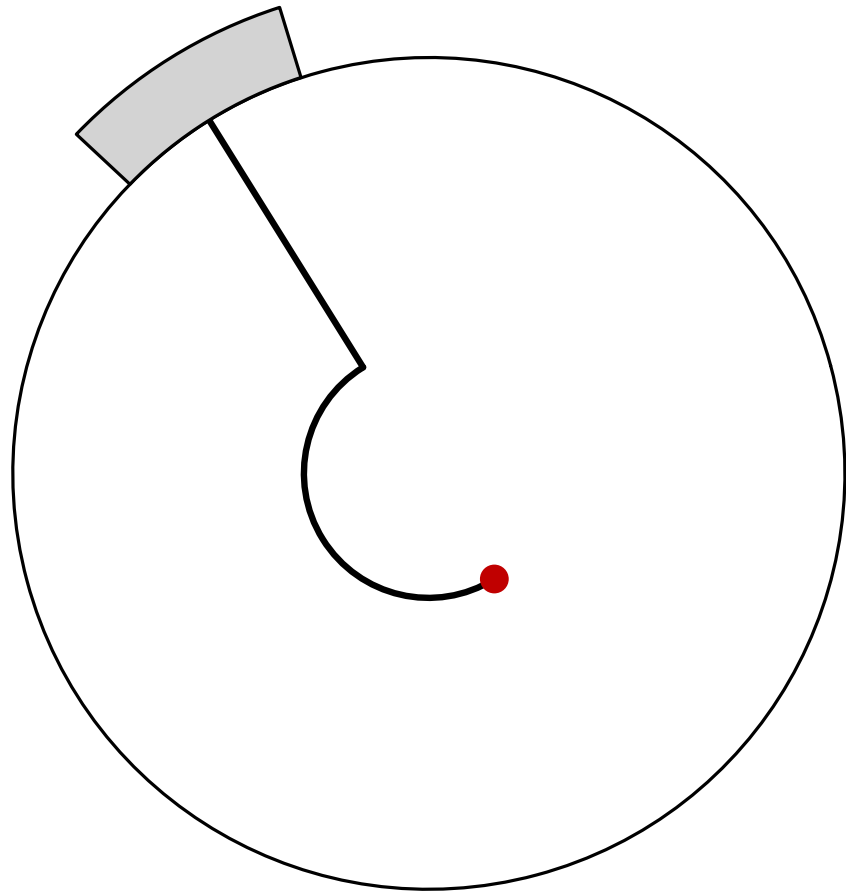
$$y = f(\alpha)$$



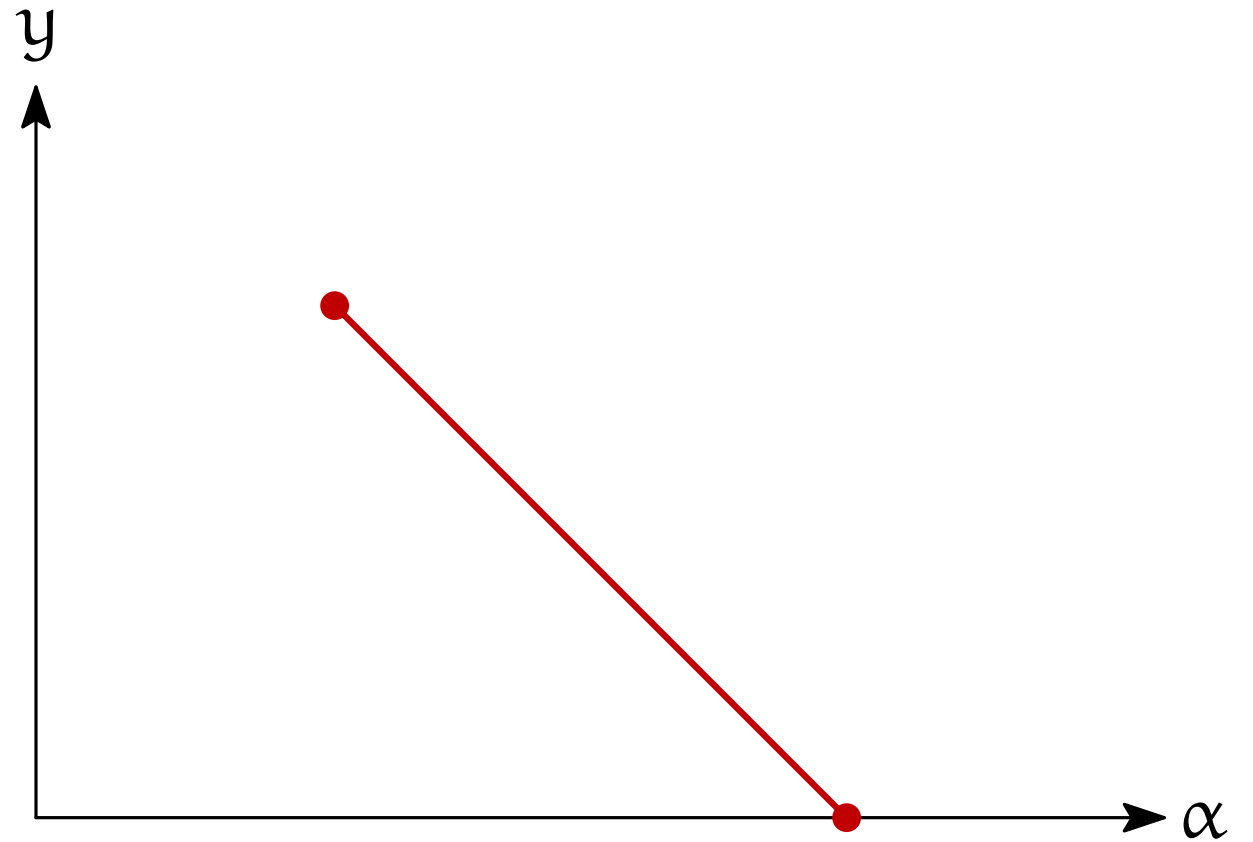
Rotating the labels



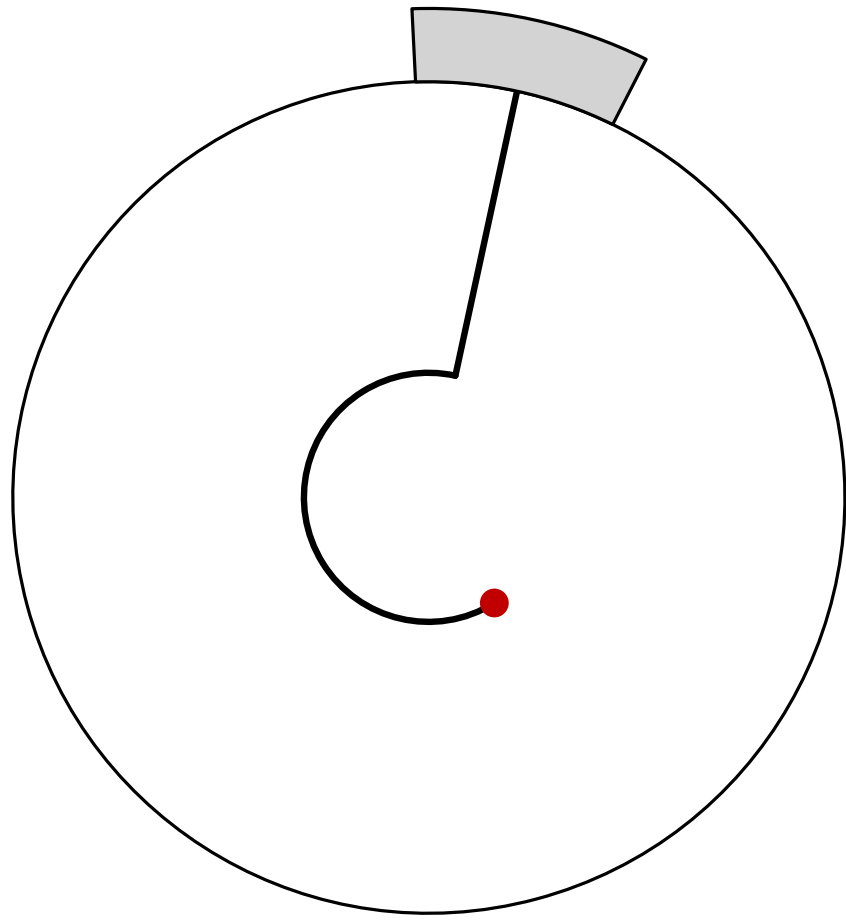
Rotating the labels



$$y = f(\alpha)$$



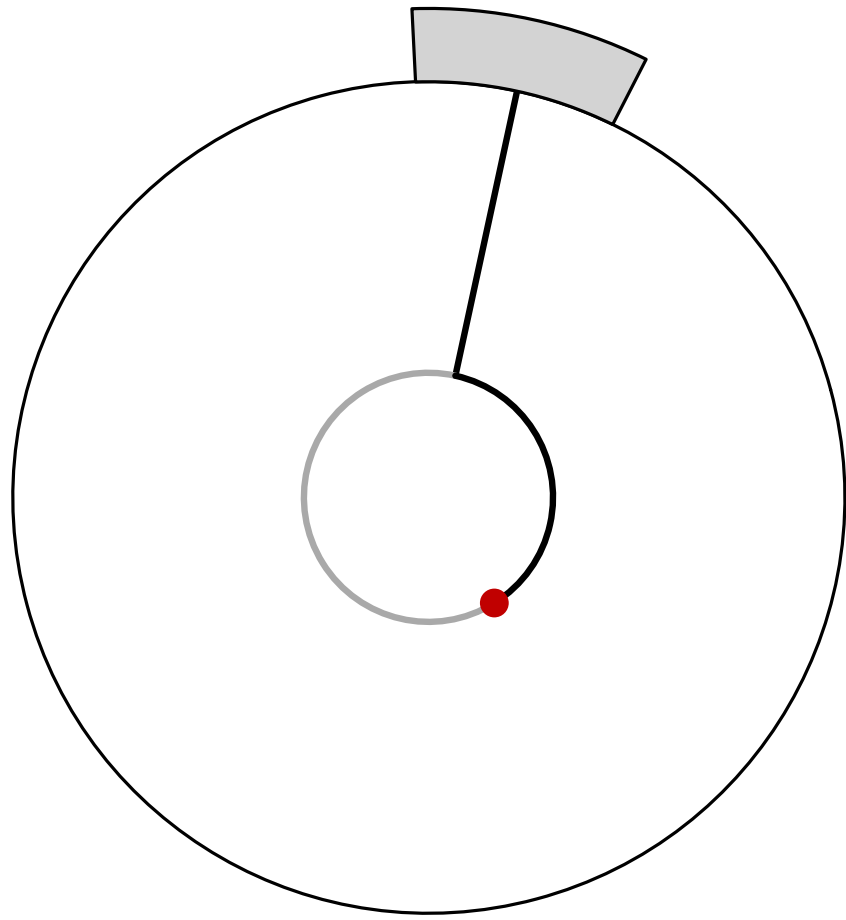
Rotating the labels



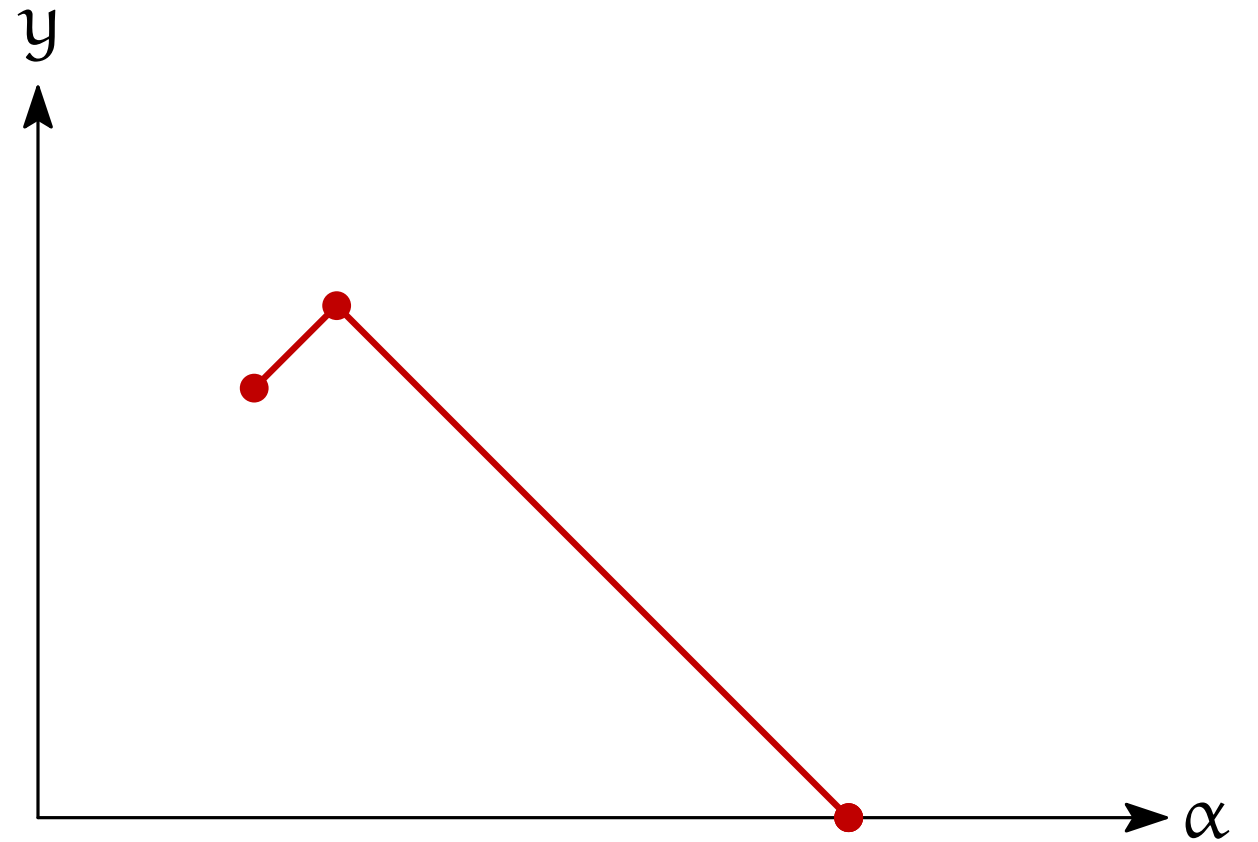
$$y = f(\alpha)$$



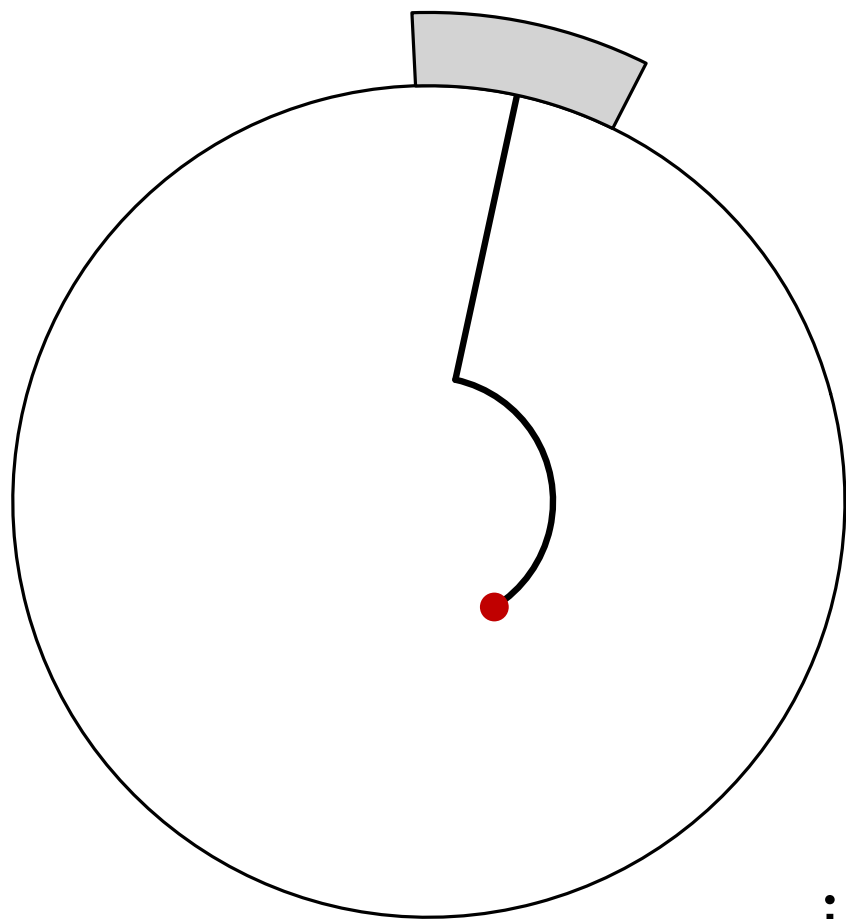
Rotating the labels



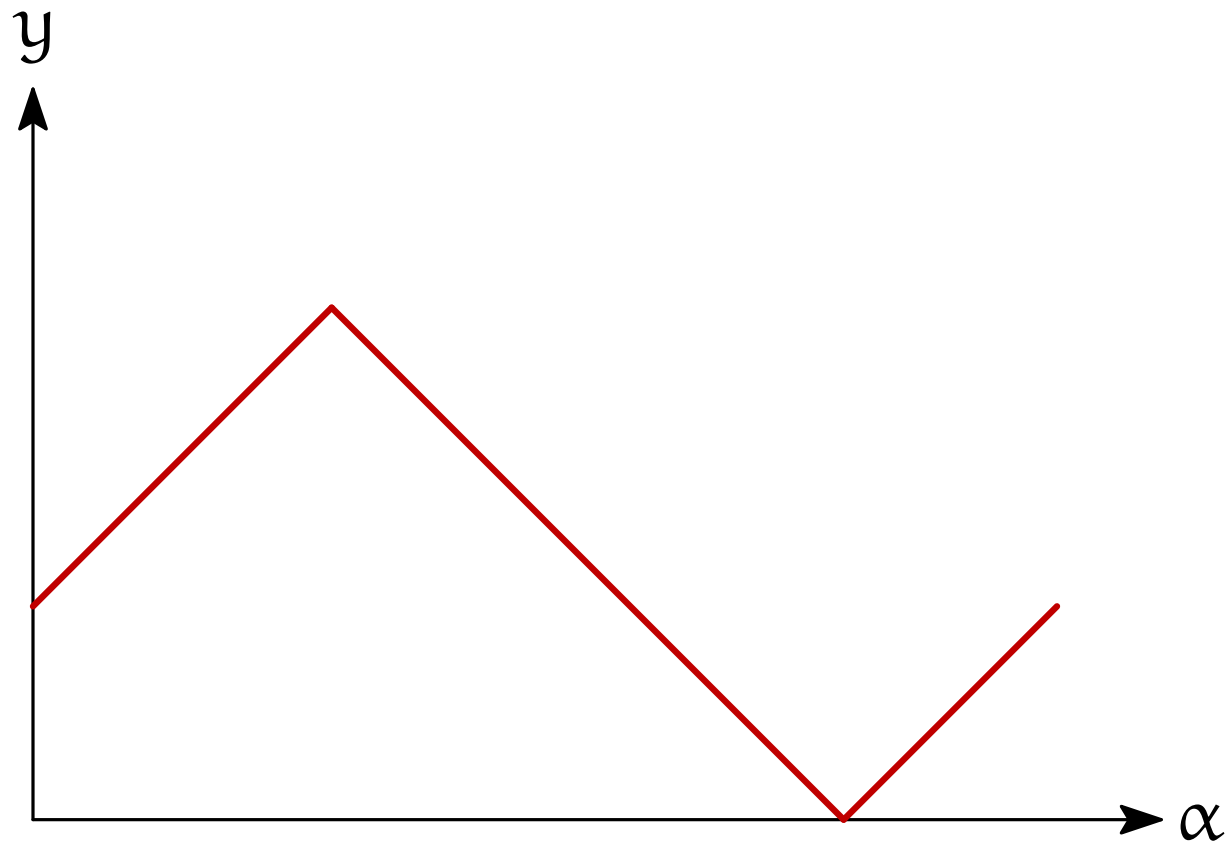
$$y = f(\alpha)$$



Rotating the labels



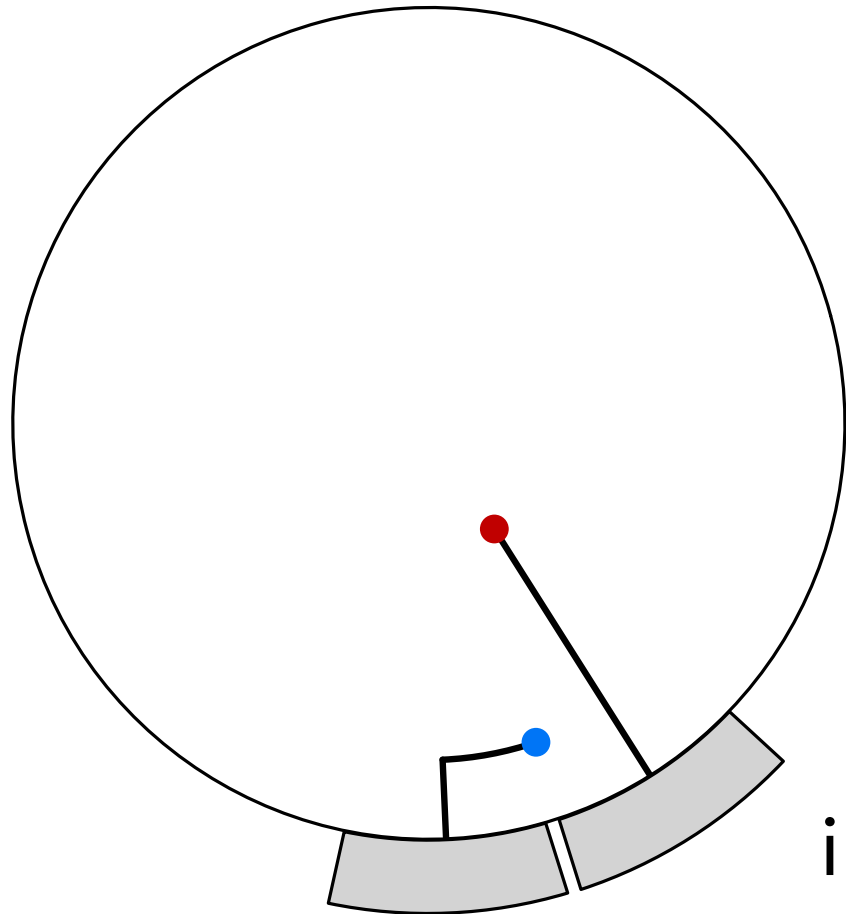
$$y = f(\alpha)$$



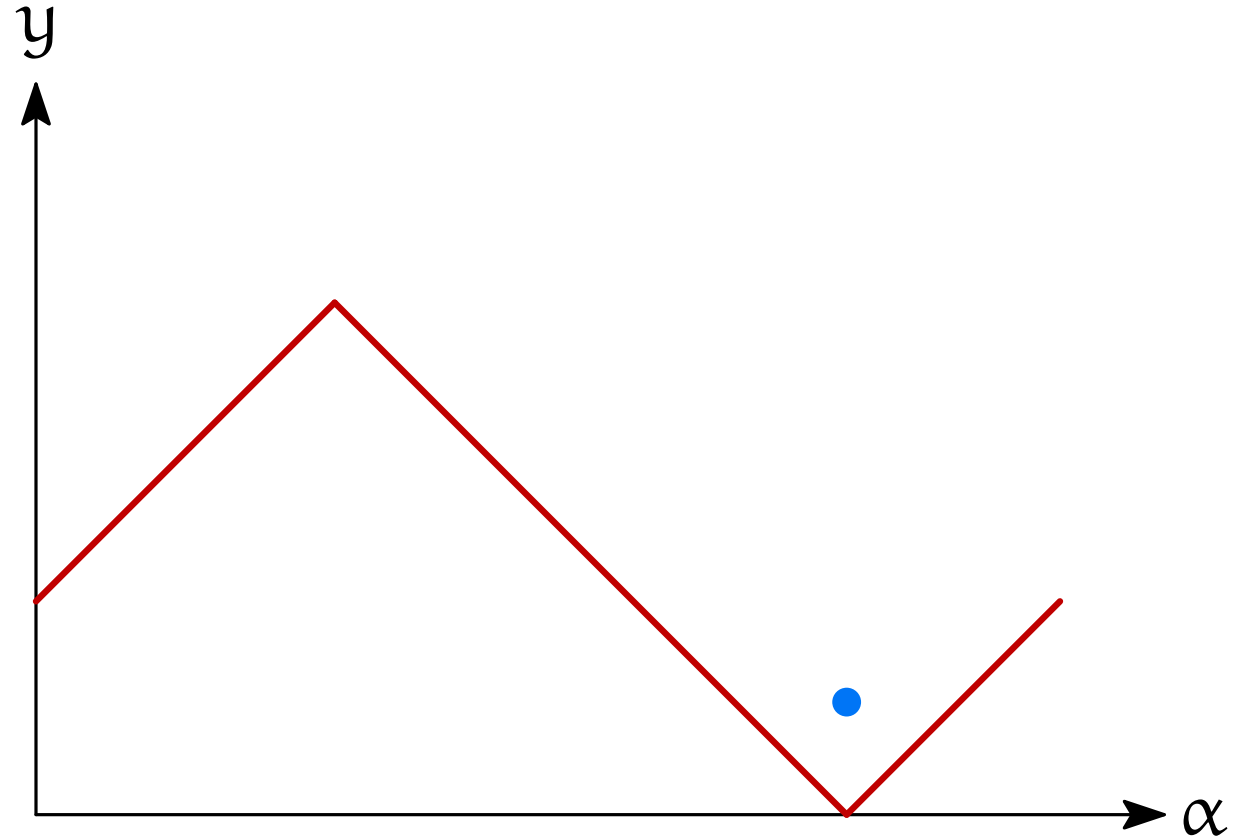
innermost:

- always shorter path

Rotating the labels



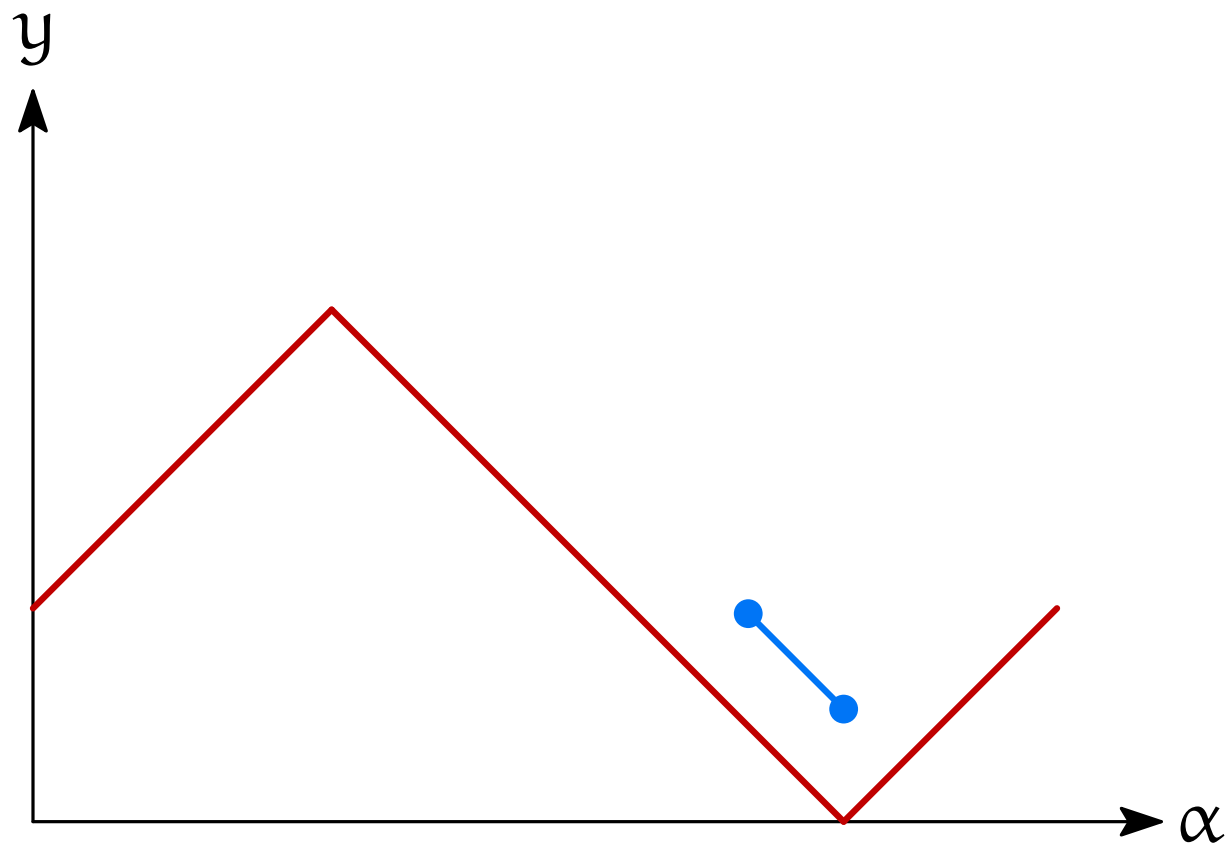
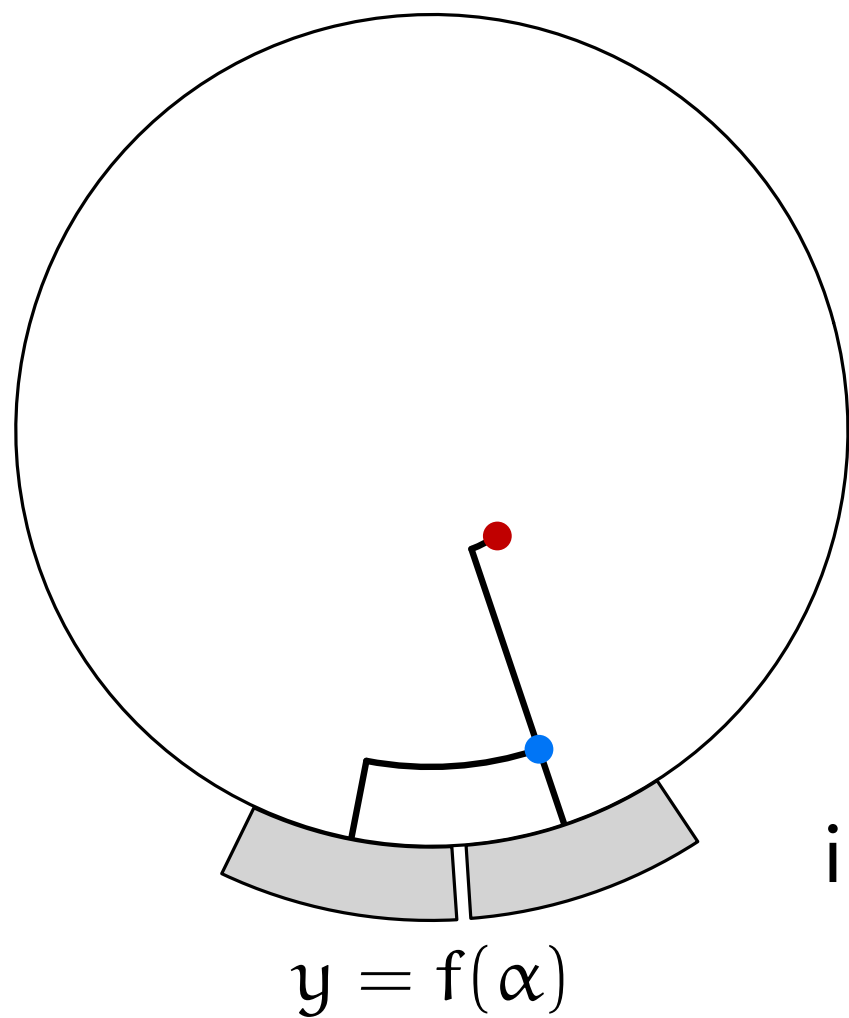
$$y = f(\alpha)$$



innermost:

- always shorter path

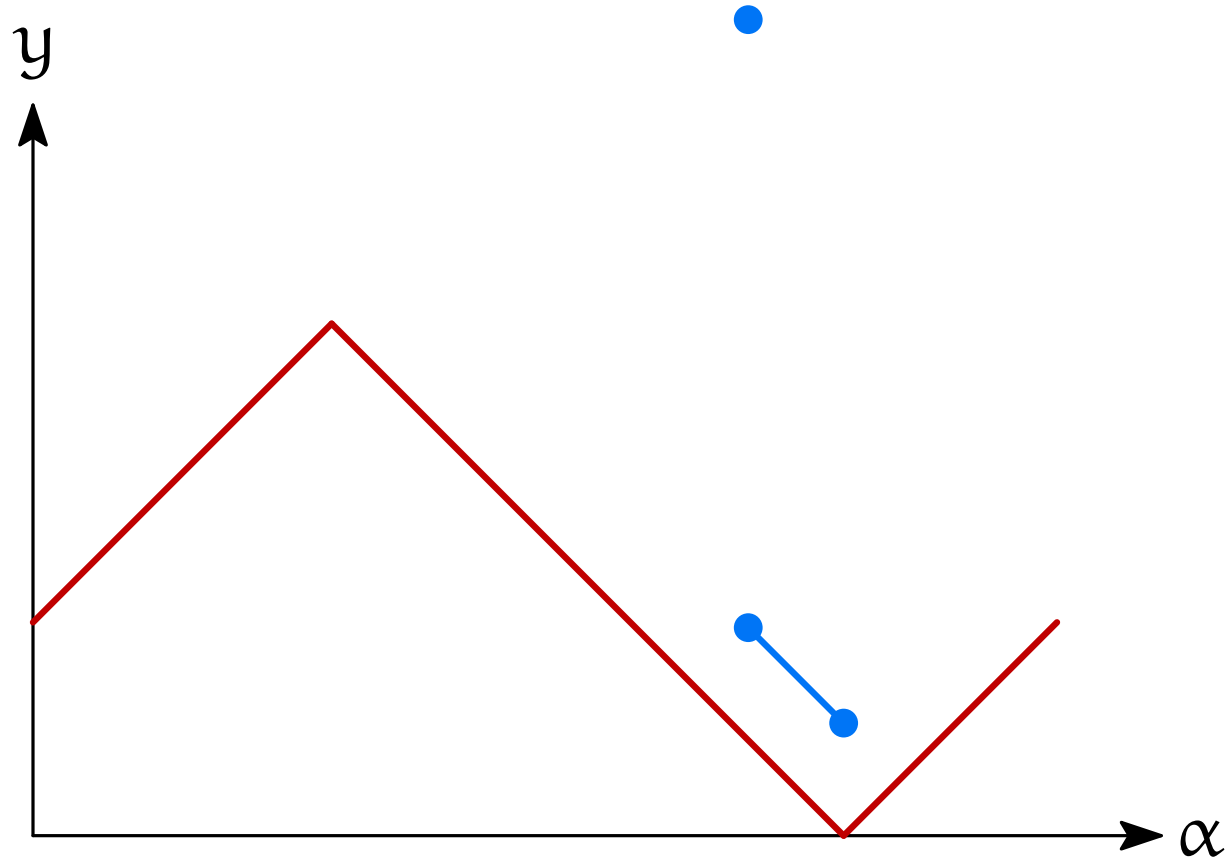
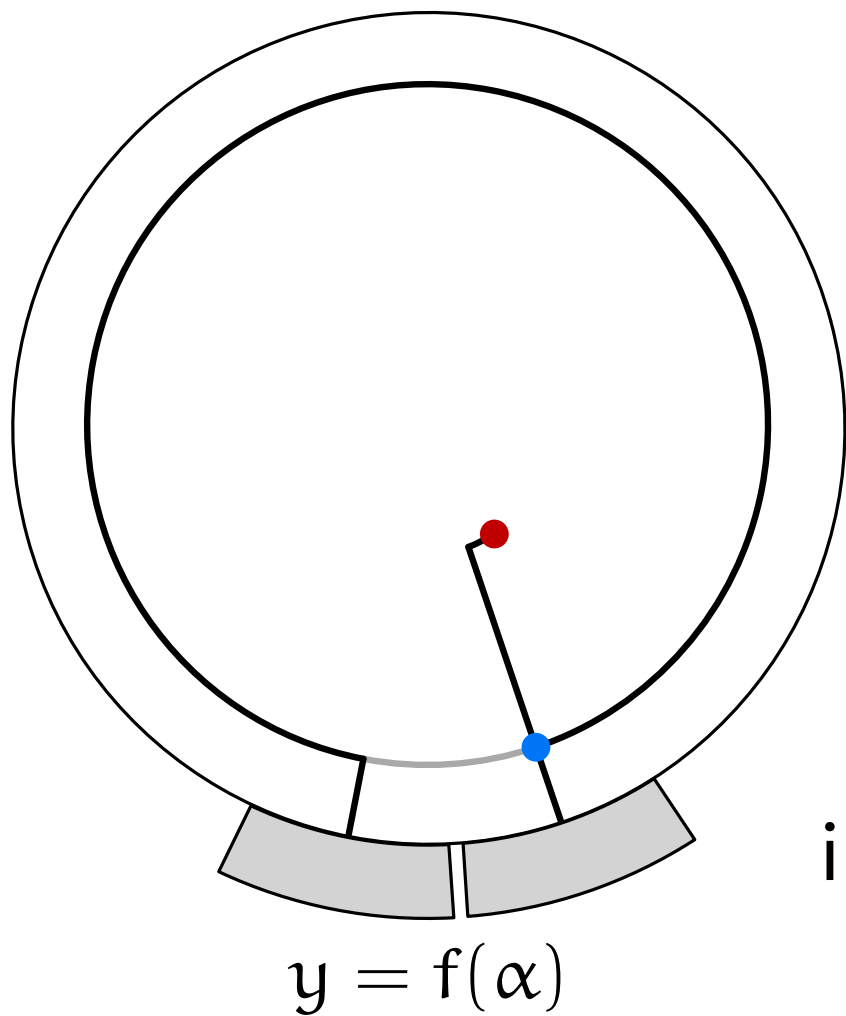
Rotating the labels



innermost:

- always shorter path

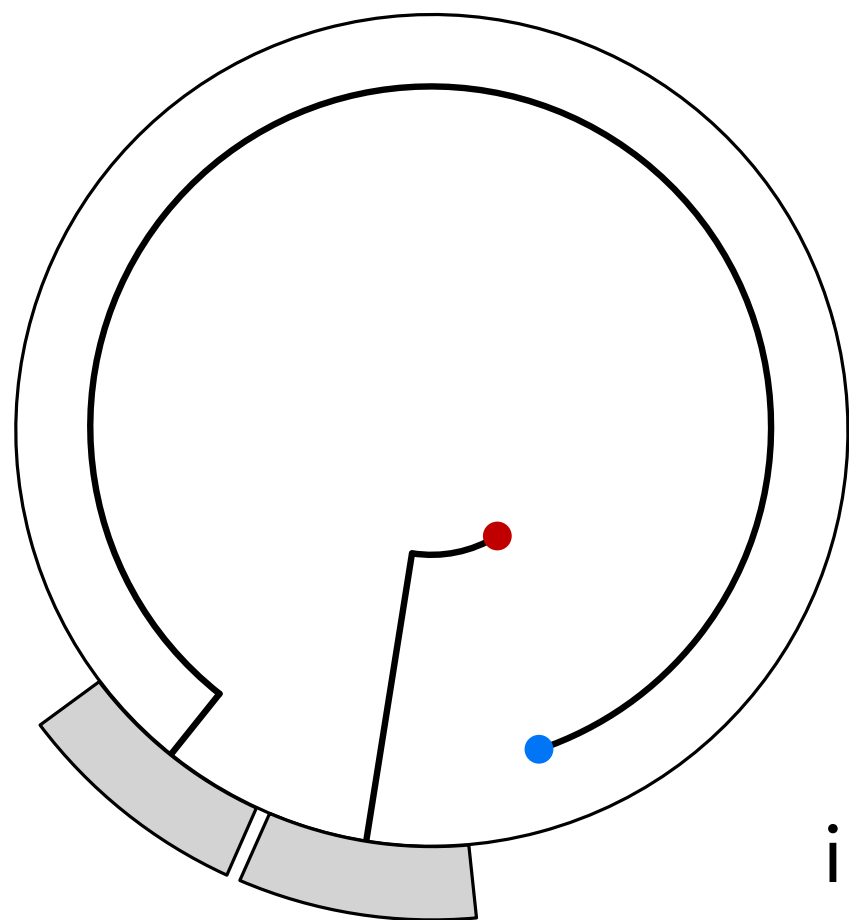
Rotating the labels



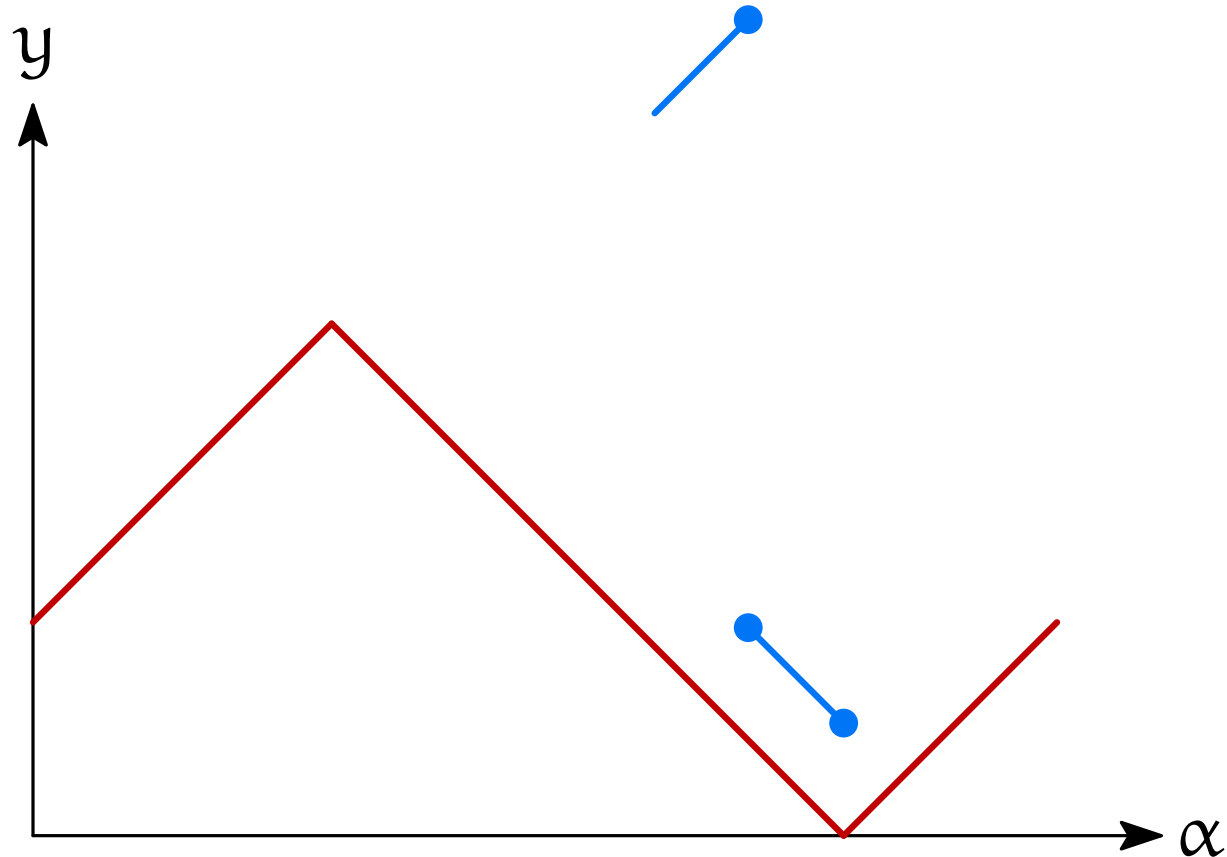
innermost:

- always shorter path

Rotating the labels



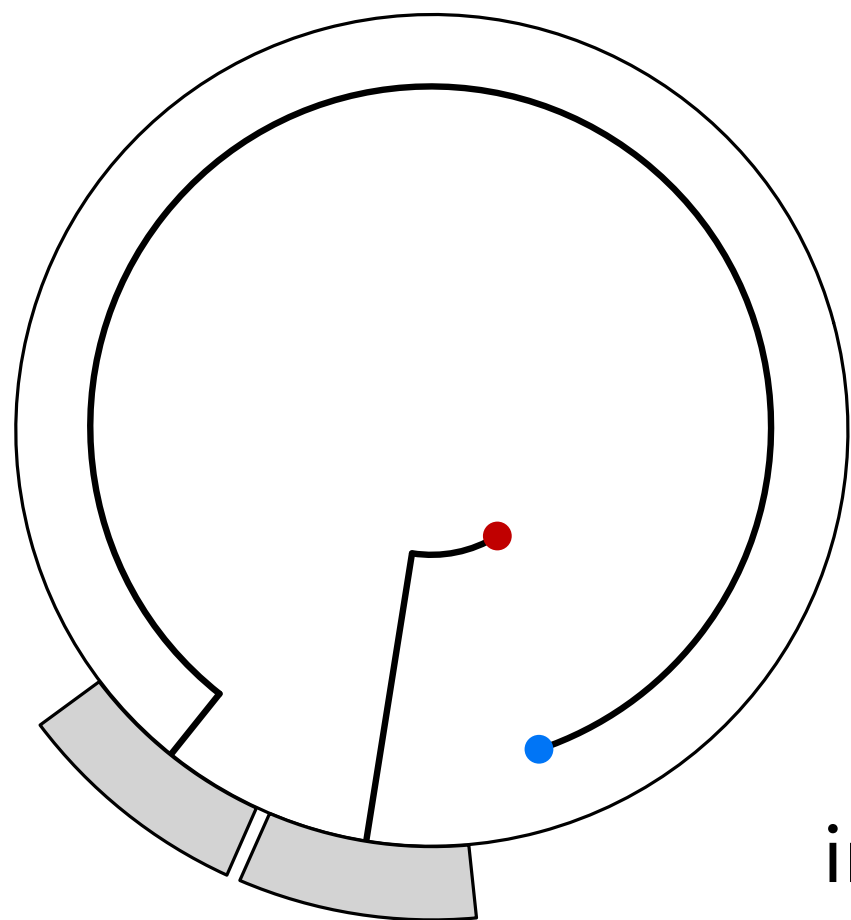
$$y = f(\alpha)$$



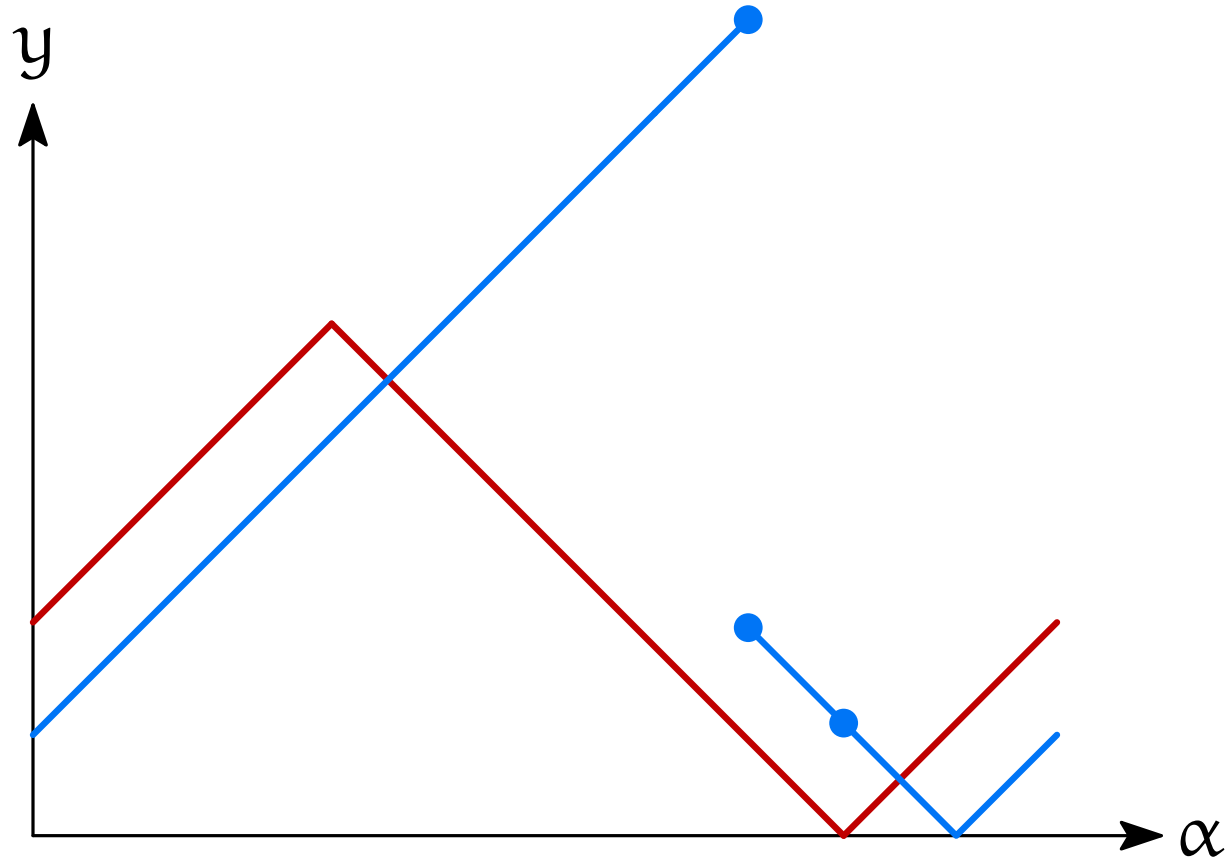
innermost:

- always shorter path

Rotating the labels



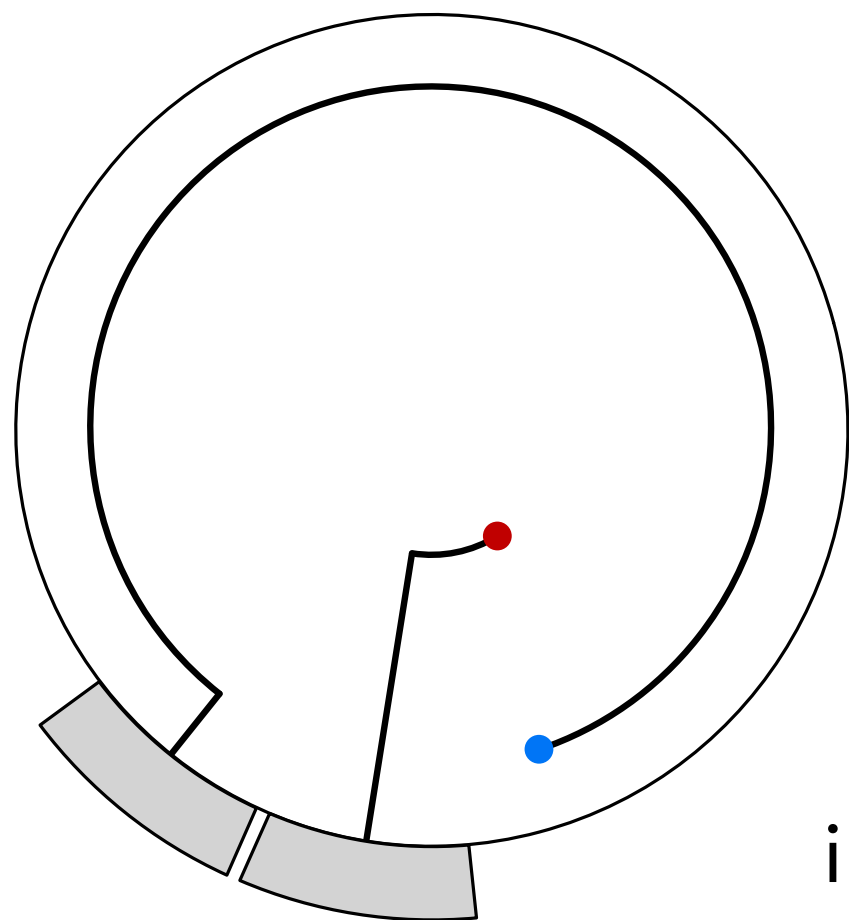
$$y = f(\alpha)$$



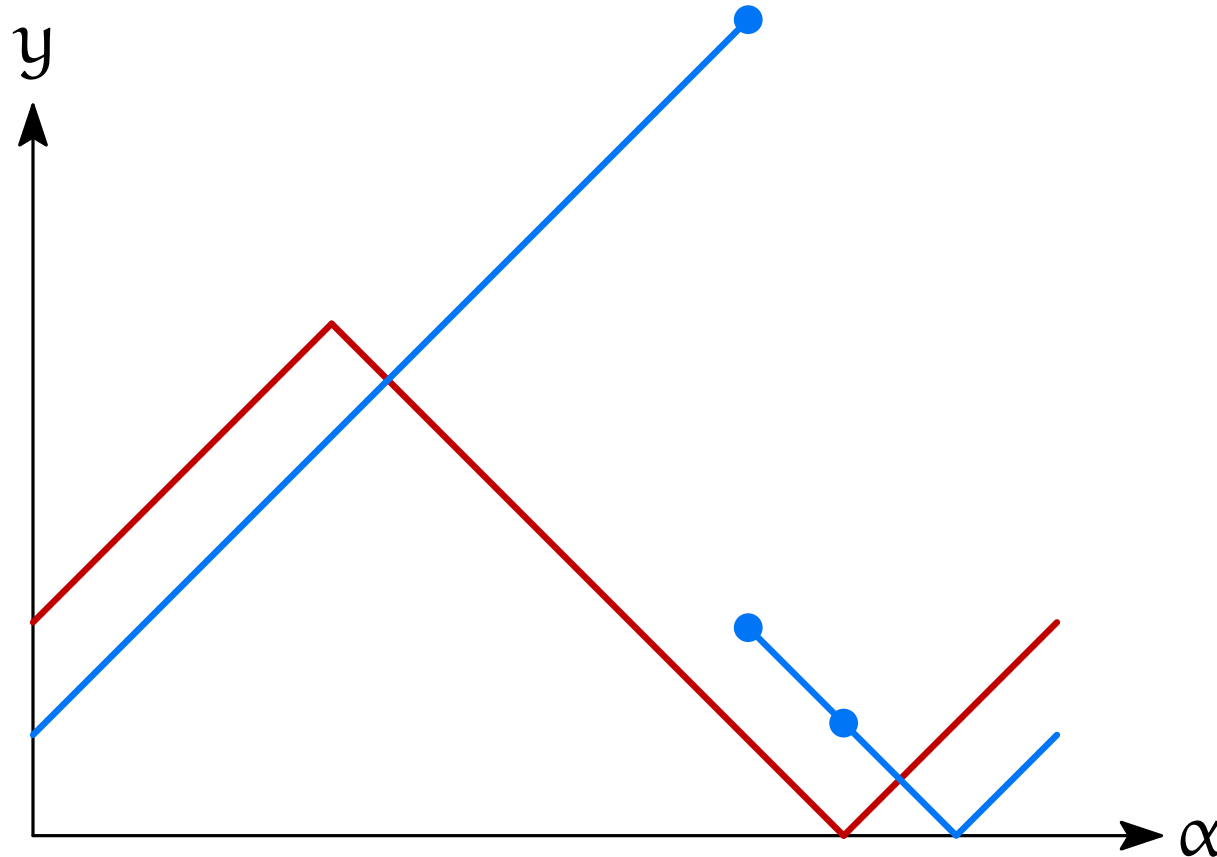
innermost:

- always shorter path

Rotating the labels



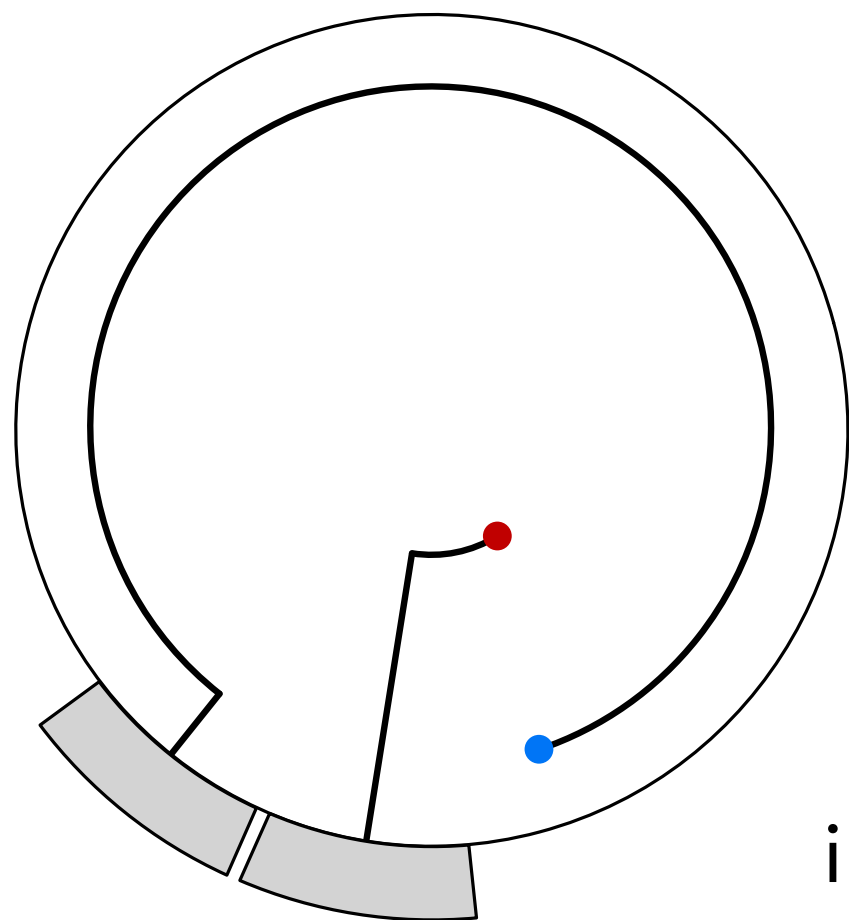
$$y = f(\alpha)$$



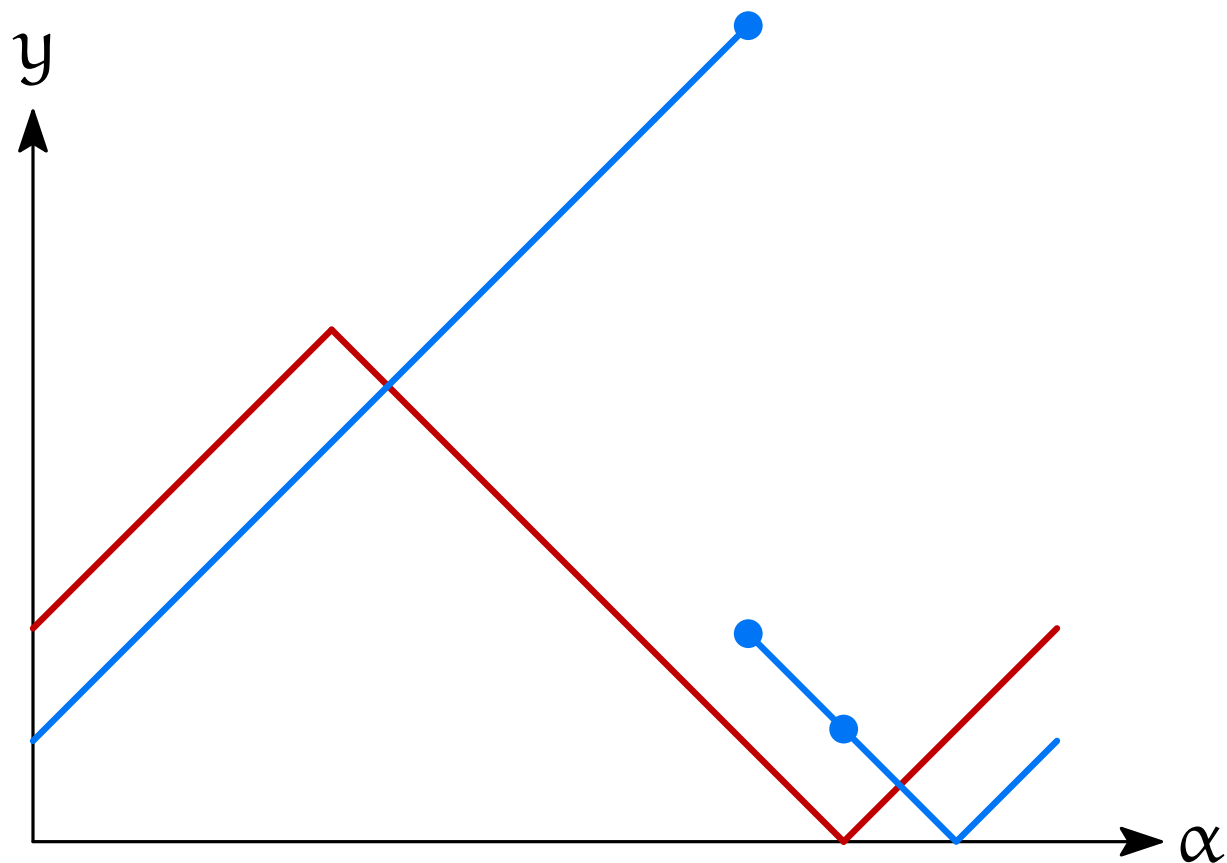
innermost:

- always shorter path
- decides all other leader directions

Rotating the labels



$$y = f(\alpha)$$

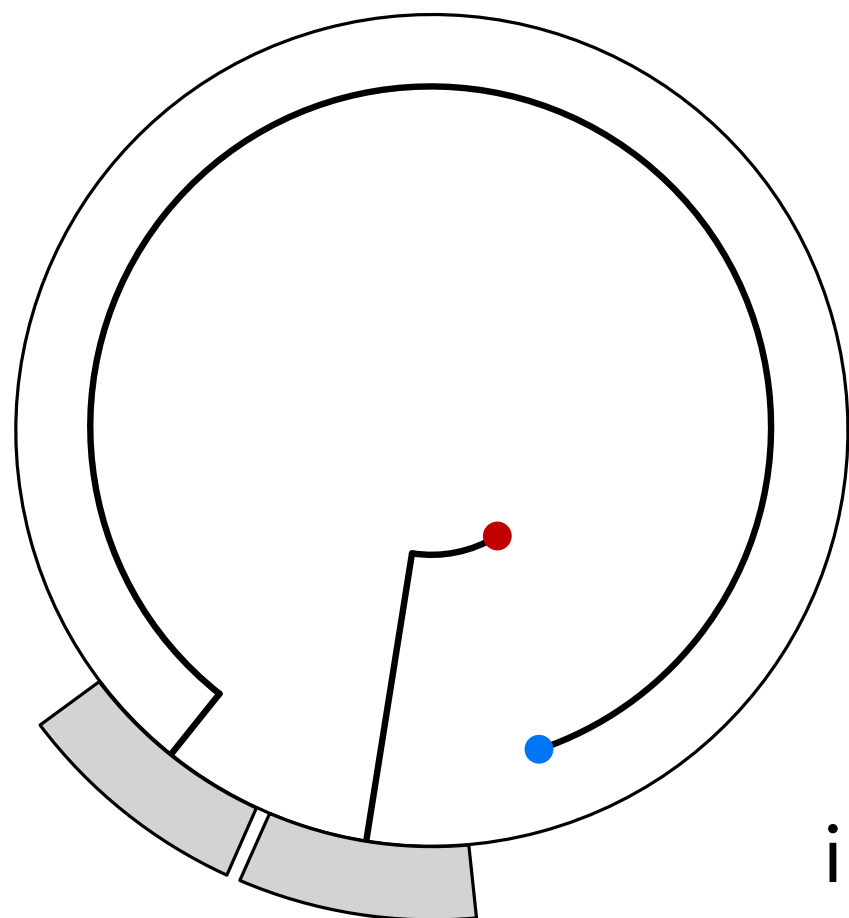


innermost:

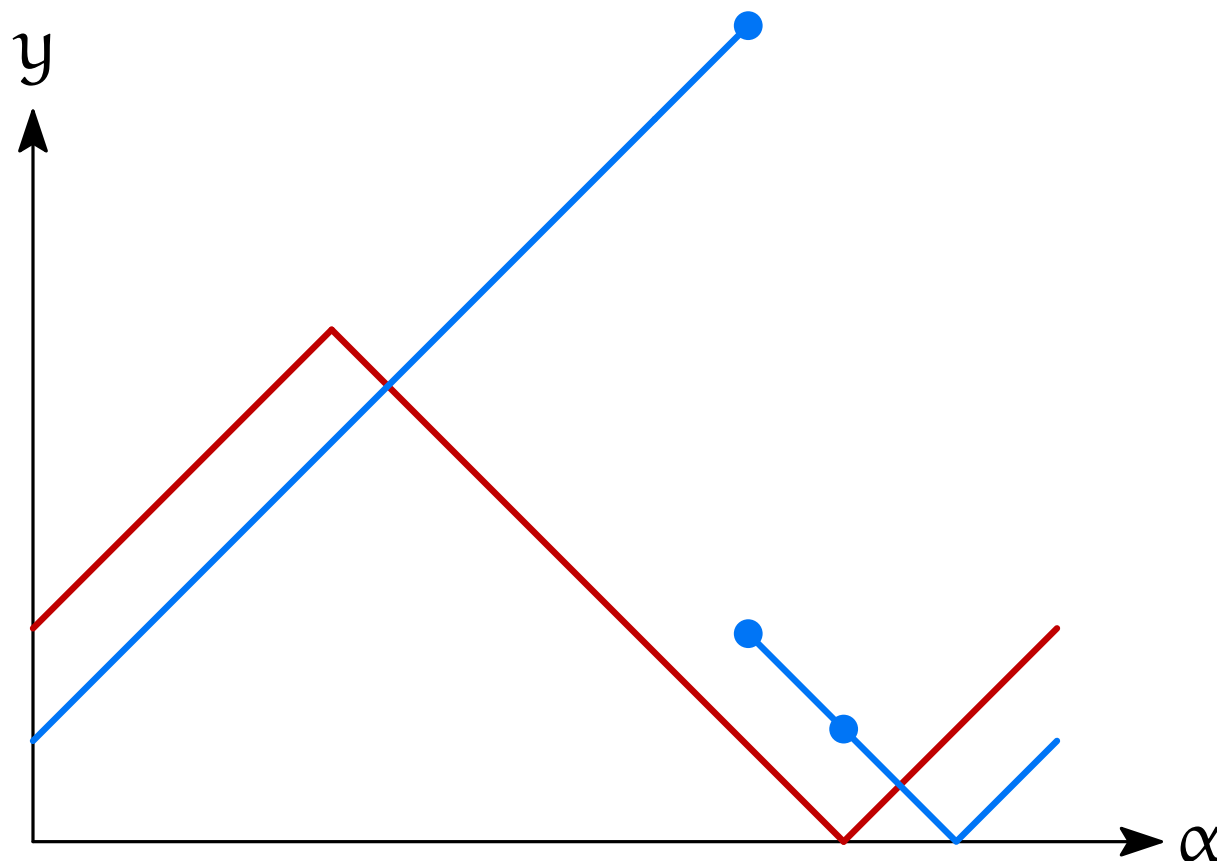
- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$

Rotating the labels



$$y = f(\alpha)$$



innermost:

- always shorter path
- decides all other leader directions

Restrict all functions to a **valid range** in $O(n^2)$

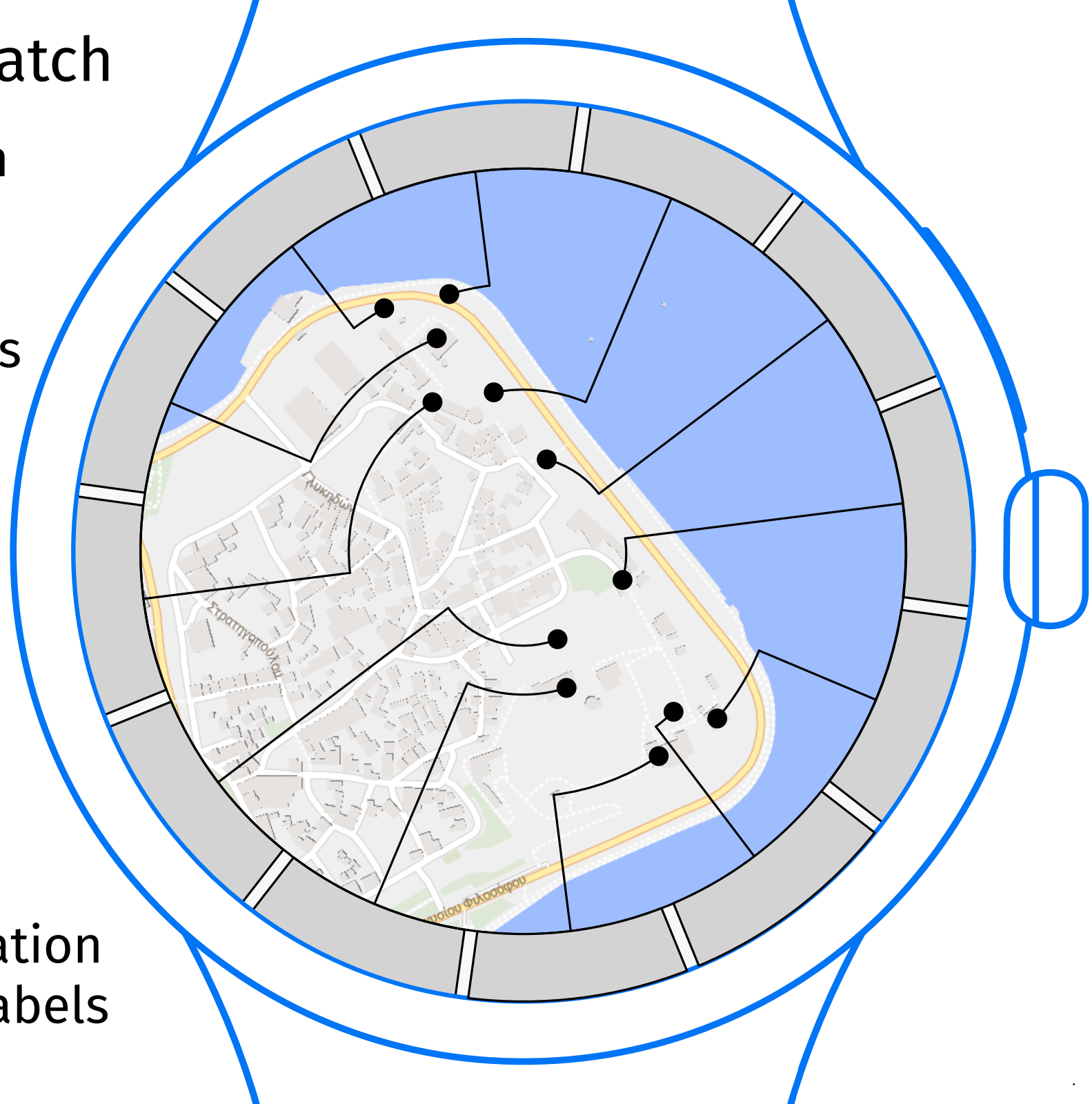
Find minimum of **sum of functions** ($2n$ piece-wise linear parts) in $O(n)$

Labeling on a smart watch

- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

- under:**
- continuous rotation
 - fixed order of labels

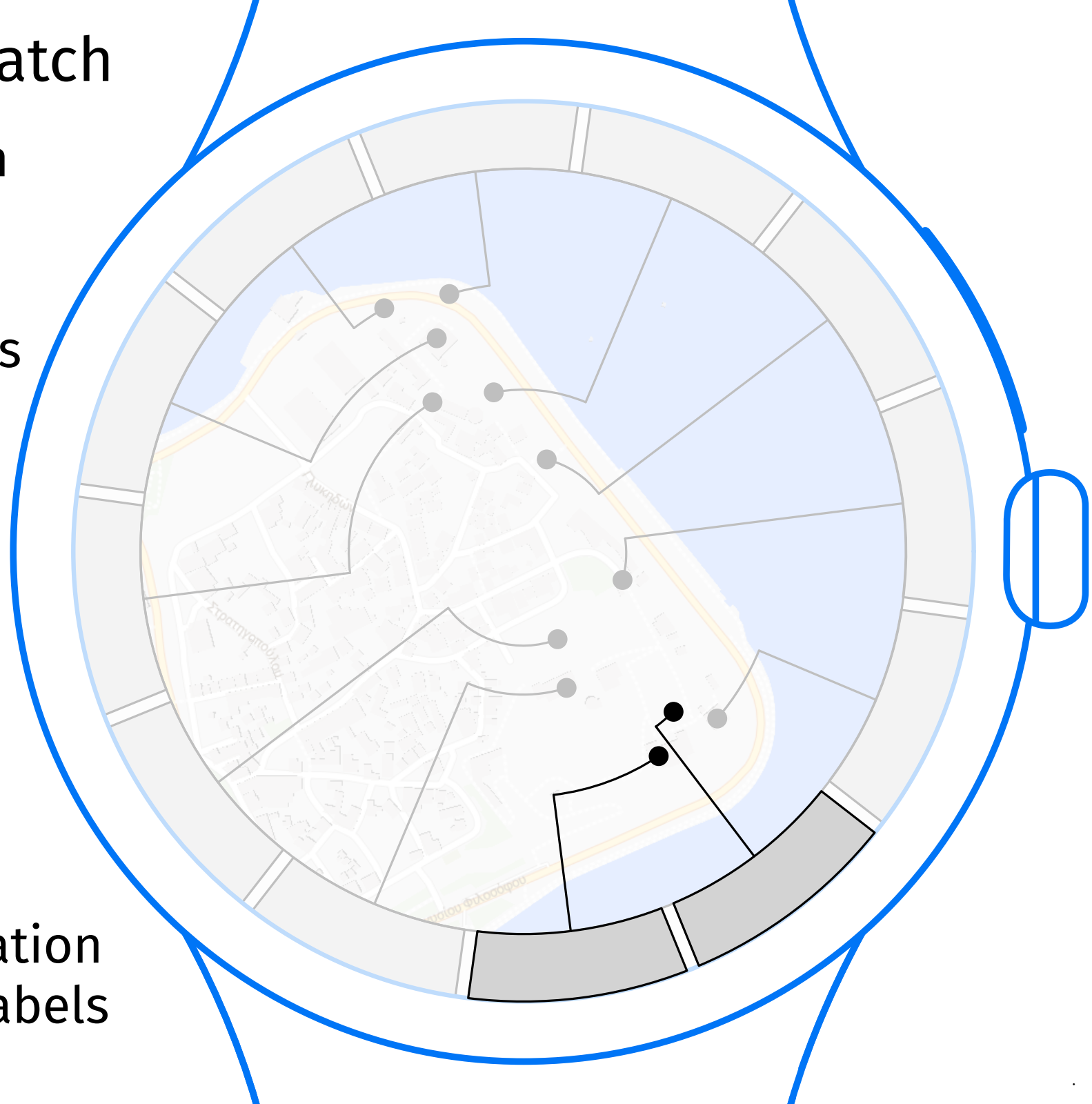


Labeling on a smart watch

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goal: minimize sum of leader lengths

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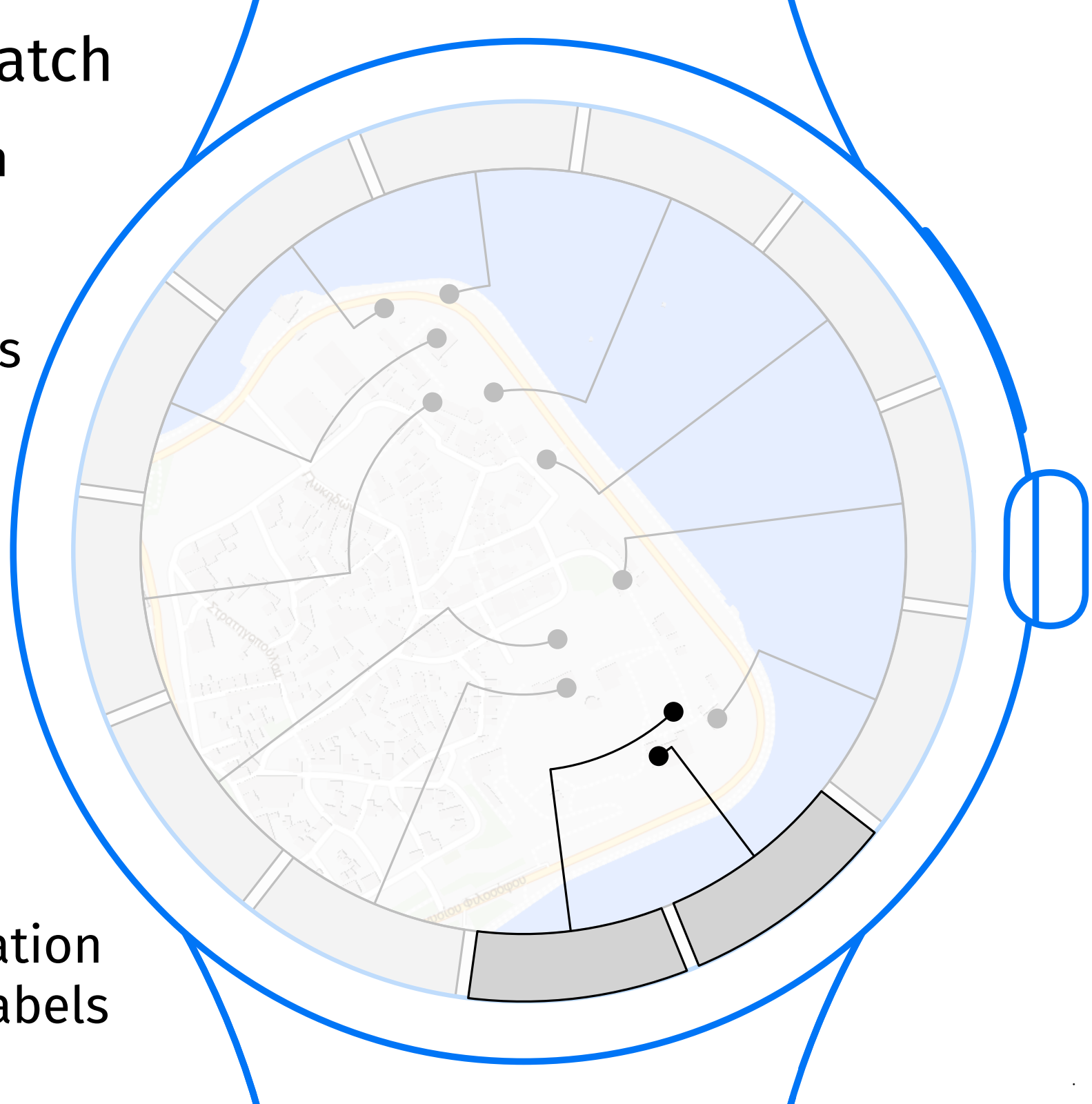


Labeling on a smart watch

- reduce map obstruction
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goal: minimize sum of leader lengths

- under:**
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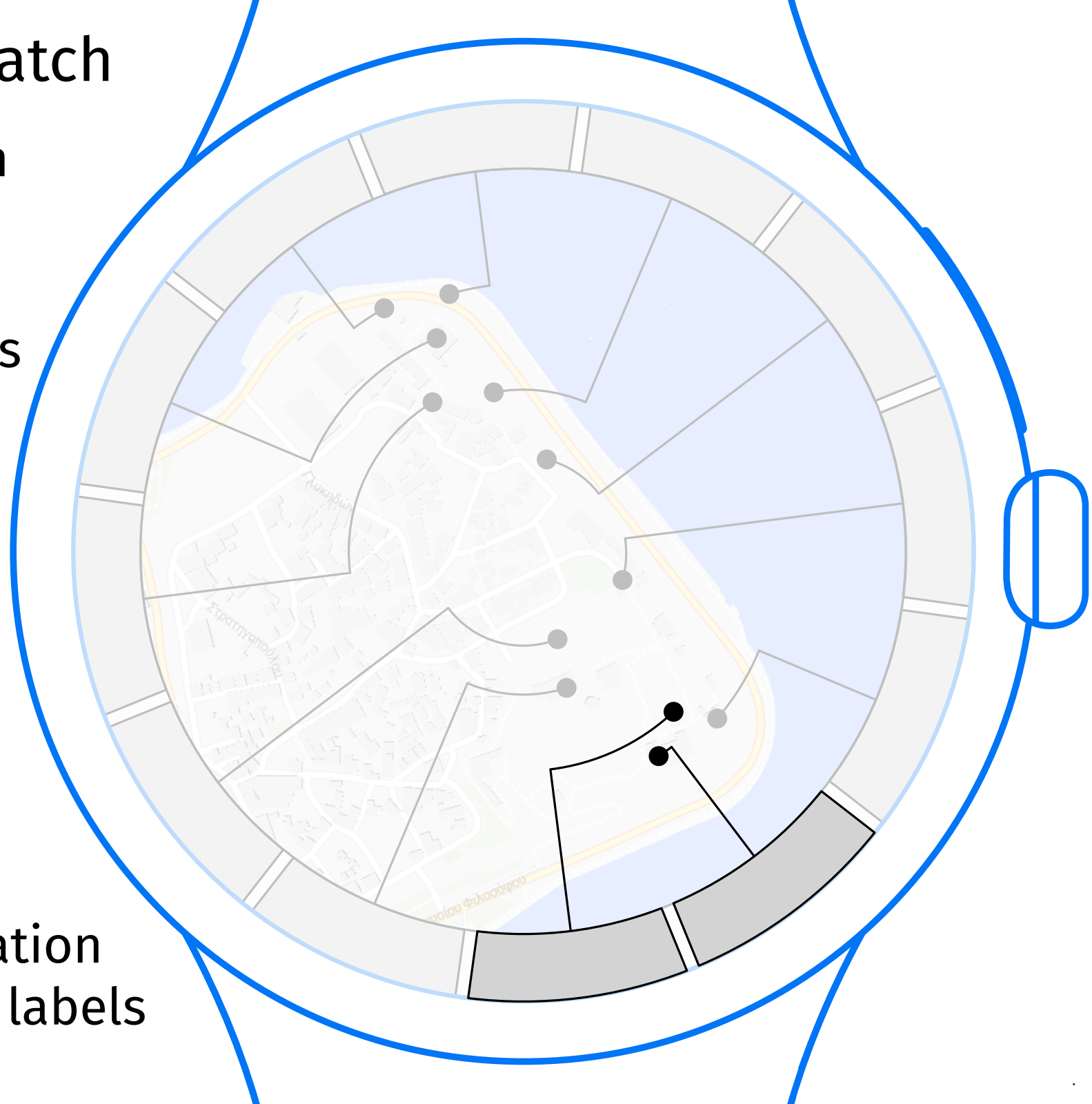


Labeling on a smart watch

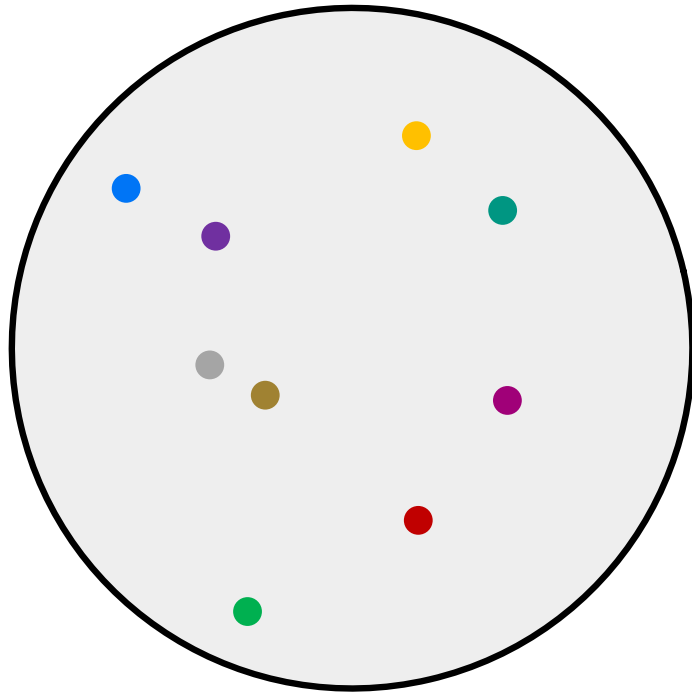
- reduce map obstruction
- avoid label overlap
- prevent leader crossings
- maximize space usage

goal: minimize sum of leader lengths

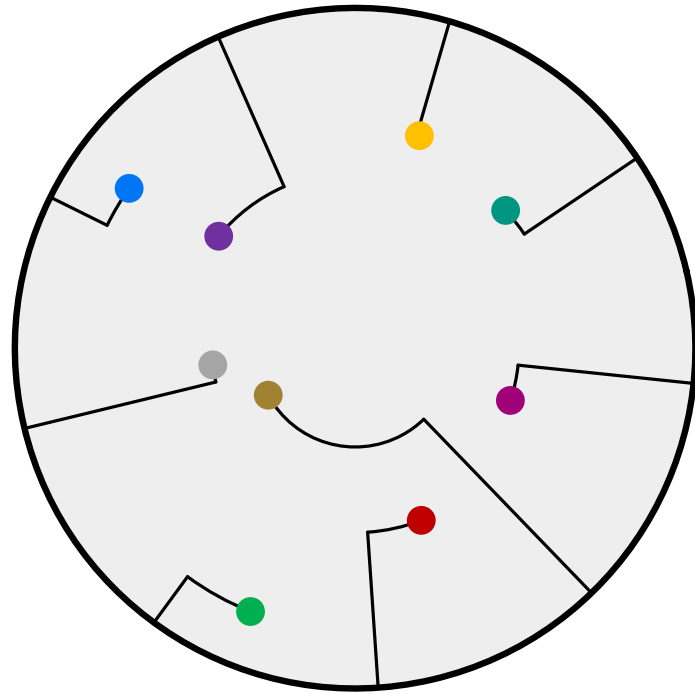
- under:**
- continuous rotation
 - **permutation** of labels



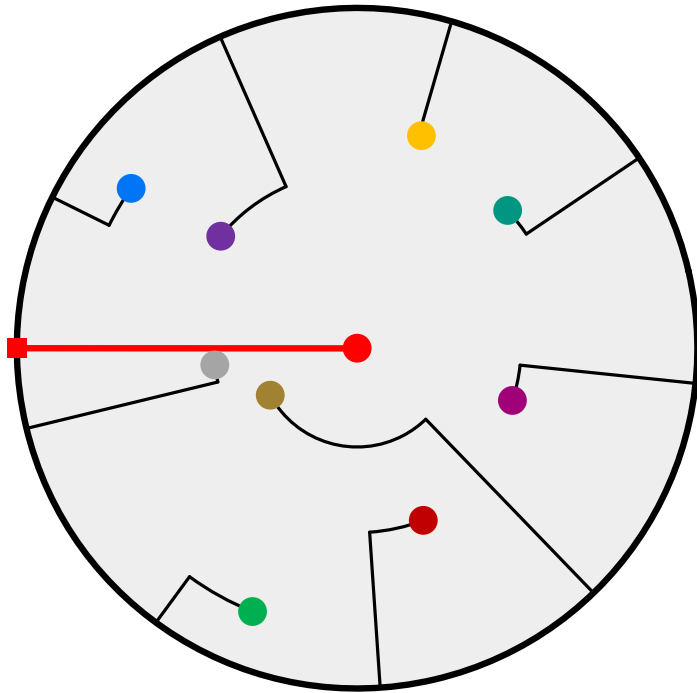
Permuting the labels



Permuting the labels

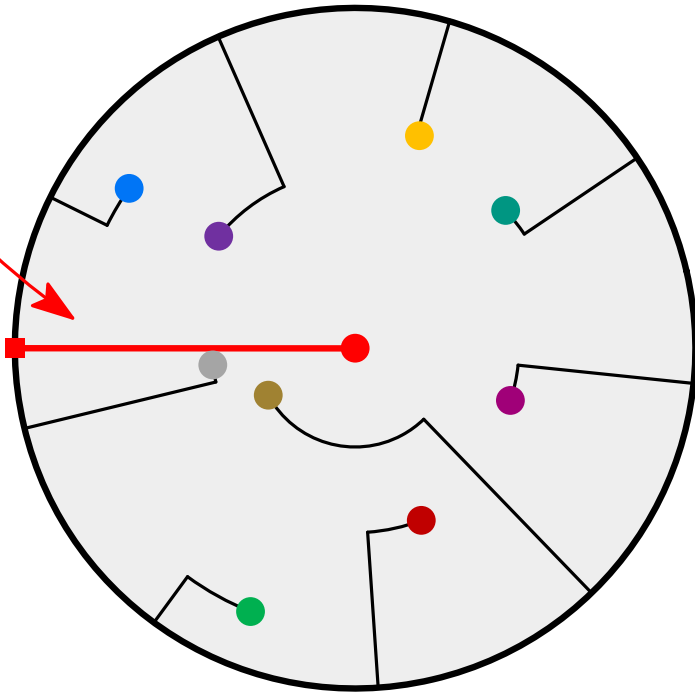


Permuting the labels



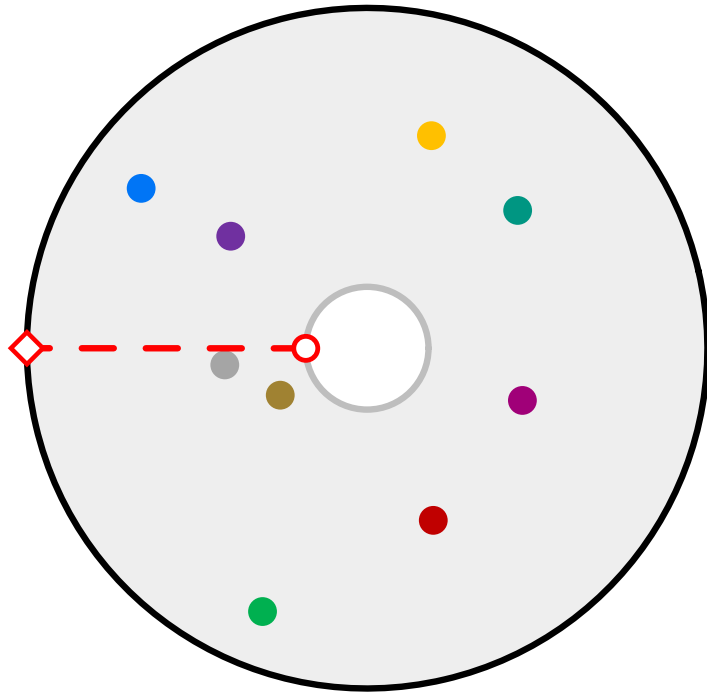
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



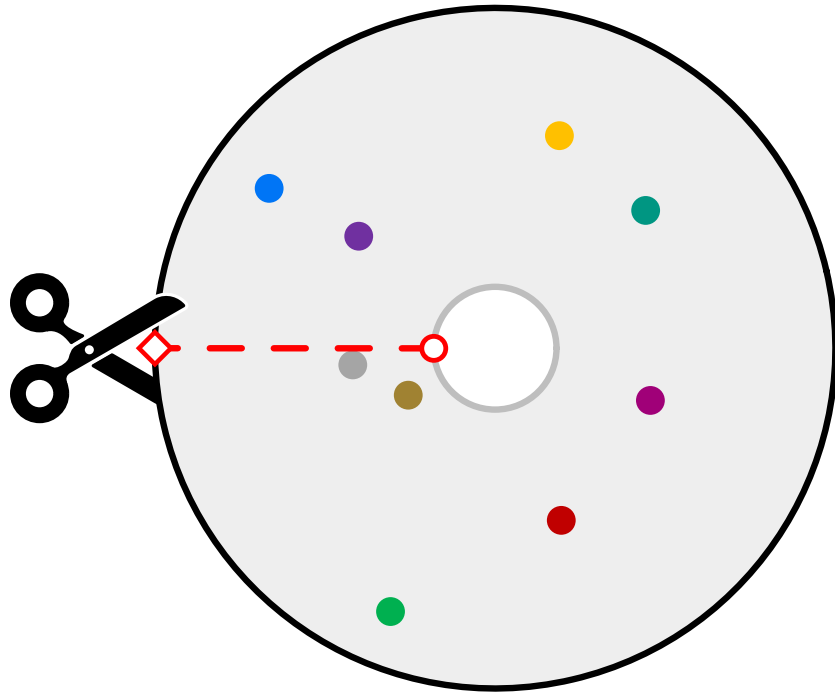
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



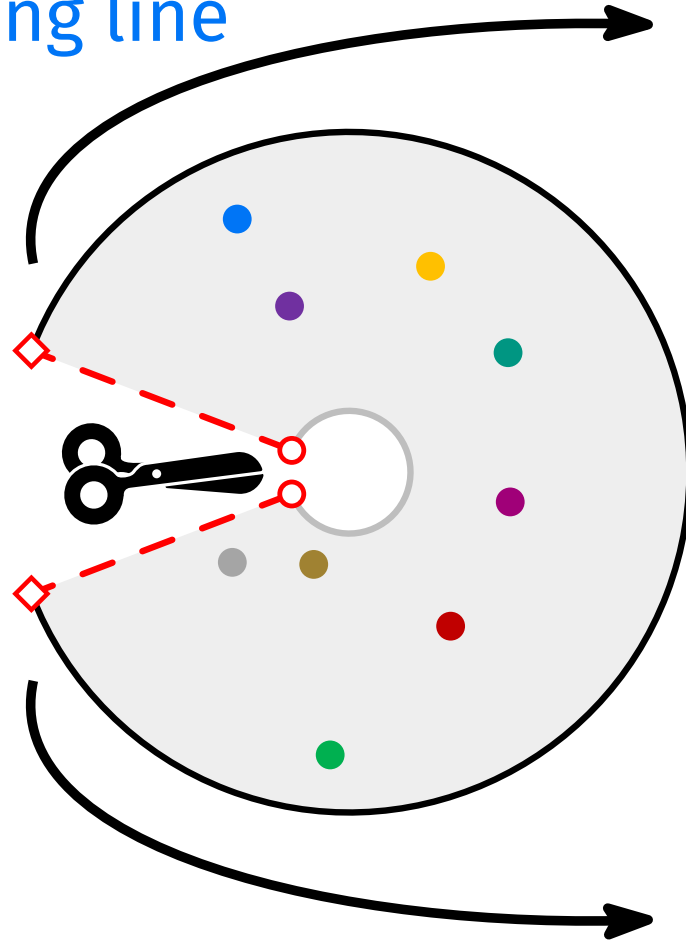
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



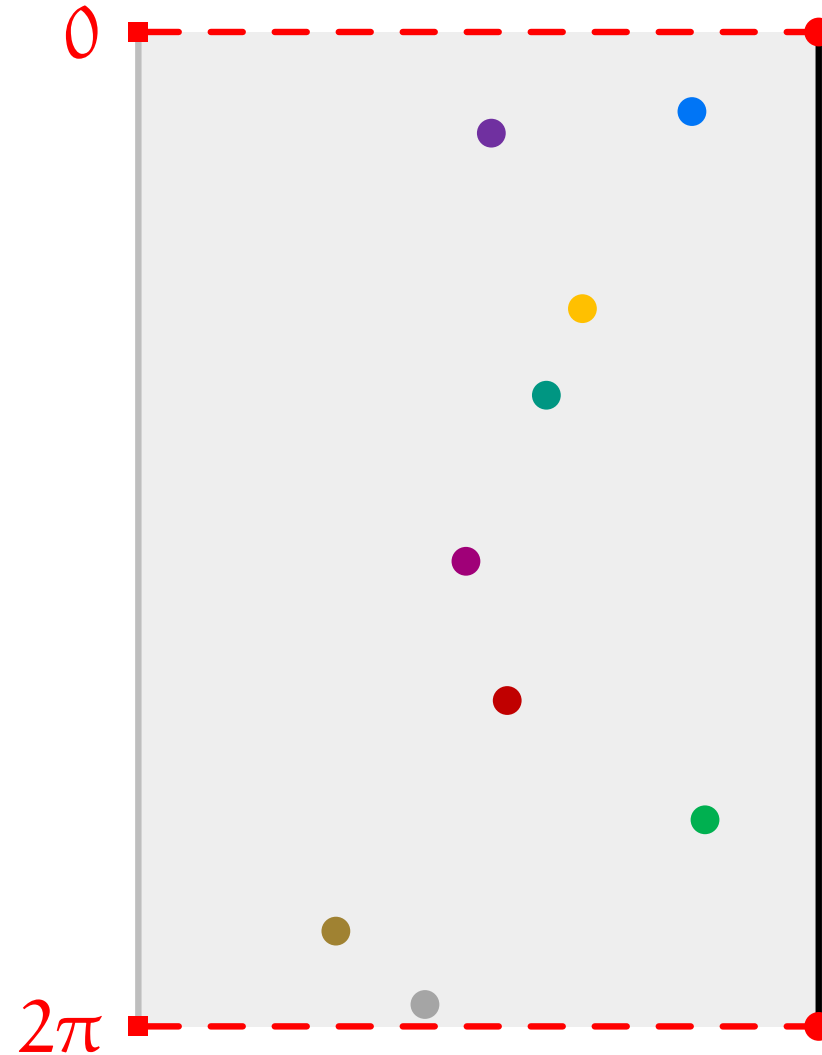
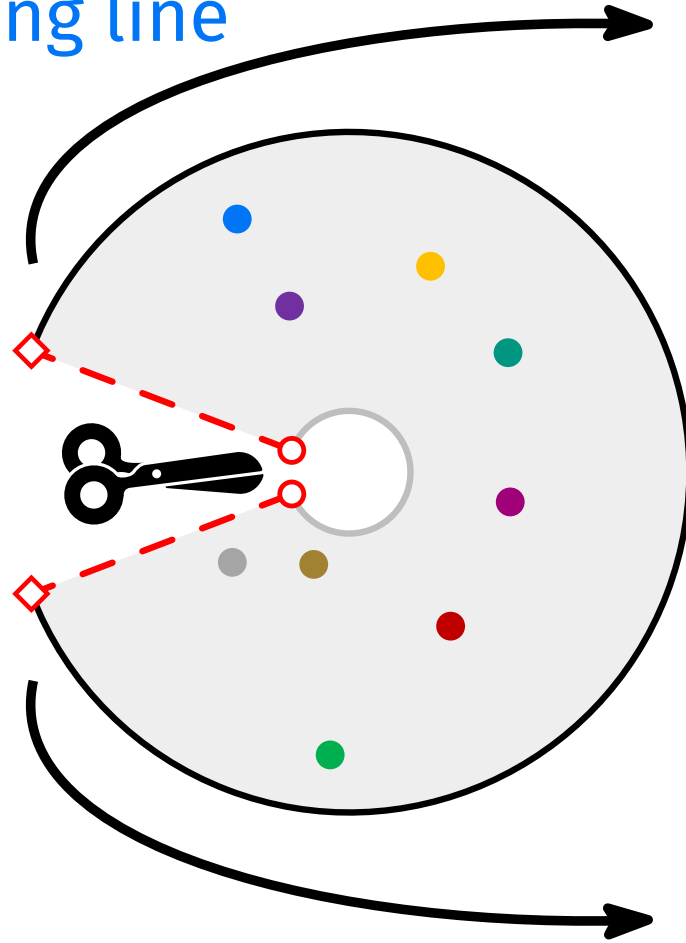
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



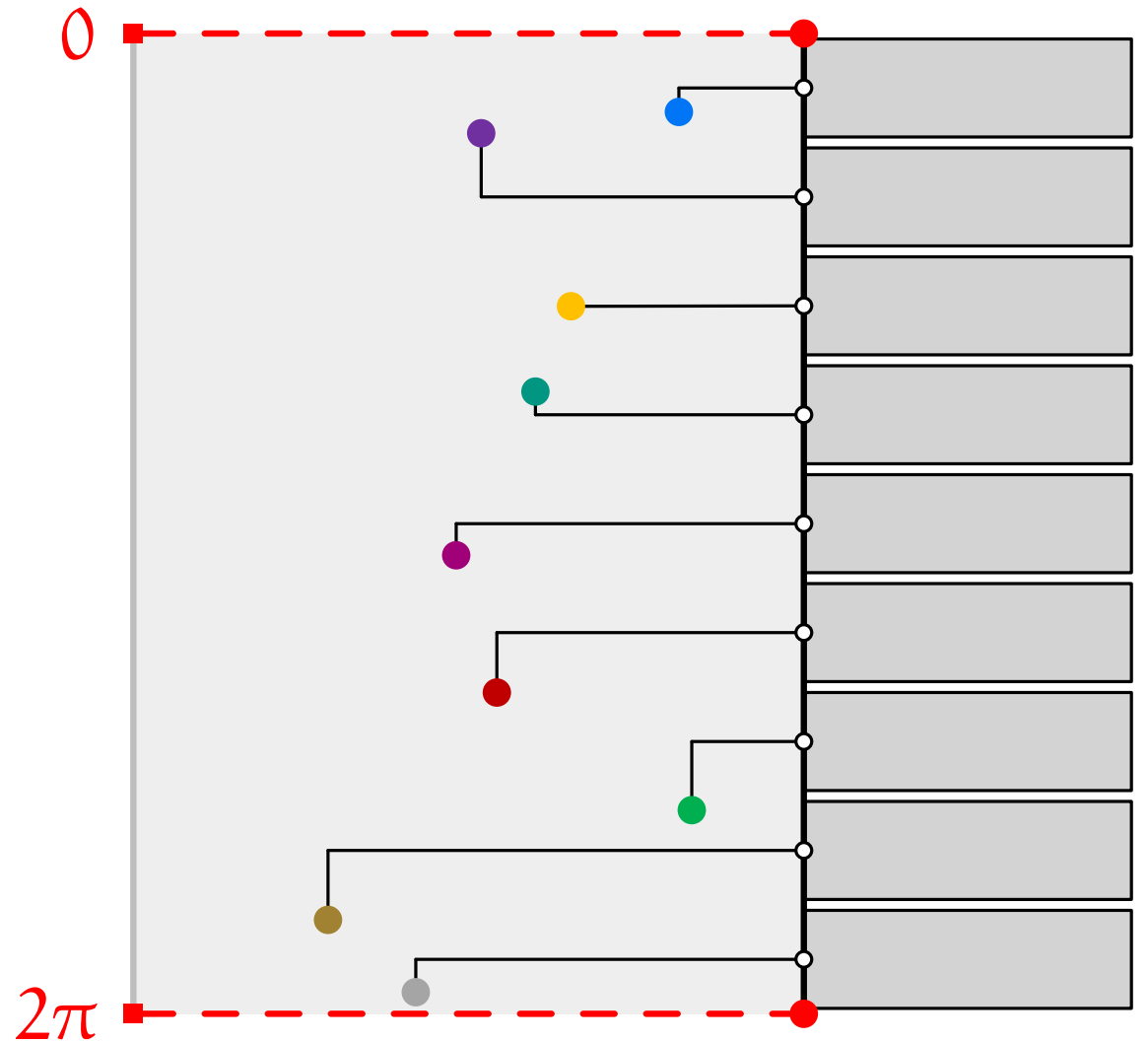
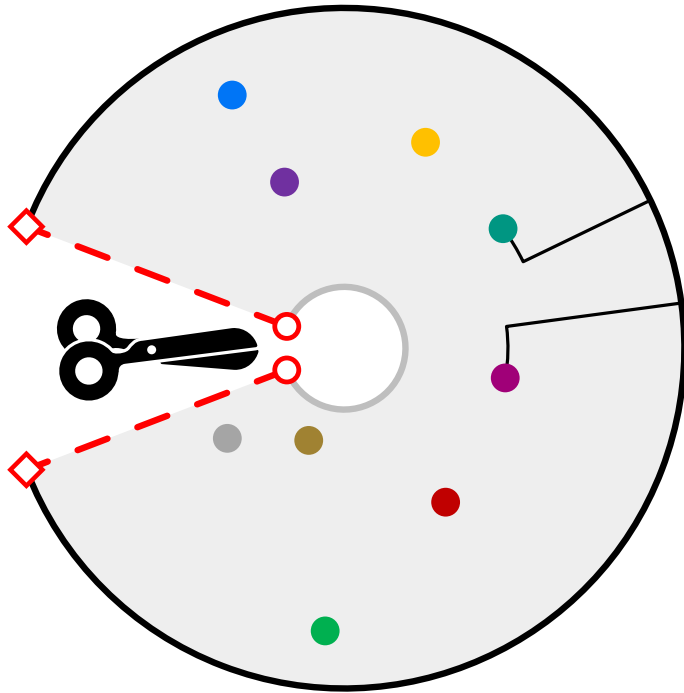
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



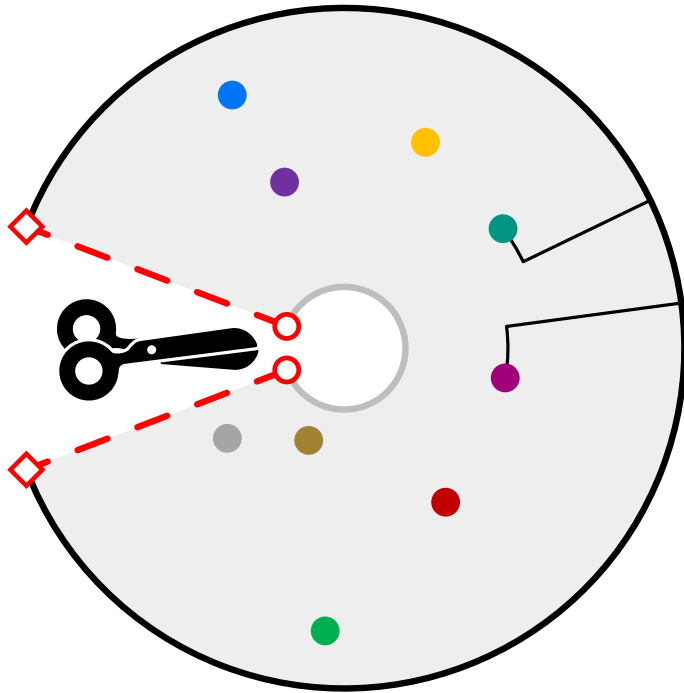
Permuting the labels

$O(n^2)$ relevant possibilities
for **splitting line**



Permuting the labels

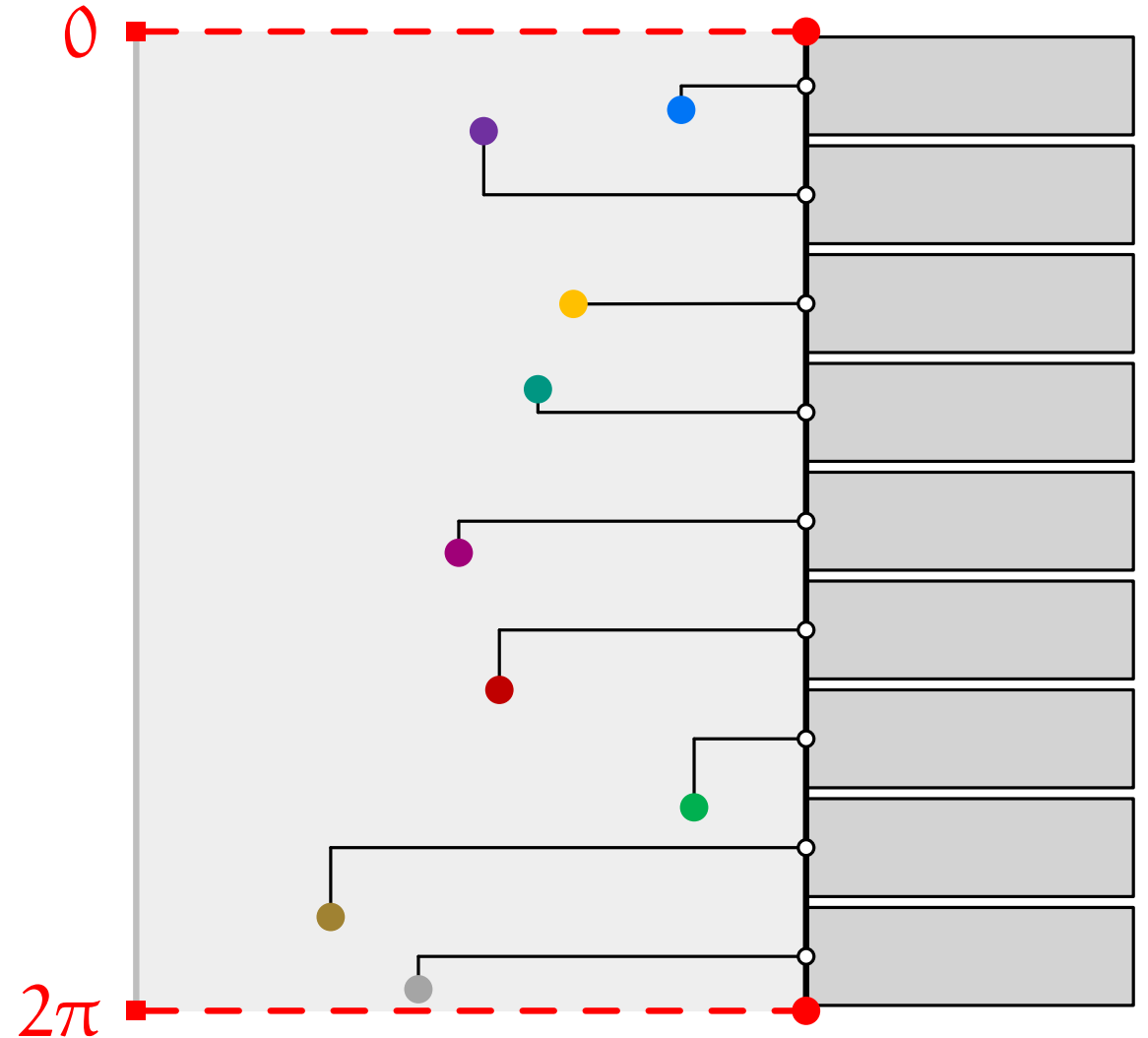
$O(n^2)$ relevant possibilities
for **splitting line**



one sided po-boundary labeling
arbitrary badness function

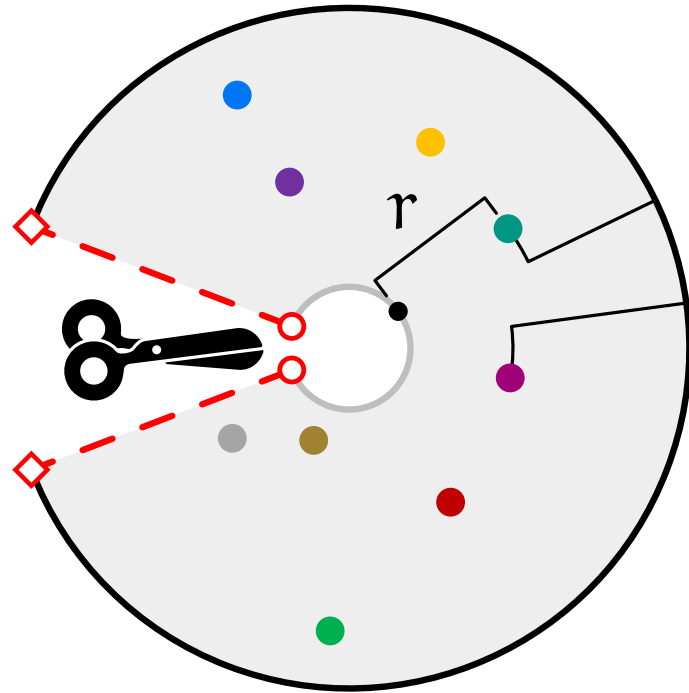
$O(n^3)$ time [Benkert et al., '09]

→ $O(n^5)$ runtime



Permuting the labels

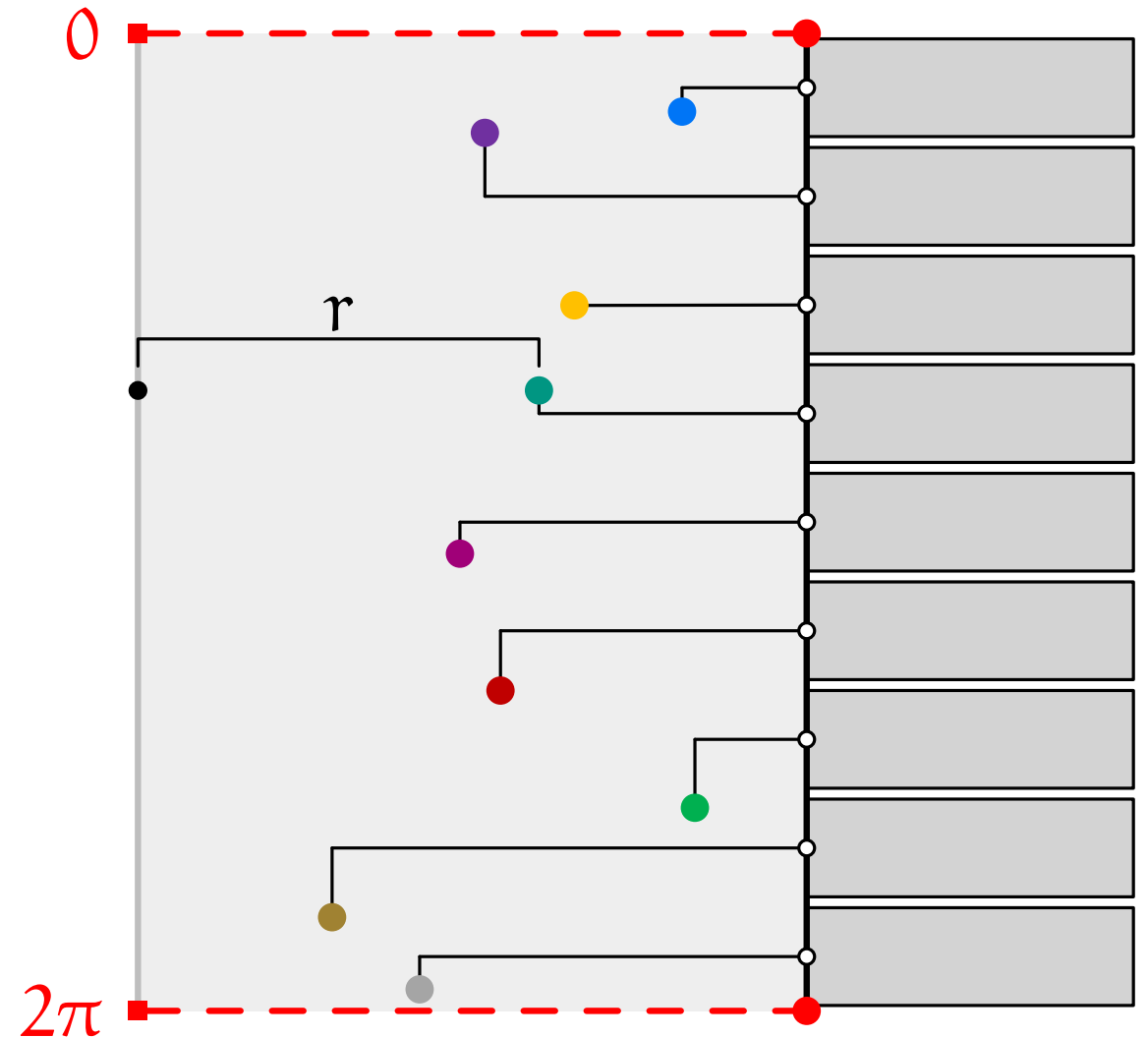
$O(n^2)$ relevant possibilities
for **splitting line**



one sided po-boundary labeling
arbitrary badness function

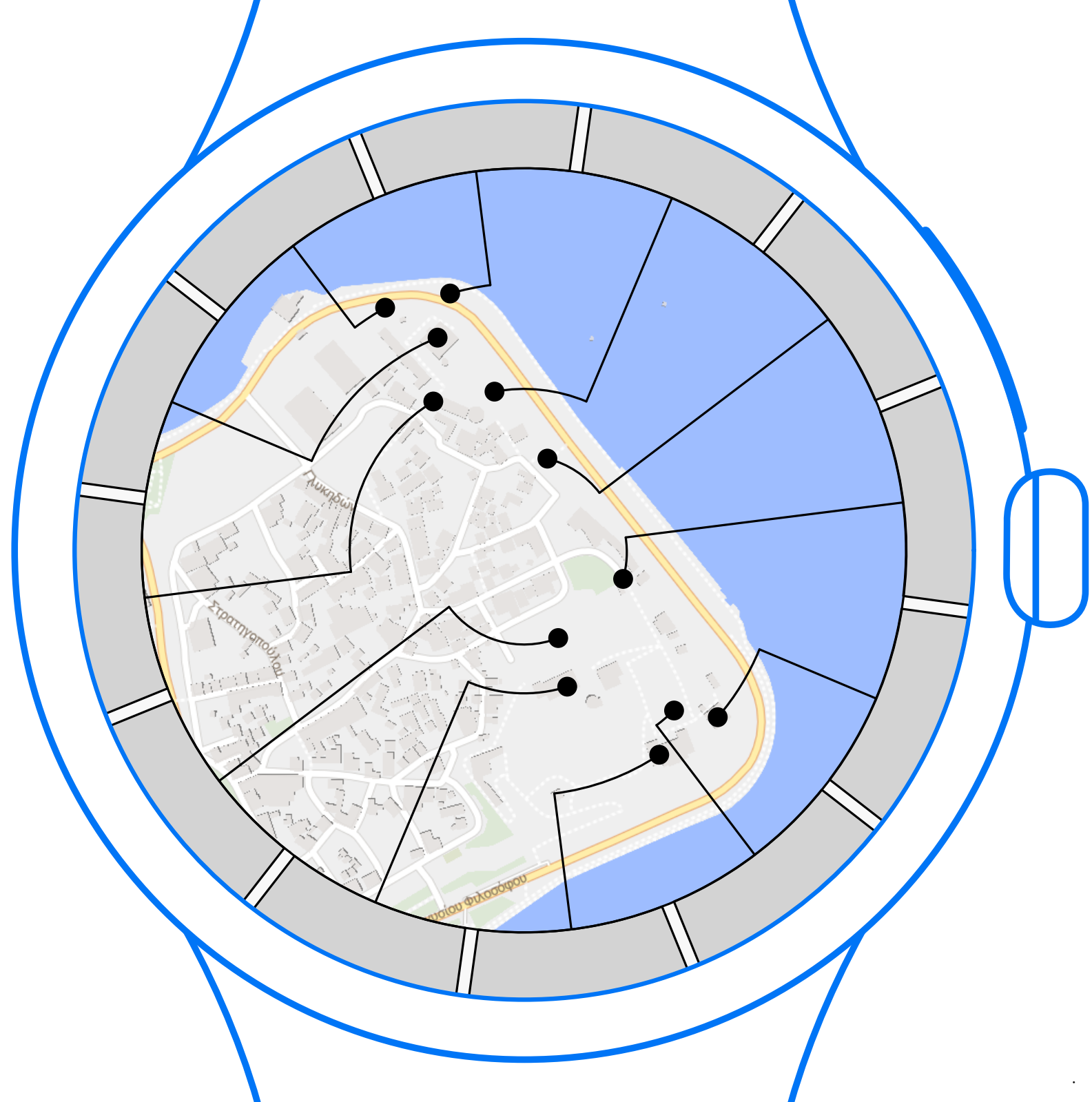
$O(n^3)$ time [Benkert et al., '09]

$\rightarrow O(n^5)$ runtime

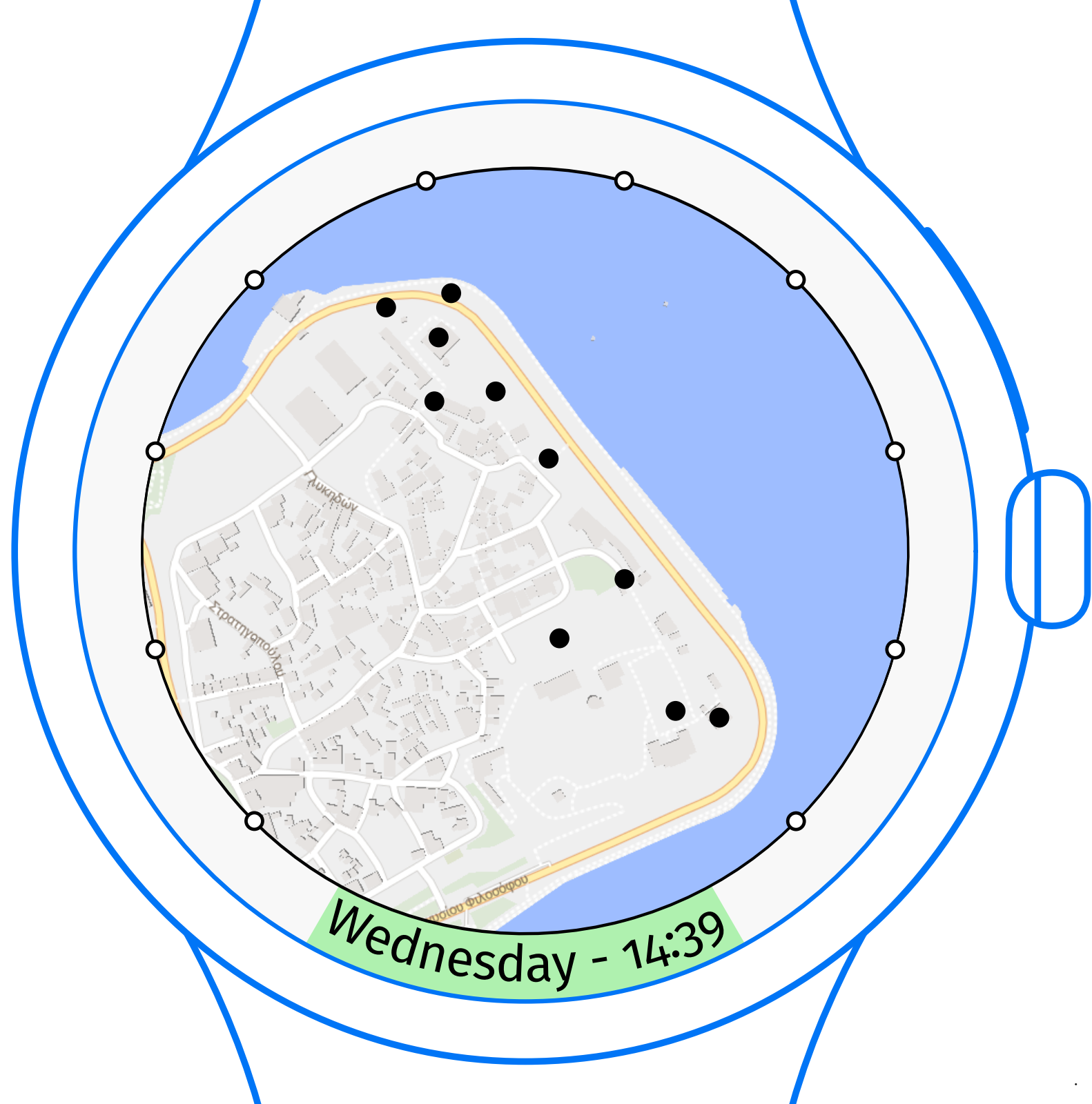


badness function depends on r

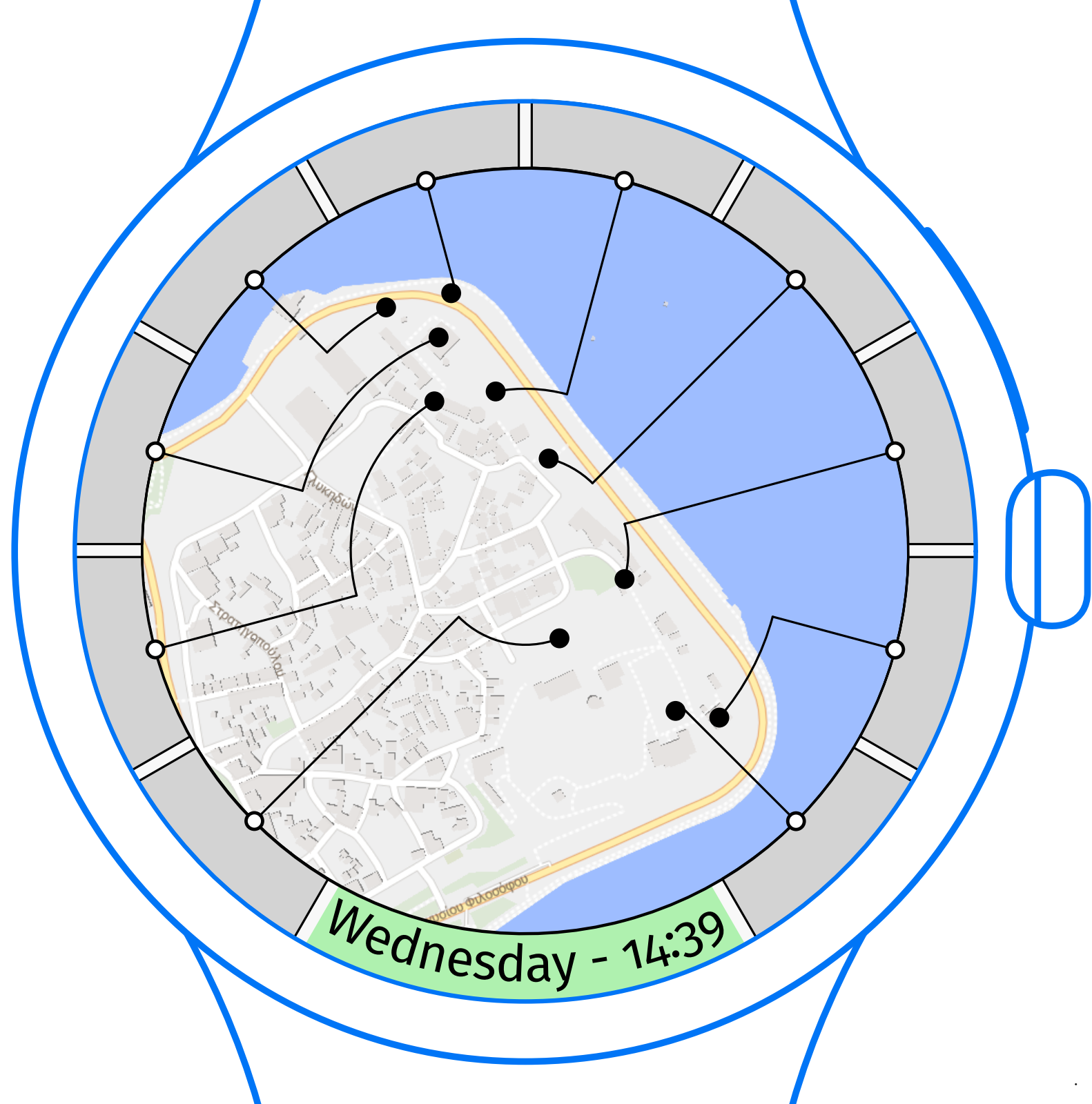
Problem variants



Problem variants





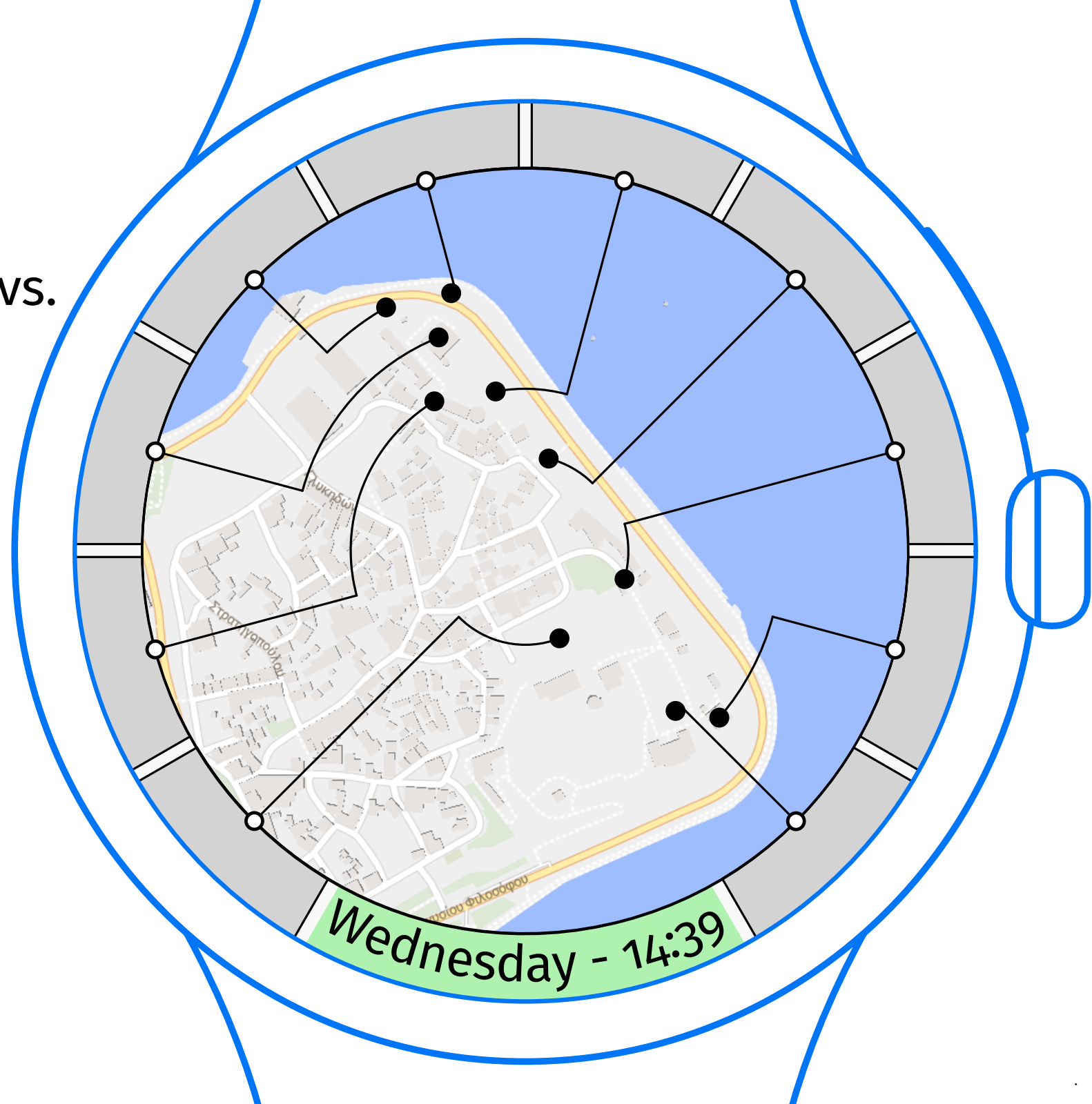
Problem variants



Problem variants



rotation:

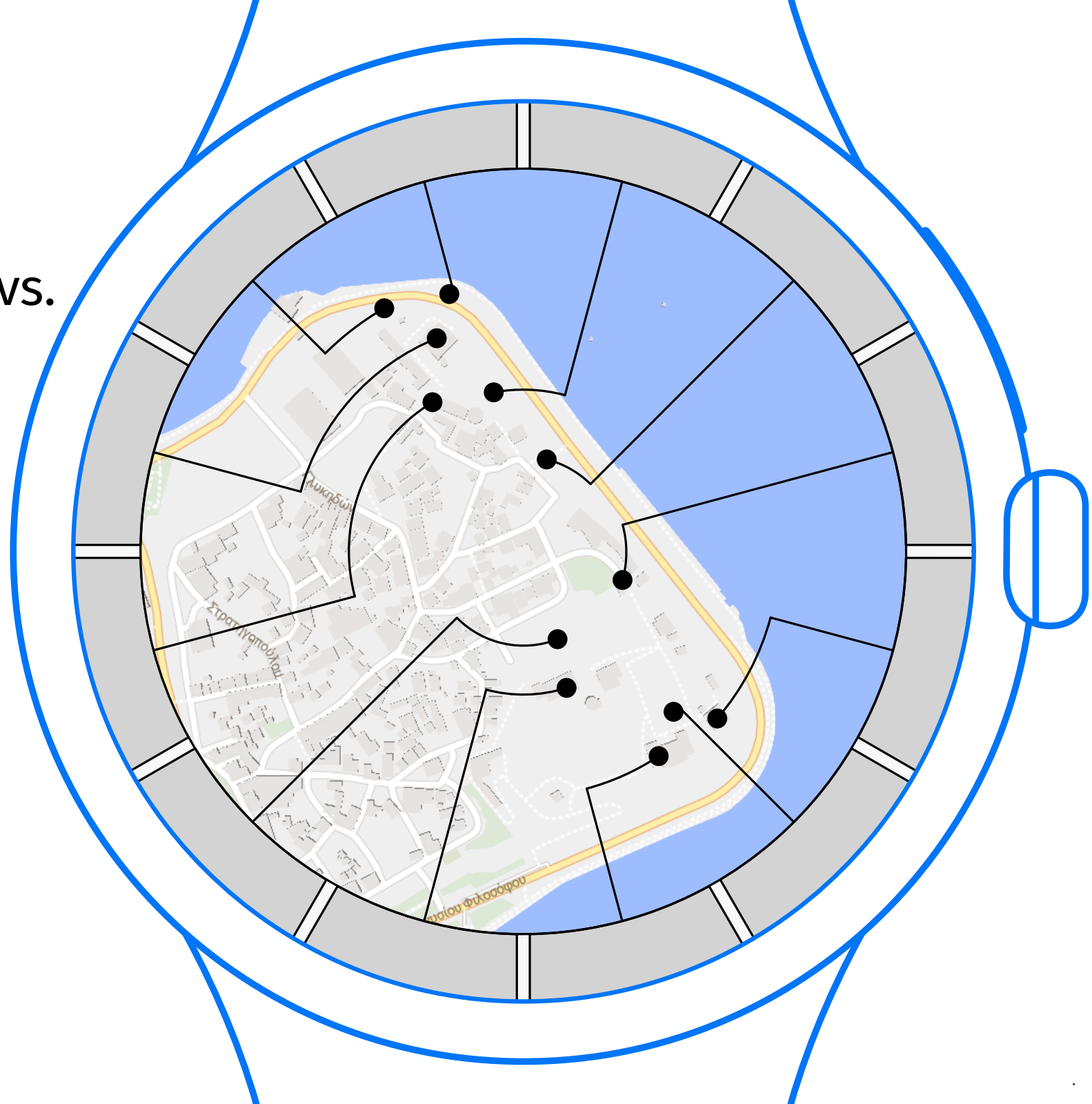
-  continuous rotation vs.
-  locked candidates C



Problem variants



rotation:

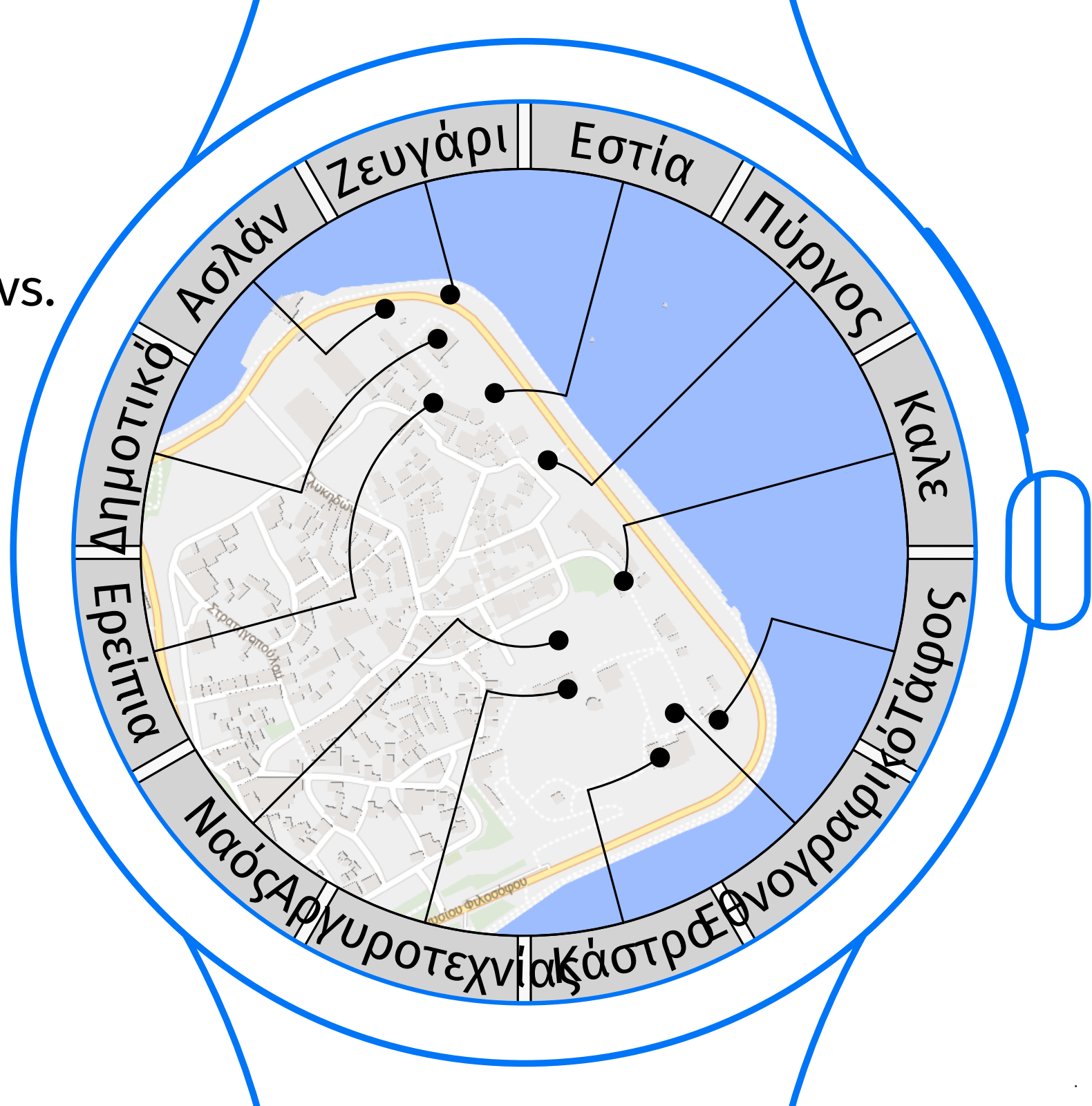
-  continuous rotation vs.
-  locked candidates C



Problem variants



rotation:

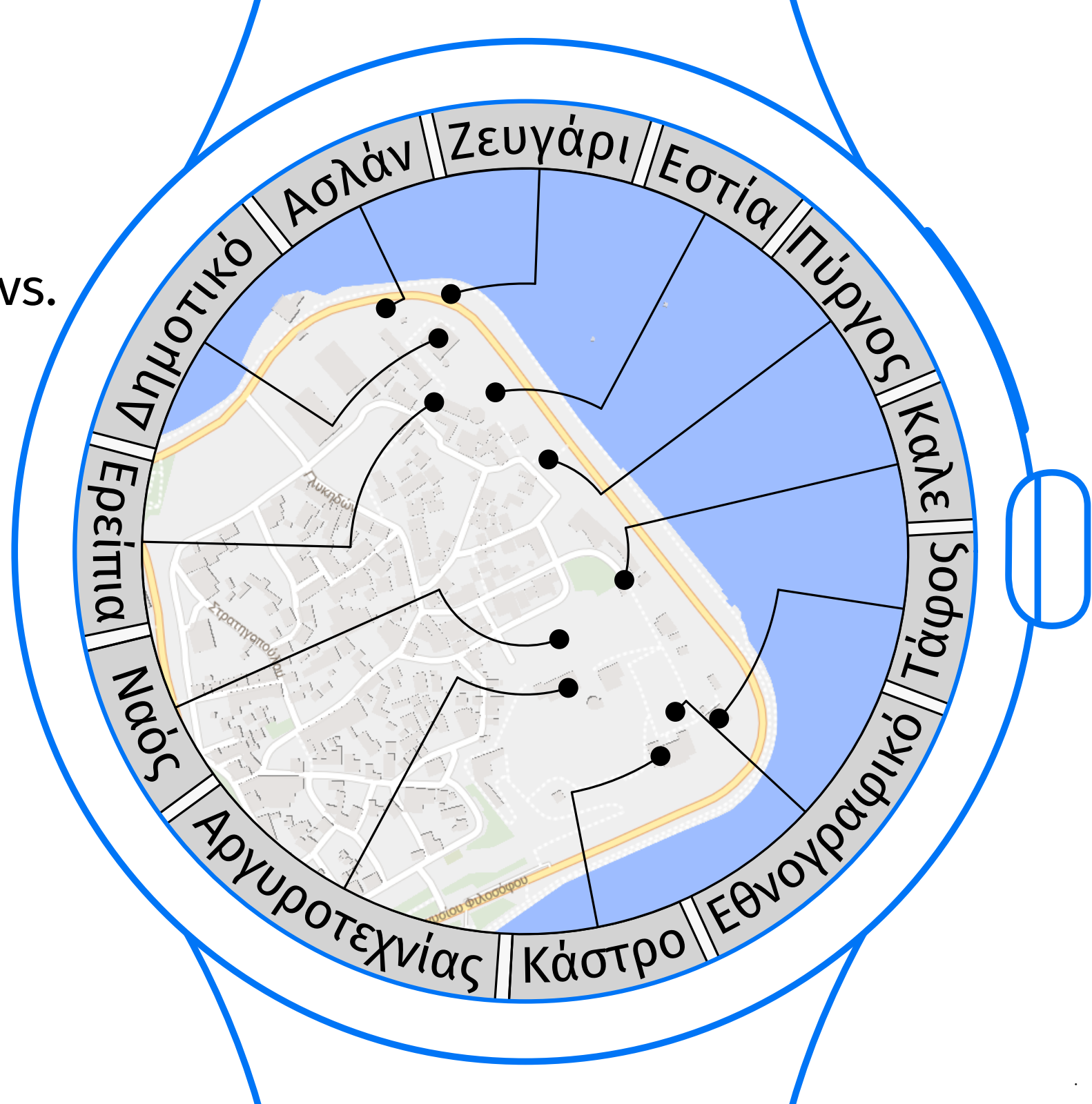
-  continuous rotation vs.
-  locked candidates C



Problem variants



rotation:

-  continuous rotation vs.
-  locked candidates C





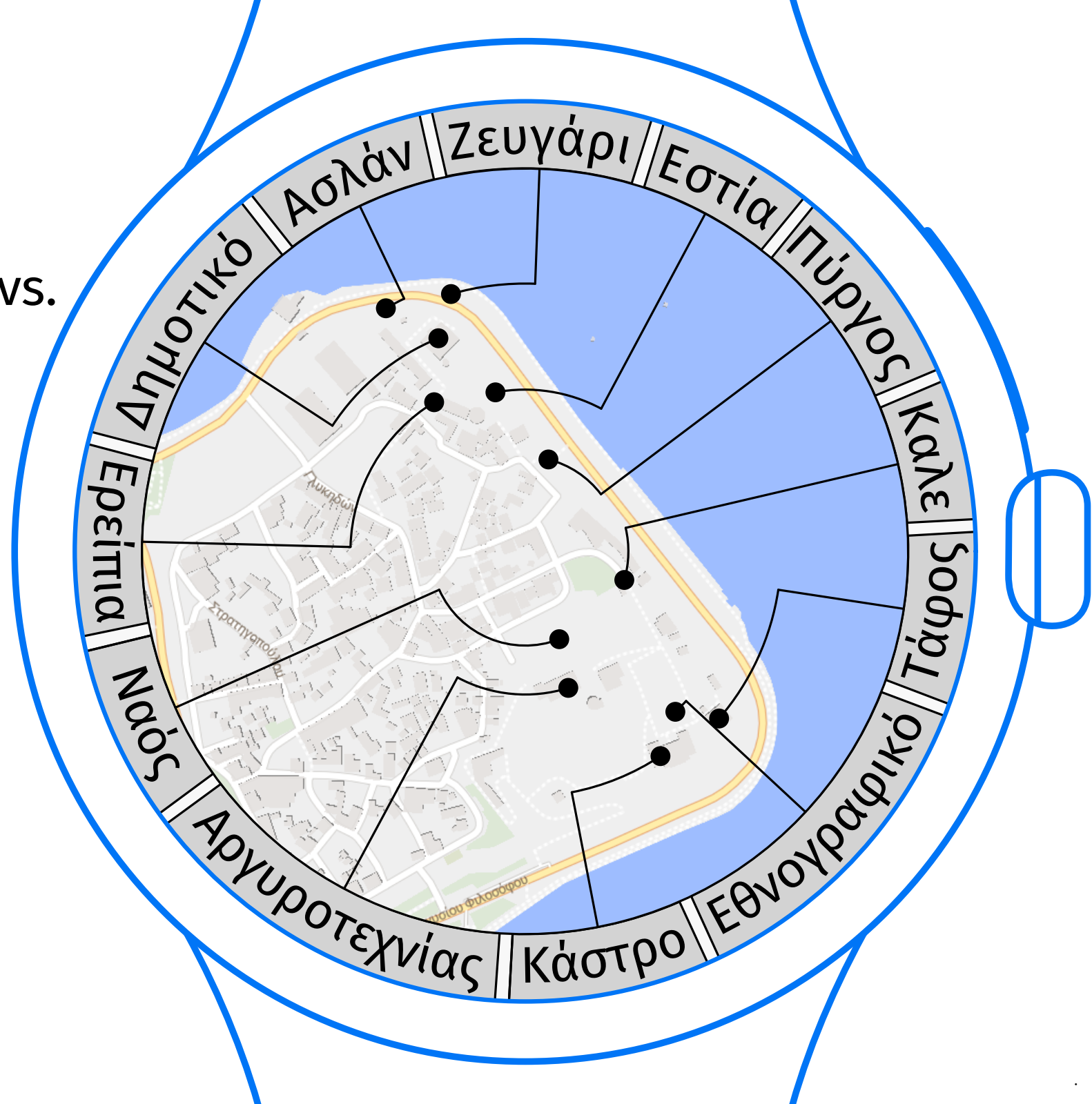
Problem variants

rotation:

-  continuous rotation vs.
-  locked candidates C



label size:

-  uniform vs.
-  non-uniform





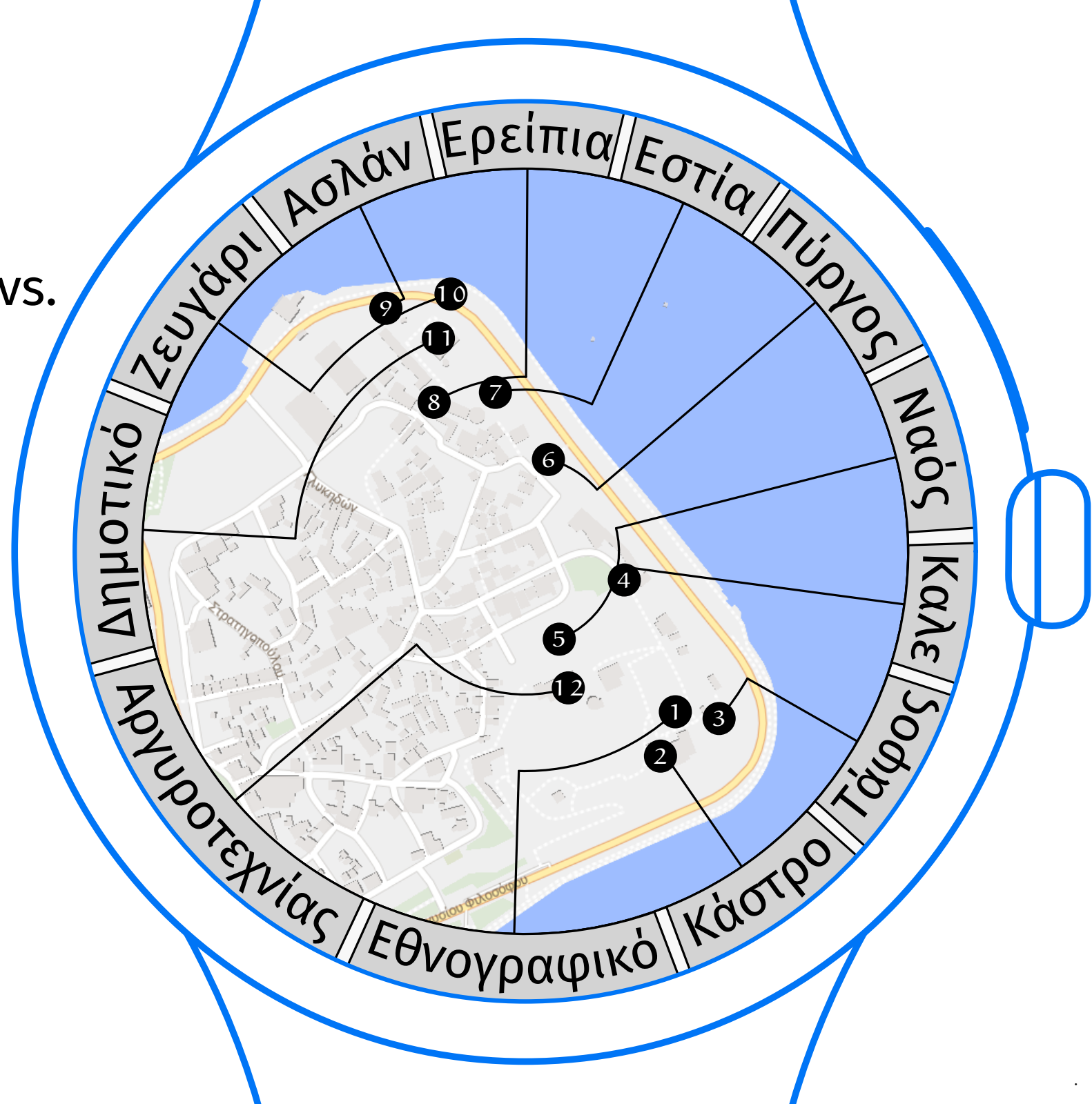
Problem variants

rotation:

-  continuous rotation vs.
-  locked candidates C



label size:

-  uniform vs.
-  non-uniform





Problem variants



rotation:

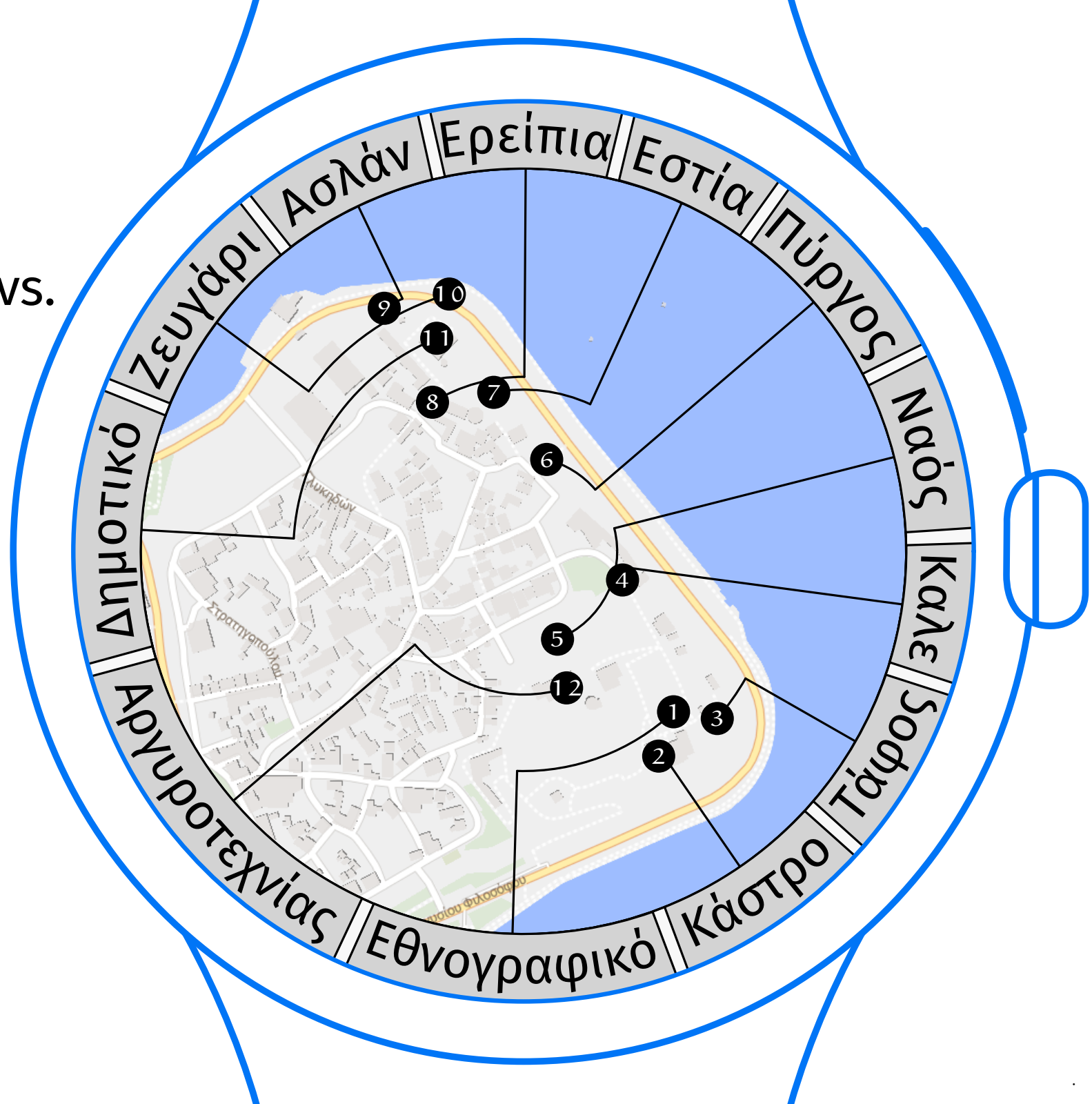
-  continuous rotation vs.
-  locked candidates C

label size:

-  uniform vs.
-  non-uniform

label order:

-  free order vs.
-  fixed order





Conclusion







rotation →

locked  (set C)









continuous 

| | locked  (set C) | | continuous  | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |










Conclusion

| | | | | |
|------------|--|--|--|--|
| rotation → | locked  (set C) | | continuous  | |
| order → | fixed  | free  | fixed  | free  |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Conclusion











| | | | | |
|---|--|--|--|--|
| rotation → | locked  (set C) | | continuous  | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | | | | |
| non-uniform  | | | | |

Conclusion

| rotation → | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | | | $O(n^2)$  | |
| non-uniform  | | | $O(n^2)$ | |

minimum of a sum
of linear functions











Conclusion

| | | | | |
|---|--|--|--|--|
| rotation → | locked  (set C) | | continuous  | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | | | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | | | $O(n^2)$ | |

minimum of a sum of linear functions

reduction to one-sided po-labeling

Conclusion











| rotation → | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | | | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | | | $O(n^2)$ | NP-complete |

minimum of a sum of linear functions

reduction to one-sided po-labeling

reduction from Partition shows weak NP-Completeness

Conclusion

| | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| rotation → | | | | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | | $O(n^2)$ | NP-complete |












brute force

minimum of a sum of linear functions

reduction to one-sided po-labeling

reduction from Partition shows weak NP-Completeness

Conclusion

| | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| rotation → | | | | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | | $O(n^2)$  | NP-complete |

brute force









minimum of a sum of linear functions

reduction to one-sided po-labeling

min-weight matching between features and ports

reduction from Partition shows weak NP-Completeness

Conclusion

| | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| rotation → | | | | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$ T | $O(n^5)$ T |
| non-uniform  | $O(n^2 C)$ | open | $O(n^2)$ T | NP-complete |

brute force












minimum of a sum of linear functions

reduction to one-sided po-labeling












min-weight matching between features and ports

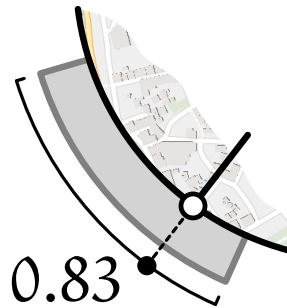
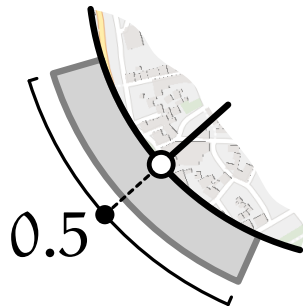
reduction from Partition shows weak NP-Completeness

Conclusion

| rotation → | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | open | $O(n^2)$  | NP-complete |












Conclusion

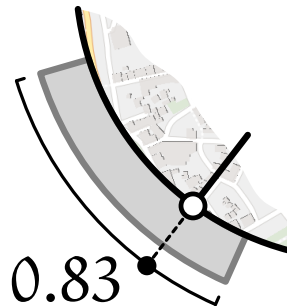
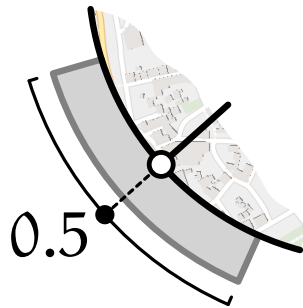
| rotation → | locked  (set C) | | continuous  | |
|---|--|--|--|--|
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | open | $O(n^2)$  | NP-complete |



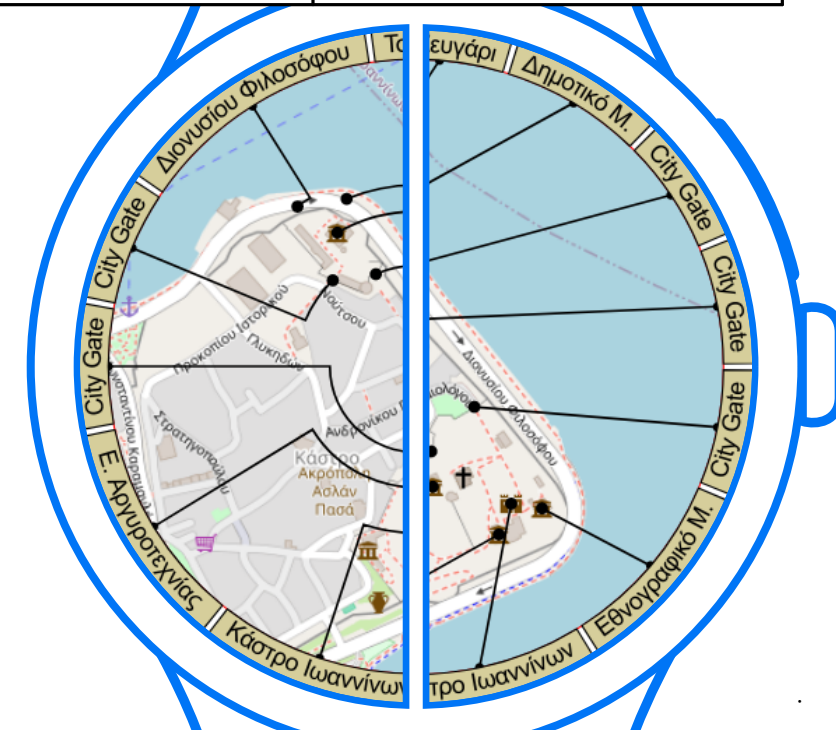
- port ratios → in particular free, non-uniform

Conclusion












| | | | | |
|---|--|--|--|--|
| rotation → | locked  (set C) | | continuous  | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | open | $O(n^2)$  | NP-complete |

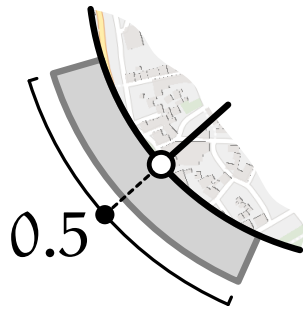


- port ratios → in particular free, non-uniform
- straight line leaders → some results extend

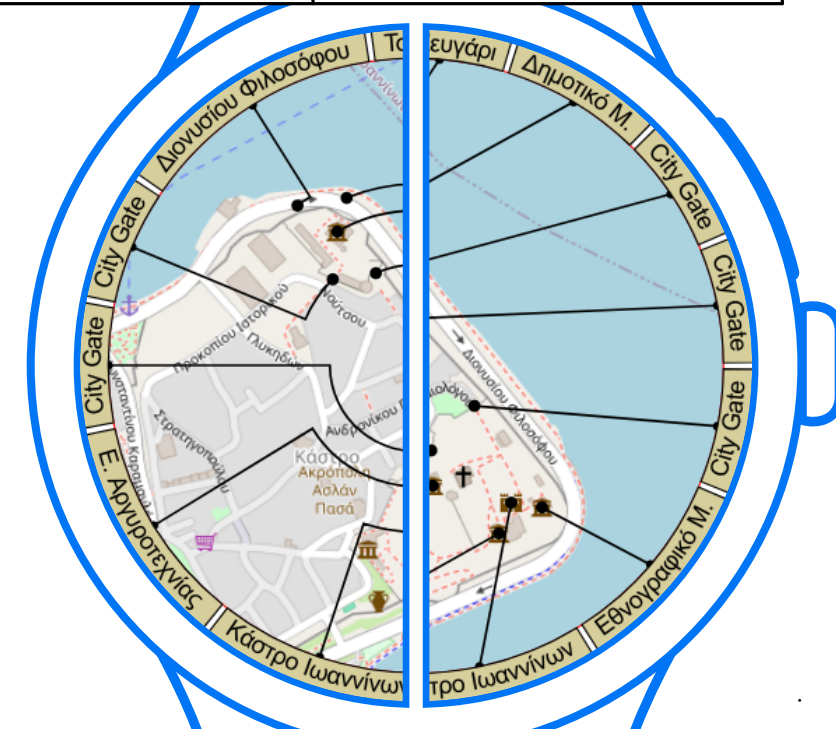


Conclusion

| | | | | |
|---|--|--|--|--|
| rotation → | locked  (set C) | | continuous  | |
| order → | fixed  | free  | fixed  | free  |
| label size ↓ | | | | |
| uniform  | $O(n^2 C)$ | $O(n C ^3)$ | $O(n^2)$  | $O(n^5)$  |
| non-uniform  | $O(n^2 C)$ | open | $O(n^2)$  | NP-complete |



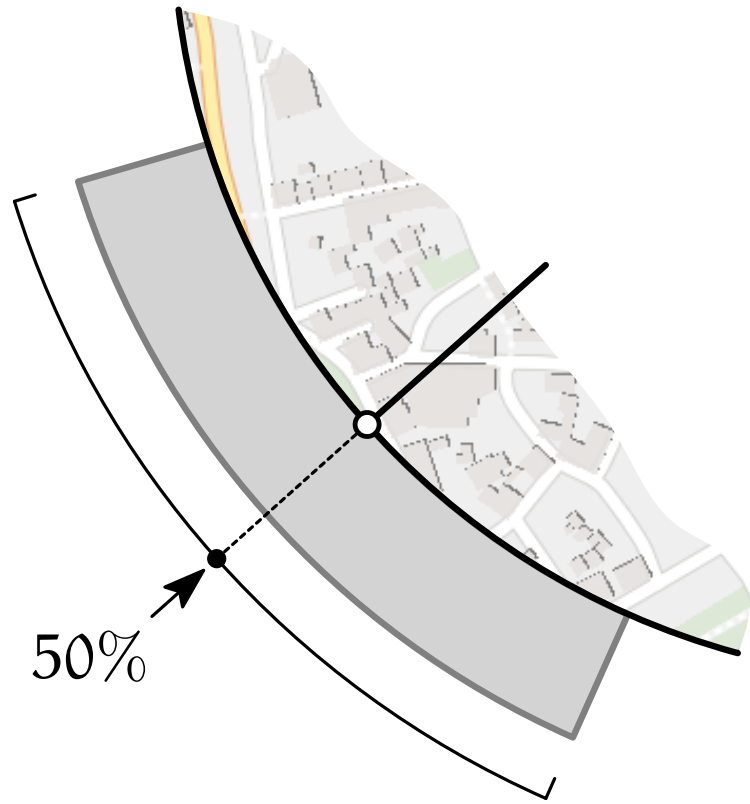
- port ratios → in particular free, non-uniform
- straight line leaders → some results extend
- user study evaluation



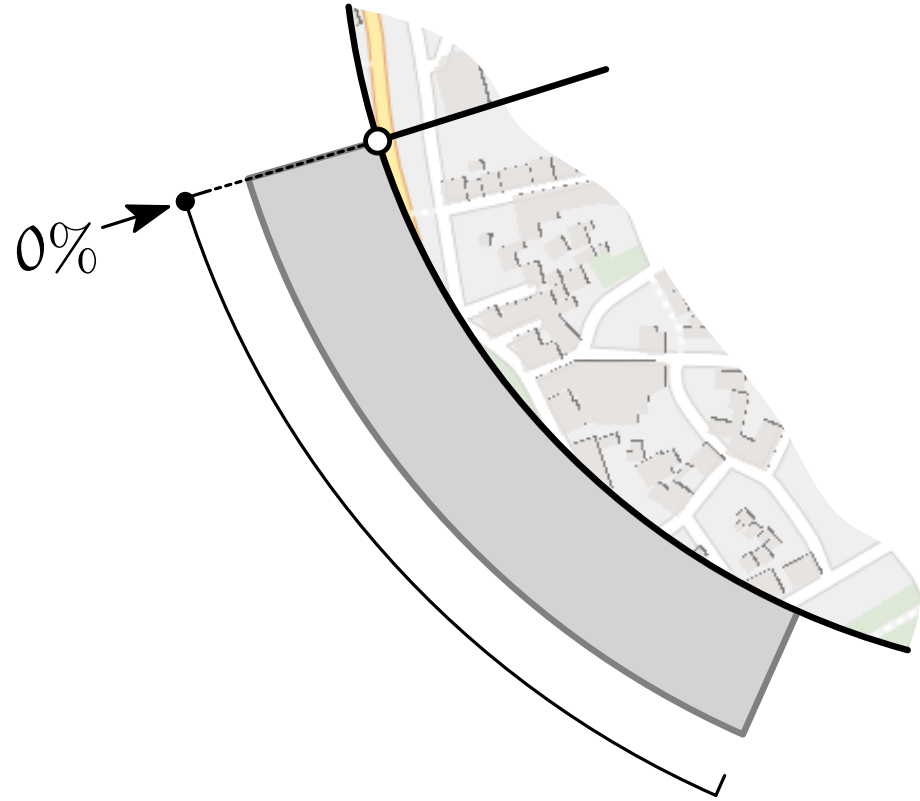
Port ratios and leader types



Port ratios and leader types



Port ratios and leader types







Port ratios and leader types



Port ratios and leader types

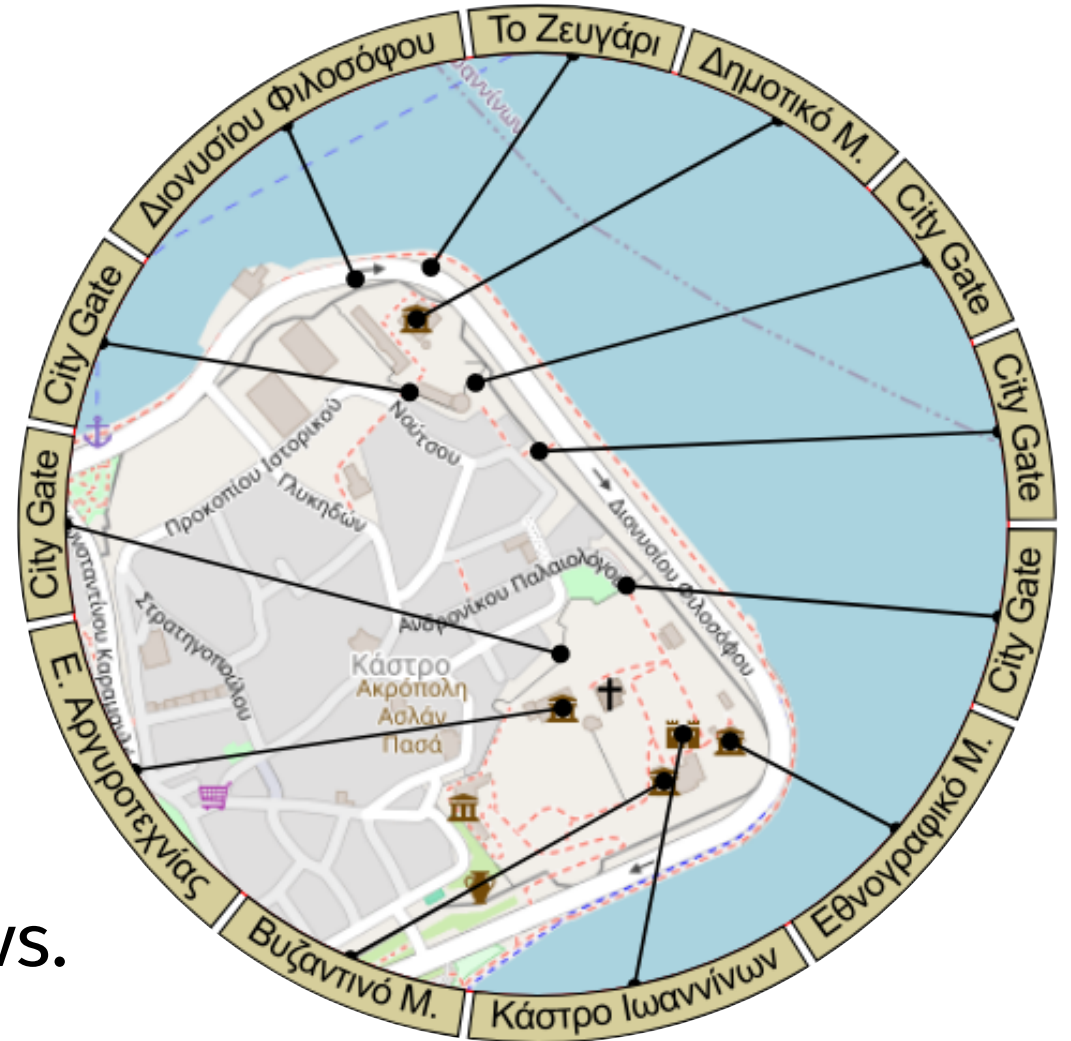






-  free vs.
 locked
-  uniform (the same for all labels) vs.
 non-uniform (individual)

Port ratios and leader types



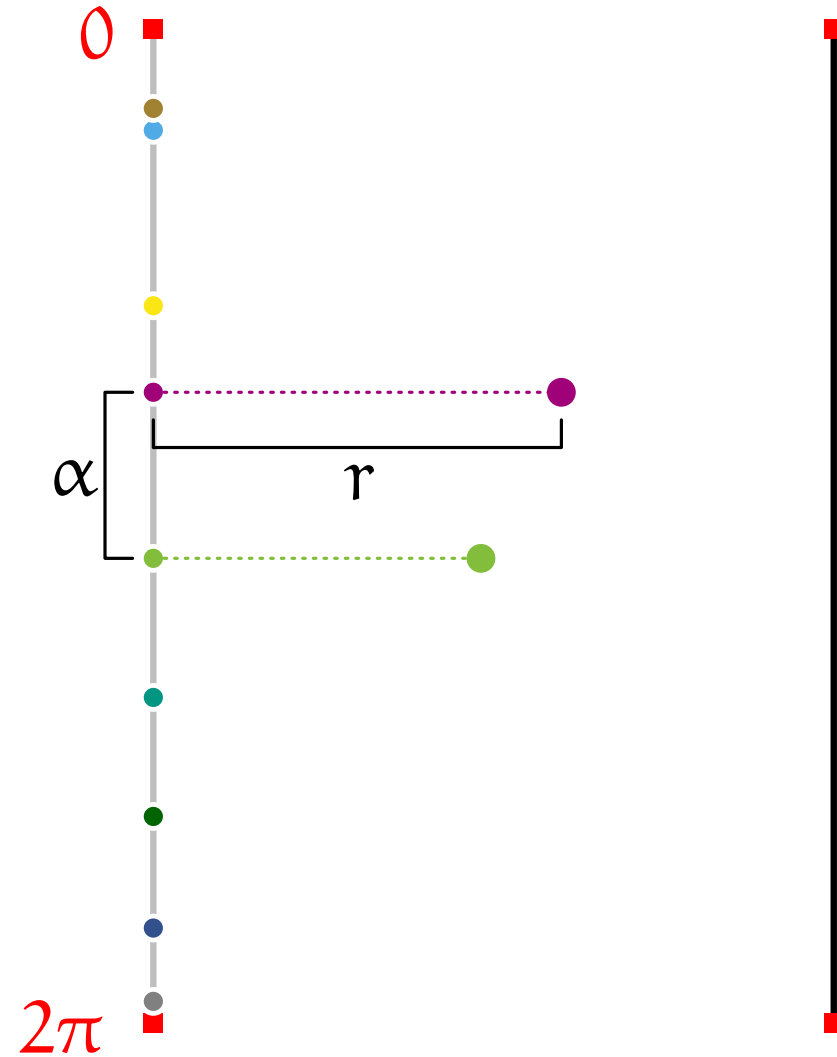
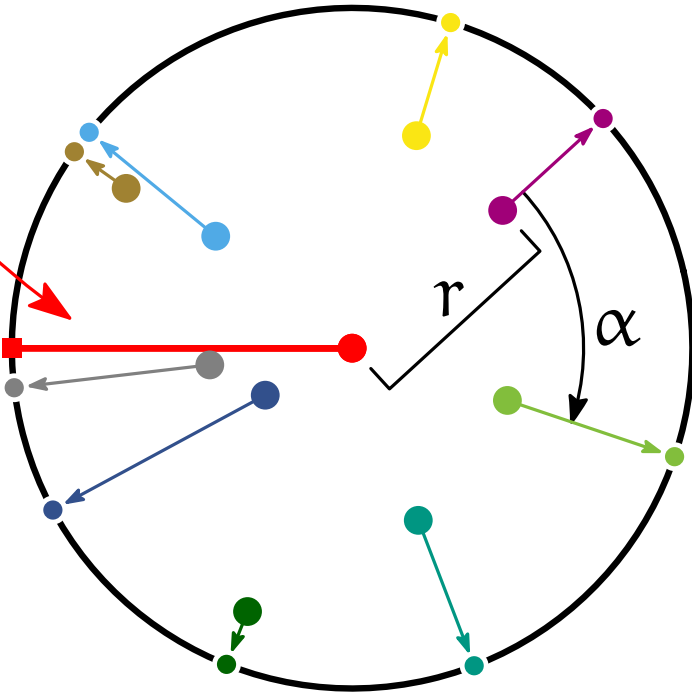
orbital-radial, straight line, ...



-  free vs.
-  locked
-  uniform (the same for all labels) vs.
-  non-uniform (individual)

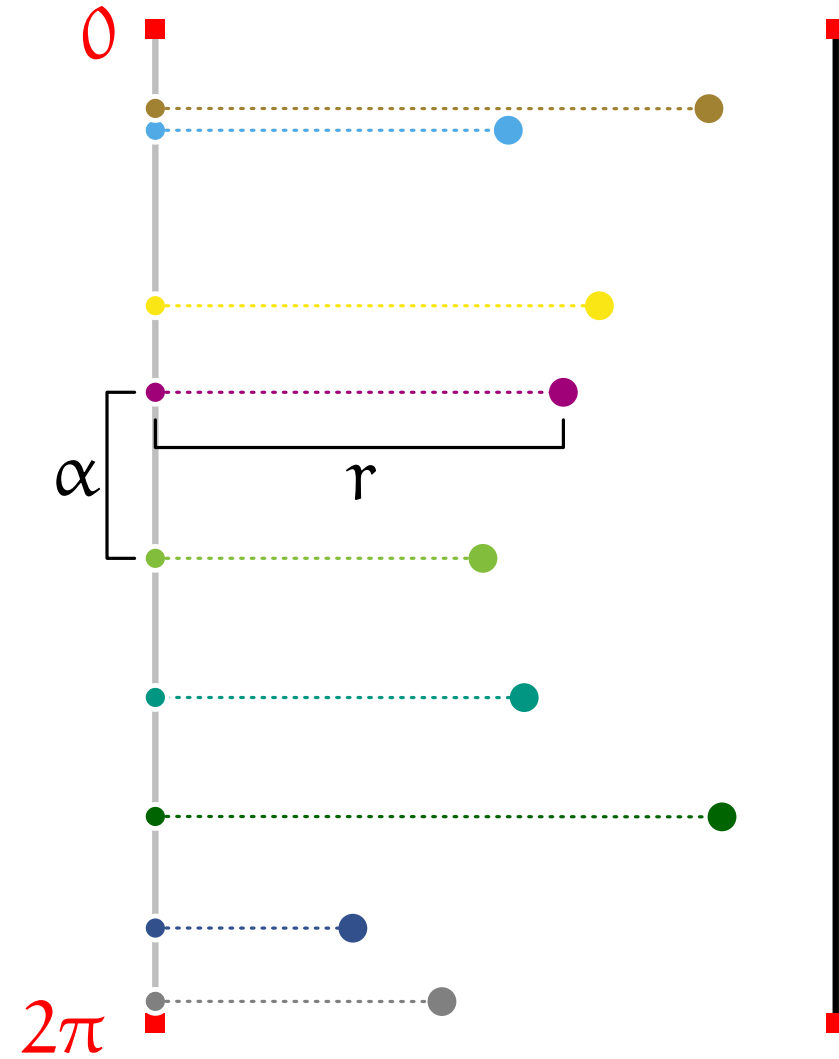
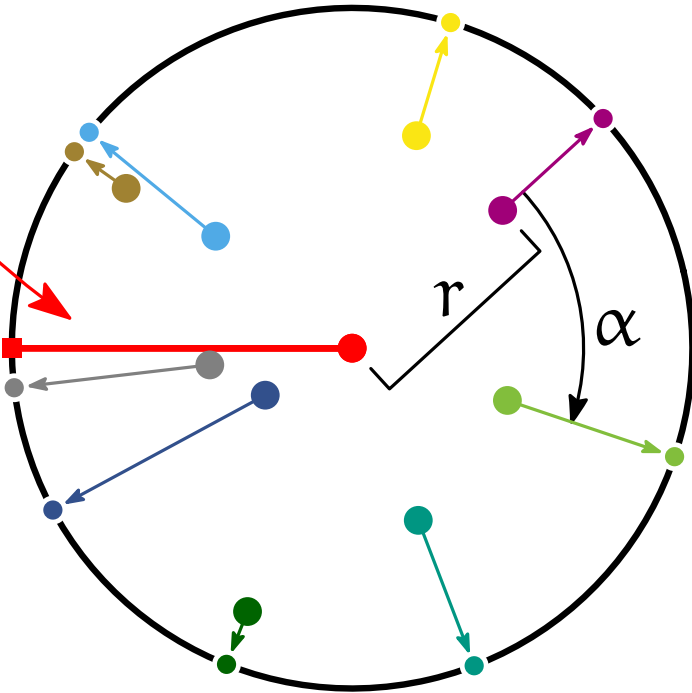
Overtime

$O(n^2)$ relevant possibilities



Overtime

$O(n^2)$ relevant possibilities



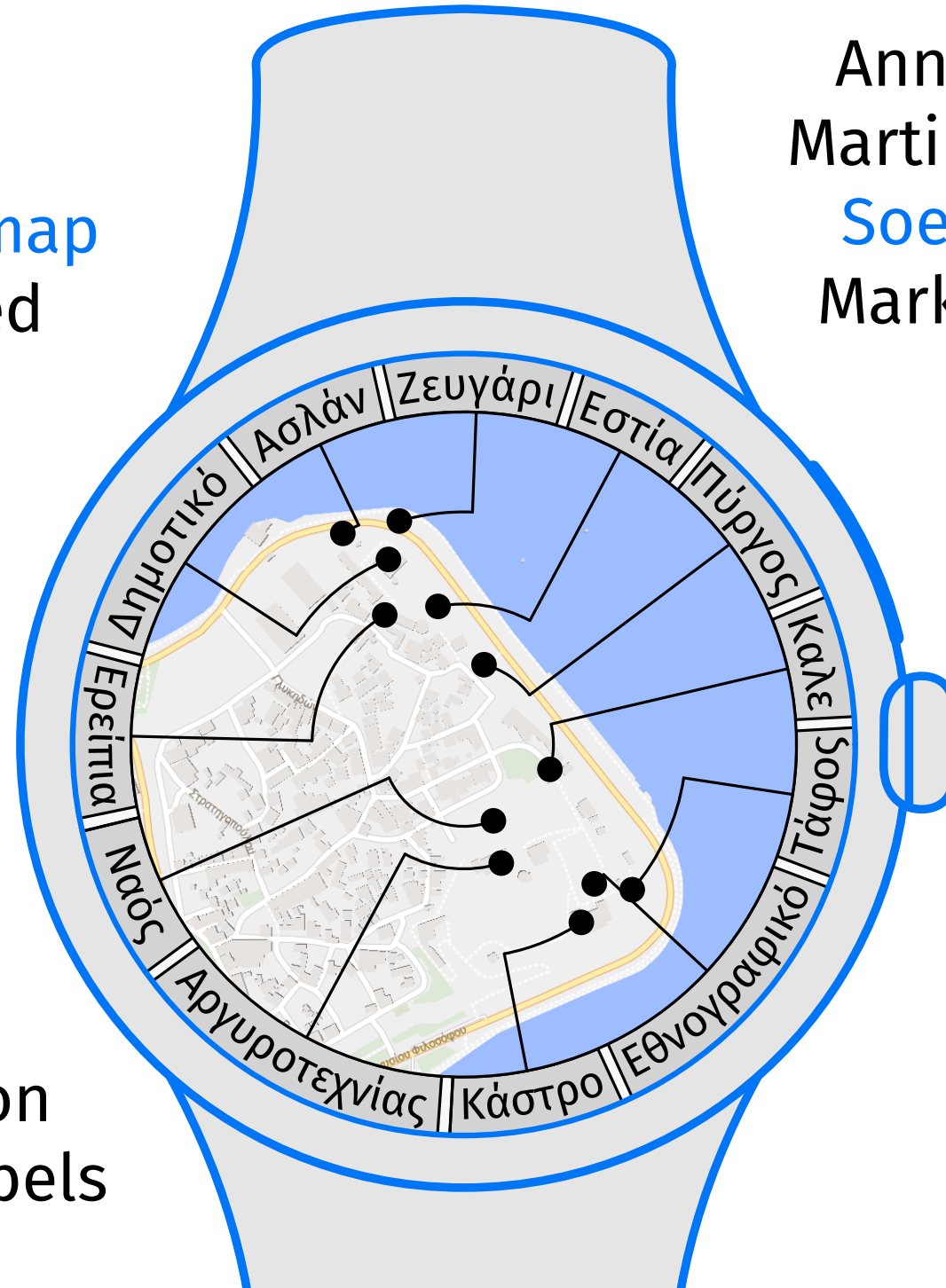
Orbital Labeling of Circular Maps

Labeling points on a **round map** with **circular labels** connected with **orbital-radial leaders**.

Goal: minimize sum of leader lengths

Under:

- continuous rotation
- permutation of labels



Annika Bonerath
Martin Nöllenburg
Soeren Terziadis
Markus Wallinger
Jules Wulms

Session **2.A**
Second talk
14:30 – 14:45